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A Broadband Beamformer Using Controllable Constraints and Minimum Variance

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Introduction

The minimum variance distortionless response (MVDR) beamformer is an optimal approach to noise reduction:

- Achieves a high output SNR.
- It has degrees of freedom (DOF) corresponding to the number of microphones minus one.
- Its output is contaminated with both residual noise and interference.

The linearly constrained minimum variance (LCMV) beamformer reduces noise, and rejects interferers using linear constraints:

- Achieves a high output SIR.
- The number of constraints may degrade the DOF.
- It may amplify background noise which causes a lower output SNR.

To achieve a trade-off between attenuation of noise and interfering sources, we proposed the controllable LCMV (C-LCMV) beamformer in the frequency-domain.

Formulation

Multi-channel observed signals at the frequency f are observed, using an array of M microphones:

$$\mathbf{y}(f) = \mathbf{d}_1(f)X_1(f) + \sum_{n=2}^N \mathbf{d}_n(f)X_n(f) + \mathbf{v}(f) = \mathbf{D}_N(f)\mathbf{x}(f) + \mathbf{v}(f), \quad (1)$$

where $\mathbf{d}_n(f) \in \mathbb{C}^M$ is the steering vector of signal source $X_n(f)$ (for $n = 1, \dots, N$), $X_1(f)$ is the signal of interest (SOI), $\mathbf{x}(f) \in \mathbb{C}^N$ is the collected N signal sources, $\mathbf{v}(f) \in \mathbb{C}^M$ is noise, and

$$\mathbf{D}_N(f) = [\mathbf{d}_1(f) \mathbf{d}_2(f) \dots \mathbf{d}_N(f)] \in \mathbb{C}^{M \times N}. \quad (2)$$

The correlation matrix of $\mathbf{y}(f)$ (assuming uncorrelated signals) is

$$\Phi_{\mathbf{y}}(f) = \mathbf{D}_N(f) \Phi_{\mathbf{x}}(f) \mathbf{D}_N^H(f) + \Phi_{\mathbf{v}}(f) = \mathbf{d}_1(f) \phi_{X_1}(f) \mathbf{d}_1^H(f) + \Phi_{\text{in}}(f), \quad (3)$$

where $\Phi_{\mathbf{x}}(f) = \text{diag}[\phi_{X_1}(f) \phi_{X_2}(f) \dots \phi_{X_N}(f)]$, $\Phi_{\text{in}}(f) = \Phi_{\mathbf{i}}(f) + \Phi_{\mathbf{v}}(f)$, and $\Phi_{\mathbf{i}}(f) = \sum_{n=2}^N \mathbf{d}_n(f) \phi_{X_n}(f) \mathbf{d}_n^H(f)$ is the interference correlation matrix.

The output variance of the beamformer $\mathbf{h}(f)$ is

$$\phi_Z(f) = \mathbf{h}^H(f) \mathbf{d}_1(f) \phi_{X_1}(f) \mathbf{d}_1^H(f) \mathbf{h}(f) + \mathbf{h}^H(f) [\Phi_{\mathbf{i}}(f) + \Phi_{\mathbf{v}}(f)] \mathbf{h}(f). \quad (4)$$

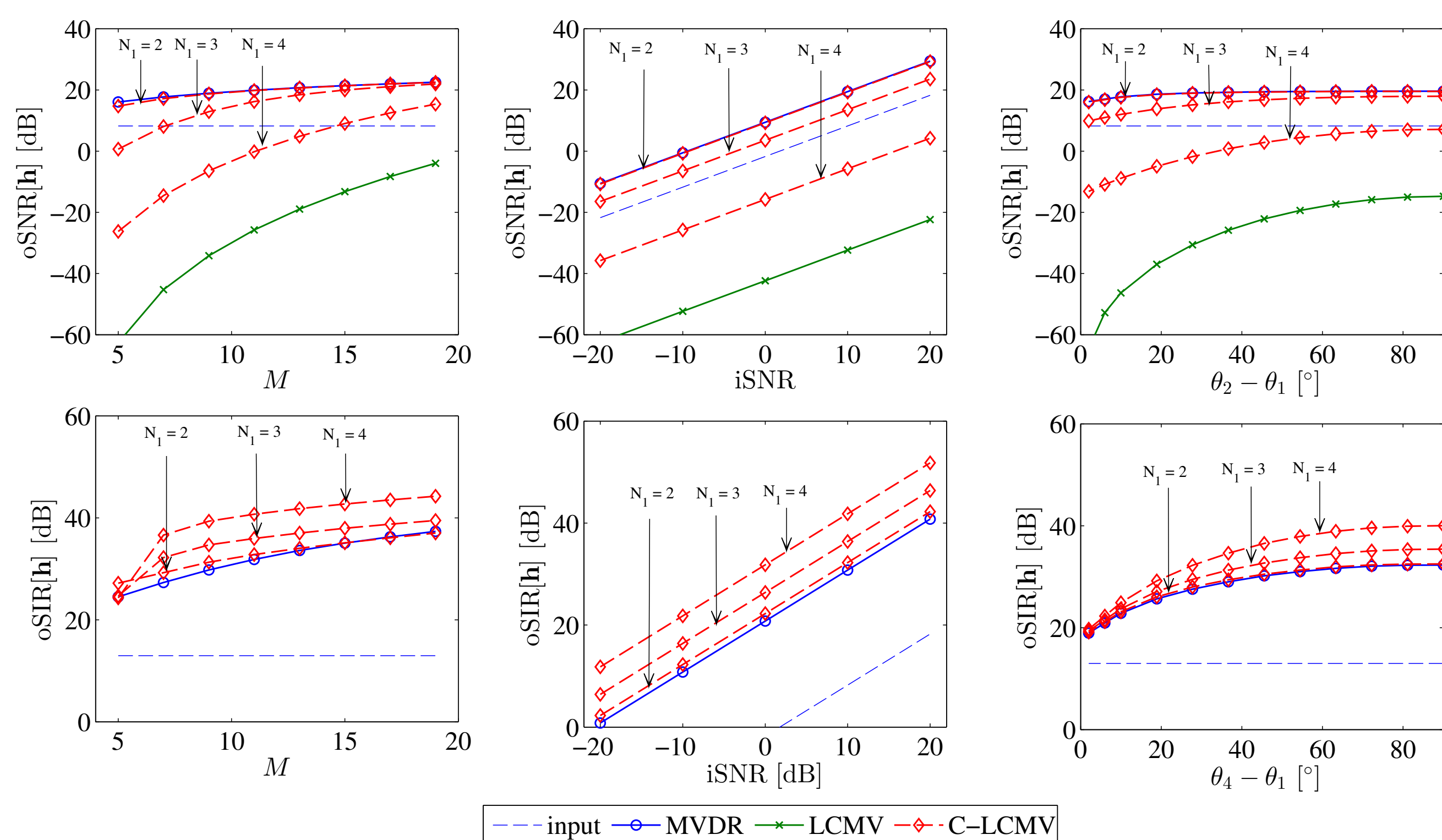
With the distortionless constraint that $\mathbf{h}^H(f) \mathbf{d}_1(f) = 1$, we can write

$$\text{oSINR}[\mathbf{h}(f)] = \frac{\phi_{X_1}(f)}{\mathbf{h}^H(f) [\Phi_{\text{in}}(f) + \Phi_{\mathbf{v}}(f)] \mathbf{h}(f)}, \quad (5)$$

$$\text{oSIR}[\mathbf{h}(f)] = \frac{\phi_{X_1}(f)}{\mathbf{h}^H(f) \Phi_{\text{in}}(f) \mathbf{h}(f)}. \quad (6)$$

Experiment: Synthetic Signal

Using a uniform linear array (ULA) with $M = 10$ microphones and Gaussian noise signal sources in $\theta_1 = 0$, $\theta_2 = \pi$, $\theta_3 = 5\pi/6$, $\theta_4 = 4\pi/6$, and $\theta_5 = \pi/2$, where $\text{iSINR} = 8$ dB and $\text{iSIR} = 13$ dB:



C-LCMV Beamformer

$$\text{MDVR:} \quad \min_{\mathbf{h}(f)} \mathbf{h}^H(f) [\Phi_{\mathbf{i}}(f) + \Phi_{\mathbf{v}}(f)] \mathbf{h}(f) \quad (7)$$

$$\text{subject to} \quad \mathbf{h}^H(f) \mathbf{d}_1(f) = 1,$$

$$\text{LCMV:} \quad \min_{\mathbf{h}(f)} \mathbf{h}^H(f) \Phi_{\mathbf{v}}(f) \mathbf{h}(f) \quad (8)$$

$$\text{subject to} \quad \mathbf{h}^H(f) \mathbf{D}_N(f) = \mathbf{i}_N^T,$$

where \mathbf{i}_N is the first column of a $N \times N$ identity matrix.

We divide N signal sources into two sets of N_1 sources, containing SOI, and $N_2 = N - N_1$ remaining signal sources;

$$\mathbf{x}(f) = [\mathbf{x}_{N_1}^T(f) \mathbf{x}_{N_2}^T(f)]^T, \text{ and} \quad (9)$$

$$\mathbf{D}_N(f) = [\mathbf{D}_{N_1}(f) \mathbf{D}_{N_2}(f)]. \quad (10)$$

Therefore,

$$\mathbf{y}(f) = \mathbf{D}_{N_1}(f) \mathbf{x}_{N_1}(f) + [\mathbf{D}_{N_2}(f) \mathbf{x}_{N_2}(f) + \mathbf{v}(f)], \quad (11)$$

and

$$\Phi_{\mathbf{y}}(f) = \mathbf{D}_{N_1}(f) \Phi_{\mathbf{x}_{N_1}}(f) \mathbf{D}_{N_1}^H(f) + \Phi_{\text{in},N_2}(f), \quad (12)$$

where $\Phi_{\text{in},N_2}(f) = \mathbf{D}_{N_2}(f) \Phi_{\mathbf{x}_{N_2}}(f) \mathbf{D}_{N_2}^H(f) + \Phi_{\mathbf{v}}(f)$ is the correlation matrix of N_2 signal sources plus background noise.

The C-LCMV beamformer is designed as

$$\min_{\mathbf{h}(f)} \mathbf{h}^H(f) \Phi_{\text{in},N_2}(f) \mathbf{h}(f) \quad (13)$$

$$\text{subject to} \quad \mathbf{h}^H(f) \mathbf{D}_{N_1}(f) = \mathbf{i}_{N_1}^T.$$

Then the solution is given like

$$\mathbf{h}_C(f) = \Phi_{\text{in},N_2}^{-1}(f) \mathbf{D}_{N_1}(f) [\mathbf{D}_{N_1}^H(f) \Phi_{\text{in},N_2}^{-1}(f) \mathbf{D}_{N_1}(f)]^{-1} \mathbf{i}_{N_1}. \quad (14)$$

► Properties:

$$\text{oSINR}[\mathbf{h}_L(f)] \leq \text{oSINR}[\mathbf{h}_C(f)] \leq \text{oSINR}[\mathbf{h}_M(f)], \quad (15)$$

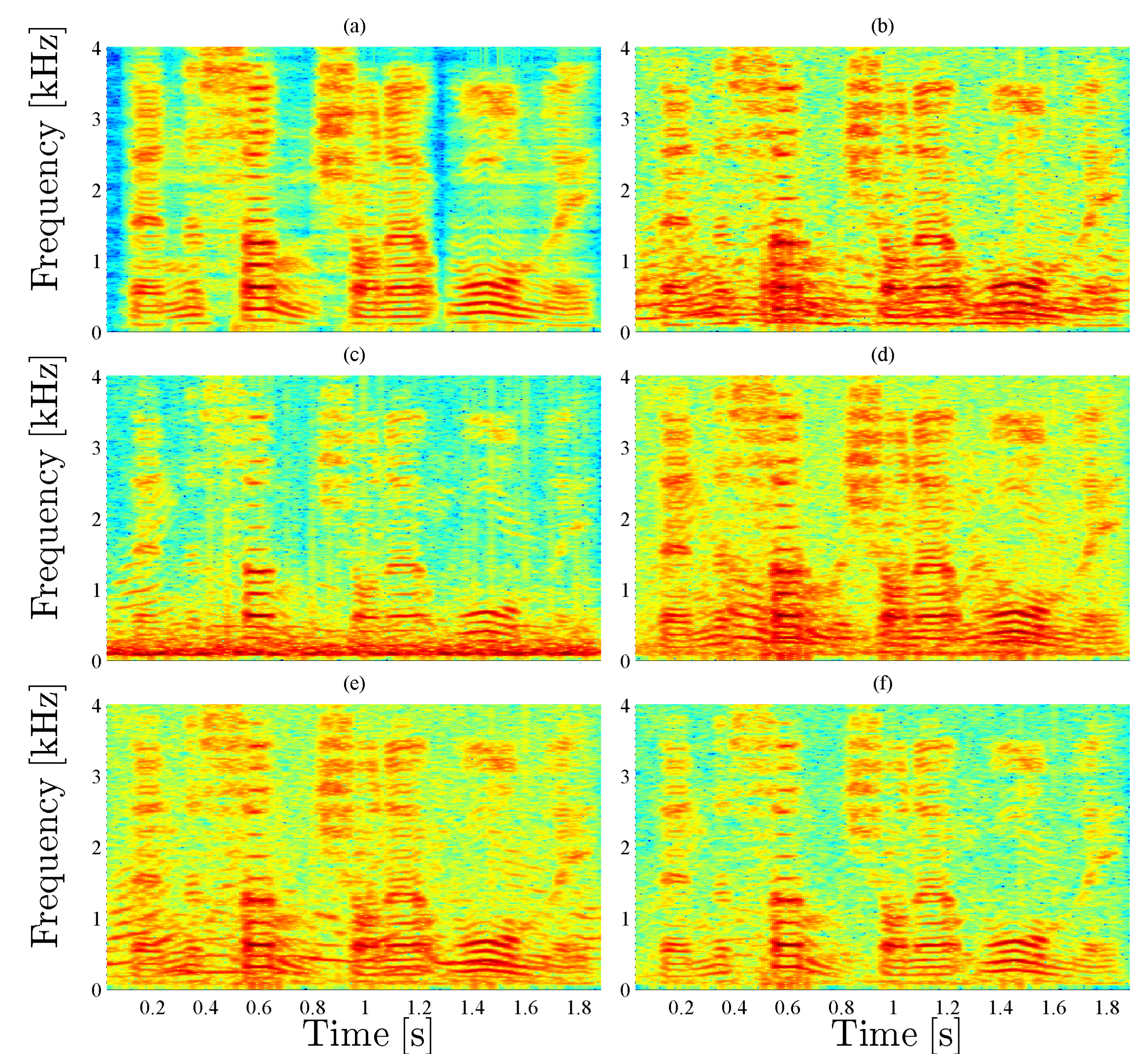
$$\text{oSIR}[\mathbf{h}_M(f)] \leq \text{oSIR}[\mathbf{h}_C(f)] \leq \text{oSIR}[\mathbf{h}_L(f)]. \quad (16)$$

► Optimal steering matrix (in practice):

$$\mathbf{D}_{N_1}^{\text{opt}}(f) = \arg \max_{\mathbf{D}_{N_1}(f)} \text{oSINR}[\mathbf{h}_C(f)]. \quad (17)$$

Experiment: Real Scenario

We simulated a room ($6 \times 7 \times 3$ m) with reverberation time $T_{60} = 0.25$ s, and $N = 3$ speech signals, and used a ULA with $M = 5$ hypercardioid microphones, $\text{SNR} = 20$ dB.



(a) SOI, (b-c) output of MVDR and LCMV beamformers, and (d-f) output of C-LCMV beamformers, where $\mathbf{D}_{N_1}(f) = [\mathbf{d}_1(f) \mathbf{d}_2(f)]$, $\mathbf{D}_{N_1}(f) = [\mathbf{d}_1(f) \mathbf{d}_3(f)]$, and $\mathbf{D}_{N_1}(f) = \mathbf{D}_{N_1}^{\text{opt}}(f)$.

Conclusion

- The C-LCMV beamformer generalizes the MVDR and LCMV beamformers with the ability to control the quality of the output signal.
- Experiment results indicate that the C-LCMV beamformer using optimal steering matrix gets better results than the other MV beamformers.