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Implementation of a Stress-Dependent Strength Material Model in PLAXIS 3D

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Abstract

To perform tests on bucket foundations, full-scale testing is rarely used since it is rather expensive. Instead small-scale testing is done to examine the static and dynamic behaviour of such structures. In the laboratory at Aalborg University, small-scale testing of offshore support structures can be performed in a pressure tank, where a pressure can be applied in order to simulate deep water situations. Since the test set-up is downscaled 15 to 30 times compared to real-life structures, stresses and strains will be downscaled too. For soils, normally a Mohr-Coulomb failure criterion is used, and in the region of small stresses, a non-linear behaviour is observed - unlike the linear behaviour normally assumed in Mohr-Coulomb. To better model this non-linearity, a stress-dependent model for the strength of the soil material is sought to be implemented in PLAXIS 3D through FORTRAN to improve the computational accuracy of small-scale tests.

1 Introduction

Small-scale testing in geotechnical engineering is very often used to simulate or clarify behaviour of support structures of various kind. Because of the scaling of these structures, it is often hard to make an accurate model - analytical or numerical - since the behaviour of the soil is very dependent on the stress state inside the soil volume. This fact has long been well known cf. Krabbenhøft et al. (2011), but traditional geotechnical models have not been able (or not needed) to take this into account when designing these structures. To ease the burden for geotechnical designers, tools such as PLAXIS 3D have been developed. Contained in PLAXIS 3D are the most commonly used material models - none of which have the ability to simulate small-scale experiments where low stresses increase the relative soil strength.

This article aims to successfully implement in PLAXIS 3D a user defined soil model (UDSM) that through stress-dependent strength in a better way reproduces real-life behaviour of soil. Firstly, a mathematical formulation is presented based on Krabbenhøft et al. (2011). After this, it is outlined how to implement this model into PLAXIS 3D using the PLAXIS 3D-interface and calculation engine. The application of the model is then tested firstly by fitting the parameters in the failure surface of the mathematical formulation to results gained from triaxial tests on Aalborg University Sand No. 1 (Ibsen and Bødker, 1994). Afterwards it is tested through comparisons between the new formulation and the existing Mohr-Coulomb formulation within PLAXIS 3D and small-scale tests performed on bucket foundations.

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2 Theory

The soil mechanics concerning the UDSM will be outlined in the following. The UDSM has the ability to calculate the strength based on the current stress state.

2.1 Failure Surface

The formulation of the failure surface is based on Krabbenhøft et al. (2011). Even though the implementation aims towards PLAXIS 3D, the default geotechnical formulation is used where tension is negative and compression is positive, which is contrary to the common finite element formulation. The failure surface f is formulated as,

$$f = k_0\sigma_3 - \sigma_1 + s_{c0}\left(1 - \exp\left(-a\frac{\sigma_3}{s_{c0}}\right)\right) = 0, \quad (1)$$

where σ_1 and σ_3 are the largest and the smallest principal stresses respectively, k_0 defines the slope of the asymptote, s_{c0} defines the intersection with the σ_1-axis and a defines the curvature. Equation (1) can then be reformulated into,

$$\sigma_1 = k_0\sigma_3 + s_{c0}\left(1 - \exp\left(-a\frac{\sigma_3}{s_{c0}}\right)\right). \quad (2)$$

The formulation of the criterion goes towards an asymptote, when σ_3 goes towards a very large positive value,

e.g. very high compression. Thus the formulation becomes,

$$\sigma_1 = k_0\sigma_3 + s_{c0} \quad , \quad \sigma_3 \to \infty. \tag{3}$$

In geotechnics the soil strength is often described by the triaxial angle of friction, since this parameter resembles a physical characteristic and is a parameter in the Mohr-Coulomb failure formulation. To link the parameters described in this failure surface to the triaxial angle of friction, it is used that,

$$k = \frac{d\sigma_1}{d\sigma_3} = k_0 + a\ \exp\left(-a\frac{\sigma_3}{s_{c0}}\right) = \frac{1+\sin\varphi}{1-\sin\varphi}, \tag{4}$$

and thus, the triaxial angle of friction is linked to the parameters of the failure surface from equation (1).

2.2 Plastic Potential

The plastic potential g is, as opposed to the yield surface, governed by the internal angle of dilation ψ. In associated plasticity, where $\psi = \varphi$, this results in $g = f$. Assuming associated plasticity results in a much simpler theoretical solution, but in reality, associated plasticity does not resemble the behaviour of soils. In this particular case, the plastic potential is given as seen in equation (5) again assuming no cohesive behaviour,

$$g = m_0\sigma_3 - \sigma_1 + s_{c0}\left(1 - \exp\left(-b\frac{\sigma_3}{s_{c0}}\right)\right) = 0. \tag{5}$$

From this, two new parameters are introduced, m_0 and b. It is assumed that it is possible to compute the internal angle of dilation ψ from the relative density I_D and σ_3 as,

$$\psi = 0.195\ I_D + 14.9\left(\sigma_3'\right)^{-0.0976} - 9.95. \tag{6}$$

Similarly to equation (4), m can be described by the internal angle of dilation ψ as,

$$m = m_0 + b\ \exp\left(-b\frac{\sigma_3}{s_{c0}}\right) = \frac{1+\sin\psi}{1-\sin\psi}, \tag{7}$$

where the parameters m_0 and b related to the dilative behaviour can be fitted to the data set calculated from equation (6). The value of s_{c0} needs to remain the same in both situations. (Ibsen et al., 2009)

2.3 Fitting of Failure Criterion

The criterion can be calibrated to be used with any material exhibiting Mohr-Coulomb-like behaviour. In this study the criterion is fitted to be used with Aalborg University Sand no. 1. In order to calibrate the parameters a series of triaxial tests are used, in which the backpressure is varied to give failure points at different stress levels. The data from these tests can be seen in table 1. Since the curvature of the criterion is dominant at low stress levels, a series of tests including very low back pressures are used. The tests are carried out at the Geotechnical Laboratory at Aalborg University and are available in the data report by Ibsen and Bødker (1994).

Table 1: Test data used for fit.

Test no.	I_D [%]	σ_3 [kPa]	q^{failure} [kPa]
9301_12	78	5	45
9301_11	81	10.1	64
9301_10	81	20.1	102
9301_04	81	39.9	189
9301_02	81	100.2	412
9301_03	80	160.7	632
9301_07	79	320.1	1218
9301_08	78	640.2	2251
9301_32	79	800.2	2714

Table 2: Parameters of the failure criterion.

k_0 [-]	s_{c0} [kPa]	a [-]	m_0 [-]	b [-]
4.3584	75.1295	2.9954	1.5507	0.31118

The calibration is done by fitting equation (2) to the failure points of each triaxial tests, represented by the coordinate set $\left(\sigma_3^{failure}, \sigma_1^{failure}\right)$. The three remaining unknown constants of equation (2) are found by a non-linear least squares regression algorithm. The data points and the fitted expression are shown in figures 1 and 2. Similarly, the parameters associated to the plastic potential are fitted through a non-linear fit. This is done by assuming that m can be described in a manner similar to k, comparing equations (4) and (7). The parameters for Aalborg University Sand no. 1 at $I_D \approx 80\%$ are listed in table 2. By using equation (4), the equivalent angle of friction can be plotted for the different stress levels, which is shown in figure 3 and by using equation (7) for the equivalent angle of dilation in figure 4. In figure 4 the data points for each of the tests are shown as well. The internal angle of dilation for these data points have been calculated using equation (6).

3 Implementation in PLAXIS 3D

To make use of the UDSM with stress-dependent strength along with the user interface and calculation engine in PLAXIS 3D, a certain procedure must be followed. The procedure will be outlined in the following.

Basically, PLAXIS 3D provides all necessary inputs for the UDSM, and it must be able to handle three objectives.

1. Initialization of needed state variables
2. Calculation of stresses using a constitutive model
3. Creation of elastic and effective stiffness matrices

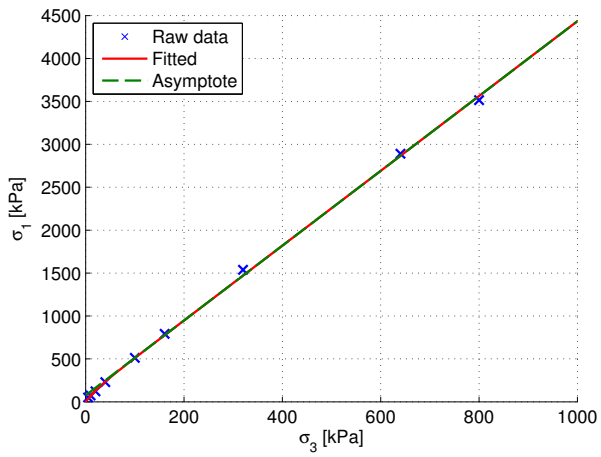

Figure 1: Fit of failure criterion to (σ_3,σ_1)-data.

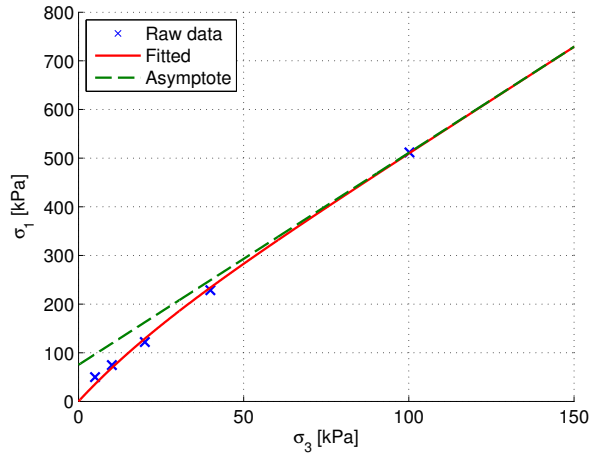

Figure 2: Zoom of figure 1. From this it is apparent that for small values of σ_3, the non-linear behaviour is important.

In this particular case, no state variables are used, however this could be e.g. the mean stress p'. The creation of the elastic stiffness matrix is done readily based on material parameter input done in the user interface in PLAXIS 3D. The creation of the effective stiffness matrix is performed by a stress return algorithm that calculates an allowable stress state for the soil, if the stress is outside the failure surface. The mechanics of these algorithms will not be of further subject in this paper.

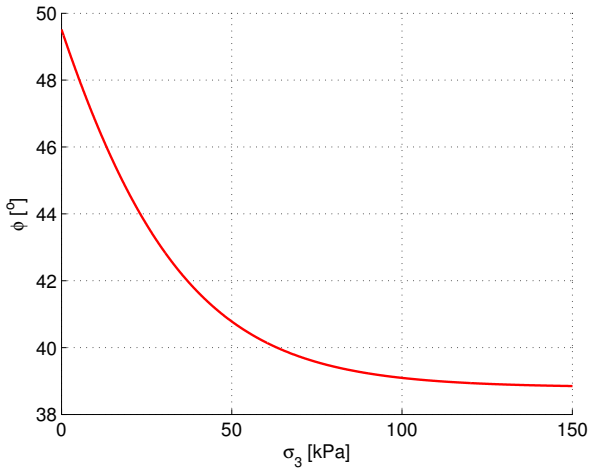

Figure 3: Internal angle of friction as a function of σ_1.

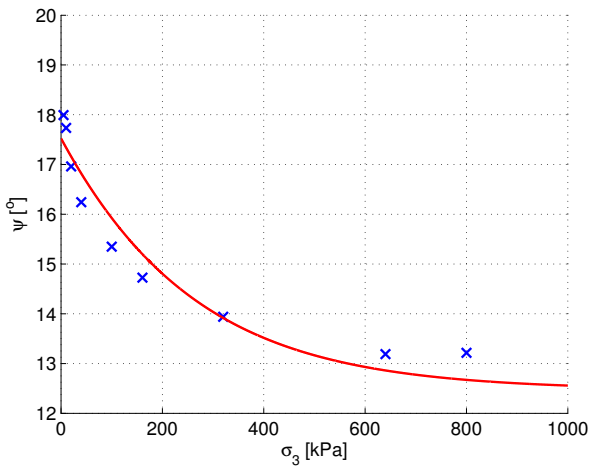

Figure 4: Internal angle of dilation as a function of σ_1.

4 Application of Material Model

In the following section, the application of the material model is tested. This is done in various ways as described below. The general method is to compare the actual test data with the results from various PLAXIS 3D models done with the linear Mohr-Coulomb model already implemented in PLAXIS 3D and the recently implemented non-linear Mohr-Coulomb model.

- The SoilTest-function in PLAXIS 3D is used to perform a triaxial test of the implemented material model with the fitted parameters on a soil volume, which is compared to triaxial tests that the material model parameters have been fitted against, and to SoilTest-results with the linear Mohr-Coulomb model.
- PLAXIS 3D is used to model a small-scale test of a bucket foundation and the results of this are compared to the actual test results. The PLAXIS 3D-model is done using both the traditional linear Mohr-Coulomb failure envelope and the non-linear Mohr-Coulomb failure envelope.

4.1 Comparison between SoilTest and Triaxial Tests.

In order to make use of the SoilTest-function in PLAXIS 3D that is able to perform triaxial tests, elastic parameters (E,ν) are needed apart from the fitted parameters defining the failure (k_0, s_{c0}, a, m_0, b).

Since the implemented model is a linear elastic-perfectly plastic model with non-linear Mohr-Coulomb failure criterion, the elastic path will not be portrayed

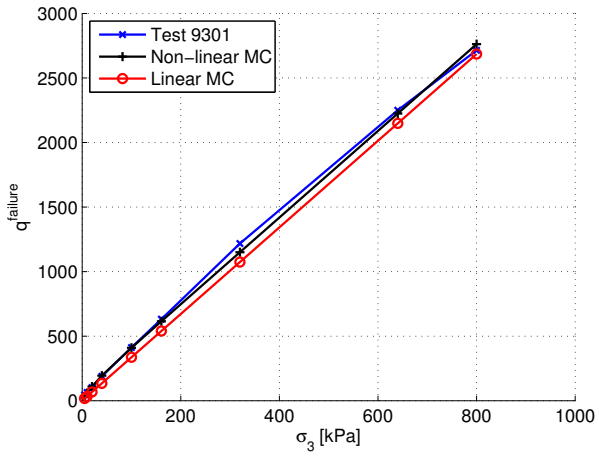

Figure 5: Plot of q_f for different σ_3 for all three approaches to determine failure.

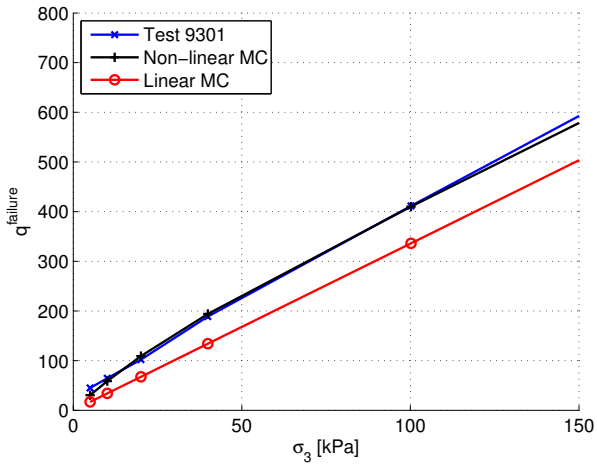

Figure 6: Zoom of figure 5. From this it is clear that the Linear MC underestimates the failure load, and that the Non-linear MC is almost similar to the test data.

properly in any case. That means in fact that only the stress at failure will be of interest.

The deviatoric stress at failure $q^{failure}$ will be examined for five triaxial tests at different σ_3. The non-linear Mohr-Coulomb has five input parameters for the failure criterion and two elastic parameters. The linear Mohr-Coulomb has three input parameters - the effective cohesion $c' = 0$ for non-cohesive materials, the internal angle of friction φ, and the internal angle of dilation ψ - and two elastic parameters. Since only $q^{failure}$ is of interest, the elastic parameters will not be mentioned any further. As the linear Mohr-Coulomb model only allows for constant values of φ and ψ, the asymptotic value of these will be used, which is $\varphi_{\text{asymp}} = 38.8°$ and $\psi_{\text{asymp}} = 12.6°$ according to the fitted expression, see also figures 3 and 4, respectively.

Figures 5 and 6 show the comparison of the different approaches to determine the deviatoric stress at which failure occurs. It is evident from the results that the linear Mohr-Coulomb underestimates $q^{failure}$ in general. The same thing applies to some degree for the non-linear Mohr-Coulomb, especially for very low σ_3. This underestimation is caused by the fact that the fitted model underestimates φ, cf. figure 2. For $\sigma \geq 5$ kPa the non-linear Mohr-Coulomb shows to accurately estimate $q^{failure}$.

4.2 Comparison between PLAXIS and Small-scale Test

In the previous section, it was shown that the non-linear Mohr-Coulomb provides a better estimate of the failure stress for a triaxial test at low backpressure than the traditional linear Mohr-Coulomb. In the following, an actual small-scale test done on a bucket foundation in the laboratory will be examined. The goal is to model the scaled bucket test in PLAXIS 3D using both the linear and the non-linear Mohr-Coulomb criterion and compare the results of the failure moment to the small-scale test results.

The static small-scale test that will be examined is described in Larsen (2008a) and documented in detail in Larsen (2008b). The test setup consists of a sandbox in which the bucket foundation is installed. Through a loading frame, vertical load can be applied. The horizontal load is applied at a distance above the sand surface to exert the bucket foundation to a moment. The test setup is seen in figure 7. All tests described in Larsen (2008a) are performed on Aalborg University Sand No. 1. In each test performed, the relative density I_D is measured. Since the failure criterion for the non-linear Mohr-Coulomb model is calibrated against triaxial tests at a certain relative density, the sand used in the small-scale test must be of similar relative density compared to the triaxial tests.

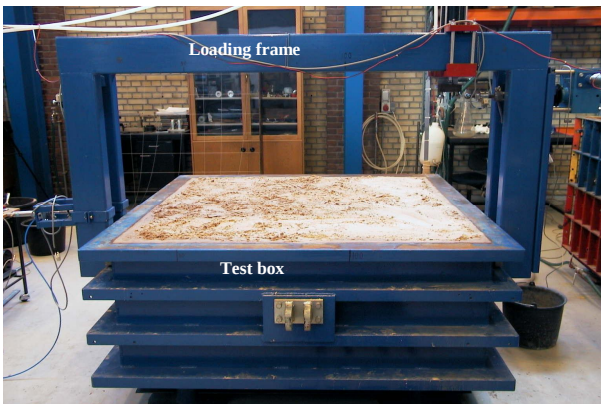

Figure 7: Test setup for scaled bucket foundations. (Larsen, 2008a)

Bucket Test No. 0104.1701

The basis for this comparison is 'Bucket test no. 0104.1701' (Larsen, 2008b). Two similar tests have been

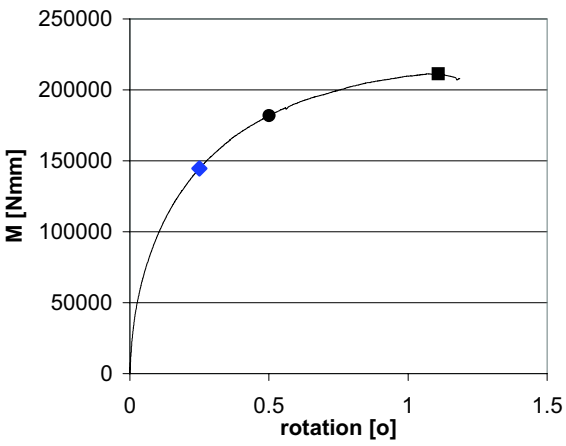

Figure 9: Moment-rotation curve for 'Bucket test no. 0104.1701'.

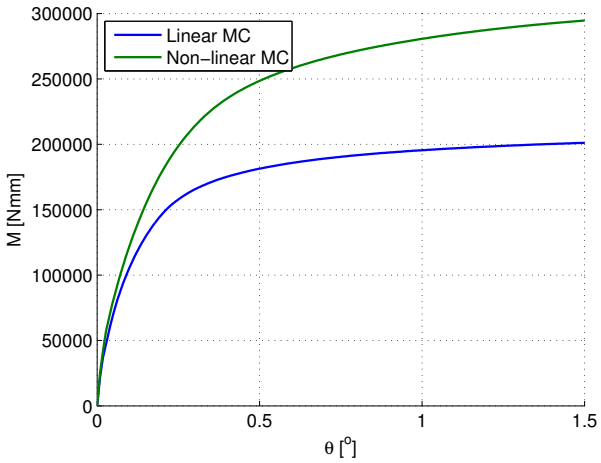

Figure 10: Moment-rotation curve for Linear MC and Non-linear MC.

executed as well, 'Bucket test no. 0104.901' and 'Bucket test no. 0104.1901'. These tests were done for a bucket with diameter $D = 300$ mm, skirt length $L = 300$ mm and with no vertical load. The horizontal load was applied in a height of 2610 mm. The relative density in the specific test is $I_D = 86\%$. This in turn means that the parameters in the non-linear Mohr-Coulomb criterion have been calibrated against a looser soil. Force and displacement are tracked in the test, which makes it possible to compute moment and rotation at sand surface. Since non-linear elasticity is not implemented in neither of the two materials models, only failure moment is examined. A schematic display of the test setup is shown in figure 8. The setup is duplicated in the numerical model using both linear and non-linear Mohr-Coulomb failure criterion. Since it is not always obvious when a finite

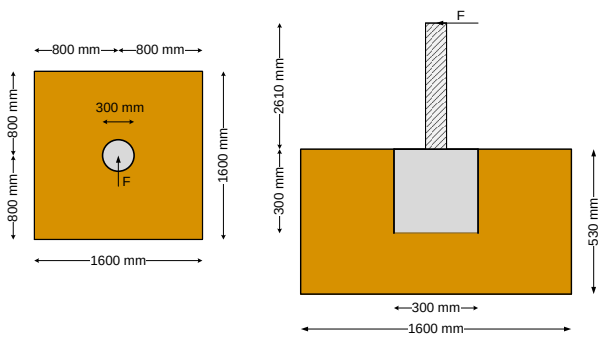

Figure 8: Test setup for 'Bucket test no. 0104.901'. (Larsen, 2008b)

element has failed, a point of failure normally needs to be chosen. In this case, the rotation of the bucket will be examined, and the failure moment will be chosen as the moment at a rotation of $\theta_{\text{failure}} = 1.11°$, which is the rotation at failure in 'Bucket test no. 0104.1701'. Figures 9 and 10 show the actual moment-rotation for the test and the finite element models, respectively.

At a rotation of $\theta_{\text{failure}} = 1.11°$, table 3 shows the moment at failure. It is evident that the failure moment for the test is higher than the linear Mohr-Coulomb model as expected, since the linear Mohr-Coulomb model does

Table 3: Comparison of failure moment at $\theta_{\text{failure}} = 1.11°$.

Test 0104.1701	Linear MC	Non-linear MC
213773 Nmm	197063 Nmm	284612 Nmm

not take the strength increase for low stresses into account. The non-linear Mohr-Coulomb model does however take this into account, and it was expected that this model would come closer to test results, although still underestimating failure moment because the non-linear failure criterion was calibrated against a looser soil than the one used in 'Bucket test no. 0104.1701'. The reason this is not the case, could perhaps be that the failure moment in the test is too low because the sand was loosened from raising the water level from the bottom of the sand container after leveling the sand. It is evident from Larsen (2008a) that this was done for some tests, while in others the water level was raised from the top of the sand container. It is however not clear in which tests, which approach was used. It could be argued that the non-linear Mohr-Coulomb model has not failed for a rotation of 1.11°, since the $M - \theta$-diagram at that point does not tend towards the asymptotic moment. Although the FE-results were not completely in line with the test results, the non-linear Mohr-Coulomb model still exhibits higher strength for low stresses present in small-scale setups like the one examined. This proves that the non-linear Mohr-Coulomb failure criterion behaves as expected. Regarding the comparison with the test results, more work should go into the modeling of the small-scale test in PLAXIS 3D.

5 Conclusion

The main goal of this study was to successfully implement a material model with stress-dependent strength in PLAXIS 3D. The stress-dependent strength was obtained through a non-linear Mohr-Coulomb failure criterion. This objective was achieved without encoun-

tering severe problems. From the implementation in PLAXIS 3D, the material model was tested in several ways, against existing test results and the linear Mohr-Coulomb failure criterion. These comparisons showed that the non-linear MC-model accurately depicted the deviatoric failure stress for a series of triaxial tests with varying backpressure. The model seemed to be less accurate for a static bucket foundation test performed in small-scale at Aalborg University (Larsen, 2008a), but the non-linear MC-model exhibited the correct behaviour compared to the linear MC-model. It was concluded that more effort should go into the built of the FE-model. Ultimately, the stress-dependent material model was implemented with success and ongoing work regarding the implementation of non-linear stiffness will result in a material well-suited for predicting the behaviour of small-scale tests.

6 Further Work

In recent years, computational methods such as the finite element method have moved to become an essential tool for every geotechnical engineer or scientist. The demand within the fields of offshore geotechnical research calls for the use of small-scale models or computer models, meaning that the demand for accurate soil models is increasing. For soils in general, the stress state within the soil volume is of great importance, which has been a well known fact for many years. In this study, the goal has been to implement a soil material model that takes the stress-dependent strength of non-cohesive soils into account into PLAXIS 3D. PLAXIS 3D has since it was published been widely used within the field of geotechnical engineering. PLAXIS 3D has been developed and improved during the years, adding more soil material models, but not a single model able to take the stress-dependent strength into account has been added.

Through a non-linear Mohr-Coulomb relationship, the stress-dependent strength is taken into account in a linear elastic-perfectly plastic soil model. The implementation has proven to be successful, and after calibration of the failure criterion of the model, the comparison with the test results showed that more work needs to be put into the FE-model of the small-scale test. The non-linear MC-model did however behave just as expected and the comparison to the triaxial test results supports this. To further improve the soil material model, non-linear elasticity needs to be implemented. This should enable the material model to take stress-dependent stiffness into account. This will enable a better representation of the path towards failure, since the currently implemented model only predicts failure. Another addition of the soil model is hardening and softening.

In this study, the material parameters have been calibrated towards nine triaxial tests for a certain sand, Aalborg University Sand No. 1. Further studies should include the calibration of the failure parameters towards more different types of sand. Since no general description of the material parameters have been developed, a general description could aim to take the relative density, the maximum or minimum void ratio, or the average grain size into account, so material model parameters could be determined in ways other than calibration towards triaxial tests.

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