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# AUDIO ANALYSIS LAB

# A Broadband Beamformer Using Controllable Constraints and Minimum Variance



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# Introduction

The minimum variance distortionless response (MVDR) beamformer is an optimal approach to noise reduction:

- Achieves a high output SNR.
- It has degrees of freedom (DOF) corresponding to the number of microphones minus one.
- Its output is contaminated with both residual noise and interference.

The linearly constrained minimum variance (LCMV) beamformer reduces noise, and rejects interferers using linear constraints:

- Achieves a high output SIR.
- ► The number of constraints may degrade the DOF.
- It may amplify background noise which causes a lower output SNR.

C-LCMV Beamformer
MDVR: $\min_{\mathbf{h}(f)} \mathbf{h}^{H}(f) \left[ \mathbf{\Phi}_{\mathbf{i}}(f) + \mathbf{\Phi}_{\mathbf{v}}(f) \right] \mathbf{h}(f) $ (7)
subject to $\mathbf{h}^{H}(f) \mathbf{d}_{1}(f) = 1,$
LCMV: $\min_{\mathbf{h}(f)} \mathbf{h}^{H}(f) \mathbf{\Phi}_{\mathbf{v}}(f) \mathbf{h}(f) $ (8)
subject to $\mathbf{h}^{H}(f) \mathbf{D}_{N}(f) = \mathbf{i}_{N}^{T}$ ,
where $\mathbf{i}_N$ is the first column of a $N \times N$ identity matrix.
We divide N signal sources into two sets of $N_1$ sources, containing SOI, and $N_2 = N - N_1$ remaining signal sources;

To achieve a trade-off between attenuation of noise and interfering sources, we proposed the controllable LCMV (C-LCMV) beamformer in the frequency-domain.

### Formulation

Multi-channel observed signals at the frequency *f* are observed, using an array of *M* microphones:

$$\mathbf{y}(f) = \mathbf{d}_1(f)X_1(f) + \sum_{n=2}^{N} \mathbf{d}_n(f)X_n(f) + \mathbf{v}(f) = \mathbf{D}_N(f)\mathbf{x}(f) + \mathbf{v}(f), \quad (1)$$

where  $\mathbf{d}_n(f) \in \mathbb{C}^M$  is the steering vector of signal source  $X_n(f)$  (for  $n = 1, \dots, N$ ),  $X_1(f)$  is the signal of interest (SOI),  $\mathbf{x}(f) \in \mathbb{C}^N$  is the collected N signal sources,  $\mathbf{v}(f) \in \mathbb{C}^M$  is noise, and

$$\mathbf{D}_{N}(f) = [\mathbf{d}_{1}(f) \mathbf{d}_{2}(f) \cdots \mathbf{d}_{N}(f)] \in \mathbb{C}^{M \times N}.$$
(2)

The correlation matrix of  $\mathbf{y}(f)$  (assuming uncorrelated signals) is  $\mathbf{\Phi}_{\mathbf{y}}(f) = \mathbf{D}_{N}(f) \mathbf{\Phi}_{\mathbf{x}}(f) \mathbf{D}_{N}^{\mathsf{H}}(f) + \mathbf{\Phi}_{\mathbf{v}}(f) = \mathbf{d}_{1}(f) \phi_{X_{1}}(f) \mathbf{d}_{1}^{\mathsf{H}}(f) + \mathbf{\Phi}_{\mathsf{in}}(f), \quad (3)$ where  $\mathbf{\Phi}_{\mathbf{x}}(f) = \operatorname{diag}[\phi_{X_{1}}(f) \phi_{X_{2}}(f) \dots \phi_{X_{N}}(f)], \ \mathbf{\Phi}_{\mathsf{in}}(f) = \mathbf{\Phi}_{\mathbf{i}}(f) + \mathbf{\Phi}_{\mathbf{v}}(f), \text{ and}$   $\mathbf{\Phi}_{\mathbf{i}}(f) = \sum_{n=2}^{N} \mathbf{d}_{n}(f) \phi_{X_{n}}(f) \mathbf{d}_{n}^{\mathsf{H}}(f)$  is the interference correlation matrix.  $\mathbf{x}(f) = [\mathbf{x}_{N_1}^{\mathsf{T}}(f) \ \mathbf{x}_{N_2}^{\mathsf{T}}(f)]^{\mathsf{T}}, \text{ and}$ (9)  $\mathbf{D}_N(f) = [\mathbf{D}_{N_1}(f) \ \mathbf{D}_{N_2}(f)].$ (10)

Therefore,

$$\mathbf{y}(f) = \mathbf{D}_{N1}(f) \, \mathbf{x}_{N_1}(f) + [\, \mathbf{D}_{N_2}(f) \, \mathbf{x}_{N_2}(f) + \mathbf{v}(f) \,], \tag{11}$$

and

$$\Phi_{\mathbf{y}}(f) = \mathbf{D}_{N_1}(f) \, \Phi_{\mathbf{x}_{N_1}}(f) \, \mathbf{D}_{N_1}^{\mathsf{H}}(f) + \Phi_{\mathsf{in},N_2}(f), \qquad (12)$$

where  $\Phi_{\text{in},N_2}(f) = \mathbf{D}_{N_2}(f) \Phi_{\mathbf{x}_{N_2}}(f) \mathbf{D}_{N_2}^{H}(f) + \Phi_{\mathbf{v}}(f)$  is the correlation matrix of  $N_2$  signal sources plus background noise.

The C-LCMV beamformer is designed as

 $\min_{\mathbf{h}(f)} \mathbf{h}^{\mathsf{H}}(f) \mathbf{\Phi}_{\mathsf{in},N_2}(f) \mathbf{h}(f)$ 

(13)

subject to  $\mathbf{h}^{\mathsf{H}}(f) \mathbf{D}_{N_1}(f) = \mathbf{i}_{N_1}^{\mathsf{T}}$ .

Then the solution is given like

 $\mathbf{h}_{\mathrm{C}}(f) = \mathbf{\Phi}_{\mathrm{in},N_2}^{-1}(f) \mathbf{D}_{N_1}(f) [\mathbf{D}_{N_1}^{\mathrm{H}}(f) \mathbf{\Phi}_{\mathrm{in},N_2}^{-1}(f) \mathbf{D}_{N_1}(f)]^{-1} \mathbf{i}_{N_1}.$ (14)

### Properties:

 $oSINR[\mathbf{h}_{L}(f)] \le oSINR[\mathbf{h}_{C}(f)] \le oSINR[\mathbf{h}_{M}(f)], \quad (15)$ 

 $oSIR[\mathbf{h}_{M}(f)] \leq oSIR[\mathbf{h}_{C}(f)] \leq oSIR[\mathbf{h}_{L}(f)].$ 

The output variance of the beamformer h(f) is

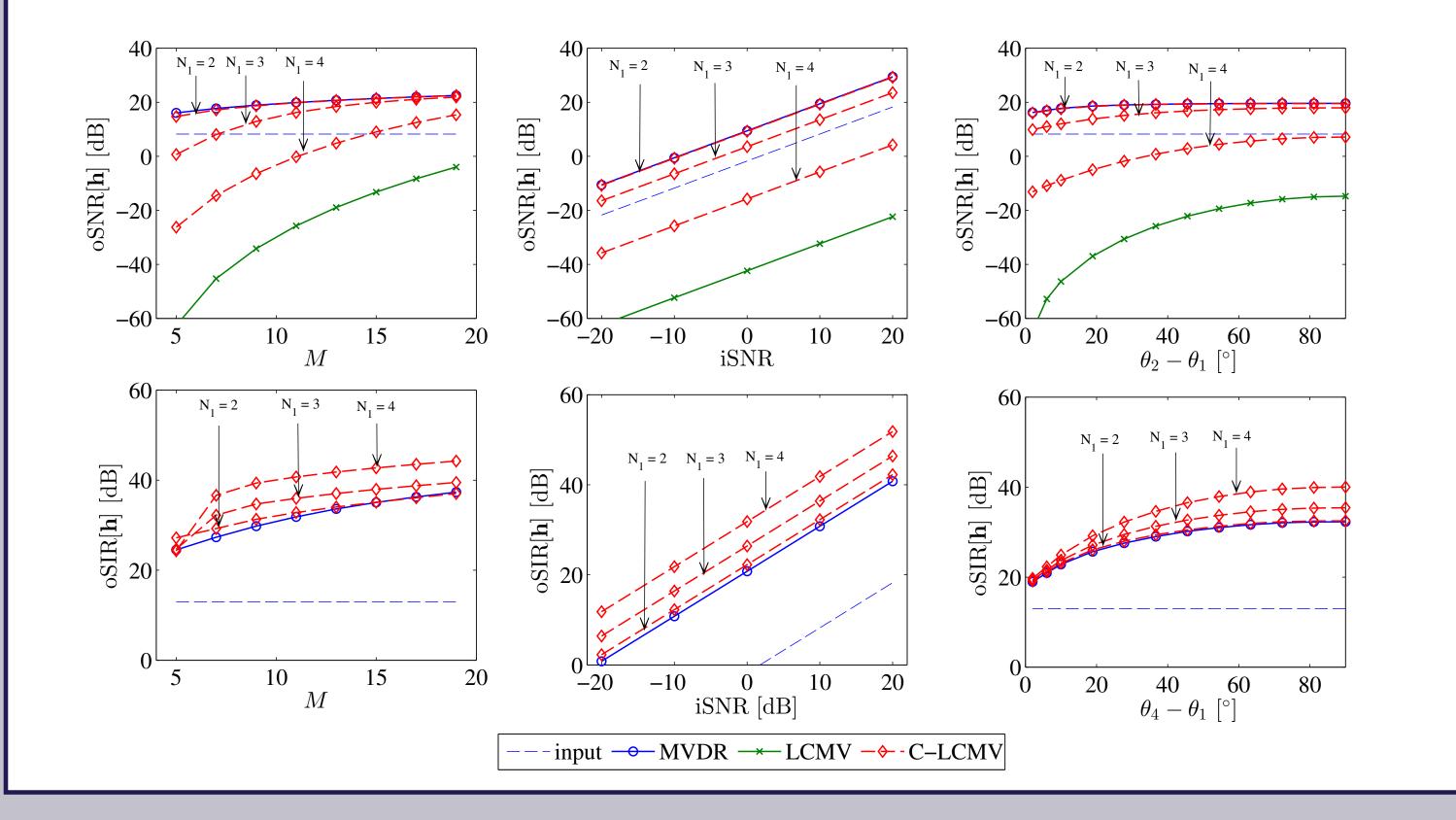
 $\phi_{Z}(f) = \mathbf{h}^{\mathsf{H}}(f) \,\mathbf{d}_{1}(f) \,\phi_{X_{1}}(f) \,\mathbf{d}_{1}^{\mathsf{H}}(f) \,\mathbf{h}(f) + \mathbf{h}^{\mathsf{H}}(f) \left[ \,\mathbf{\Phi}_{\mathbf{i}}(f) + \,\mathbf{\Phi}_{\mathbf{v}}(f) \,\right] \mathbf{h}(f). \tag{4}$ 

With the distortionless constraint that  $\mathbf{h}^{H}(f)\mathbf{d}_{1}(f) = 1$ , we can write

$$oSINR[\mathbf{h}(f)] = \frac{\phi_{X_1}(f)}{\mathbf{h}^{\mathsf{H}}(f) \left[ \mathbf{\Phi}_{\mathsf{in}}(f) + \mathbf{\Phi}_{\mathbf{v}}(f) \right] \mathbf{h}(f)},$$
(5)  
$$oSIR[\mathbf{h}(f)] = \frac{\phi_{X_1}(f)}{\mathbf{h}^{\mathsf{H}}(f) \mathbf{\Phi}_{\mathsf{in}}(f) \mathbf{h}(f)}.$$
(6)

## **Experiment: Synthetic Signal**

Using a uniform linear array (ULA) with M = 10 microphones and Gaussian noise signal sources in  $\theta_1 = 0$ ,  $\theta_2 = \pi$ ,  $\theta_3 = 5\pi/6$ ,  $\theta_4 = 4\pi/6$ , and  $\theta_5 = \pi/2$ , where iSINR = 8 dB and iSIR = 13 dB:



• Optimal steering matrix (in practice):  $\mathbf{D}_{i}^{opt}(f) = \arg\max oSINB[\mathbf{h}_{o}(f)]$ 

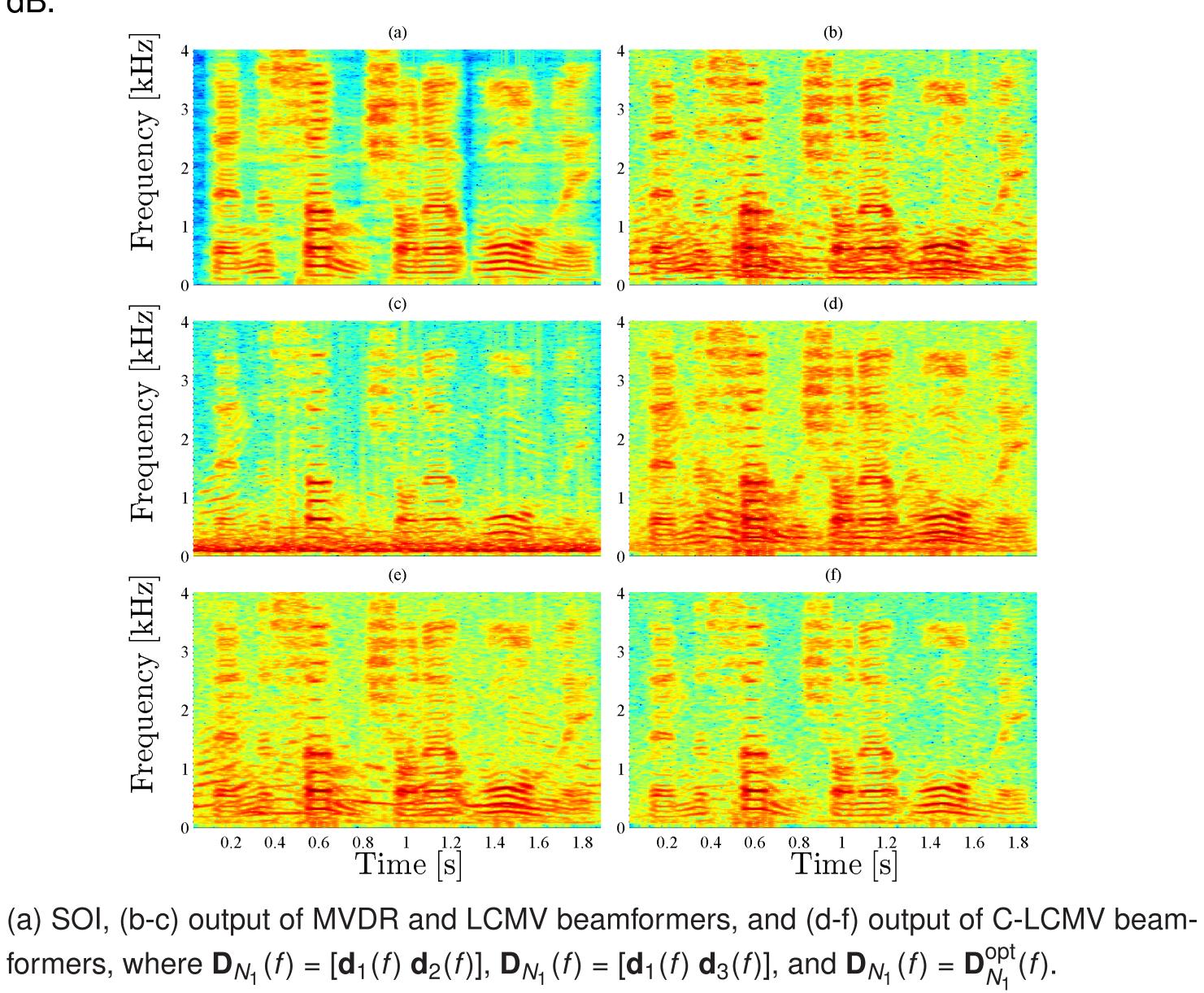
# $\mathbf{D}_{N_1}^{\text{opt}}(f) = \arg \max_{\mathbf{D}_{N_1}(f)} \operatorname{oSINR}\left[\mathbf{h}_{\mathrm{C}}(f)\right].$

### (17)

(16)

## **Experiment: Real Scenario**

We simulated a room  $(6 \times 7 \times 3 \text{ m})$  with reverberation time  $T_{60} = 0.25 \text{ s}$ , and N = 3 speech signals, and used a ULA with M = 5 hypercardioid microphones, SNR= 20 dB.



# Conclusion

The C-LCMV beamformer generalizes the MVDR and LCMV beamformers with the ability to control the quality of the output signal.

Experiment results indicate that the C-LCMV beamformer using optimal steering matrix gets better results than the other MV beamformers.