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## Size Effects on the Bending Behaviour of Reinforced Concrete Beams

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## APPENDIX A:

## TEST PROGRAMME FOR BENDING FAILURE OF REINFORCED CONCRETE BEAMS OF DIFFERENT SCALE

Prepared by M.S. Henriksen, R. Brincker and G. Heshe

*Abstract*

In this appendix a brief summary of experiments on reinforced concrete beams in three-point bending performed at Aalborg University is given. The aim of the investigation is to determine the full load-deflection curves for different beam sizes, different types of concrete and different amounts and types of reinforcement, and to calculate the rotational capacity for all beams. The rotational capacity is here defined as the non-dimensional area under the load-deflection curve, and the results are shown as functions of the reinforcement ratio.

*Introduction*

This appendix gives a summary of the experimental results from tests of reinforced concrete beams in three-point bending carried out in the Structural Research Laboratory at the Department of Building Technology and Structural Engineering, Aalborg University. The experiments were performed in the period of August 1994 to June 1995 in connection with a Round Robin on "Scale Effects and Transitional Failure Phenomena of Reinforced Concrete Beams in Flexure" initiated by the European Structural Integrity Society - Technical Committee 9. The test programme has been designed according to the proposals given by Bosco and Carpinteri [1]. The aim of the investigations was to verify the scale dependency of plastic rotational capacity and minimum reinforcement and the existence of transitional phenomena of failure.

The test programme at Aalborg University has involved 114 reinforced concrete beams for determination of load-deflection curves, 54 plain concrete beams for determination of the fracture energy  $G_F$  and 324 concrete cylinders for determination of strength parameters. For the reinforced concrete beams four different parameters were varied. The slenderness ratios were 6, 12 and 18 and the beam depths were 100 mm, 200 mm and 400 mm giving a total of nine different geometries. In order to fulfill the requirements for the Round Robin 6 reinforcement ratios were chosen between 0.06 % and 1.57 % giving a lightly and more heavily reinforced regime. For the concrete both a normal strength and a high strength concrete were chosen with a compressive strength of approximately 60 MPa and 100 MPa. All experiments were repeated three times. In Henriksen et al. [2] a more detailed description of the experiments is given.

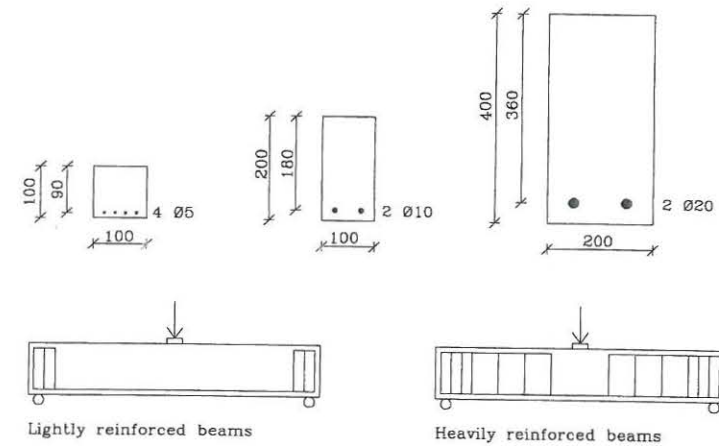


Figure A1. Scaling of cross-section and design of a typical test beam. Units in mm.

*Test Specimens*

The geometry of the beams was chosen in accordance with the requirements given by Bosco and Carpinteri [1], and the different geometries and main reinforcements for all beams are given in Table A1. The reinforcement ratios (the steel area divided by cross-section of the beam) were varying from 0.06 % to 1.57 % for the normal strength concrete beams and 0.14 % to 0.39 % for the high strength concrete beams.

The reinforcement of the beams was designed so that all beams failed in bending. Some of the highly reinforced beams were reinforced with stirrups to avoid anchorage and shear failure. Furthermore, to avoid influence on the compression failure, stirrups and compressive reinforcement were not placed in the mid zone of the beams.

The geometry of the cross-sections of the beams is shown in Figure A1. The distance from the top of the beam to the middle of the reinforcement bars were in all cases equal to  $0.9h$ , thus  $h_{ef} = 0.9h$  for all beams. Note that the beams and the size of the reinforcement bars are scaled to preserve geometrical similitude.

*Concrete*

Two types of concrete were used for the experiments. The mix of concrete was prepared in accordance with the requirements given by Bosco and Carpinteri [1] and the recipes are listed in Table A2. Due to the large amount of concrete for each casting, a commercial manufacturer (ISO 9002 certificate) delivered the concrete. Totally 18 castings have been carried out. The test beams were cast in steel moulds.

Table A1: Overview of test beams, units are in [mm]

$b \times h$	$l/h$	NSC and HSC					
		NSC 0.06 %	NSC 0.14 %	NSC 0.25 %	HSC 0.39 %	NSC 0.78 %	NSC 1.57 %
100 × 100	6					4 $\phi$ 5	8 $\phi$ 5
100 × 100	12		1 $\phi$ 4	2 $\phi$ 4	2 $\phi$ 5	4 $\phi$ 5	8 $\phi$ 5
100 × 100	18					4 $\phi$ 5	8 $\phi$ 5
100 × 200	6					2 $\phi$ 10	4 $\phi$ 10
100 × 200	12		1 $\phi$ 6	1 $\phi$ 8	1 $\phi$ 10	2 $\phi$ 10	4 $\phi$ 10
100 × 200	18					2 $\phi$ 10	4 $\phi$ 10
200 × 400	6					2 $\phi$ 20	4 $\phi$ 20
200 × 400	12	1 $\phi$ 8	1 $\phi$ 12	1 $\phi$ 16	1 $\phi$ 20	2 $\phi$ 20	4 $\phi$ 20
200 × 400	18					2 $\phi$ 20	4 $\phi$ 20

Table A2: Mix proportions of the two types of concrete

Content	Normal strength concrete		High strength concrete	
	Units	$kg/m^3$	$kg/m^3$	$kg/m^3$
Cement		350	466	148
Water		160	146	146
Silica		0	36.1	16.4
Plasticiser 1		3.84	1.80	1.5
Plasticiser 2		0	12.6	10.4
Sand (0-2 mm)		898	899	324
Gravel (4-8 mm)		896	899	324
Air		0	0	0
Density		2307 $kg/m^3$	2399 $kg/m^3$	

The mechanical properties of the concrete were obtained by standard tests. The tensile splitting strength, the compressive strength, the compression softening and the modulus of elasticity were determined on cylinders (diameter 100 mm and depth 200 mm). The bending tensile strength and the bending fracture energy were determined from tests on notched RILEM beams with a span of 800 mm, a thickness of 100 mm and a depth of 100 mm. The material parameters are listed in Table A3.

For both normal strength and high strength concrete experiments on cylinders (100 × 200 mm) to obtain the stress-strain relations in uniaxial compression have been carried out by Dr. Henrik Stang at the Technical University of Denmark. Typical compression softening curves are shown in Figure A2. Note the more brittle behaviour of the high strength concrete.

#### Reinforcement

In order to have six different reinforcement ratios and to reinforce beams of different scale without changing the geometry of the cross-sections it was necessary to use reinforcement with 8 different diameters. Two types of ribbed reinforcement bars were used with approximately the same type of ribs. The  $\phi$ 4 mm and  $\phi$ 5 mm steel bars were cold deformed and had a relatively small deformation

Table A3: Mechanical properties of the two types of concrete

	Units	Normal strength concrete		High strength concrete	
		Mean	S. Dev.	Mean	S. Dev.
Compressive strength	[MPa]	64.0	6.12	98.5	6.60
Splitting strength	[MPa]	4.09	0.54	6.06	0.28
Modulus of elasticity	[MPa]	4.23E4	0.31E4	4.55E4	0.23E4
Bending tensile strength	[MPa]	5.51	0.34	7.16	0.29
Bending fracture energy	[J/m <sup>2</sup> ]	126	8.30	118	5.24

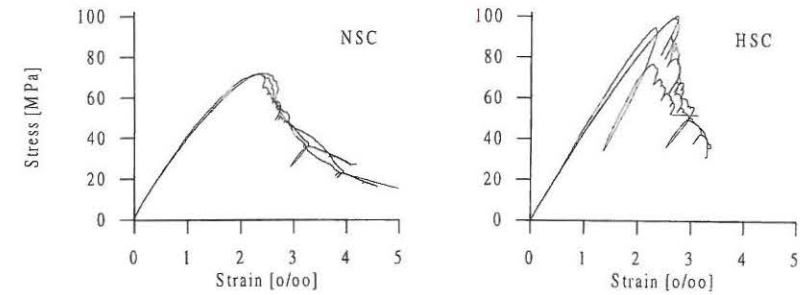


Figure A2. Compression tests with softening for normal strength concrete (left) and high strength concrete (right). The test specimen is a 100 × 200 ( $d \times h$ ) mm cylinder.

capacity, while the  $\phi$ 6 mm,  $\phi$ 8 mm,  $\phi$ 10 mm,  $\phi$ 12 mm,  $\phi$ 16 mm and  $\phi$ 20 mm steel bars had a large yield capacity and a clear strain hardening.

The mechanical properties of the steel were determined on 500 mm long specimens subjected to uniaxial tension. The results given as nominal values are summarized in Table A4. It is observed from Table A4 that the bars with a large yield capacity fulfil the Eurocode 2 requirements for high ductility reinforcement  $f_u/f_y < 1.08$  and  $\epsilon_{su} > 5\%$ . Some typical curves from the tensile test of the ribbed steel bars are shown in Figure A3. Note the difference in the behaviour for the two types of steel.

#### Test Set-up

All beams were subjected to three-point bending. The tests were carried out using the same specially designed servo-hydraulic testing system allowing for testing of beams of different size. At both supports horizontal displacements and rotations were allowed for and, at one support, also rotations around the beam axis were allowed. At the load point rotations were allowed around all axes. The load was transferred through a square steel plate with sides equal to the beam width.

The stroke (the displacement of the piston of the hydraulic actuator) was measured using the

Table A4: Mechanical properties of the reinforcement bars.

Steel type	Young's Modulus $E_s$ [MPa]	Yield strength $f_y$ [MPa]	Yield capacity $\Delta\epsilon_y$ [%]	Ultimate strength $f_u$ [MPa]	Ultimate strain $\epsilon_{su}$ [%]	Ultimate to yield strength ratio $f_u/f_y$ [-]
ø4	2.01E5	740	0	740	1.41	-
ø5 charge 1	1.94E5	701	0	701	2.20	-
ø5 charge 2	2.01E5	708	0	708	2.94	-
ø6	2.09E5	600	3.44	664	13.1	1.11
ø8	2.05E5	604	2.29	685	10.7	1.13
ø10	2.06E5	611	2.27	681	11.2	1.11
ø12	2.01E5	555	2.56	642	11.5	1.16
ø16	1.96E5	531	1.60	630	11.0	1.19
ø20	1.82E5	531	1.13	624	9.27	1.18

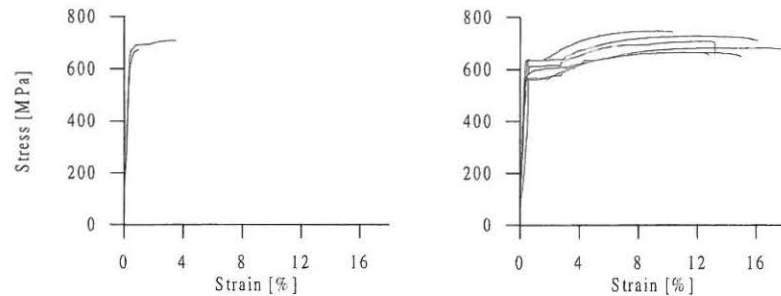


Figure A3. Typical stress-strain relations for ribbed steel bars subjected to tensile testing. Typical cold deformed bars (left) and typical normal ribbed bars (right).

built-in LVDT (Linear Variable Displacement Transformer). The vertical deflection of the beams was measured at eight points along the beam axis. The rotations of the ends of the beams were measured using two LVDT's at each end.

The mutual rotations of the cross-sections at different points along the beam axis were measured using a number of specially designed measuring frames. Dividing these rotations by the distance between the measuring frames gives a direct estimate of the average curvature between the frames.

#### Main Test Results

The main results for the normal strength and high strength concrete beams with slenderness ratio 12 are shown in Figures A4-A6. The results are given as load-deflection curves for the mid point of the beam.

Typical distributions of vertical displacement and of the curvature along the beam axis at different load levels (indicated at the load deflection curve) are shown in Figure A7 for a normal strength concrete beam with dimensions  $200 \times 400 \times 4800$  mm ( $b \times h \times l$ ) reinforced with one ø20 mm ribbed steel bar corresponding to a reinforcement ratio of 0.39 %.

#### Rotational capacity

For each beam the rotational capacity was obtained by calculating the total plastic work as the area under the load-deflection curve and dividing this work by the maximum yield moment (the maximum moment at the yielding regime for each load-deflection curve). This value is a direct estimate of the total plastic rotation of the beam ends.

Results for the rotational capacity as a function of the reinforcement ratio are given in Figure A8 for normal strength concrete and in Figure A9 for high strength concrete.

The results for slenderness ratios 6 and 18 in Figure A8 are too few to indicate any clear size effect. In fact, the small beams have the smallest rotational capacity. This is believed to be due to the low deformation capacity of the reinforcement used for the smallest beams. The results for slenderness ratio 12, however, show a clear size effect for the two larger beam sizes. The smallest beam has a significantly lower rotational capacity, again indicating the large influence from the limited deformation capacity of the cold deformed reinforcement.

The results for high strength concrete in Figure A9 are too limited to indicate any clear size effect. Again the limited deformation capacity of the cold deformed reinforcement causes the small beams to have a significantly lower rotational capacity.

#### References

1. Bosco, C. and A. Carpinteri (1993): Proposals for a Round Robin on Scale Effects and Transitional Failure Phenomena of Reinforced Concrete Beams in Flexure. European Structural Integrity Society, Technical Committee 9 on Concrete, ESIS-TC9.
2. Henriksen, M.S., J.P. Ulfkjær, and R. Brincker (1996): Scale Effects and Transitional Failure Phenomena of reinforced Concrete Beams in Flexure. Part 1. Fracture & Dynamics Paper No. 81. Department of Building Technology and Structural Engineering, Aalborg University, Denmark.

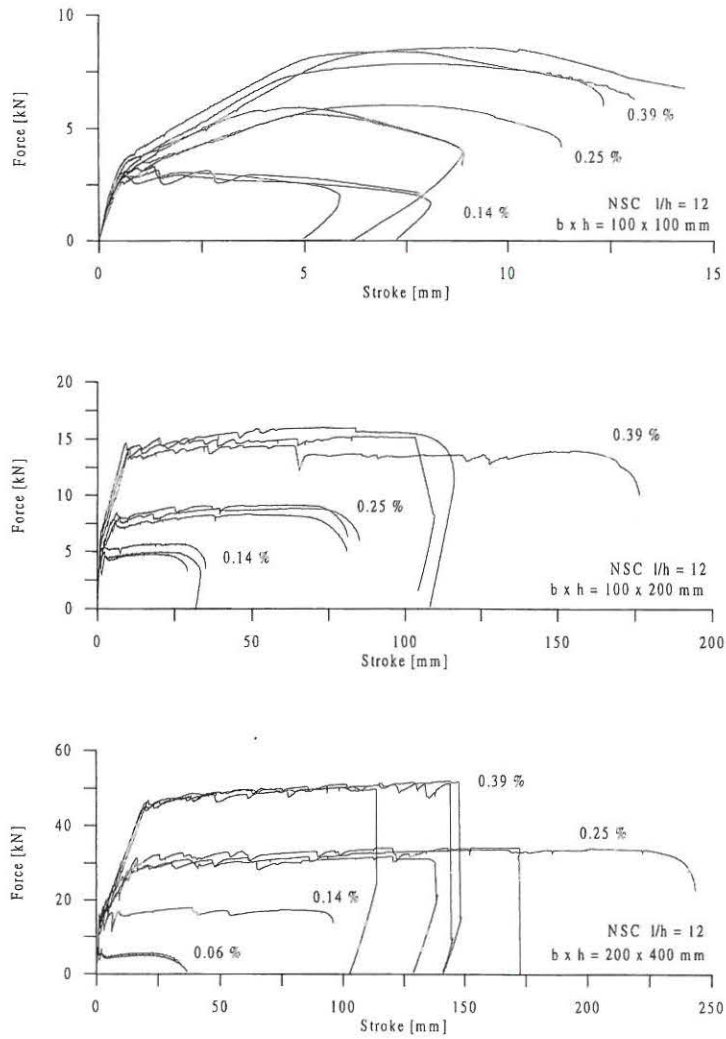


Figure A4. Load-displacements curves for normal strength concrete (NSC) beams with reinforcement ratios 0.06 %, 0.14 %, 0.25 % and 0.39 % and slenderness number 12.

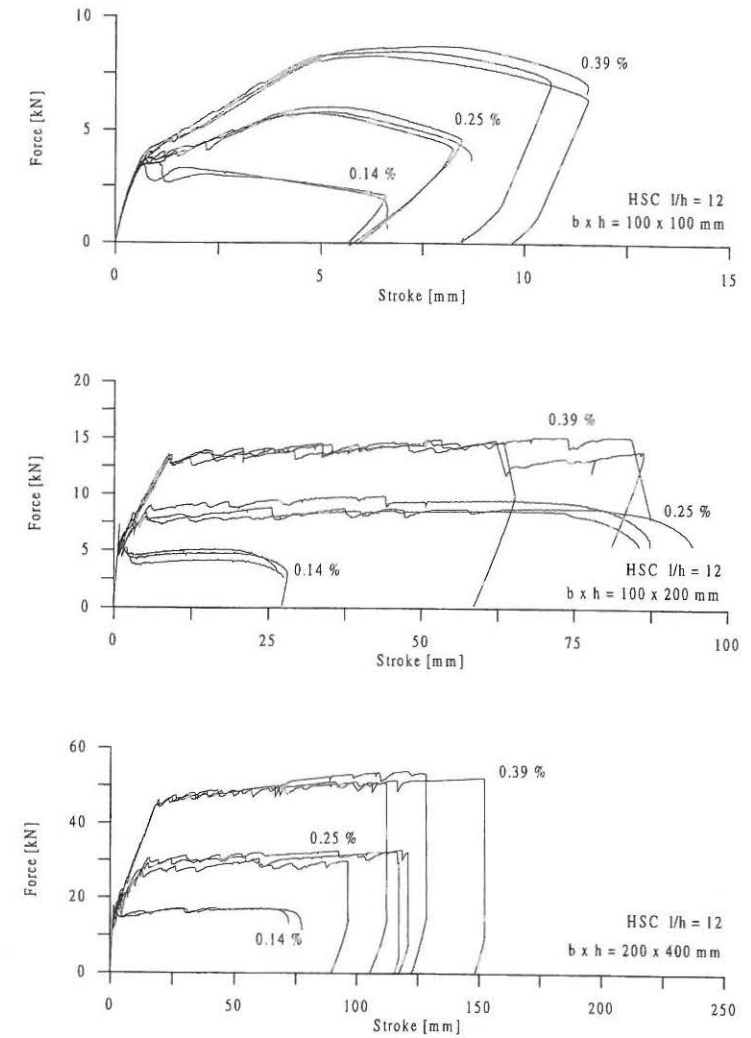


Figure A5. Load-displacements curves for high strength concrete (HSC) beams with reinforcement ratios 0.14 %, 0.25 % and 0.39 % and slenderness number 12.

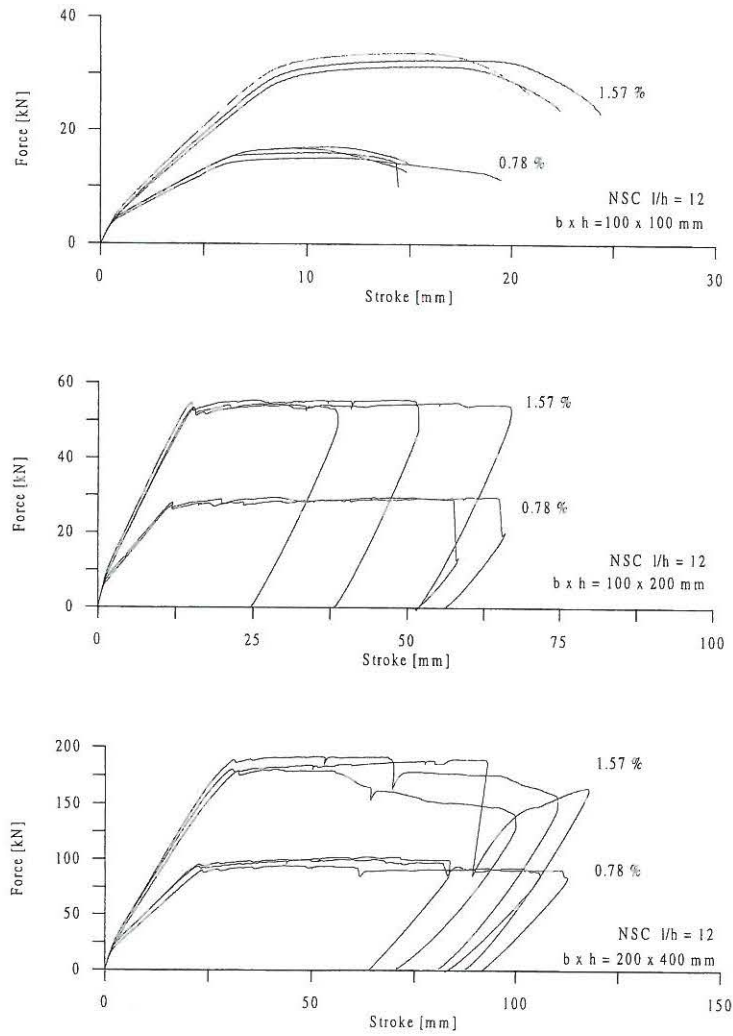


Figure A6. Load-displacements curves for normal strength concrete (NSC) beams with reinforcement ratios 0.78 % and 1.57 % and slenderness number 12.

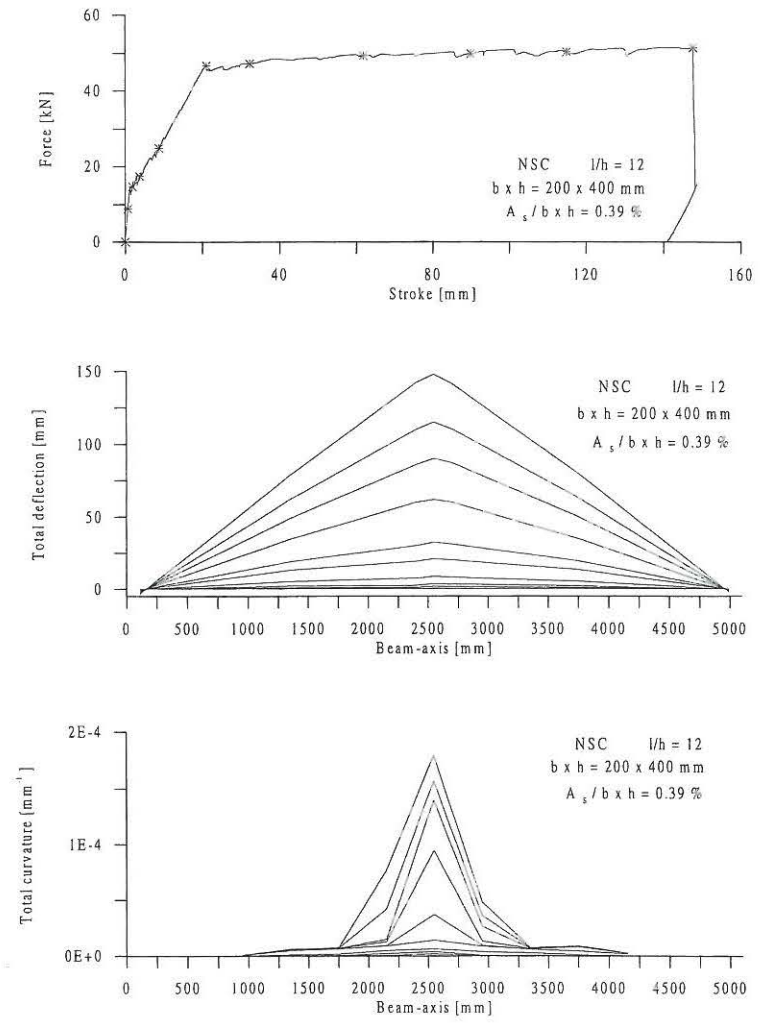


Figure A7. Load-deflection curve (top), vertical distribution (mid) and distribution of curvature (bottom) along the beam axis for a normal strength concrete beam (200 × 400 × 4800 mm) with a reinforcement ratio of 0.39 %. The load levels are marked with asterisk (top).

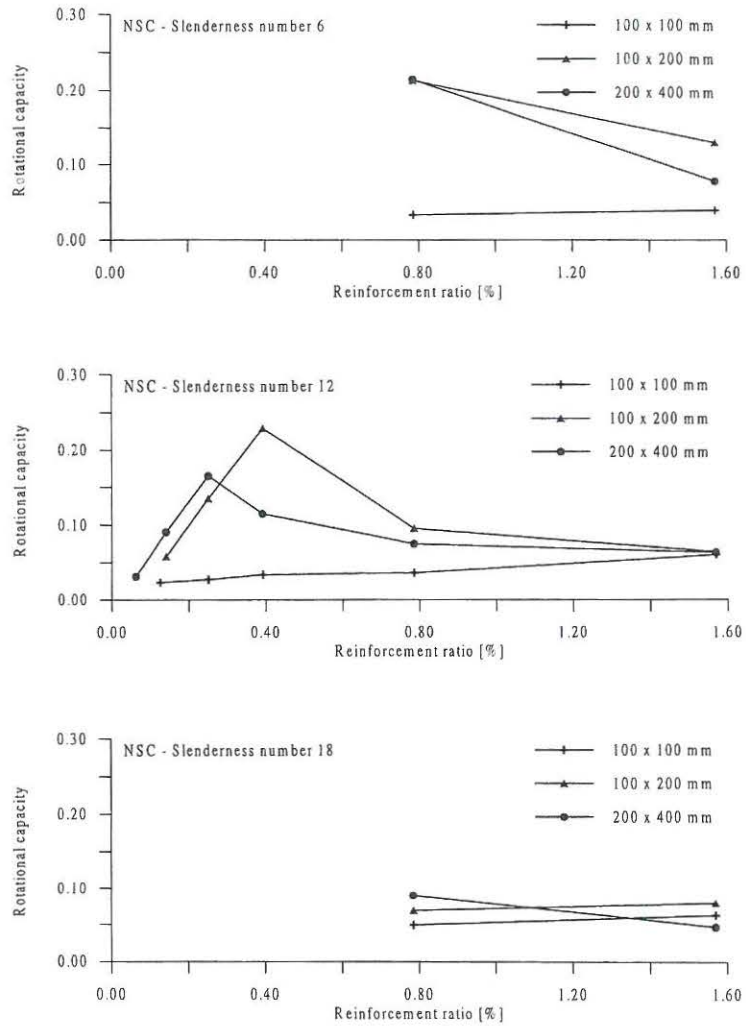


Figure A8. Rotational capacity as a function of reinforcement ratio for normal strength concrete (NSC) beams with slenderness numbers 6, 12 and 18.

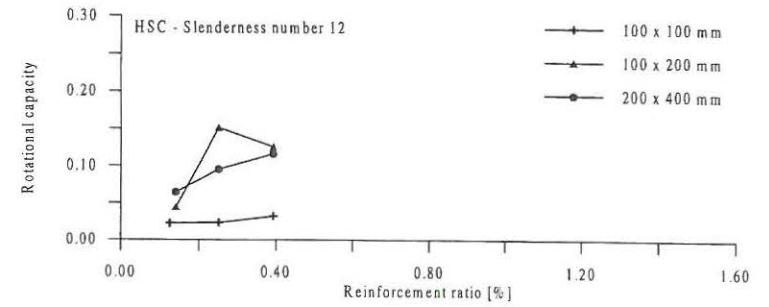


Figure A9. Rotational capacity as a function of reinforcement ratio for high strength concrete (HSC) beams with slenderness number 12.