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Fixed-Lag Smoothing for Low-Delay Predictive Coding with Noise Shaping for Lossy Networks

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Abstract

We consider linear predictive coding and noise shaping for coding and transmission of auto-regressive (AR) sources over lossy networks. We generalize an existing framework to arbitrary filter orders and propose use of fixed-lag smoothing at the decoder, in order to further reduce the impact of transmission failures. We show that fixed-lag smoothing up to a certain delay can be obtained without additional computational complexity by exploiting the state-space structure. We prove that the proposed smoothing strategy strictly improves performance under quite general conditions. Finally, we provide simulations on AR sources, and channels with correlated losses, and show that substantial improvements are possible.

I. Introduction

In coding of source signals for transmission across lossy networks, it is traditionally the job of a subsequent error correcting code (ECC) to ensure robustness against transmission losses. In [1] we have presented a method for design of linear predictive coding (LPC) with noise shaping optimized for transmission losses, which does not rely on ECCs in order to achieve the desired degree of robustness towards losses. The coding problem is formulated and solved as a state estimation problem and the coding performance is improved through optimization for the known transmission loss statistics over part of a state-space model representing the source and the encoder.

Kalman estimation is employed for state estimation at the decoder. The technique for handling lost measurements has been described in [2], [3]. However, [2], [3] do not consider the optimization of a coding system for such losses. Optimization of a LPC system for transmission losses has been considered in [4], [5]. The approach in [4], [5] is, however, quite different from the approach presented in this paper and the decoding is based on fixed filters alternating between loss and non-loss states and are as such not linear minimum mean-squared error (LMMSE)-optimal estimators. [4], [5] work with a flexible Markov loss model. An optimal estimator for such Markov jump linear systems is found in [6]. The estimator type employed in [4], [5] is more akin to the approach described in [7].

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In this paper, we propose to use fixed-lag smoothing at the decoder, in order to reduce the impact of transmission failures, at the expense of a small decoder delay. We show that if the desired smoothing lag is less than or equal to the order of the predictor, fixed-lag smoothing can be obtained without additional computational complexity. Moreover, under quite general conditions, we prove that our proposed smoothing strategy strictly improves performance. Finally, we provide simulations on higher order auto-regressive (AR) sources, and channels with correlated losses, and show that substantial improvements are possible. In particular, at loss rates around $10\,\%$, a reduction of about $2\,\mathrm{dB}$ in distortion is observed, with only three samples delay at the decoder.

II. Coding Framework

This section describes the coding framework. The framework and design method have been presented in [1]. In the following, we summarize and generalize important results of [1], which will be needed in order to establish our main results in Section III.

A. Source Encoder

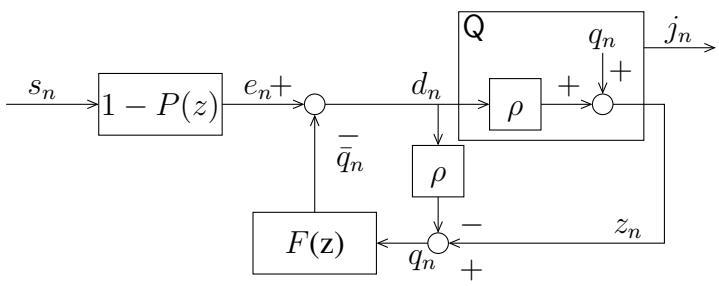


Fig. 1. Source encoder with linear additive noise quantizer model.

The source signal s is modeled as outcomes of an AR process [1]

$$s_n = \sum_{i=1}^{N} \alpha_i s_{n-i} + r_n, \tag{1}$$

of order N, driven by zero-mean stationary white Gaussian noise r with known variance σ_r^2. The source process is assumed stable.

The encoder (shown in Figure 1) is defined by the following equations [1]

$$d_n = e_n - \bar{q}_n, \qquad e_n = s_n - \sum_{i=1}^{p} a_i s_{n-i}, \qquad \bar{q}_n = \sum_{i=1}^{f} b_i q_{n-i}, \tag{2}$$

where e_n is the prediction error, $\bar{q}_n$ is the filtered quantization noise feedback, d_n is the input to the quantizer, and q_n is the additive component of the quantization error; p is the predictor order; $a_i,\ i = 1, \ldots, p$ are the predictor coefficients; f is the noise feedback filter order and $b_i,\ i = 1, \ldots, f$ are the noise feedback filter coefficients.

The output transmitted to the decoder is quantization indices, j, for the quantized prediction error, $z_n = Q(d_n)$. The encoder's scalar quantization $Q(\cdot)$ is approximated by the following linear model with additive noise

$$z_n = \rho d_n + q_n, \tag{3}$$

where $\rho \in [0, 1]$, and q_n is a stationary zero-mean white Gaussian noise, independent of d_n, with variance $\sigma_q^2 = k \operatorname{var}\{d_n\}$. The quantization noise variance is proportional, by a constant k, to the variance of the input to the quantizer, d_n. ρ and k are given by the coding loss, β, of the quantizer:

$$\rho = 1 - \beta \qquad\qquad k = \beta(1 - \beta),$$

where β is the inverse of the quantizer coding gain [8]. See [1] for a discussion of this linear model of the quantizer.

The quantization noise feecback is de-correlated from d_n by scaling d_n by ρ in the path subtracting the quantizer input from the quantizer output. This ensures that only the additive part of the quantization noise $q_n = z_n - \rho d_n$ is fed back. This allows us to model the fed back noise as white Gaussian which simplifies the optimization of the encoder. The design of Lloyd-Max quantizers for use in the encoder is treated in [1].

B. Kalman Filter-Based Decoder

The decoder uses LMMSE (Kalman) estimation of the source signal s based on the measurements z obtained from the partially received sequence of quantization indices j. The Kalman estimator $\hat{s}_n$ of s_n is derived as in, e.g., [9] from a state-space model of the source and encoder. The process equation encompasses both the production of the source signal as well as the evolution of the states of the encoder filters P and F. The measurement equation represents the encoder's filters and the linear model of the quantizer such that the measurements are the quantized prediction errors from the encoder. The following descriptions of the state-space model and decoder equations are a generalization from the state-space model in [1].

The process equation, (4), and the measurement equation, (5), constitute the state-space model modeling the source and the encoder.

$$\mathbf{x}_{n+1} = \mathbf{F}\mathbf{x}_n + \mathbf{G}\mathbf{w}_n \tag{4}$$

$$z_n = \mathbf{h}^{\mathrm{T}}\mathbf{x}_n + q_n \tag{5}$$

The state $\mathbf{x}_n$ is composed of the current signal sample and the states of the predictor and noise feedback filter.

$$\mathbf{x}_n = \left[\begin{array}{c|ccc|ccc} s_n & s_{n-1} & \cdots & s_{n-(M-1)} & q_{n-1} & \cdots & q_{n-f} \end{array}\right]^{\mathrm{T}}, \tag{6}$$

where $M = \max\{N, p+1\}$. The state transition matrix $\mathbf{F}$ is defined as

$$\mathbf{F} = \left[\begin{array}{c|c|c} \alpha_{1\times N} & \mathbf{0}_{1\times(M-N)} & \mathbf{0}_{M\times f} \\ \hline \mathbf{I}_{M-1} & \mathbf{0}_{(M-1)\times 1} & \\ \hline \multicolumn{2}{c|}{\mathbf{0}_{f\times M}} & \begin{array}{c|c} \multicolumn{2}{c}{\mathbf{0}_{1\times f}} \\ \hline \mathbf{I}_{(f-1)} & \mathbf{0}_{(f-1)\times 1} \end{array} \end{array}\right], \tag{7}$$

where $\alpha = [\alpha_1 \cdots \alpha_N]$ are the coefficients of the source AR process, $\mathbf{I}_x$ is an $x \times x$ identity matrix, and $\mathbf{0}$ is an all-zero matrix. The subscripts in (7) denote the dimensions of the individual components.

The process noise $\mathbf{w}_n$ is stationary zero-mean white Gaussian with covariance $\mathbf{Q}$

$$\mathbf{w}_n = \begin{bmatrix} r_n \\ q_n \end{bmatrix} \qquad\qquad \mathbf{Q} = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_q^2 \end{bmatrix}. \tag{8}$$

The measurement noise is the additive part of the quantization noise, q_n, so the measurement noise covariance in typical Kalman filter notation is $\mathbf{R} = \sigma_q^2$. The connection between quantization noise in the state originating from $\mathbf{w}_n$, and q_n added to the measurement is captured by defining the covariance, $\mathbf{S}$, between process and measurement noise as follows:

$$\mathbf{S} = \mathrm{E}\left\{ \begin{bmatrix} r_n \\ q_n \end{bmatrix} q_n \right\} = \begin{bmatrix} 0 \\ \sigma_q^2 \end{bmatrix}. \tag{9}$$

We use a formulation of the Kalman filter taking the covariance $\mathbf{S}$ into account. $\mathbf{G}$ is a transform enabling the process noise $\mathbf{w}_n$ to be defined in a compact form

$$\mathbf{G} = \begin{bmatrix} 1 & 0 \\ \mathbf{0}_{(M-1)\times 2} \\ 0 & 1 \\ \mathbf{0}_{(f-1)\times 2} \end{bmatrix}, \text{ or } \mathbf{G} = \begin{bmatrix} 1 & 0 \\ \mathbf{0}_{(M-1)\times 2} \end{bmatrix}, \text{ if } f = 0. \tag{10}$$

The measurement vector $\mathbf{h}$ represents the filters P and F as well as the scaling ρ from (3).

$$\mathbf{h} = \rho\tilde{\mathbf{h}} \qquad \tilde{\mathbf{h}} = \begin{bmatrix} 1 & -a_1 \cdots - a_p & \mathbf{0}_{1\times(M-p-1)} & -b_1 \cdots - b_f \end{bmatrix}^{\mathrm{T}}, \tag{11}$$

where $\tilde{\mathbf{h}}$ contains the coefficients of the prediction error and noise feedback filters, cf. (2). The zeros fill the measurement vector to match the necessary size of the state in case P is chosen with order $p < N - 1$.

Following the approach described in [1], (12)–(15) are obtained for the Kalman estimator $\hat{\mathbf{x}}_n$ of $\mathbf{x}_n$ in the case of channel erasures.

$$\hat{\mathbf{x}}_n = \hat{\mathbf{x}}_n^- + \gamma_n \mathbf{P}_n^- \mathbf{h} \left(\mathbf{h}^{\mathrm{T}} \mathbf{P}_n^- \mathbf{h} + \mathbf{R}\right)^{-1} \left(z_n - \mathbf{h}^{\mathrm{T}} \hat{\mathbf{x}}_n^-\right) \tag{12}$$

$$\mathbf{P}_n = \mathbf{P}_n^- - \gamma_n \mathbf{P}_n^- \mathbf{h} \left(\mathbf{h}^{\mathrm{T}} \mathbf{P}_n^- \mathbf{h} + \mathbf{R}\right)^{-1} \mathbf{h}^{\mathrm{T}} \mathbf{P}_n^{-\mathrm{T}} \tag{13}$$

$$\hat{\mathbf{x}}_{n+1}^- = \mathbf{F}\hat{\mathbf{x}}_n + \gamma_n \mathbf{G}\mathbf{S}\mathbf{R}^{-1} \left(z_n - \mathbf{h}^{\mathrm{T}} \hat{\mathbf{x}}_n\right) \tag{14}$$

$$\begin{aligned} \mathbf{P}_{n+1}^- = & \left(\mathbf{F} - \gamma_n \mathbf{G}\mathbf{S}\mathbf{R}^{-1}\mathbf{h}^{\mathrm{T}}\right) \mathbf{P}_n \left(\mathbf{F} - \gamma_n \mathbf{G}\mathbf{S}\mathbf{R}^{-1}\mathbf{h}^{\mathrm{T}}\right)^{\mathrm{T}} \\ & + \mathbf{G}\left(\mathbf{Q} - \gamma_n \mathbf{S}\mathbf{R}^{-1}\mathbf{S}^{\mathrm{T}}\right)\mathbf{G}^{\mathrm{T}}, \end{aligned} \tag{15}$$

where

$$\hat{\mathbf{x}}_n = \mathrm{E}\left\{\mathbf{x}_n | z_0 \ldots z_n\right\} \qquad \hat{\mathbf{x}}_{n+1}^- = \mathrm{E}\left\{\mathbf{x}_{n+1} | z_0 \ldots z_n\right\}$$

$$\mathbf{P}_n = \mathrm{E}\left\{\tilde{\mathbf{x}}_n \tilde{\mathbf{x}}_n^{\mathrm{T}}\right\}, \ \tilde{\mathbf{x}}_n = \mathbf{x}_n - \hat{\mathbf{x}}_n \quad \mathbf{P}_{n+1}^- = \mathrm{E}\left\{\left(\mathbf{x}_{n+1} - \hat{\mathbf{x}}_{n+1}^-\right)\left(\mathbf{x}_{n+1} - \hat{\mathbf{x}}_{n+1}^-\right)^{\mathrm{T}}\right\}.$$

III. Fixed-Lag Smoothing

As seen in (6), the state $\mathbf{x}_n$ contains both the source sample s_n as well as $M - 1$ previous source samples $s_{n-1} \ldots s_{n-(M-1)}$. This means that in addition to providing an estimate of the current source sample, $\hat{s}_{n|n}$, the state estimate also provides the delayed source signal estimates $\hat{s}_{n-1|n} \ldots \hat{s}_{n-(M-1)|n}$. In this section we show how these estimates provide fixed-lag smoothing.

In [10], the authors describe a fixed-lag smoothing approach with an example of a state-space-model somewhat similar (models an autoregressive source) to the one considered in this paper in Section II-B. This smoothing approach does not apply to the signal model presented here due to the requirements of [10, eq. (5)].

A different fixed-lag smoothing approach using Kalman filters is found in, e.g., [9], [11]. This smoothing approach is based on an augmentation of the state-space model and requires extensions to the Kalman estimator.

Fixed-lag smoothing with a state-space model similar to the one in Section II-B is discussed in [12]. The model in [12] is simpler, e.g., does not contain a separate noise feedback part and the paper does not present mathematical details of estimation improvement by smoothing.

The fixed-lag smoothing approach made possible with our presented state-space model comes at no additional computational cost as it is already an inherent part of the state-space model, i.e., the smoothed (delayed) estimates are readily available from the state estimate.[1] In the following, we show under which conditions the delayed estimates $\hat{s}_{n-k|n}$ provide an improvement over $\hat{s}_{n-k|n-k}$ for $k \in [1, M-1]$.

From (14) we see that

$$\begin{aligned}\left[\ \hat{s}^-_{n|n-1} \mid \hat{s}^-_{n-1|n-1} \dots \hat{s}^-_{n-(M-1)|n-1} \mid \hat{q}^-_{n-1|n-1} \dots \hat{q}^-_{n-f|n-1}\ \right]^{\mathrm{T}} = \\ \mathbf{F}\left[\ \hat{s}_{n-1|n-1} \mid \hat{s}_{n-2|n-1} \dots \hat{s}_{n-M|n-1} \mid \hat{q}_{n-2|n-1} \dots \hat{q}_{n-f-1|n-1}\ \right]^{\mathrm{T}} + \\ \gamma_n \mathbf{G}\mathbf{S}\mathbf{R}^{-1}\left(z_n - \mathbf{h}^{\mathrm{T}}\hat{\mathbf{x}}_n\right). \quad (16)\end{aligned}$$

By the structure of $\mathbf{G}$, $\mathbf{S}$, and $\mathbf{R}$ we see that

$$\mathbf{G}\mathbf{S}\mathbf{R}^{-1} = \begin{cases} \left[\mathbf{0}_{1\times M}\ 1\ \mathbf{0}_{1\times(f-1)}\right]^{\mathrm{T}} & \text{for } f > 0 \\ \left[\mathbf{0}_{1\times M}\right]^{\mathrm{T}} & \text{for } f = 0, \end{cases} \quad (17)$$

meaning that the right summand in (16) affects only $\hat{q}^-_{n-1}$. From the structure of $\mathbf{F}$, see (7), we thus conclude that

$$\hat{s}^-_{n-k|n-1} = \hat{s}_{n-k|n-1}, \text{ for } k \in [1, M-1]. \quad (18)$$

It follows from (13) and (18) that in order for the estimate $\hat{s}_{n-k|n}$ (in $\hat{\mathbf{x}}_n$) to be better than $\hat{s}_{n-k|n-1}$ (in $\hat{\mathbf{x}}_{n-1}$), we require elements $2 \dots M$ on the diagonal of $\mathbf{P}_n$ to be smaller than the corresponding elements on the diagonal of $\mathbf{P}^-_n$. Examining (13) it is trivial to see that this is not the case in the event of loss ($\gamma_n = 0$). In the event of correct arrival ($\gamma_n = 1$) we see that elements $2 \dots M$ on the diagonal $\mathbf{d} = \mathrm{diag}\left(\mathbf{P}^-_n \mathbf{h}\mathbf{h}^{\mathrm{T}} \mathbf{P}^{-\mathrm{T}}_n\right)$ in (13) must be positive in order to fulfill the requirement.

$$\mathbf{d} = [d_1 \dots d_{2N+1}]^{\mathrm{T}} = \mathrm{diag}\left(\mathbf{P}^-_n \mathbf{h}\mathbf{h}^{\mathrm{T}} \mathbf{P}^{-\mathrm{T}}_n\right). \quad (19)$$

By construction, $\mathbf{h}\mathbf{h}^{\mathrm{T}}$ is a positive semidefinite matrix. By definition of a positive semidefinite matrix we see that the elements d_i on the diagonal $\mathbf{d}$, are $d_i \geq 0,\ \forall i$. Thus it is proved that the estimates $\hat{s}_{n-k|n}$ are never worse than $\hat{s}_{n-k|n-1}$. In fact, improvement is generally guaranteed under the conditions stated in Theorem 1.

Theorem 1. Given at least one of the following conditions:

1) The encoder prediction error filter is $1 - P(z) \neq H^{-1}_{\text{src}}(z)$, i.e., $1 - P(z)$ does not whiten the source completely.

[1] We have not been able to prove that our proposed smoothing technique is optimal. However, we have implemented existing smoothing techniques based on augmented state-space models [9], [11] and through simulations observed that the results are indeed identical.

2) There is noise feedback in the encoder: $F(z) \neq 0$.

Then

$$\mathrm{E}\left\{\left(s_{n-k} - \hat{s}_{n-k|n}\right)^2\right\} < \mathrm{E}\left\{\left(s_{n-k} - \hat{s}_{n-k|n-1}\right)^2\right\}, \text{ for } k \in [1, M-1].$$

Proof of Theorem 1: Consider the diagonal elements d_i from (19). Observe from (13) that

$$\mathrm{E}\left\{\left(s_{n-k} - \hat{s}_{n-k|n}\right)^2\right\} = \mathrm{E}\left\{\left(s_{n-k} - \hat{s}_{n-k|n-1}\right)^2\right\} - \frac{d_{1+k}}{\mathbf{h}^{\mathrm{T}}\mathbf{P}_n^{-}\mathbf{h} + \mathbf{R}}, \text{ for } k \in [1, M-1].$$

We prove the theorem by showing that $d_i > 0$ for $i = 2 \ldots M$ for conditions 1 and 2. Considering $\gamma_n = 1$, (15) reduces to

$$\mathbf{P}_n^{-}\mathbf{h} = \left(\mathbf{F} - \mathbf{G}\mathbf{S}\mathbf{R}^{-1}\mathbf{h}^{\mathrm{T}}\right)\mathbf{P}_n\left(\mathbf{F} - \mathbf{G}\mathbf{S}\mathbf{R}^{-1}\mathbf{h}^{\mathrm{T}}\right)^{\mathrm{T}}\mathbf{h} + \mathbf{G}\left(\mathbf{Q} - \mathbf{S}\mathbf{R}^{-1}\mathbf{S}^{\mathrm{T}}\right)\mathbf{G}^{\mathrm{T}}\mathbf{h}. \quad (20)$$

Referring to (7) and (17) ($-\mathbf{G}\mathbf{S}\mathbf{R}^{-1}\mathbf{h}^{\mathrm{T}}$ subtracts $\mathbf{h}^{\mathrm{T}}$ from the $(M+1)$*th* row of $\mathbf{F}$) we see that

$$\left(\mathbf{F} - \mathbf{G}\mathbf{S}\mathbf{R}^{-1}\mathbf{h}^{\mathrm{T}}\right)_{(i,j)} = \mathbf{F}_{(i,j)}, \text{ for } i = 2 \ldots M,\ j = 1 \ldots M+f. \quad (21)$$

From (8)–(10) we see that

$$\left\{\mathbf{G}\left(\mathbf{Q} - \mathbf{S}\mathbf{R}^{-1}\mathbf{S}^{\mathrm{T}}\right)\mathbf{G}^{\mathrm{T}}\right\}_{i,j} = \begin{cases} \sigma_r^2 & \text{for } i = j = 1 \\ 0 & \text{for } i \neq 1,\ j \neq 1. \end{cases} \quad (22)$$

Let $p_{i,j}$ denote the (i,j)*th* element of $\mathbf{P}_n^{-}$, $p_{i,j} = p_{j,i}$. Then

$$\begin{aligned}\left\{\mathbf{P}_n^{-}\mathbf{h}\right\}_{(i)} &= \left\{\left(\mathbf{F} - \mathbf{G}\mathbf{S}\mathbf{R}^{-1}\mathbf{h}^{\mathrm{T}}\right)\mathbf{P}_n\left(\mathbf{F} - \mathbf{G}\mathbf{S}\mathbf{R}^{-1}\mathbf{h}^{\mathrm{T}}\right)^{\mathrm{T}}\right\}_{(i,1\ldots(M+f))}\mathbf{h} \\ &\quad + \underbrace{\left\{\mathbf{G}\left(\mathbf{Q} - \mathbf{S}\mathbf{R}^{-1}\mathbf{S}^{\mathrm{T}}\right)\mathbf{G}^{\mathrm{T}}\right\}_{(i,1\ldots(M+f))}}_{\mathbf{0},\text{ cf. (22)}}\mathbf{h}, \text{ for } i = 2 \ldots M \\ &= \Bigg[\underbrace{\left(\sum_{j=1}^{N}\alpha_j p_{i,j}\right)\ p_{i,1}\ \ldots\ p_{i,M-1}}_{\text{(a)}}\ \underbrace{\left(\sum_{j=1}^{M+f}\mathbf{h}_{(j)}p_{i,j}\right)\ p_{i,(M+1)}\ \ldots\ p_{i,(M+f)}}_{\text{(b)}}\Bigg]\mathbf{h}, \\ &\qquad \text{for } i = 2 \ldots N+1. \quad (23)\end{aligned}$$

It follows from (19) that $d_i = \left(\left\{\mathbf{P}_n^{-}\mathbf{h}\right\}_{(i)}\right)^2$. Observe that under the converse of condition 1, $a_i = \alpha_i$, for $i = 0 \ldots N$ and $a_i = 0$, for $i = N+1 \ldots M$. Thus under condition 1,

$$\exists i \text{ such that } a_i \neq \begin{cases} \alpha_i & i = 0 \ldots N \\ 0 & i = N+1 \ldots M. \end{cases} \quad (24)$$

By the structure of $\mathbf{h}$, cf. (11), and (23), part (a), (24) guarentees that $d_i > 0$.

Under condition 2,

$$\exists i \text{ such that } b_i \neq 0. \quad (25)$$

By the structure of $\mathbf{h}$ and (23), part (b), (25) guarentees that $d_i > 0$. ■

Remark 1. From the state-space model it may seem that it is not possible to obtain smoothed estimates $\hat{s}_{n-l|n}$ for lags $l > M-1$, i.e. that one cannot obtain smoothed estimates beyond the chosen order of the predictor, p. It is however possible to obtain smoothed estimates for arbitrary lags l without increasing the predictor order p, by defining

$$\bar{a}_i = \begin{cases} a_i & i = 0 \dots p \\ 0 & i = p+1 \dots l, \end{cases}$$

where $\{\bar{a}_i\}$ and l are used in place of $\{a_i\}$ and p in the encoder and decoder equations in Sections II-A and II-B.

Smoothed estimates at lags $l > M-1$ come at additional computational cost since they require extension of the signal model beyond what is required to model the encoder and source.

IV. Simulations

We present results for coding of a stationary source with transmission across an erasure channel. Section IV-A presents the decoding performance results and Section IV-B presents the encoder filters designed for the loss statistics under consideration (using the filter design algortihm in [1]) and exemplifies how these filters generally satisfy the conditions in Theorem 1.

A. Performance of Fixed-Lag Smooting

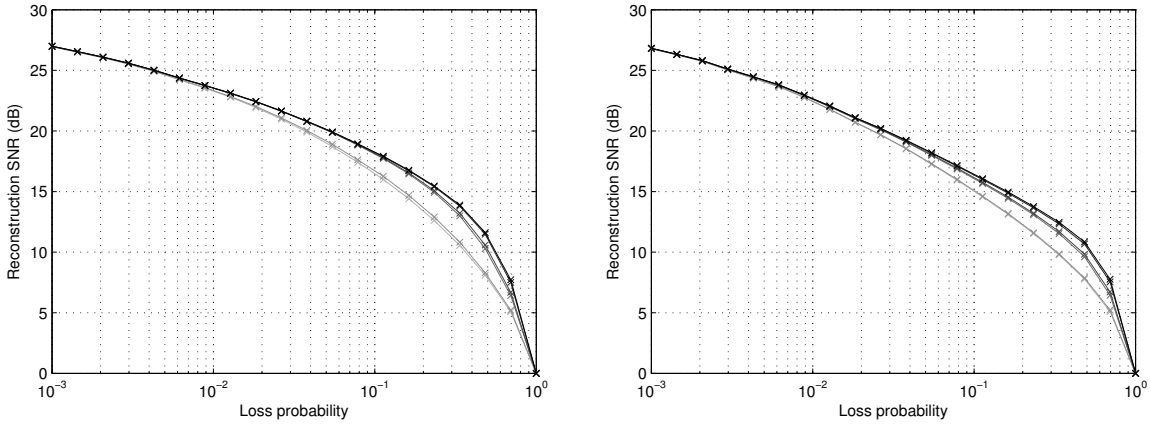
A set of Monte Carlo simulations were performed for the order $N = 5$ stationary source defined by the parameters $\alpha_1 = -0.2948,\ \alpha_2 = -0.9527,\ \alpha_3 = -0.0032,\ \alpha_4 = 0.0040,\ \alpha_5 = 0.1995$.

We consider channel erasure probabilities between 1×10^{-3} and 1 and simulate i.i.d. losses as well as Gilbert-Elliot (GE) losses with a mean error burst length 3, denoted "GE-3". Simulations have been performed for identical source sequences of length 1×10^6 at the simulated erasure probabilities. The quantizer is a Lloyd-Max quantizer at 4 bits/sample. In this example, we set the encoder filter orders to $p = f = N$.

Signal-to-noise ratios (SNRs) of the decoded signals are shown in Figure 2a for i.i.d. channel erasures and in Figure 2b for GE-3. The figures clearly show substantial improvement in SNR when using smoothing. In the i.i.d. loss case, the improvement by smoothing is up to $3.5\,\mathrm{dB}$ (at $48.3\,\%$ loss prob.) and approximately $2\,\mathrm{dB}$ at $10\,\%$. The improvement is most pronounced at high loss rates at which the decoding error continues to decrease significantly up to lag 4 out of 5 shown in the figure. In the GE-3 loss case, the decoding error is evidently worse due to the correlated losses. However an improvement in SNR of up to $3.0\,\mathrm{dB}$ is still achievable in this case (at $48.3\,\%$ loss prob.) and approximately $1.5\,\mathrm{dB}$ at $10\,\%$.

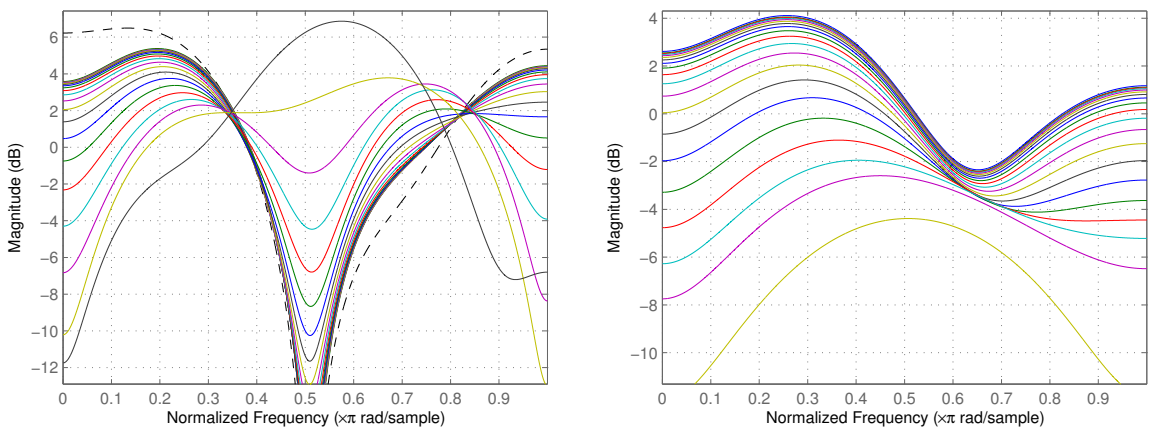
B. Examples of Encoder Filters

Figure 3a depicts the magnitude spectra of the encoder prediction error filters $1-P(z)$ designed for each of the channel erasure probabilities considered in Section IV-A. The encoder noise feedback filter spectra are depicted in Figure 3b. The inverse of the source magnitude spectrum is plotted as a dashed line. Figure 3 was included to emphasize that the filter design algorithm generally produces encoder filters that fullfil both of the requirements in Theorem 1. Figure 3a shows that the prediction error filters do not



(a) (b)

Fig. 2. Examples of coding: (a) i.i.d. channel erasures, (b) GE-3 channel erasures. Lloyd-Max quantization at 4 bits/sample. Results for increasing smoothing lags (0-5 samples) are plotted (from bottom to top) in increasingly darker shades of grey.



(a) (b)

Fig. 3. Encoder filters designed for the example in Section IV-A: (a) $1 - P(z)$ magnitude spectra for the erasure probabilities simulated in the example, the dashed line depicts the magnitude spectrum of the inverse of the source model; (b) accompanying $F(z)$ magnitude spectra. The spectra are designed for increasing loss probability from top to bottom.

equal the inverse of the source model and thus fulfill Theorem 1, condition 1. Figure 3b shows that the noise feedback filters are generally non-zero and thus fulfill Theorem 1, condition 2.

V. Conclusions

We have presented a generalization of the source coding and filter design framework previously introduced in [1], providing fixed-lag smoothing up to arbitrary lengths as well as encoder prediction error and noise feedback filters of arbitrary orders. These properties were constrained to the source model order in our previous work.

We have examined the fixed-lag smoothing properties of the coding framework and pointed out that smoothed (delayed) estimates at the decoder for lags up to at least

$\max\{N-1, p\}$ (source model and predictor orders N and p, resp.) are readily available at no additional computational cost which is not the case for other more general smoothing approaches associated with Kalman filtering. We have provided proof that the estimation error of these delayed estimates is guaranteed to decrease under simple conditions fulfilled by the filter design approach of the coding framework.

We have provided simulation results that demonstrate how the described smoothing approach can provide substantial improvements in estimation accuracy. Furthermore we have shown accompanying filter design examples which support the theoretical foundations for the observed improvements in estimation accuracy by smoothing.

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