AALBORG UNIVERSITY

DENMARK

A Broadband Beamformer Using Controllable Constraints and Minimum Variance

Karimian-Azari, Sam; Benesty, Jacob; Jensen, Jesper Rindom; Christensen, Mads Græsbøll

A Broadband Beamformer Using Controllable Constraints and Minimum Variance

Sam Karimian-Azari[†,1], Jacob Benesty[*], Jesper Rindom Jensen[†,2], and Mads Græsbøll Christensen[†]

email: [†]{ska, jrj, mgc}@create.aau.dk, and [*]benesty@emt.inrs.ca

AUDIO ANALYSIS LAB

AALBORG UNIVERSITY
DENMARK

Introduction

The minimum variance distortionless response (MVDR) beamformer is an optimal approach to noise reduction:
► Achieves a high output SNR.
► It has degrees of freedom (DOF) corresponding to the number of microphones minus one.
► Its output is contaminated with both residual noise and interference.

The linearly constrained minimum variance (LCMV) beamformer reduces noise, and rejects interferers using linear constraints:
► Achieves a high output SIR.
► The number of constraints may degrade the DOF.
► It may amplify background noise which causes a lower output SNR.

To achieve a trade-off between attenuation of noise and interfering sources, we proposed the controllable LCMV (C-LCMV) beamformer in the frequency-domain.

Formulation

Multi-channel observed signals at the frequency f are observed, using an array of M microphones:

$$\mathbf{y}(f) = \mathbf{d}_1(f)X_1(f) + \sum_{n=2}^{N} \mathbf{d}_n(f)X_n(f) + \mathbf{v}(f) = \mathbf{D}_N(f)\mathbf{x}(f) + \mathbf{v}(f), \quad (1)$$

where $\mathbf{d}_n(f) \in \mathbb{C}^M$ is the steering vector of signal source $X_n(f)$ (for $n = 1, \cdots, N$), $X_1(f)$ is the signal of interest (SOI), $\mathbf{x}(f) \in \mathbb{C}^N$ is the collected N signal sources, $\mathbf{v}(f) \in \mathbb{C}^M$ is noise, and

$$\mathbf{D}_N(f) = [\,\mathbf{d}_1(f)\,\mathbf{d}_2(f)\,\cdots\,\mathbf{d}_N(f)\,] \in \mathbb{C}^{M \times N}. \quad (2)$$

The correlation matrix of $\mathbf{y}(f)$ (assuming uncorrelated signals) is

$$\mathbf{\Phi_y}(f) = \mathbf{D}_N(f)\mathbf{\Phi_x}(f)\mathbf{D}_N^H(f) + \mathbf{\Phi_v}(f) = \mathbf{d}_1(f)\phi_{X_1}(f)\mathbf{d}_1^H(f) + \mathbf{\Phi}_{in}(f), \quad (3)$$

where $\mathbf{\Phi_x}(f) = \operatorname{diag}[\,\phi_{X_1}(f)\ \phi_{X_2}(f)\ \ldots\ \phi_{X_N}(f)\,]$, $\mathbf{\Phi}_{in}(f) = \mathbf{\Phi_i}(f) + \mathbf{\Phi_v}(f)$, and $\mathbf{\Phi_i}(f) = \sum_{n=2}^{N} \mathbf{d}_n(f)\phi_{X_n}(f)\mathbf{d}_n^H(f)$ is the interference correlation matrix.

The output variance of the beamformer $\mathbf{h}(f)$ is

$$\phi_Z(f) = \mathbf{h}^H(f)\mathbf{d}_1(f)\phi_{X_1}(f)\mathbf{d}_1^H(f)\mathbf{h}(f) + \mathbf{h}^H(f)[\,\mathbf{\Phi_i}(f) + \mathbf{\Phi_v}(f)\,]\mathbf{h}(f). \quad (4)$$
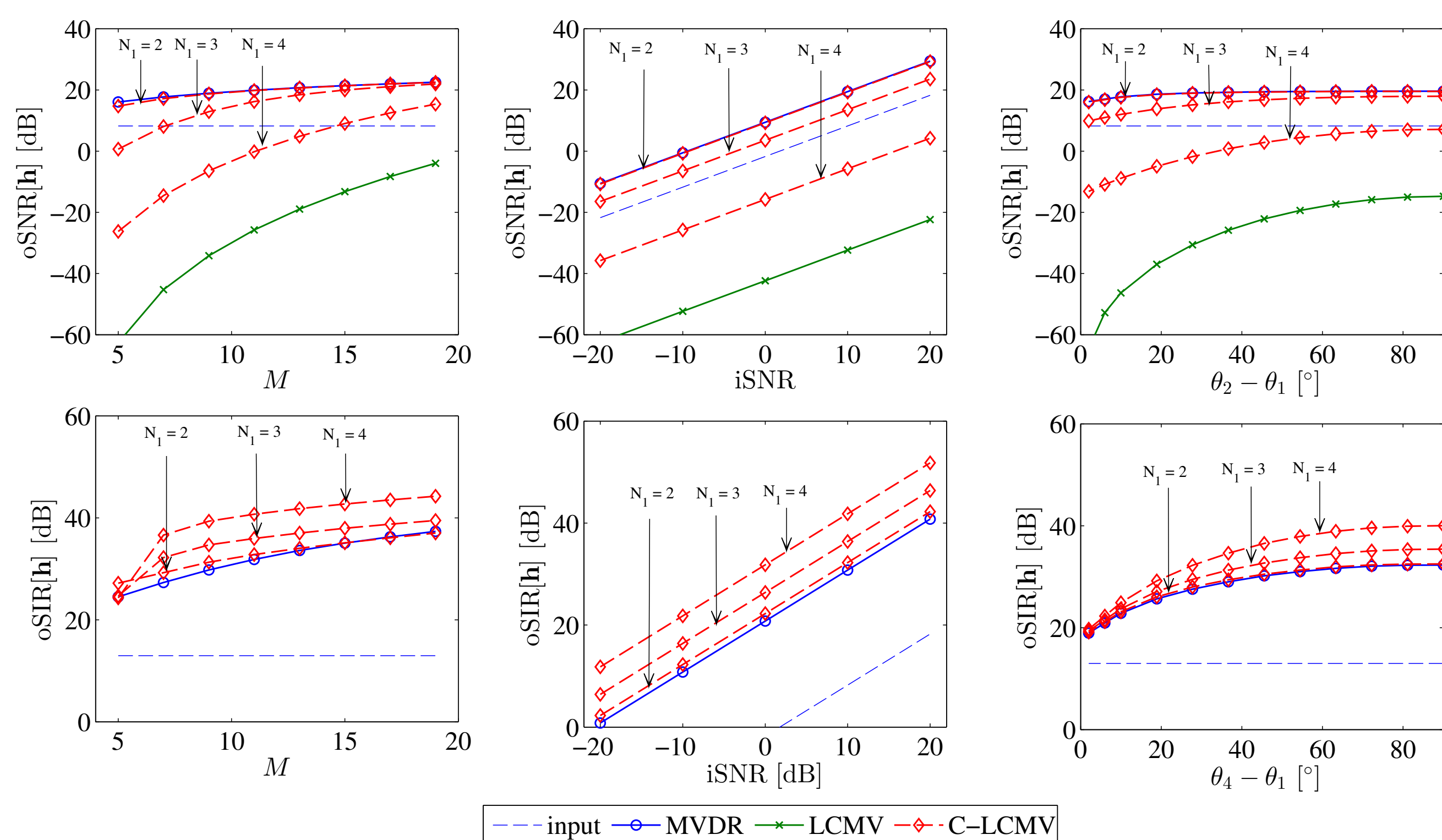
With the distortionless constraint that $\mathbf{h}^H(f)\mathbf{d}_1(f) = 1$, we can write

$$\mathrm{oSINR}[\mathbf{h}(f)] = \frac{\phi_{X_1}(f)}{\mathbf{h}^H(f)[\,\mathbf{\Phi}_{in}(f) + \mathbf{\Phi_v}(f)\,]\mathbf{h}(f)}, \quad (5)$$

$$\mathrm{oSIR}[\mathbf{h}(f)] = \frac{\phi_{X_1}(f)}{\mathbf{h}^H(f)\mathbf{\Phi}_{in}(f)\mathbf{h}(f)}. \quad (6)$$

Experiment: Synthetic Signal

Using a uniform linear array (ULA) with $M = 10$ microphones and Gaussian noise signal sources in $\theta_1 = 0$, $\theta_2 = \pi$, $\theta_3 = 5\pi/6$, $\theta_4 = 4\pi/6$, and $\theta_5 = \pi/2$, where iSINR = 8 dB and iSIR = 13 dB:

C-LCMV Beamformer

MDVR:
$$\min_{\mathbf{h}(f)} \quad \mathbf{h}^H(f)[\,\mathbf{\Phi_i}(f) + \mathbf{\Phi_v}(f)\,]\mathbf{h}(f) \quad (7)$$
$$\text{subject to} \quad \mathbf{h}^H(f)\mathbf{d}_1(f) = 1,$$

LCMV:
$$\min_{\mathbf{h}(f)} \quad \mathbf{h}^H(f)\mathbf{\Phi_v}(f)\mathbf{h}(f) \quad (8)$$
$$\text{subject to} \quad \mathbf{h}^H(f)\mathbf{D}_N(f) = \mathbf{i}_N^T,$$

where $\mathbf{i}_N$ is the first column of a $N \times N$ identity matrix.

We divide N signal sources into two sets of N_1 sources, containing SOI, and $N_2 = N - N_1$ remaining signal sources;

$$\mathbf{x}(f) = [\,\mathbf{x}_{N_1}^T(f)\ \mathbf{x}_{N_2}^T(f)\,]^T, \text{ and} \quad (9)$$

$$\mathbf{D}_N(f) = [\,\mathbf{D}_{N_1}(f)\ \mathbf{D}_{N_2}(f)\,]. \quad (10)$$

Therefore,

$$\mathbf{y}(f) = \mathbf{D}_{N1}(f)\mathbf{x}_{N_1}(f) + [\,\mathbf{D}_{N_2}(f)\mathbf{x}_{N_2}(f) + \mathbf{v}(f)\,], \quad (11)$$

and

$$\mathbf{\Phi_y}(f) = \mathbf{D}_{N_1}(f)\mathbf{\Phi}_{\mathbf{x}_{N_1}}(f)\mathbf{D}_{N_1}^H(f) + \mathbf{\Phi}_{in,N_2}(f), \quad (12)$$

where $\mathbf{\Phi}_{in,N_2}(f) = \mathbf{D}_{N_2}(f)\mathbf{\Phi}_{\mathbf{x}_{N_2}}(f)\mathbf{D}_{N_2}^H(f) + \mathbf{\Phi_v}(f)$ is the correlation matrix of N_2 signal sources plus background noise.

The C-LCMV beamformer is designed as

$$\min_{\mathbf{h}(f)} \quad \mathbf{h}^H(f)\mathbf{\Phi}_{in,N_2}(f)\mathbf{h}(f) \quad (13)$$
$$\text{subject to} \quad \mathbf{h}^H(f)\mathbf{D}_{N_1}(f) = \mathbf{i}_{N_1}^T.$$

Then the solution is given like

$$\mathbf{h}_C(f) = \mathbf{\Phi}_{in,N_2}^{-1}(f)\mathbf{D}_{N_1}(f)[\,\mathbf{D}_{N_1}^H(f)\mathbf{\Phi}_{in,N_2}^{-1}(f)\mathbf{D}_{N_1}(f)\,]^{-1}\mathbf{i}_{N_1}. \quad (14)$$

► Properties:

$$\mathrm{oSINR}[\mathbf{h}_L(f)] \le \mathrm{oSINR}[\mathbf{h}_C(f)] \le \mathrm{oSINR}[\mathbf{h}_M(f)], \quad (15)$$
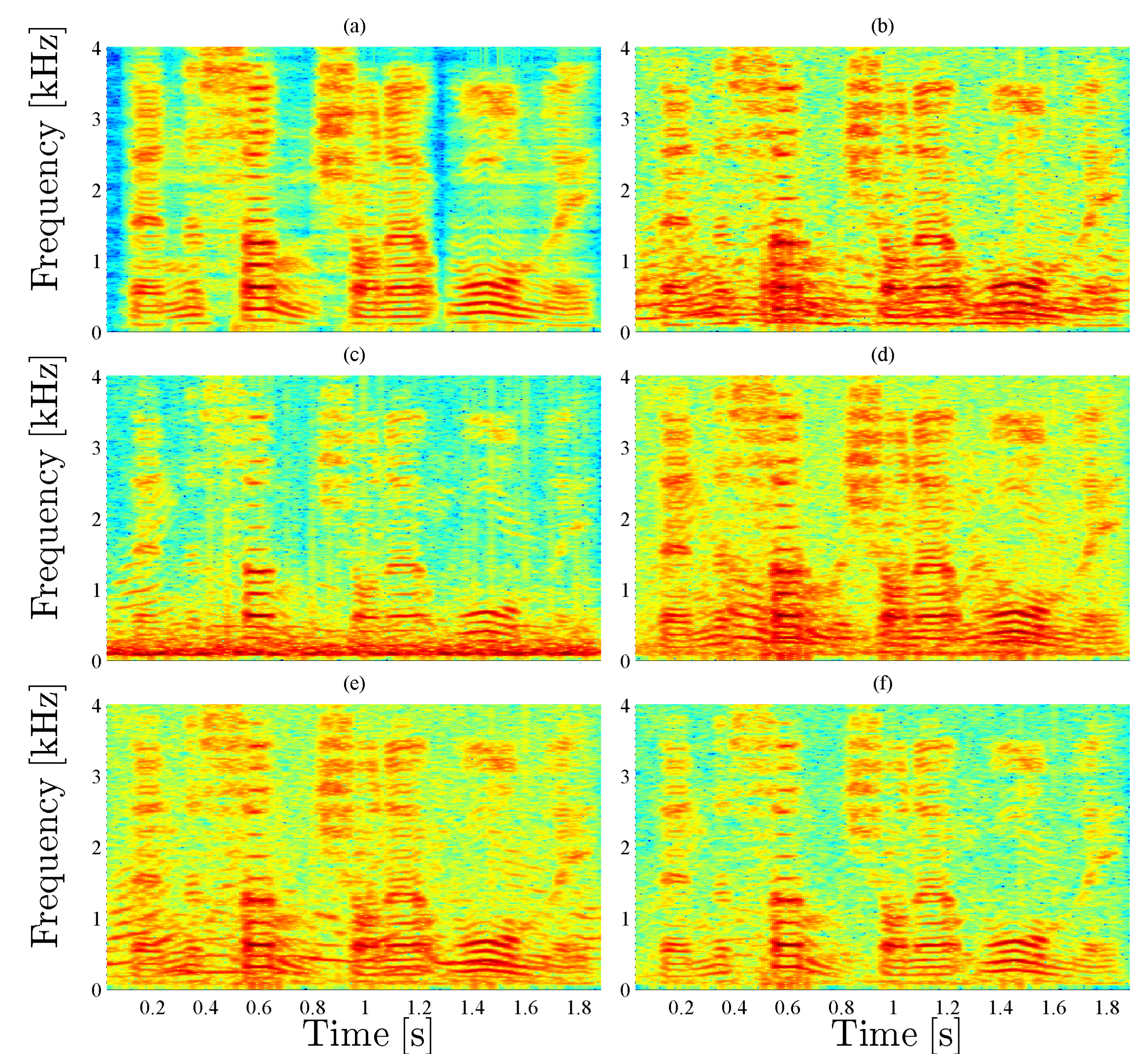
$$\mathrm{oSIR}[\mathbf{h}_M(f)] \le \mathrm{oSIR}[\mathbf{h}_C(f)] \le \mathrm{oSIR}[\mathbf{h}_L(f)]. \quad (16)$$

► Optimal steering matrix (in practice):

$$\mathbf{D}_{N_1}^{opt}(f) = \arg\max_{\mathbf{D}_{N_1}(f)} \mathrm{oSINR}[\mathbf{h}_C(f)]. \quad (17)$$

Experiment: Real Scenario

We simulated a room ($6 \times 7 \times 3$ m) with reverberation time $T_{60} = 0.25$ s, and $N = 3$ speech signals, and used a ULA with $M = 5$ hypercardioid microphones, SNR= 20 dB.

(a) SOI, (b-c) output of MVDR and LCMV beamformers, and (d-f) output of C-LCMV beamformers, where $\mathbf{D}_{N_1}(f) = [\mathbf{d}_1(f)\ \mathbf{d}_2(f)]$, $\mathbf{D}_{N_1}(f) = [\mathbf{d}_1(f)\ \mathbf{d}_3(f)]$, and $\mathbf{D}_{N_1}(f) = \mathbf{D}_{N_1}^{opt}(f)$.

Conclusion

► The C-LCMV beamformer generalizes the MVDR and LCMV beamformers with the ability to control the quality of the output signal.
► Experiment results indicate that the C-LCMV beamformer using optimal steering matrix gets better results than the other MV beamformers.