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# Physical Layer Network Coding: An Outage Analysis in Cellular Network

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***Abstract*—Physical layer network coding (PLNC) has been proposed to improve throughput of the two-way relay channel, where two nodes communicate with each other, being assisted by a relay node. Most of the works related to PLNC are focused on a simple three-node model and they do not take into account the impact of interference from other transmissions. Unlike these conventional studies, in this paper, we apply PLNC to a large-scale cellular network in the presence of intercell interference (ICI). In cellular networks, a terminal and a Base Station (BS) have different transmission power, which causes different impact of ICI on downlink (DL) and uplink (UL) phase. We theoretically derive outage probability with a tractable approach based on stochastic geometry which accurately models ICI. Moreover, we compare the performance of PLNC with Direct and conventional Relay scheme. With the obtained numerical results, we discuss how the interference and the difference of transmission power affect outage probability achieved by PLNC.**

## I. INTRODUCTION

Physical layer network coding (PLNC) offers performance improvement in wireless networks for two-way (or multi-way) communications flows, and has been extensively studied in literature [1]-[7]. With PLNC, two nodes simultaneously transmit packets to a relay. The relay processes the received signal and broadcasts the result to the end nodes. The end nodes extract the desired packets by using the signal forwarded by the relay, information on the packet previously transmitted by themselves, and channel state information (CSI) of the relayed links. PLNC appears in several flavors, depending on the operation done at the relay, such as Amplify-and-Forward (AF) [1], Denoise-and-Forward (DNF) [2], Decode-and-Forward (DF) [3], etc.

Most of the initial works related to PLNC were focused on a simple three-node model with two-way relaying. In recent years, some of the studies have attempted to employ PLNC in wireless networks of a larger scale, taking into account the fact that there can be other neighboring nodes that cause interference. For instance, in [4][5], distributed medium access control (MAC) protocols for PLNC have been introduced, and the impact of interference from neighboring nodes has been analyzed. On the other hand, several studies applied PLNC to cellular networks without considering the impact of intercell interference (ICI) [6][7]. The other works considered the impact of ICI on PLNC[8], but not in a cellular setting. In this work, we consider PLNC in cellular networks by taking into account the ICI. In our model, a Terminal and a Base Station (BS), which have different transmission (Tx) power, employ PLNC in order to exchange their packets. In order to accurately quantify the interference from neighboring nodes, a model of positioning of interference nodes is necessary. Recently, spatial point processes have been suggested for modeling the placement of wireless network nodes [9]. Specifically, homogeneous Poisson Point Process (PPP) is known as a point process that accurately models the positioning of nodes in urban area, and has been employed in many works since it offers a tractable way of dealing with ICI [10]. Here we use PPP to theoretically derive outage probability for several transmission schemes in cellular networks. The numerical results provide insights into how PLNC is affected by interference as well as difference if the Tx power in uplink/downlink.

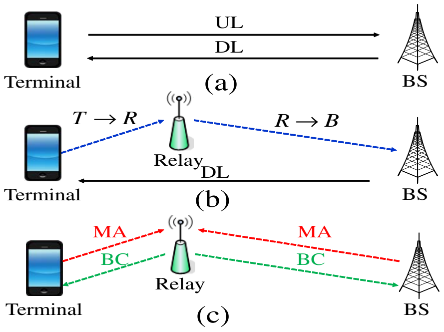


Fig. 1. Transmission Schemes (a) Direct (b) Relay (c) PLNC

## II. SYSTEM AND CHANNEL MODEL

Fig. 1 illustrates three transmission schemes considered in this paper: Direct, Relay and PLNC. Fig. 1(a) and (b) show the Direct and Relay scheme, respectively. In both schemes, BS transmits a packet to Terminal in the downlink (DL) phase without using the Relay. In the uplink (UL) phase if the Direct scheme, the Terminal transmits a packet to BS without Relay node; while in the UL of the Relay scheme, also called Relay phase, the Terminal transmits a packet to BS through the Relay node. Fig. 1(c) shows the PLNC scheme. In step 1, called multiple access (MA) phase, both Terminal and BS transmit signals simultaneously, and the signals are added at Relay node through MA channel. In this paper, we focus on PLNC with DF operation, such that Relay node decodes the received signals and then uses bitwise XOR to generate the signal to be forwarded in the next broadcast (BC) phase. Terminal and BS attempt to derive their desired signal by applying bitwise XOR operation to the received signal in BC phase.

The channel model consists of a path loss exponent $\alpha$ and fading between that is assumed to be independent identically distributed (i.i.d.) Rayleigh fading with a mean of 1. Moreover,

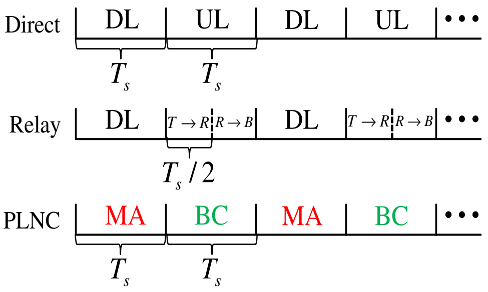


Fig. 2. Slot Allocation

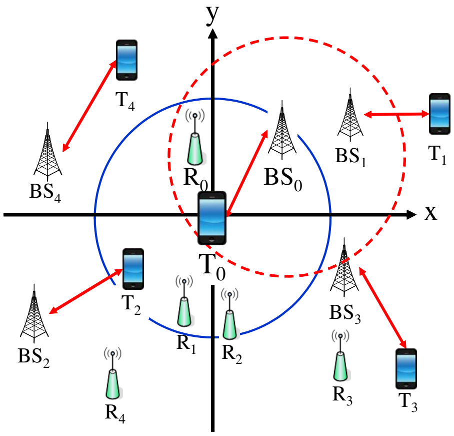


Fig. 3. Terminal, BS and Relay Deployment

we employ constant Tx power at the BS, Relay and Terminal denoted by $P_B$ $(= \mu_B^{-1})$, $P_R$ $(= \mu_R^{-1})$ and $P_T$ $(= \mu_T^{-1})$, respectively. The interference power at a receiver is the sum of the received powers from all the undesired transmitters. The additive white noise has a power of $\sigma^2$. The capacity of link is denoted as:

$$C(SINR) = \log_2(1 + SINR) \ \text{[bit/s/Hz]}. \qquad (1)$$

where SINR is the Signal-to-Interference plus Noise Ratio. In this work, all the nodes transmit with a rate that satisfies a two-way end-to-end rate requirement of $R$ [bit/s/Hz]. Let $T_s$ [s] be one slot length, as shown in Fig. 2. We allocate the same slot length to DL, UL, Relay, MA and BC phases. In the Relay phase, the Terminal first transmits to Relay in the first half of its phase (duration $T_s/2$ [s]), followed by the transmission from Relay to BS in the second half.

We apply PLNC to a cellular network, where many nodes (Terminals, BSs and Relays) access the shared channel. Fig. 3 shows a network model used for the analysis in this paper. The cellular network consists of BSs arranged according to homogeneous PPP of intensity $\lambda$ $[BSs/km^2]$ in the Euclidean plane. The set of BSs is denoted by $\Phi_B$. In this work, we consider a Terminal ($T_0$) located at the origin. All the other Terminals are located according to homogeneous PPP of intensity $\lambda$ $[Terminals/km^2]$. We assume that each Terminal is associated with the closest BS. In Fig. 3, as $T_0$ is associated with $BS_0$ with their distance of $r$, the other BSs are located outside the circle with the radius of $r$, which is shown by the solid line. We assume that all Terminals and BSs excluding $T_0$ and $BS_0$ transmit with the Direct scheme. This excludes the interference from the other relay nodes that may occur during the UL phase in the Direct/Relay scheme and the Broadcast phase of PLNC. Therefore, in DL and MA phases, ICI is caused by all BSs but $BS_0$. Similarly, in UL, Relay, and BC phase, all Terminals but $T_0$ act as interferers. Moreover, we assume that $T_0$ is the closest Terminal to $BS_0$, such that the other Terminals are located outside the circle shown by the dotted line in Fig. 3. Furthermore, Relay nodes are also deployed according to homogeneous PPP of intensity $\lambda_R$ $[Relays/km^2]$. We denote $R_0$ as the closest Relay to $T_0$. Here, we define the interference power at $T_0$, $BS_0$, and $R_0$ as $I_T$, $I_B$, and $I_R$, respectively. The SINRs at links $B_0$-$T_0$, $T_0$-$B_0$, $R_0$-$B_0$, $T_0$-$R_0$, and $R_0$-$T_0$ are denoted as $\gamma_{BT}$, $\gamma_{TB}$, $\gamma_{RB}$, $\gamma_{TR}$, and $\gamma_{RT}$, respectively.

## III. DERIVATION OF THE OUTAGE PROBABILITY

We focus on the transmission between $T_0$ and $BS_0$, and analyze the outage probability of Direct, Relay, and PLNC schemes. Before we calculate outage probability of two-way end-to-end transmission, we calculate outage probability for one-way transmission of DL, UL, and Relay phase. An outage of one-way transmission occurs when the transmission rate is larger than the link capacity.

### A. Distance to the Closest BS

An important quantity is the distance $r$ which is the distance between $T_0$ and $BS_0$ [9]. Since each Terminal communicates with the closest BS, no other BS can be located closer than $r$. In other words, all the interfering BSs must be located farther than $r$. The probability density function (pdf) of $r$ can be derived by using null probability, defined as the probability that no BS other than $BS_0$ is closer than $L$ and is expressed as $P[r > L] = \exp(-\lambda\pi L^2)$. Then, the cumulative distribution function (cdf) is calculated as $P[r \le L] = F_r(L) = 1 - \exp(-\lambda\pi L^2)$. Therefore, the pdf of $r$ can be calculated as

$$f_r(r) = \frac{dF_r(r)}{dr} = 2\pi\lambda r \exp(-\lambda\pi r^2)\,. \qquad (2)$$

### B. One-way Transmission

For calculating outage probability, we first evaluate coverage probability, $p_c$. In one-way transmission, coverage probability is the probability that transmission rate becomes smaller than the capacity. Then, we calculate outage probability as $1 - p_c$. First, we calculate coverage probability of DL averaged over the plane, $p_c^d$, conditioning on the closest BS being at a distance $r$ from $T_0$ as follows [10]:

$$\begin{aligned}
p_c^d(R, \mu_B, \alpha) &= \mathbb{E}_r[\mathbb{P}[C(\gamma_{BT}) > R|r]] \\
&= \int_0^\infty \mathbb{P}[C(\gamma_{BT}) > R|r] f_r(r)dr \\
&= \int_0^\infty \mathbb{P}\left[\frac{g_B r^{-\alpha}}{\sigma^2 + I_T} > b\middle| r\right] f_r(r)dr \\
&= \int_0^\infty \mathbb{P}\big[g_B > br^\alpha(\sigma^2 + I_T)|r\big] f_r(r)dr \\
&\overset{(a)}{=} \int_0^\infty \mathbb{E}_{I_T}\big[\exp(-\mu_B br^\alpha(\sigma^2 + I_T))|r\big] f_r(r)dr
\end{aligned}$$

$$\overset{(b)}{=} \int_0^{\infty} \exp(-\mu_B b r^{\alpha} \sigma^2) \mathcal{L}_{I_T}(\mu_B b r^{\alpha}) f_r(r) dr, \qquad (3)$$

where $g_B$ is a random variable following an exponential distribution with mean $\mu_B^{-1}$, and $b$ is $2^R - 1$. $(a)$ follows from the fact that $g_B$ follows exponential distribution. $(b)$ follows from the definition of Laplace transform $\mathcal{L}_{I_T}(s) = \mathbb{E}_{I_T}[\exp(-sI_T)]$. Here, we define $V_i$ as the distance between $T_0$ and the $i$-th interfering BS, and $g_{Bi}$ as the random variable following an exponential distribution with mean $\mu_B^{-1}$. With the assumption on the i.i.d. distribution of fading random variables, we can calculate $\mathcal{L}_{I_T}(s)$ as follows:

$$\begin{aligned}
\mathcal{L}_{I_T}(s) &= \mathbb{E}_{I_T}[\exp(-sI_T)] \\
&= \mathbb{E}_{\Phi_B, g_{Bi}}\left[\exp\left(-s \sum_{i \in \Phi_B \setminus BS_0} g_{Bi} V_i^{-\alpha}\right)\right] \\
&= \mathbb{E}_{\Phi_B, g_{Bi}}\left[\prod_{i \in \Phi_B \setminus BS_0} \exp\left(-s g_{Bi} V_i^{-\alpha}\right)\right] \\
&\overset{(a)}{=} \mathbb{E}_{\Phi_B}\left[\prod_{i \in \Phi_B \setminus BS_0} \frac{\mu_B}{\mu_B + s V_i^{-\alpha}}\right] \\
&\overset{(b)}{=} \exp\left(-2\pi\lambda \int_r^{\infty} \left(1 - \frac{\mu_B}{\mu_B + s v_i^{-\alpha}}\right) v dv\right) \\
&= \exp\left(-2\pi\lambda \int_r^{\infty} \frac{1}{1 + \frac{\mu_B v^{\alpha}}{s}} v dv\right), \qquad (4)
\end{aligned}$$

where $(a)$ follows from the moment generating function (MGF) of exponential distribution, and $(b)$ follows from the probability generating functional (PGFL) of the 2-D PPP[9], which states for some function $f(t)$ that

$$G[f] \triangleq \mathbb{E}\left(\prod_{t \in \Phi} f(t)\right) = \exp\left(-\int_{\mathbb{R}^2} \left(1 - f(t)\right) \Lambda(dt)\right), \qquad (5)$$

where $\Lambda$ is an intensity measure. Moreover, as we employ the 2-D homogeneous PPP, $\Lambda(dt) = \lambda dx dy$. Plugging in $s = \mu_B b r^{\alpha}$ and changing the variable $u = b^{-\frac{2}{\alpha}} \left(\frac{v}{r}\right)^2$ result in

$$\mathcal{L}_{I_T}(\mu_B b r^{\alpha}) = \exp\left(-\pi\lambda r^2 \rho(b, \alpha)\right), \qquad (6)$$

where

$$\rho(b, \alpha) = b^{\frac{2}{\alpha}} \int_{b^{-\frac{2}{\alpha}}}^{\infty} \frac{1}{1 + u^{\frac{\alpha}{2}}} du. \qquad (7)$$

Combining (2), (3) and (6), and employing the change of variable $r^2 = v$, we derive coverage probability as

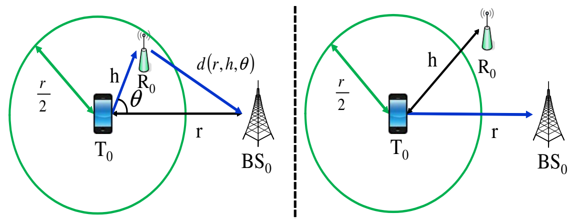
$$p_c^d(R, \mu_B, \alpha) = \pi\lambda \int_0^{\infty} \exp\left(-\mu_B b \sigma^2 v^{\frac{\alpha}{2}} - \pi\lambda v\left(1 + \rho(b, \alpha)\right)\right) dv. \qquad (8)$$

With $\alpha = 4$, we can derive this coverage probability with a quasi-closed form as follows:

$$\begin{aligned}
&p_c^d(R, \mu_B) \triangleq p_c^d(R, \mu_B, 4) \\
&= \frac{\pi^{\frac{3}{2}}\lambda}{\sqrt{4\mu_B b \sigma^2}} \exp\left(\frac{\left(\pi\lambda(1 + \rho(b))\right)^2}{4\mu_B b \sigma^2}\right) \operatorname{erfc}\left(\frac{\pi\lambda(1 + \rho(b))}{\sqrt{4\mu_B b \sigma^2}}\right), \qquad (9)
\end{aligned}$$

where $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-t^2) dt$ and $\rho(b)$ is defined as

$$\rho(b) \triangleq \rho(b, 4) = \sqrt{b}\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{\sqrt{b}}\right)\right). \qquad (10)$$



(a) (b)

Fig. 4. (a)Transmission with $R_0$ (b)Transmission without $R_0$

Hereafter, we assume a path loss exponent of 4. Next, we evaluate coverage probability of UL in a similar way to DL. In UL phase, interference is caused by the other Terminals which transmit signal with Tx power of $\mu_T^{-1}$. Therefore, coverage probability is $p_c^u(R, \mu_T) = p_c^d(R, \mu_T)$.

For the coverage probability of Relay phase, we denote $h$ as the distance between $T_0$ and the closest Relay node, $R_0$, as shown in Fig. 4. We assume that $T_0$ is the closest Terminal to $R_0$, and $BS_0$ is the closest BS to $R_0$. We denote the distance between $R_0$ and $BS_0$: $d^2(r, h, \theta) = r^2 + h^2 - 2rh\cos\theta$. Hereafter, we describe $d(r, h, \theta)$ simply as $d$. If $R_0$ is located far from $T_0$, the transmission through a Relay node does not improve coverage probability and it is better for $T_0$ to transmit without relaying. Therefore, we introduce an area within which $T_0$ should transmit to $BS_0$ through Relay node. We assume that if $h$ is smaller than $r/2$, that is, if $R_0$ is located within a circle of radius $r/2$, $T_0$ transmits through $R_0$, as shown in Fig. 4(a). Otherwise, $T_0$ transmits without Relay node (Direct scheme), as shown in Fig. 4(b). When $h$ is smaller than $r/2$, coverage probability is expressed as

$$\begin{aligned}
p_c^{r1}(R, \mu_T) &= \mathbb{E}_{r,h,\theta}[\mathbb{P}[C(\gamma_{TR}) > 2R \cap \\
&\qquad C(\gamma_{RB}) > 2R | r, h, \theta, h \le r/2] P[h \le r/2]] \\
&= \int_0^{\infty} \int_0^{\frac{r}{2}} \int_0^{2\pi} exp(-\mu_T b_R \sigma^2 h^4 - \mu_R b_R \sigma^2 d^4) \mathcal{L}_{I_R}(\mu_T b_R h^4) \\
&\qquad \mathcal{L}_{I_B}(\mu_R b_R d^4) P[h \le r/2] f_{r,h,\theta}(r, h | h \le r/2) d\theta dh dr, \qquad (11)
\end{aligned}$$

where $b_R$ is $2^{2R} - 1$, $\mathcal{L}_{I_R}(s) = \mathbb{E}_{I_R}[\exp(-sI_R)]$, $\mathcal{L}_{I_B}(s) = \mathbb{E}_{I_B}[\exp(-sI_B)]$, $P[h \le r/2] = 1 - \exp(-\lambda_R \pi r^2/4)$ is the cumulative probability of $h$ and $f_{r,h,\theta}(r, h | h \le r/2)$ is the conditional joint probability density function (cjpdf) which is expressed as [12]:

$$f_{r,h,\theta}(r, h | h \le r/2) = \begin{cases} \frac{2\pi\lambda\lambda_R r h \exp(-\lambda\pi r^2 - \lambda_R \pi h^2)}{1 - \exp(-\lambda_R \pi r^2/4)} & \text{if } h \le r/2 \\ 0 & \text{otherwise} \end{cases} \qquad (12)$$

In order to satisfy the same end-to-end rate with Direct scheme, when $T_0$ transmits through $R_0$, $T_0$ and $R_0$ need to transmit with the rate, $2R$. On the other hand, if $h$ is larger than $r/2$, coverage probability is expressed as

$$\begin{aligned}
p_c^{r2}(R, \mu_T) &= \mathbb{E}_{r,h,\theta}[\mathbb{P}[C(\gamma_{TB}) > R | r, h, \theta, h > r/2] P[h > r/2]] \\
&= \int_0^{\infty} \int_{\frac{r}{2}}^{\infty} \int_0^{2\pi} exp(-\mu_T b \sigma^2 r^4) \mathcal{L}_{I_B}(\mu_T b r^4) \\
&\qquad P[h > r/2] f_{r,h,\theta}(r, h | h > r/2) d\theta dh dr, \qquad (13)
\end{aligned}$$

where $P[h > r/2] = \exp(-\lambda_R \pi r^2/4)$ and $f_{r,h,\theta}(r, h | h >$

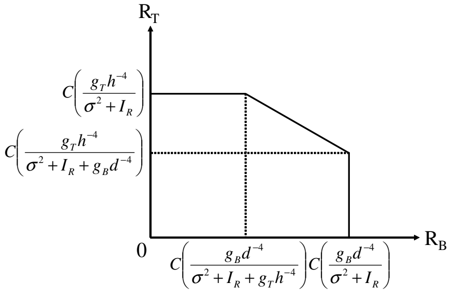


Fig. 5. The capacity region of MA channel

$r/2)$ is expressed as

$$f_{r,h,\theta}(r,h|h>r/2)=\begin{cases}\frac{2\pi\lambda\lambda_R rh\exp(-\lambda\pi r^2-\lambda_R\pi h^2)}{\exp(-\lambda_R\pi r^2/4)} & \text{if } h>r/2\\ 0 & \text{otherwise}\end{cases} \tag{14}$$

Therefore, coverage probability of Relay, $p_c^r$, is calculated as $p_c^r(R,\mu_T)=p_c^{r1}(R,\mu_T)+p_c^{r2}(R,\mu_T)$.

### C. Two-way Transmission

Based on coverage probability of one-way transmission derived in the previous subsection, we derive the coverage probability of two-way transmission, defined as the probability that transmission rate becomes smaller than the capacity in both DL and UL, DL and Relay, MA and BC phases. Then we calculate outage probability by subtracting the coverage probability from 1. First, we calculate coverage probability of Direct scheme, $p_c^D$. As we employ the fixed slot model, $p_c^D$ can be calculated as

$$\begin{aligned}p_c^D(R,\mu_B,\mu_T)&=\mathbb{E}_r[\mathbb{P}[C(\gamma_{BT})>R\cap C(\gamma_{TB})>R|r]]\\&=\int_0^\infty \mathbb{P}[C(\gamma_{BT})>R\cap C(\gamma_{TB})>R|r]f_r(r)dr\\&=\frac{\pi^{\frac{3}{2}}\lambda}{\sqrt{4\mu b\sigma^2}}\exp\left(\frac{\big(\pi\lambda\big(1+2\rho(b)\big)\big)^2}{4\mu b\sigma^2}\right)\operatorname{erfc}\left(\frac{\pi\lambda\big(1+2\rho(b)\big)}{\sqrt{4\mu b\sigma^2}}\right),\end{aligned} \tag{15}$$

where $\mu$ is $\mu_B+\mu_T$. Next, we derive coverage probability of Relay scheme. When $h<r/2$, the coverage probability is expressed as

$$\begin{aligned}p_c^{R1}(R,\mu_B,\mu_T)=\mathbb{E}_{r,h,\theta}[\mathbb{P}[C(\gamma_{BT})>R\cap C(\gamma_{TR})>2R\cap\\ C(\gamma_{RB})>2R|r,h,\theta,h\le r/2]P[h\le r/2]].\end{aligned} \tag{16}$$

When $h$ is larger than $r/2$, it is calculated as

$$\begin{aligned}p_c^{R2}(R,\mu_B,\mu_T)=\mathbb{E}_{r,h,\theta}[\mathbb{P}[C(\gamma_{BT})>R\cap\\ C(\gamma_{TB})>R|r,h,\theta,h>r/2]P[h>r/2]].\end{aligned} \tag{17}$$

Therefore, coverage probability of Relay scheme, $p_c^R$, is calculated as $p_c^R(R,\mu_B,\mu_T)=p_c^{R1}(R,\mu_B,\mu_T)+p_c^{R2}(R,\mu_B,\mu_T)$.

Finally, we evaluate coverage probability of PLNC scheme. As for the Relay scheme, we assume that PLNC is conducted when $h$ is smaller than $r/2$. Otherwise, $T_0$ and $BS_0$ transmit with Direct scheme. As shown in Fig. 5, the capacity region of MA is a convex region. It is the set, $S$, which is composed of all the rates $(R_B,R_T)$ satisfying the following three constraints [11]

TABLE I
SYSTEM PARAMETERS

| Bandwidth | 10 [MHz] |
|---|---|
| Terminal density: $\lambda$ | 0.24[Terminals/km$^2$] |
| BS density: $\lambda$ | 0.24[BSs/km$^2$] |
| Relay station density: $\lambda_R$ | $5\lambda$ [Relays/km$^2$] |
| BS Tx power: $P_B$ | 45 [dBm] |
| Terminal Tx power: $P_T$ | 23 [dBm] |
| Relay Tx power: $P_R$ | 30 [dBm] |
| Noise power density | -174 [dBm/Hz] |

$$\begin{cases}R_B<C\Big(\frac{g_B d^{-4}}{\sigma^2+I_R}\Big)\\ R_T<C\Big(\frac{g_T h^{-4}}{\sigma^2+I_R}\Big)\\ R_B+R_T<C\Big(\frac{g_B d^{-4}+g_T h^{-4}}{\sigma^2+I_R}\Big)\end{cases} \tag{18}$$

where $g_T$ is a random variable following an exponential distribution with mean $\mu_T^{-1}$, $R_B$ and $R_T$ are the transmission rates of $BS_0$ and $T_0$, respectively[1]. Then the conditional coverage probability of MA phase is calculated as [14]

$$\begin{aligned}&\mathbb{P}[(R_B,R_T)\in S|r,h,\theta,h\le r/2]=\mathbb{P}\Big[R_B<C\Big(\frac{g_B d^{-4}}{k}\Big)\cap R_T<\\&C\Big(\frac{g_T h^{-4}}{k}\Big)\cap R_B+R_T<C\Big(\frac{g_B d^{-4}+g_T h^{-4}}{k}\Big)\Big|r,h,\theta,h\le r/2\Big]\\&=\mathbb{E}_k\Big[\frac{\mu_B d^4}{\mu_B d^4-\mu_T h^4}\exp\big((-\mu_T h^4 b_{TB}+\mu_T h^4 b_T-\mu_B d^4 b_B)k\big)\\&-\frac{\mu_T h^4}{\mu_B d^4-\mu_T h^4}\exp\big((-\mu_B d^4 b_{TB}+\mu_B d^4 b_B-\mu_T h^4 b_T)k\big)\Big],\end{aligned} \tag{19}$$

where $k$ is $\sigma^2+I_R$, $b_B$, $b_T$ and $b_{TB}$ are $2^{R_B}-1$, $2^{R_T}-1$ and $2^{R_B+R_T}-1$, respectively. Putting $R_B=R_T=R$ we get:

$$\begin{aligned}&\mathbb{P}[(R,R)\in S|r,h,\theta,h\le r/2]\\&=\frac{\mu_B d^4\exp(-t\sigma^2)\mathcal{L}_{I_R}(t)-\mu_T h^4\exp(-u\sigma^2)\mathcal{L}_{I_R}(u)}{\mu_B d^4-\mu_T h^4},\end{aligned} \tag{20}$$

where $t=2^R\mu_T h^4 b+\mu_B d^4 b$ and $u=2^R\mu_B d^4 b+\mu_T h^4 b$. Therefore, coverage probability with PLNC scheme considering both MA and BC phase is calculated as

$$\begin{aligned}p_c^{P1}(R,\mu_B,\mu_T)=\mathbb{E}_{r,h,\theta}[\mathbb{P}[(R,R)\in S\cap C(\gamma_{RT})>R\cap\\ C(\gamma_{RB})>R|r,h,\theta,h\le r/2]P[h\le r/2]].\end{aligned} \tag{21}$$

When $h\ge r/2$, $T_0$ and $BS_0$ communicate with Direct scheme. Then, coverage probability is same as (17), that is $p_c^{P2}(R,\mu_B,\mu_T)=p_c^{R2}(R,\mu_B,\mu_T)$. Therefore, coverage probability of PLNC scheme, $p_c^P$, is calculated as $p_c^P(R,\mu_B,\mu_T)=p_c^{P1}(R,\mu_B,\mu_T)+p_c^{P2}(R,\mu_B,\mu_T)$.

## IV. NUMERICAL RESULTS

The system parameters used for evaluations are given in Table I, which are taken from studies analyzing performance of outage probability considering ICI for an LTE-based cellular network [10][15].

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[1] Strictly speaking, the maximum rate of each node should be calculated with $C'(SINR)=log_2(1/2+SINR)$ instead of eq. (1)[13], however, at high SINR region, the difference between $C'(SINR)$ and $C(SINR)$ is negligible, so we employ eq. (1) for simplicity.

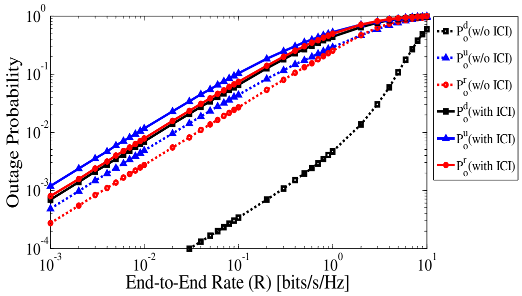

Fig. 6. Outage probability of one-way transmission

Fig. 6 shows outage probability of DL, UL and Relay against End-to-End rate, $R$. In this figure, the dotted lines represent the outage probability when ICI is neglected, while the solid lines indicate outage probability considering ICI as derived in Sec. III-B. Without ICI, the outage probability of DL is much lower than that of UL and Relay. In general, as shown in Table I, Tx power of BS, $P_B$, is much larger than $P_T$ and $P_R$. This means that SNR (Signal-to-Noise Ratio) at a DL receiver becomes larger than SNR at a UL and Relay receiver. The outage probability for Relay is lower than that of UL. This is because the Tx power of Relay, $P_R$, is larger than $P_T$. Moreover, as $R_0$ is deployed closer to $T_0$ than $BS_0$, despite small Tx power of Terminal, the received power at $R_0$ becomes large. Therefore, though $T_0$ and $R_0$ need to transmit with higher rate, $2R$, Relay outperforms UL. When ICI is considered, outage probability on DL is extremely deteriorated. This is because, in DL, ICI is caused by the other BSs which transmit with large Tx power. On the other hand, in UL and Relay, ICI is caused by Terminals which transmit with lower Tx power than BSs. This means that the impact of ICI on DL is larger than that on UL and Relay. Even when ICI is considered, Relay is still superior to UL. This shows the usefulness of Relay node.

Fig. 7 shows the outage probability of two-way transmission derived in Sec. III-C. Dotted lines represent outage probability without ICI, while the solid lines with ICI. It can be observed that, without ICI, PLNC outperforms the other schemes, as already shown in the prior work. Interestingly, when ICI is considered, the superiority of PLNC vanishes. In the MA phase, $T_0$ and $BS_0$ transmit to $R_0$ simultaneously while interference is caused by the other BSs. Here, in PLNC, $R_0$ needs to decode both signals transmitted by $T_0$ and $BS_0$. However, as explained above, the impact of ICI caused by BSs is large, and Tx power of $T_0$, $P_T$, is lower than Tx power of BSs, $P_B$. Therefore, the coverage probability on MA phase derived from (20) becomes small, which results in larger outage probability for PLNC compared to the other schemes. This result clearly demonstrates the importance of the interference on the schemes with PLNC.

## V. CONCLUSIONS

In this paper, we have analyzed outage probability of Direct, Relay and PLNC schemes considering the impact of ICI in a large-scale cellular network. Without ICI condition, PLNC is superior to the other schemes, which is the observation shown in many literatures. However, when ICI is considered, the superiority of PLNC is vanished. These results clearly demonstrate the importance of analysis of PLNC considering the impact of interference.

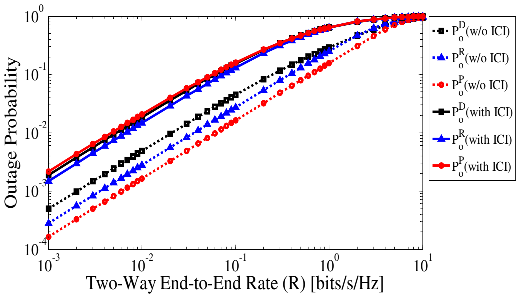

Fig. 7. Outage probability of two-way transmission

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