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## Size Effects on the Bending Behaviour of Reinforced Concrete Beams

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after the concrete tension failure peak seems to be well estimated by the model, indicating that it seems reasonable to assume a constant frictional shear stress in the debonded zones at each side of the crack.

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#### APPENDIX C:

#### MULTIPLE CRACKING AND ROTATIONAL CAPACITY OF LIGHTLY REINFORCED BEAMS

Prepared by F.A. Christensen, M.S. Henriksen and R. Brincker

#### Abstract

In this appendix a model is formulated for the rotational capacity of reinforced concrete beams assuming rebar tension failure. The model is based on a classical approach and establishes the load-deflection curve of a reinforced concrete beam. The rotational capacity is then obtained as the area under the load-deflection curve divided by the yield moment of the beam. In calculating the load deflection curve, the cracking process of the concrete is ignored. By assuming that all cracks are fully opened, the energy dissipated during cracking of the concrete is taken into account by simply adding the total tensile fracture energy to the total plastic work obtained by the classical analysis.

#### Model Formulation

Before cracking of the concrete both the concrete and the reinforcement are assumed to behave elastically, and no slip is assumed between concrete and reinforcement. Assuming a linear variation of the normal beam strain over the cross-section, the stress distribution is obtained by classical beam theory.

When the tensile strength is reached at the tensile side of the beam, the concrete is assumed to crack. Further, cracks are assumed to be formed during constant bending moment (no decrease of the bending moment) and are allowed to extend until the level of the neutral axis. The tension force from the reinforcement is balanced by compression stresses in the concrete. The size of the compression zone is obtained by assuming a uniform distribution of the compression stresses and using an equilibrium equation. At the cracked section the tensile force in the reinforcement is transferred to the surrounding concrete by assuming a formation of two debonded zones around the crack with constant shear friction  $\tau_f$ . In Figure C1 the stress distribution at a cracked section of the beam is shown immediately before and after formation of a crack.

#### Calculation Procedure

The crack development is initiated when the tensile strength is reached at the tensile side of the beam in the cross section with maximum bending moment. This corresponds to the situation shown in Figure C1. As the load increases, cracks might form in neighbour sections. If the bending moment is equal to the cracking moment at section II a new crack will be formed at section II. If the bending moment is less than the cracking moment, the load is increased causing the debonded zones to extend. By repeating this procedure cracks are formed one by one until tensile

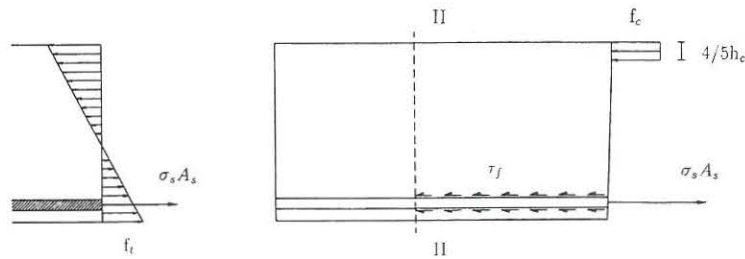


Figure C1. Stress distribution in cracked and uncracked cross-section. The debonded zone with constant shear friction stress ends at section II.

failure of the reinforcement bar. At the time a new crack is formed it is assumed that the strain at section II in Figure C1 is the same in the concrete and in the reinforcement and that the strain is equal to the tensile fracture strain of the concrete ( $\epsilon = f_t/E_c$ ).

The load-deflection curve is found by integrating the curvature over the length of the beam. A typical load-deflection curve is shown in Figure C2. The curvature is determined as the ratio between the reinforcement strain and the distance from the reinforcement to the neutral axis at the cracked sections.

The rotational capacity is calculated by integrating the load deflection curve and adding the energy dissipated in the crack formation process estimated as  $nA_c G_F$  where  $n$  is the number of cracks,  $A_c$  is the cross sectional area of the beam and  $G_F$  is the fracture energy of the concrete.

#### Model Properties

Investigating the model properties, results have been derived using the following values for the material parameters:

$f_y$	: 500 MPa
$f_u$	: 575 MPa
$\tau_f$	: 5.0 MPa
$f_t$ (NSC)	: 3.0 MPa
$f_t$ (HSC)	: 5.0 MPa
$f_c$ (NSC)	: 60.0 MPa
$f_c$ (HSC)	: 90.0 MPa
$G_F$	: 0.120 N/mm
$\epsilon_y$	: 2.0 %
$\epsilon_u$	: 15.0 %
$E_s$	: 210,000 MPa

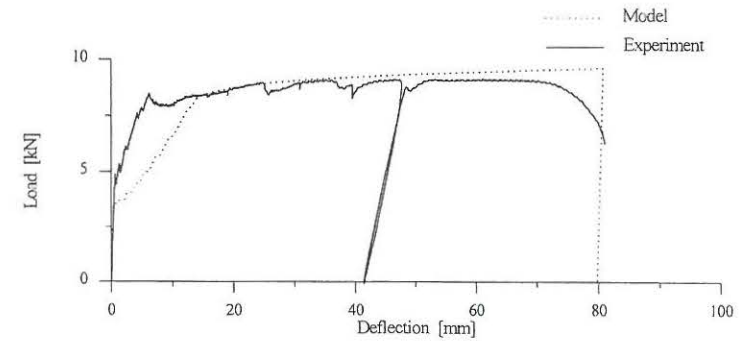


Figure C2. Failure response curve for  $100 \times 200 \times 2400$  mm beam with 0.14 % reinforcement compared with typical experimental result.

representative for the two types of concrete used in the experimental investigation, see Appendix A.

For this kind of model, one of the most important parameters is the amount of reinforcement strain hardening described by the ratio  $f_u/f_y$ . If no strain hardening is present, i.e. if the ratio  $f_u/f_y = 1$ , at each crack, only one point (the point situated just between the concrete tensile failure crack faces) can be in the state of yielding. Thus, since the length of the zone over which yielding takes place tends to zero when the ratio tends to 1, the rotational capacity tends to zero. Furthermore, in this case only one crack will be formed reducing the possibilities of energy dissipation even further. The results clearly support these considerations. Figure C3 shows that the rotational capacity is highly dependent upon the ratio  $f_u/f_y$ .

One would expect the results of the model to be rather sensitive to the value of the shear friction stress  $\tau_f$ . However, this is not the case, Figure C4. Generally it can be stated that the rotational capacity decreases with increasing friction stress, although the influence is small. The energy dissipation due to yielding and debonding decreases with increasing values of the shear friction stress, but at the same time the number of cracks increases, and thus, the contribution from dissipation of energy in the concrete tensile cracks increases.

Figure C5 shows the results for a normal strength concrete and a high strength concrete. As it appears from the figure, increasing the concrete strength decreases the rotational capacity for all values of the reinforcement ratio.

For the value of shear friction stress used here,  $\tau_f = 5$  MPa, the model only shows a small size effect. For larger values of  $\tau_f$ , introducing a relatively larger contribution from concrete tensile

fracture energy dissipation, somewhat larger size effects are observed, see the results in the main part of the chapter. However, the results in Figure C6 clearly show that the rotational capacity is non-sensitive to the size, but highly sensitive to the deformation capacity of the steel. For the low deformation capacity steel the value of the ultimate strain was reduced to  $\epsilon_u = 2.0\%$ , approximately equal to the failure strain of the cold deformed bars used in the experimental investigation, see Appendix A.

All Figures C3-C6 show how the rotational capacity is influenced by the reinforcement ratio. When reinforcement tensile failure controls the failure of the beam the rotational capacity is increasing with increasing reinforcement ratio.

#### Comparing with Experimental Results

Figure C7 and C8 show how the model compares with experimental results for the rotational capacity. In Figure C7 the case of normal strength concrete is shown.

In case of reinforcement ratios of 0.14% and 0.25% the model results fit the experiments well, whereas in some cases of 0.39% reinforcement ratio the experiments show that reinforcement tensile failure is no longer the dominating failure mode and therefore the model over-estimates the rotational capacity. The experimental results show, as the model, that the rotational capacity is higher in the case of normal strength concrete.

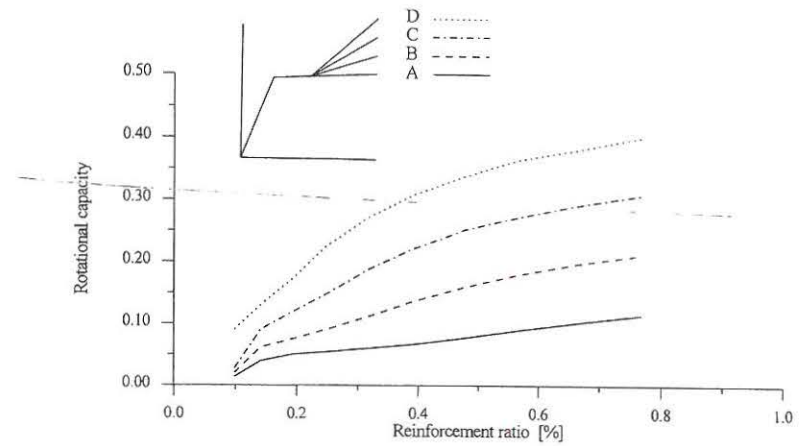


Figure C3. Rotational capacity as a function of the reinforcement ratio for different strain hardening properties A), B), C) and D) of the reinforcement.

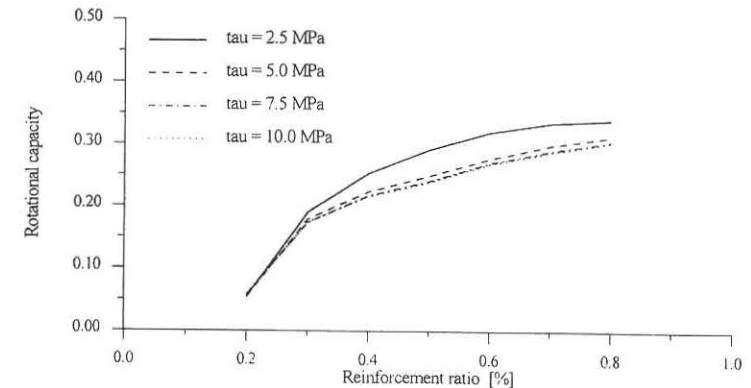


Figure C4. Rotational capacity as a function of the reinforcement ratio for different values of the debonding shear friction stress  $\tau_f$ .

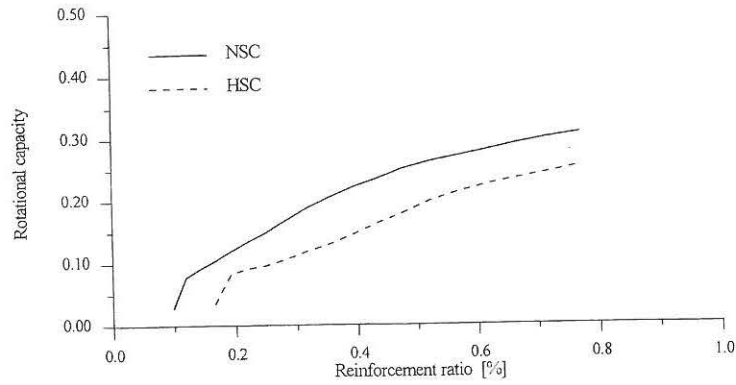


Figure C5. Rotational capacity as a function of the reinforcement ratio for normal strength concrete and high strength concrete.

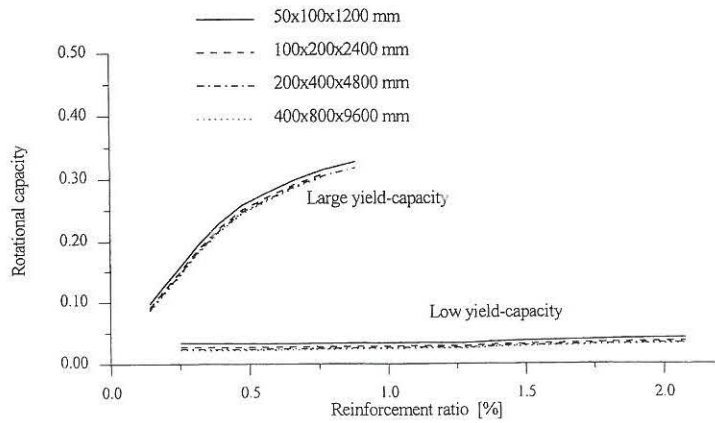


Figure C6. Size effect on the rotational capacity as a function of the reinforcement ratio for different reinforcement types: a low deformation capacity type, and a high deformation capacity type.

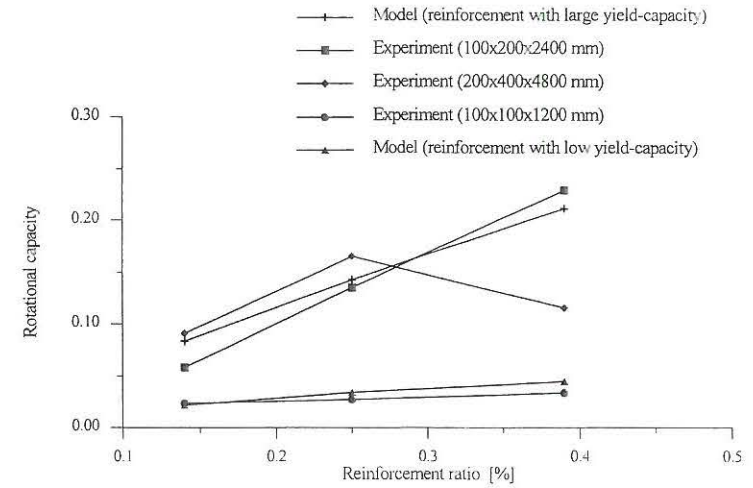


Figure C7. Comparison between rotational capacities obtained from model and from experiments (normal strength concrete).

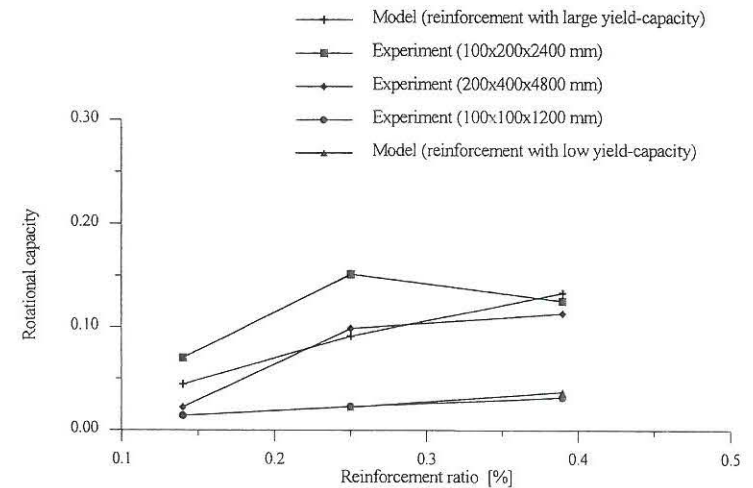


Figure C8. Comparison between rotational capacities obtained from model and from experiments (high strength concrete).

## APPENDIX D:

## FRACTURE MECHANICAL MODEL FOR ROTATIONAL CAPACITY OF HEAVILY REINFORCED CONCRETE BEAMS

Prepared by M.S. Henriksen, R. Brincker and G. Heshe

## Abstract

In this appendix the flexural behaviour of reinforced concrete beams is investigated by analytical methods originally introduced by A. Hillerborg. A simple analytical model is presented which describes the bending moment-curvature relation for normal and over-reinforced beams taking into account the strain localization within the compression zone of the concrete. The strain softening part of the stress-strain curve for the concrete is described as a stress-deformation relation which is dependent on the length over which the compression failure extends along the beam axis. On the basis of the moment-curvature relation estimated by the model, the load-deflection curve is calculated, and the rotational capacity is obtained as the total plastic work divided by the yield moment of the beam. The results of the model are obtained assuming a linear compression softening curve with different values of the critical compression deformation and the fracture zone length. The results are compared with experiments.

## Introduction

In the past ten years a lot of research has been carried out in the field of compression failure of reinforced concrete structures. After development of different crack models for the fracture in tension like the Fictitious Crack Model, Hillerborg was one of the researchers that began to investigate the softening behaviour of the fracture in compression [1],[2] and [3].

Hillerborg had already shown that the softening behaviour in tension was size-dependent, and inspired by the work of van Mier [4], Hillerborg got the idea of using the model for tension softening on compression failure. This led to a simple model describing the uniaxial stress-strain relation for concrete based on fracture mechanical concepts, Figure D1. The strain localization within the compression zone was taken into account by defining a characteristic length dependent on the depth of the compression zone.

Van Mier and Vonk at the Stevin Laboratory carried out different experimental studies on the full range behaviour in compressive loading of different concrete cubes as well as performed micro-mechanical modelling of the compression softening [4],[5]. Recent studies of the behaviour in compression of both normal strength concrete (45 MPa) and high strength concrete (90 MPa) have been presented by Jansen and Shah [6], who performed experimental investigations on cylinders with constant diameter and different depths to examine the effect of specimen depth on compressive strain softening of concrete, see Figure D2. Their results show that the post-peak

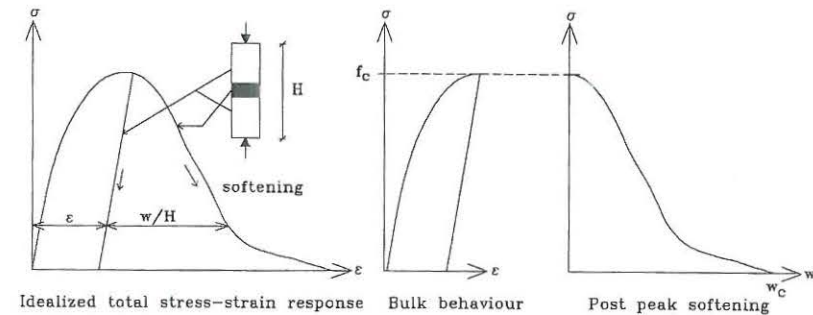


Figure D1. Basic idea of stress-strain relation in compression by Hillerborg [1].

behaviour including the post peak energy dissipation is relatively insensitive to the depth of the cylinder specimen.

Several researchers have examined the influence on compression softening behaviour when changing the depth of test cylinders and when using intermediate layers between the loading plate and the cylinder, but it seems to be a lack of investigations on the influence of changing the diameter. A so-called Compressive Damage Zone model has also been established by Markset taking into account also localized shear deformation and deformation due to splitting cracks [7].

In this investigation, the length, over which the compression failure extends along the beam axis, is introduced as a characteristic length proportional to the depth of the compression zone  $l_{ch} = \beta h_c$  and the softening is assumed to be linear. Thus, the model contains two parameters describing the basic fracture mechanical properties of the model: the characteristic length parameter  $\beta$  and a critical softening deformation  $w_c$ . In the following the influence of the parameters  $w_c$  and  $\beta$  on the full range behaviour of different model beams is analysed, and, based on the load-deflection curves, the rotational capacity is estimated as the total plastic work divided by the yield moment of the beam.

## Basic Assumptions of the State of Compression Failure

When a reinforced concrete beam is loaded to ultimate compression failure and an unloading starts taking place, the critical cross-section is assumed to pass through three different states of failure, see Figure D3.

The continuum state for the critical cross-section describes an elastic state for the concrete where the concrete stresses  $\sigma_c < f_c$  for all points in the cross-section. Varying the concrete strain  $\varepsilon_c$  from zero to the peak strain  $\varepsilon_0$ , the depth of the compression zone  $h_c$  will be constant. In this phase the reinforcement is assumed to be in an elastic state corresponding to  $\sigma_s < f_y$ .

The condition of fracture zone growth is satisfied when the concrete stress in the compressed edge

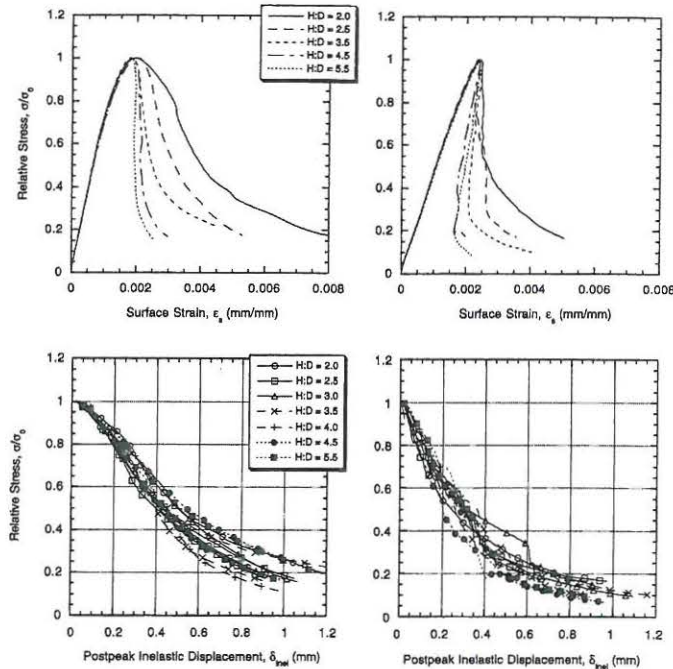


Figure D2. Influence of specimen length on concrete cylinder uniaxial compressive stress-strain curve, (diameter 100 mm). Left: Normal strength concrete (45 MPa). Right: High strength concrete (90 MPa). According to Jansen and Shah [6].

of the cross-section reaches the concrete compression strength. At this condition a fracture zone will start developing. When the fracture zone is fully developed, i.e. when the compression stress at the top of the beam has dropped to zero, the length of the fracture zone along the beam axis is assumed to be  $l_{ch}$ , where  $l_{ch}$  is defined as a characteristic length. The material within the fracture zone follows a softening branch and outside this zone an unloading takes place. The characteristic length could be assumed to be dependent on either the depth of the compression zone or on the width of the cross-section.

It is well known that the final compression failure of cylinders is often a so-called "cone-failure" where the concrete fails in a compression-shear mode with the development of slip-planes at an angle  $\gamma$  typically around  $\gamma \cong 30^\circ$ . Thus, for a cylinder with radius  $r$ , the characteristic length might be defined as  $l_{ch} \tan(\gamma) = 2r$ , and then taking the approximation  $\tan(\gamma) \cong 0.5$  we get  $l_{ch} \cong 4r$ , see Figure D4.

Following this idea, the failure mode of the compression zone of a beam is assumed to be a similar compression-shear mode with the development of slip-planes at a certain angle to horizontal. Now, assuming that the slip-planes will start at the point where the strain is zero, the characteristic length becomes proportional to the depth  $h_c$  of the compression zone, thus  $l_{ch} = \beta h_c$ . Assuming

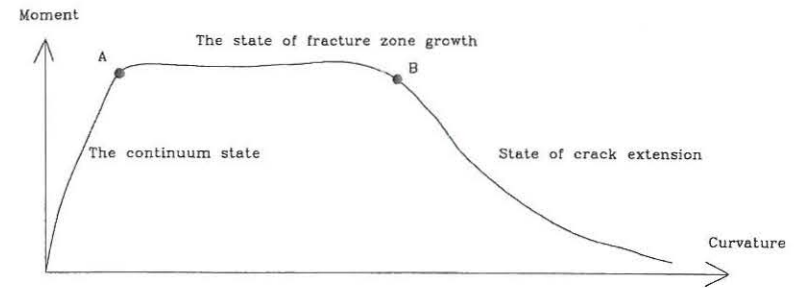


Figure D3. Full range behaviour for a reinforced concrete beam.

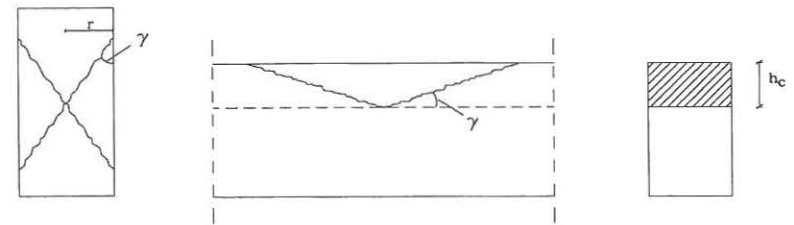


Figure D4. Definition of the parameter  $\beta$  describing the length of the failure zone. Left: Cone failure of a cylinder in compression. Right: Assumed failure mode in the compression zone of a beam in bending.

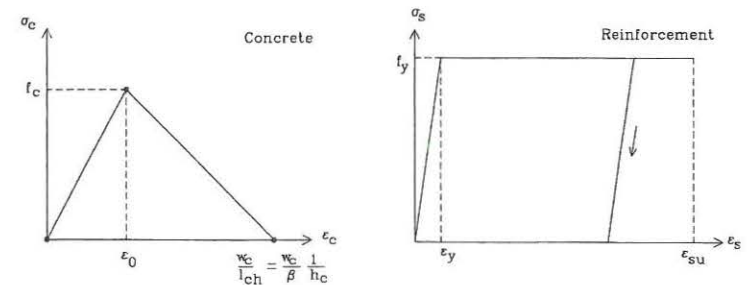


Figure D5. Simplified analytical stress-strain curves for the reinforcement and the concrete.





The parameter  $\theta$  is a direct measure of the rotational capacity of the beam.

Results for the rotational capacity are shown in Figures D9-D12 for different values of  $w_c$  and  $\beta$ . As expected, the rotational capacity is strongly sensitive to both parameters. Note that since the ultimate strain of the reinforcement is incorporated in the analysis, some of the test results correspond to reinforcement tensile failure. The reason for incorporating this rather simple tension failure in this model (no bond-slip is modeled) is to have a rough check on the capability of the model to show a reasonable switch between the two modes of failure. The part of the rotational capacity curves in Figures D9-D12, where the rotational capacity is increasing with the reinforcement ratio, corresponds to reinforcement tension failure. This part of the results should be acknowledged as less interesting (modelling for this part is given in Appendix C) than the rest of the curve corresponding to concrete compression failure.

More details of the modelling are given in Henriksen and Brincker [8].

#### Comparison with Experimental Results

The values of the rotational capacity estimated by the model are shown in Figures D13-D16 as a function of the reinforcement ratio and compared with experimental results from Appendix A for normal strength concrete beams with a cross-section of  $100 \times 100 \text{ mm}$ ,  $100 \times 200 \text{ mm}$  and  $200 \times 400 \text{ mm}$  and slenderness number 12. As it appears, the best agreement is obtained for the values  $w_c = 4 \text{ mm}$  and  $\beta = 8$ .

Figure D16 shows that using the values  $w_c = 4 \text{ mm}$  and  $\beta = 8$ , the rotational capacity estimated by the model compares reasonably well with experimental results for all the three beam sizes.

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#### Size Effects on the Bending Behaviour of Reinforced Concrete Beams

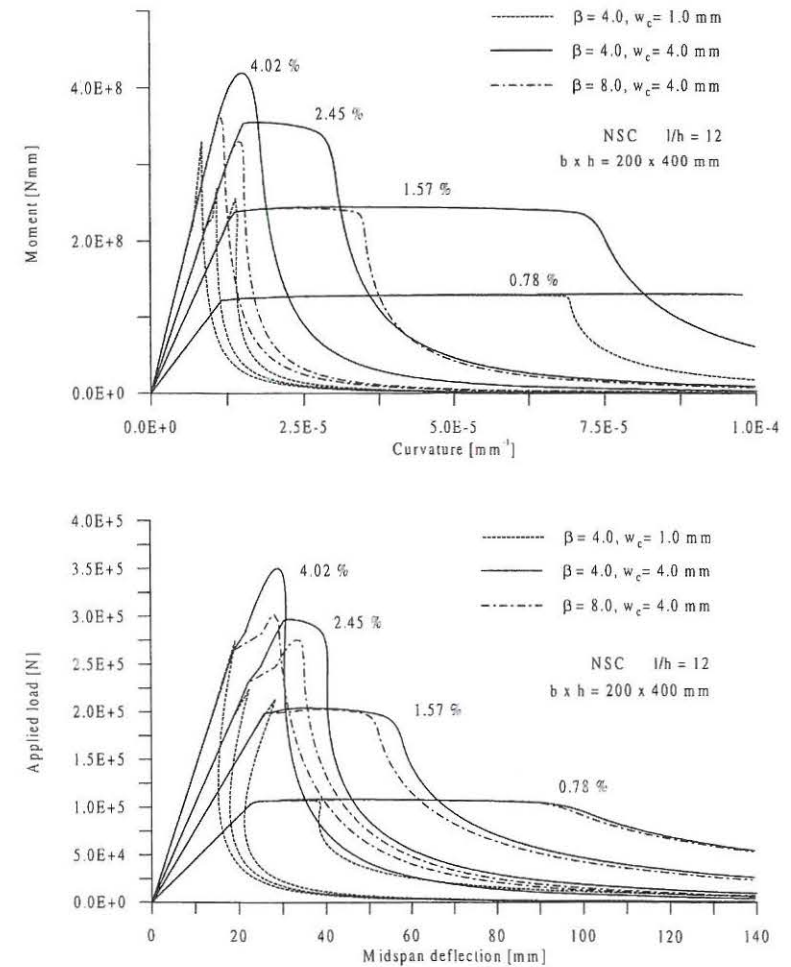


Figure D7. Model results for moment-curvature (top) and load-deflection (bottom) relation for a normal strength concrete beam ( $200 \times 400 \times 4800 \text{ mm}$ ) and different values of  $w_c$  and  $\beta$ .

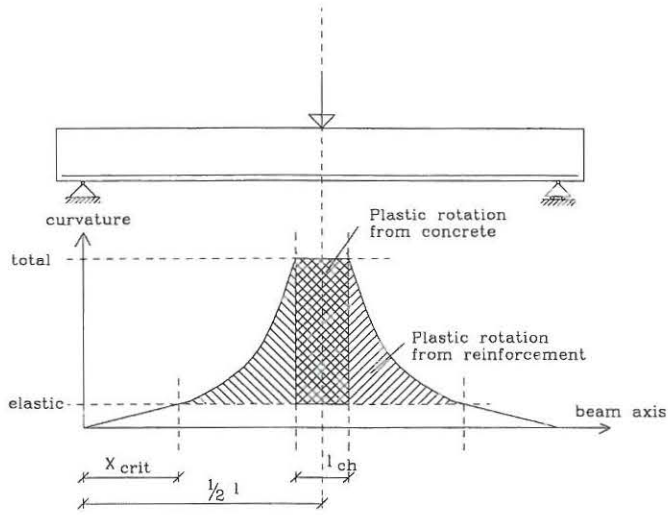


Figure D8. Typical curvature distribution along the beam axis for a beam at ultimate load.

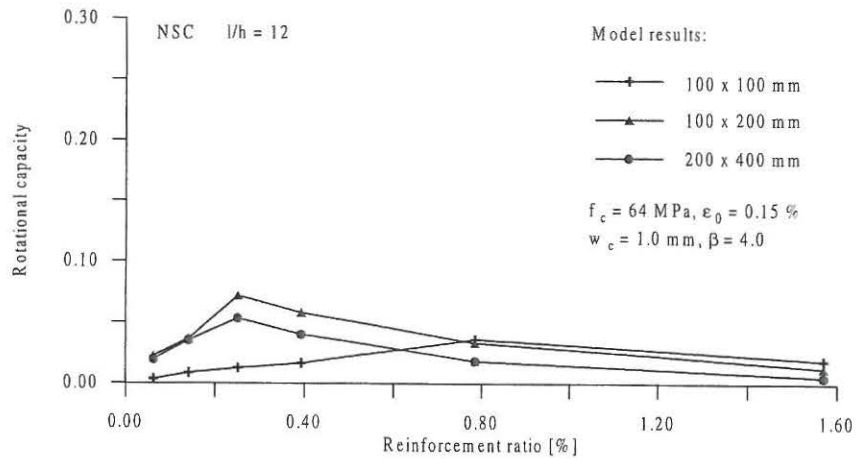


Figure D9. Model results for normal strength concrete beams with  $w_c = 1.0 \text{ mm}$  and  $\beta = 4.0$ .

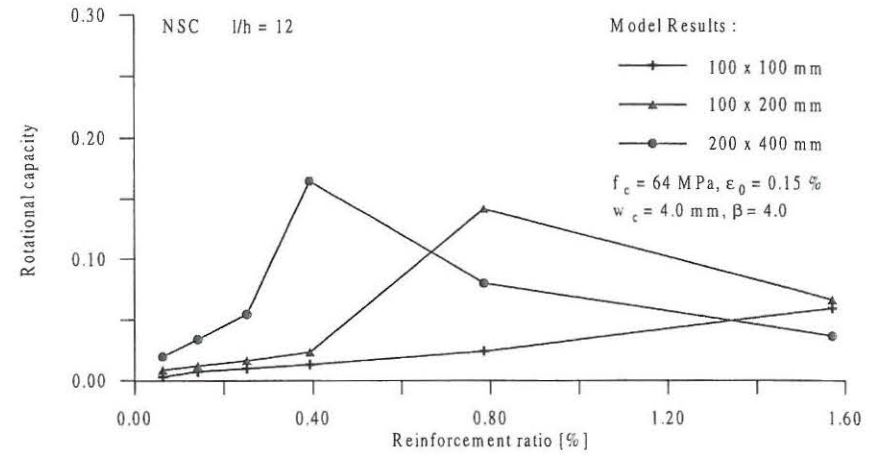


Figure D10. Model results for normal strength concrete beams with  $w_c = 4.0 \text{ mm}$  and  $\beta = 4.0$ .

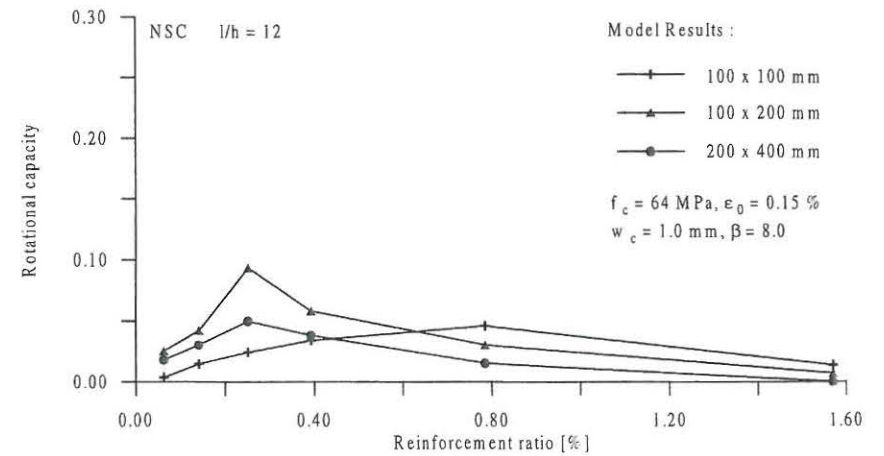


Figure D11. Model results for normal strength concrete beams with  $w_c = 1.0 \text{ mm}$  and  $\beta = 8.0$ .

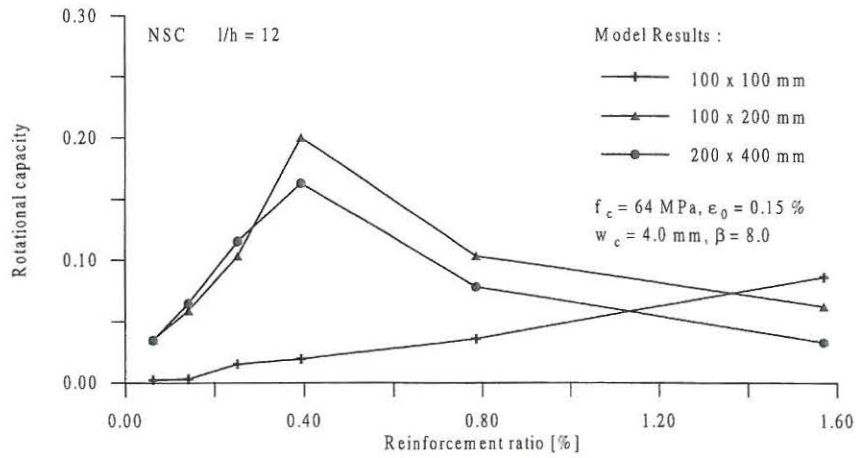


Figure D12. Model results for normal strength concrete beams with  $w_c = 4.0 \text{ mm}$  and  $\beta = 8.0$ .

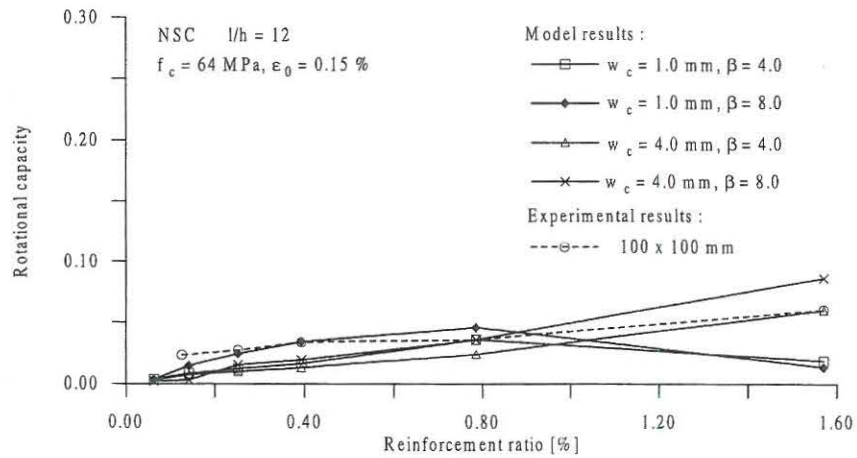


Figure D13. Results for  $100 \times 100 \times 1200 \text{ mm}$  normal strength concrete beams.

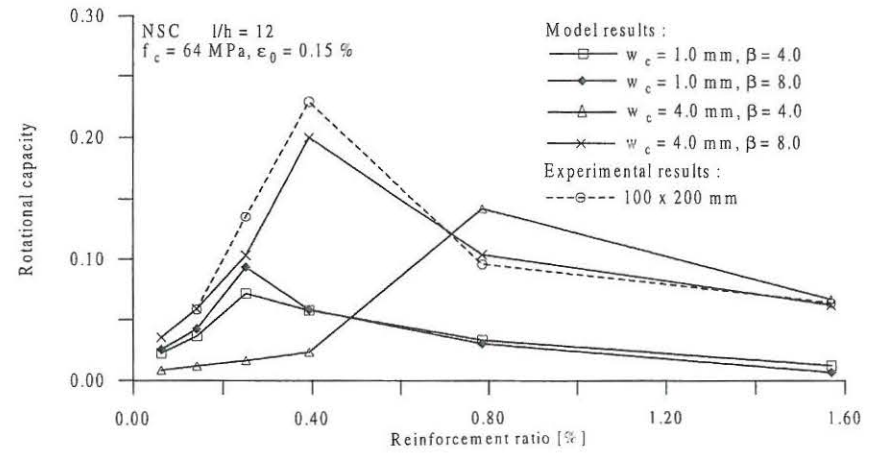


Figure D14. Results for  $100 \times 200 \times 2400 \text{ mm}$  normal strength concrete beams.

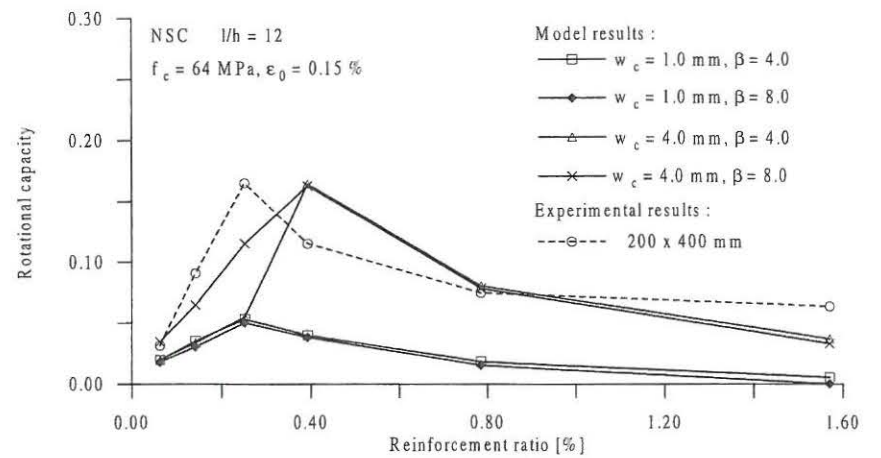


Figure D15. Results for  $200 \times 400 \times 4800 \text{ mm}$  normal strength concrete beams.

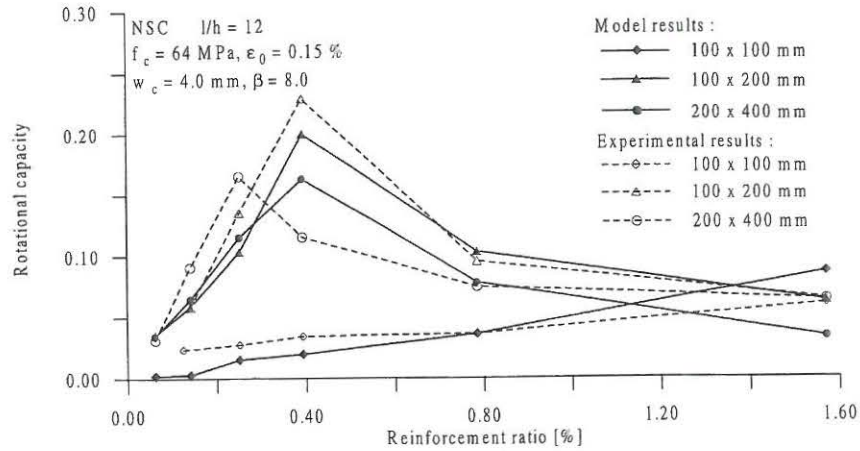


Figure D16. Results for normal strength concrete beams with  $w_c = 4.0 \text{ mm}$  and  $\beta = 8.0$ .

## MINIMUM REINFORCEMENT REQUIREMENT FOR RC BEAMS

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### ABSTRACT

In the present paper different aspects of the requirement for the minimum reinforcement ratio are studied and discussed. The influence of the beam depth is investigated in more detail. Numerical analysis for reinforced concrete beams of different sizes is carried out using plane finite element code *MASA2* which is based on the nonlocal mixed constrained microplane model. Presently, an extensive test project for reinforced concrete beams in which the material and geometrical properties are varied is in progress. Currently available test results are compared with the numerical results. It is concluded that the requirement on the minimum reinforcement depends on the beam size but also on the material properties as well as on the amount and type of the distributed reinforcement. To define the dependency between the minimum reinforcement and geometrical as well as material parameters in more detail, further theoretical and experimental studies are needed.

### KEYWORDS

Minimum reinforcement, RC beams, nonlocal microplane model, energy criteria.

### INTRODUCTION

In engineering practice RC beams of different sizes and with different reinforcement ratios are often used. They are normally designed such that the internal forces as well as their distribution over the cross section are calculated according to the elastic beam theory. On the contrary, the dimensioning is performed using a limit state procedure. Obviously, this is in contradiction. Therefore, significant efforts have recently been made in order to develop consistent tools and recommendations for the nonlinear structural analysis and dimensioning according to the limit state procedure. In order to provide enough structural safety and to make a redistribution of internal forces possible, RC beams must be designed such that they fail in a ductile manner. Some recent fracture mechanics studies [1-5] indicated that larger beams are more brittle than smaller. Consequently, the question is whether the same material laws and design rules may be used for RC beams of different sizes.

An important condition for ductile failure of RC beams is the minimum reinforcement requirement. The minimum reinforcement must assure a stable and ductile beam response after the concrete tensile strength at the beam tensile zone is reached. Presently, in almost all design codes the minimum reinforcement requirement is independent of the beam depth. For example, according to [6] the typical