Papers

Volume 7: 2004-2008

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Publication date:
2008

Document Version
Publisher's PDF, also known as Version of record

Link to publication from Aalborg University

Citation for published version (APA):
Thoft-Christensen, P. (2008). *Papers: Volume 7: 2004-2008*. Department of Civil Engineering, Aalborg University.

CHAPTER 128

3D-MODELLING OF CORROSION CRACK OPENING [1]

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ABSTRACT

It is of great interest to investigate whether it is possible to estimate the reliability of a given reinforced concrete structure by observations of corrosion crack on the surface of the structure. In the paper recent progress in modelling of the deterioration of reinforced concrete structures is presented with special emphasis on the corrosion crack opening. Experiments and 2-dimensional FEM analysis seem to support that the relation between the reduction of the reinforcement diameter ΔD_{bar} and the corresponding increase in crack width Δw_{crack} in a given time interval Δt measured on the surface of the concrete specimen can be approximated by a linear function; Thoft-Christensen [1]. More recently the crack-corrosion index $\gamma = \Delta w_{crack} / \Delta D_{bar}$ is further studied with respect to its dependency on the diameter of the rebar and the concrete cover; Thoft-Christensen [2]. In the paper it is shown that the above-mentioned work can be extended to more realistic beam configurations when a 3-dimensional FEM modelling is used. The crack corrosion index may then be evaluated for a 3-dimensional beam by assuming a concentrated corrosion area of the reinforcement. In this way it is possible not only to investigate homogeneous corrosion, but also pit corrosion.

[1] Proceedings of the IFIP WG 7.5 Conference on “Reliability and Optimization of Structural Systems”, Aalborg, Denmark, May 22-25, 2005. In “Advances in Reliability and Optimization of Structural Systems”, Balkema/Taylor & Frances, 2006, pp. 163-170.

1. **INTRODUCTION**

Life-cycle assessment of the structural reliability of a reinforced concrete structure is based on modelling of the deterioration of the concrete. The most serious deterioration is often corrosion of the reinforcement due to chloride penetration of the concrete. A fully satisfactory modelling of the corrosion of the reinforcement has not yet been established. Modelling of the corrosion initiation is often based on Fick's law of diffusion; see e.g. Thoft-Christensen [3]. After initiation of corrosion in the reinforcement it is often assumed that the cross-section of the reinforcement decreases with time. By this modelling it is simple to perform a deterministic or stochastic estimate of the so-called reliability profile that is the capacity or reliability as functions of time; see e.g. Thoft-Christensen [3].

The reliability profile consists of six parts:

1. Chloride penetration
2. Corrosion initiation
3. Corrosion evolution
4. Initial cracking
5. Crack propagation
6. Spalling.

Deterioration steps 1-3 are well understood, and are presented in numerous papers, e.g. Thoft-Christensen [4]. Step 4 has been addressed in Thoft-Christensen [5], but steps 5 and 6 have only recently been investigated in this connection, Thoft-Christensen [6].

Using diffusion modelling of the chloride penetration of the concrete it is shown based on simulation data that the corrosion initiation time for a considered example may be modelled by a Weibull distribution. This approach based on diffusion theory seems to have reached general acceptance among researchers in this field. The time to crack initiation is estimated by calculating the amount of corrosion products and estimate the space needed for these products. Initially the rust products will fill the interconnected porous zone completely and then result in an expansion of the concrete near the reinforcement. A result of this is that tensile stresses are initiated in the concrete. After some time with increasing corrosion the tensile stresses will reach a critical value, when the reinforcement is relatively close to the surface of the concrete, and corrosion cracks will be developed. After formation of the initial crack the rebar cross-section is further reduced due to the continued corrosion, and the width of the crack is increased. Experiments show that the function between the reduction of the rebar diameter ΔD_{bar} and the corresponding increase in crack width Δw_{crack} measured on the surface of the concrete specimen can be approximated by a linear function $\Delta w_{crack} = \gamma \, \Delta D_{bar}$, where the factor γ is of the order 1.5 to 5.

2. CRACK-CORROSION INDEX

Several researchers have investigated the evolution of corrosion cracks in reinforced concrete beams experimentally. After formation of the initial crack the rebar cross-section is further reduced due to the continued corrosion, and the width of the crack is increased. An impressed current are normally used to artificially corrode the reinforcement. The loss of bar sections is then monitored and the corresponding crack evolution is measured by the use of strain gauges attached to the surface of the beams.

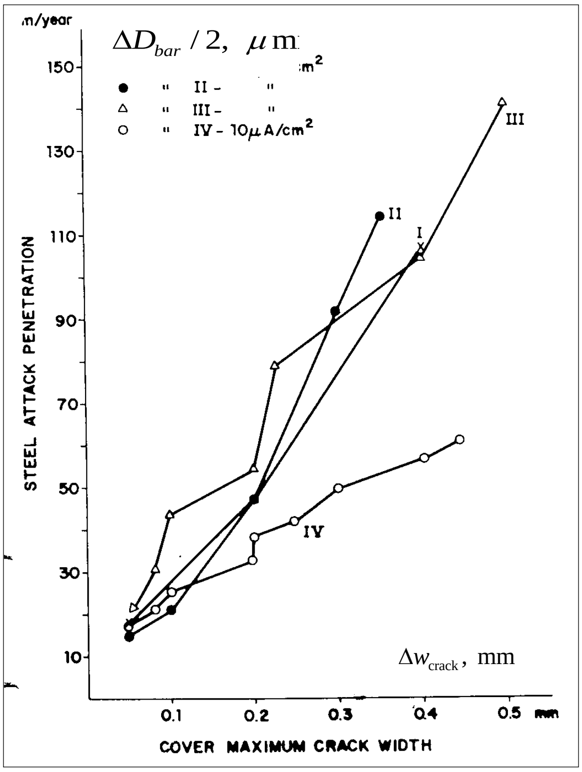


Figure 1. Loss in rebar diameter ΔD_{bar} versus the increase in crack width Δw_{crack}, Andrade, Alonso & Molina [6].

In most experiments the function between the reduction of the rebar diameter and the maximum crack width measured in the surface of the concrete specimen can be approximated by a linear function, see Andrade, Alonso & Molina [7]. Based on this linearization the increased crack width with time may be modelled.

The opening of corrosion cracks in reinforced concrete beams has been experimentally investigated; see e.g. Andrade, Alonso & Molina [7]. After formation of the initial crack the rebar cross-section is further reduced due to the continued corrosion, and the width of the crack is increased. In the paper four simple test specimens have been investigated. The specimens are simplified small reinforced concrete beams with only a single rebar and 20 or 30 mm of cover. An impressed current artificially corrodes the beams. The loss of bar sections is monitored and the corresponding crack evolution is measured by the use of strain gauges attached to the surface of the beams. In all four experiments the function between the reduction of the rebar diameter and the maximum crack width measured in the surface of the concrete specimen can be approximated by a linear function, see figure 1. Let Δw_{crack} be the increase in crack width in the time interval Δt and let the corresponding loss of rebar diameter be ΔD_{bar}. Then, $\Delta w_{crack} = \gamma \, \Delta D_{bar}$, where the crack corrosion index γ is of the order 1.5 to 5 in the experiments reported in Andrade, Alonso & Molina [7].

2. 2D-MODDELING

The first FEM estimation of the crack-corrosion index $\gamma = \Delta w_{crack} / \Delta D_{bar}$ was presented at the IFIP TC7 Conference on "System Modelling and Optimization" in Sophia Antipolis, France, July 2003 using FEMLAB/MATLAB; see Thoft-Christensen [1]. A rectangular beam cross-section with only one reinforcement bar was considered, see figure 2. The diameter of the hole around the rebar at the time of crack initiation is D_{hole} = 20 mm and that the cover is c = 10 mm. The initial crack width is 0.01 mm.

In the FEM modelling the rectangular cross-section in a long beam (plain strain) is assumed to have a hole at the location of the reinforcement and a crack (0.01 mm) from the hole to the boundary. The material is assumed to be linear elastic with the elasticity module $E = 25 \times 10^9$ Pa. It is assumed that the pressure on the boundary of the hole from the increasing corrosion products can be modelled as a uniform loading (pressure) $p = 1 \times 10^8$ N/m at the boundary of the hole.

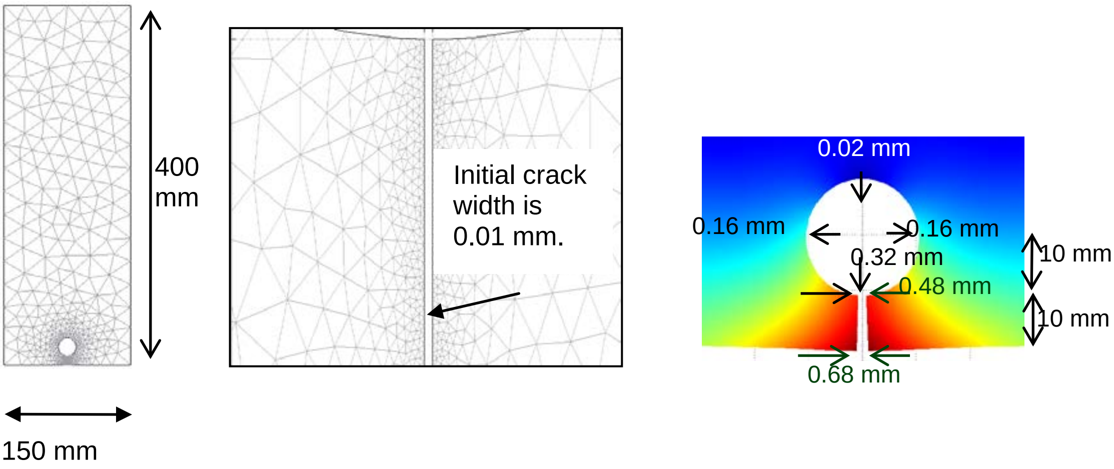


Figure 2. FEM net. The total net is shown to the left, the local net near the crack is shown in the middle, and the displacements are shown at the right; Thoft-Christensen [1].

The result of the analysis is shown in figure 2. The increase in the crack width Δt is Δw_{crack} = 0.67 mm and the average increase in the hole diameter is ΔD_{hole} = 0.31 mm.

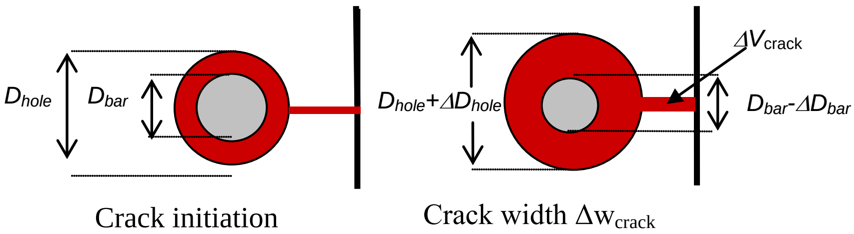


Figure 3. Crack width increase from crack initiation (0.1 mm) to Δw_{crack}; Thoft-Christensen [2].

An approximate relation between the decrease in the steel bar ΔD_{bar} and the increase in the hole ΔD_{hole} may be obtained easily by considering the volume of the produced rust products $\pi \Delta D_{bar}\ D_{bar}\ \alpha$ ($\alpha = \rho_{steel}/\rho_{rust}$ is the relation between the densities of the steel and the rust product; see table 1) and the volume due to the expansion of the hole $\pi \Delta D_{bar}\ D_{bar} + \pi \Delta D_{hole}\ D_{hole} + \Delta V_{crack}$; see figure 3. By equalizing these two volumes and assuming that $D_{bar} \approx D_{hole}$ one gets

$$(\alpha - 1)\Delta D_{bar} \approx \Delta D_{hole} + \Delta V_{crack}/\pi D_{hole} \tag{1}$$

and

$$\gamma = \Delta w_{crack}/\Delta D_{bar} \approx (\alpha - 1)\Delta w_{crack}/(\Delta D_{hole} + \Delta V_{crack}/\pi D_{hole}) \tag{2}$$

For the example shown in figure 2 one gets

$$\gamma \approx (\alpha - 1)0.67/(0.31 + 5.7/20\pi)) = 1.67(\alpha - 1) \tag{3}$$

or γ equal to 1.8 and 5.3 for black and brown rust respectively, in good agreement with the values obtained by the experiments in chapter 2.

The results of a similar FEM analysis of a cross-section like the one illustrated in figure 2 are presented for 10 different combinations of the cover c and the diameter D_{bar} in Thoft-Christensen [8]. The conclusion is that γ increases with the cover c for a fixed rebar diameter D_{bar}, and that γ also increases with the diameter D_{bar} for a fixed cover c.

A similar analysis with 30 combinations of D_{bar} and c but with a different cross-section confirms partly this conclusion; see Thoft-Christensen [2]. The γ values for the 30 combinations are shown in table 2. It follows from table 2 that the crack-corrosion index γ for the used parameter combinations increases with the diameter D_{bar} and decreases with the concrete cover c. However, crack-corrosion index γ is in this case estimated by a different numerical procedure.

Corrosion product	Colour	α
Fe_3O_4	Black	2.1
$Fe(OH)_2$	White	3.8
$Fe(OH)_3$	Brown	4.2
$Fe(OH)_3, 3H_2$	Yellow	6.4

Table 1. $\alpha = \rho_{steel}/\rho_{rust}$ for different corrosion products.

In the same paper it is also investigated whether the distance b from the crack to the nearest uncracked vertical side of the beam is important for the estimate of crack-corrosion index γ. The same cross-section is used, but the hole and the crack are placed at different distances from the vertical side of the beam. The conclusion is that the distance b to the vertical side of the beam is only significant if b is smaller than twice the rebar diameter. Otherwise, γ with a good approximation seems to be independent of b.

Diameter D_{bar}, mm			Cover c,mm		
	20	25	30	35	40
10	3.72	3.53	3.39	3.30	3.25
12	3.93	3.76	3.63	3.55	3.50
14	4.09	3.94	3.83	3.75	3.71
16	4.20	4.08	3.99	3.92	3.88
18	4.26	4.19	4.11	4.06	4.03
20	4.28	4.27	4.21	4.18	4.15

Table 2. Estimates of $\gamma(D, a)$.

3. 3D-modelling

In this section the procedure used in 2-D modelling of corrosion crack evolution is extended to a 3D- modelling. The 2D-moddeling in section 2 is based on several

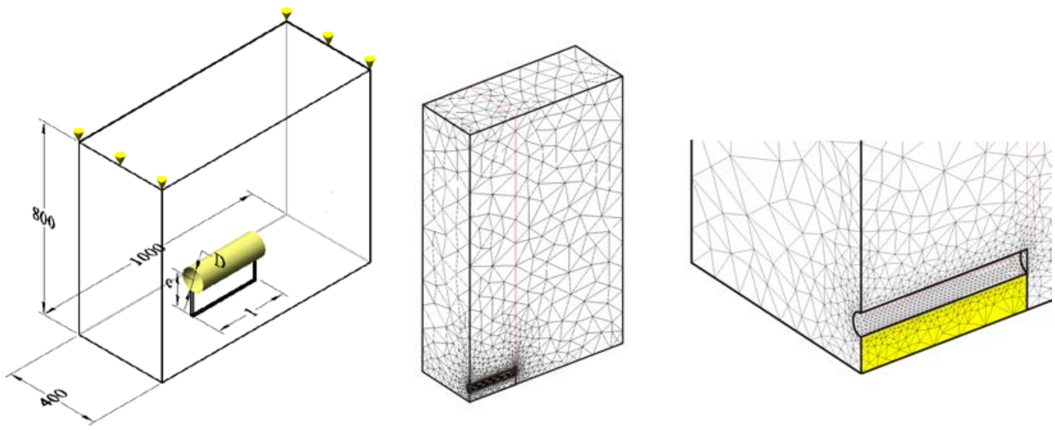


Figure 4. Beam element with corroded part of the reinforcement and the initial crack from the crack to the surface of the beam is shown to the left. In the middle the FEM-net is shown for one quarter of the beam, and to the right the FEM-net near the reinforcement and the crack is shown.

assumptions e.g. that the movement of corrosion products is restricted to the considered cross-section. In a 3D-modelling the corrosion products may also move in the direction of the reinforcement. Further it is in a 3-D modelling possible to consider situations where only parts of the reinforcement are corroded.

A beam element with the dimensions 400×800×1000 mm is considered. It contains only one rebar with the diameter D_{bar} = 20 mm, and the cover *c*, see figure 4. Corrosion is supposed to take place in the central part with the length *l* ($0 < l < 1000$ mm) of the reinforcement. An initial crack connects the corroded part of the reinforcement with the surface of the beam as shown in figure 4.

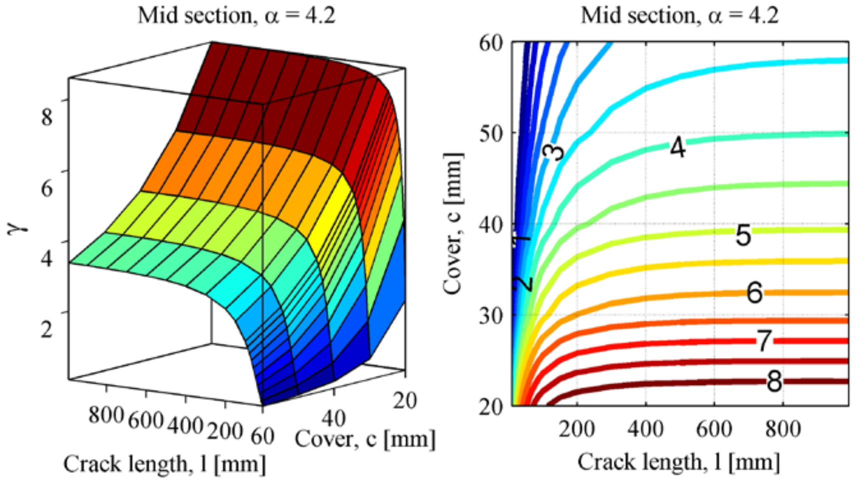


Figure 5. The crack-corrosion index $\gamma(l,c)$ estimated on basis of the mid section of the corroded part of the

In the FEM model the beam is assumed to have a cylindrical hole at the location of the reinforcement and a crack (0 mm thick) from the hole to the boundary. The material outside the hole and the crack is assumed to be linear elastic with the elasticity module $E = 25 \times 10^9$ Pa. It is assumed that the pressure on the boundary of the hole from the increasing corrosion products can be modelled as a uniform loading (pressure) $p = 1 \times 10^8$ N/m^2 normal to the boundary of the hole. The FEM analysis is due to symmetries based on only on quarter of the considered beam element. The FEM analysis is performed with 5 covers *c* = 20, 30, 40, 50, and 60 for the corrosion length *l* = 10, 20, 30, 40, 50, 100, 200, 300, 400, 500, 600, 700, 800 900, and 1000 mm. The crack-corrosion index $\gamma = \Delta w_{crack} / \Delta D_{bar}$ is calculated for each combination of the above mentioned values of the cover *c* and the corrosion (crack) length *l* using equation (2).

The calculated displacements and are based on the cross-section trough the midpoint of the corroded part of the reinforcement, see figure 5. The same estimations of the crack-corrosion index $\gamma(l,c)$ estimated on basis of the mid section is shown in figure 6 to the left. A similar estimation but based on the entire part of the corroded rebar is shown in the same figure 6 to the right. The crack-corrosion index $\gamma(l,c)$,estimated on basis of the mid section, is slightly higher than for the entire crack but of the same order.

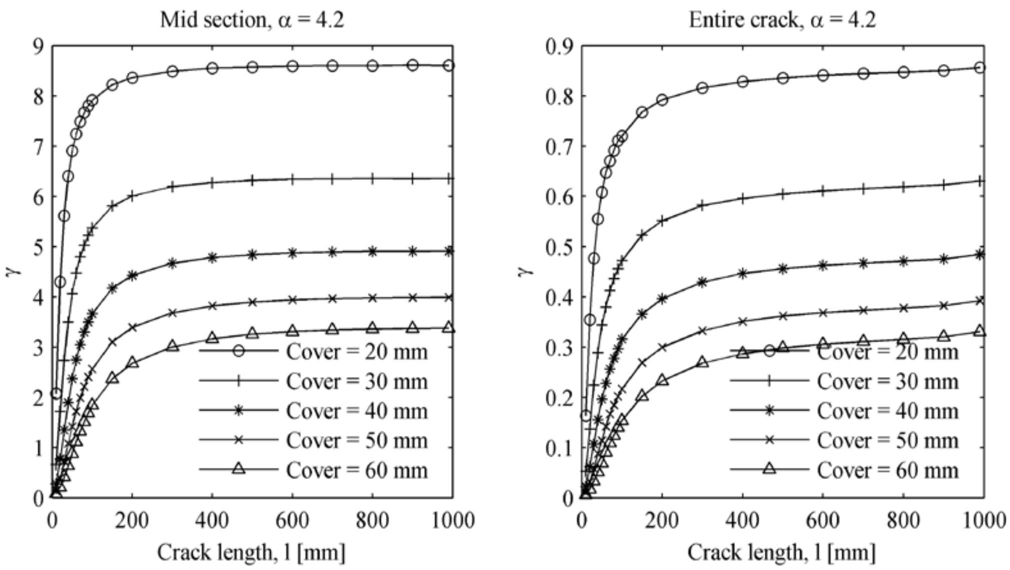
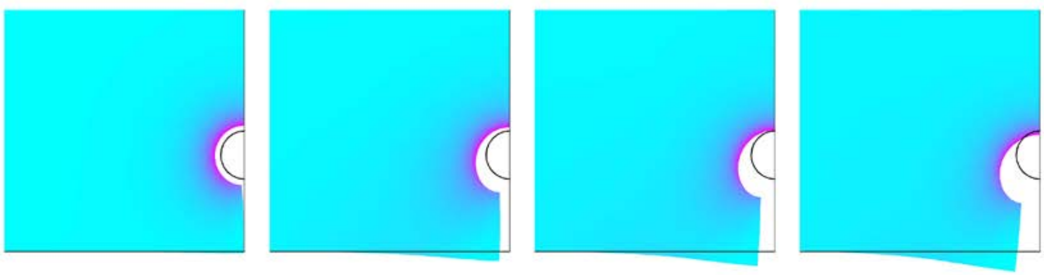


Figure 6. The crack-corrosion index γ as a function of the length of the corroded part l of the rebar for the midpoint approach to the left and the entire crack approach to the right. D_{bar}=20 mm.

It follows from figure 6 that the crack-corrosion index γ decreases with decreased length l especially for small lengths of l. $\gamma \to 0$ in the case of pit corrosion ($l \to 0$). Also notice that decreases with increased values of the cover like in table 2.



c = 40 mm	c = 40 mm	c = 40 mm	c = 40 mm
l = 20 mm	l = 100 mm	l = 200 mm	l = 1000

Figure 7. Illustration of the hole and the crack profile for 4 different lengths of the corroded part of the rebar. D_{bar}=20 mm. Magnification factor = 6.

l, mm	20	100	200	1000
Crack bottom, mm	0.123	1.514	2.457	3.48
crack top, mm	0.425	1.441	1.970	2.58
	0.377	0.617	0.72	0.823
,	0.159	0.413	0.551	0.709
ΔD_{hole}, mm	0.78	3.66	4.42	4.91

Table 3. Data related to the 4 parameter set used in figure 7.

ΔD_{bar}, mm

γ

The crack openings for 4 selected crack lengths are illustrated in figure 7 and the corresponding data are shown in table 3. The crack openings at the surface Δw_{crack} and near the hole Δw_{hole} as a function of the crack length l are shown in figure 8. It is interesting to note that Δw_{crack} - Δw_{hole} is positive for l greater than 90 mm and negative for l smaller than 90 mm.

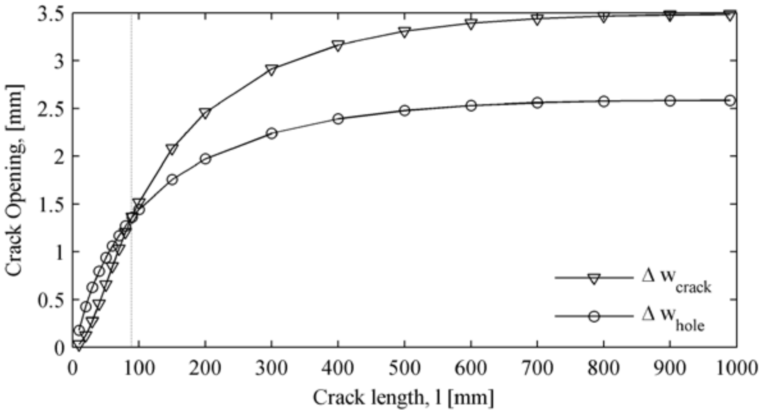


Figure 8. The crack openings at the surface Δw_{crack} and near the hole Δw_{hole} as a function of the crack length l. D_{bar}=20 mm.

In figure 9 is shown the displacements for the relative short crack length l = 20 mm (pit corrosion), D_{bar} = 20 mm, and the cover is 40 mm. The volume of the crack ΔV_{crack} (see figure 3) is small compared with the space around the corroded part of the rebar. Accordingly, the crack-corrosion index γ is small as shown in table 3. For even smaller cracks (pit corrosion) the crack-corrosion index γ is close to 0.

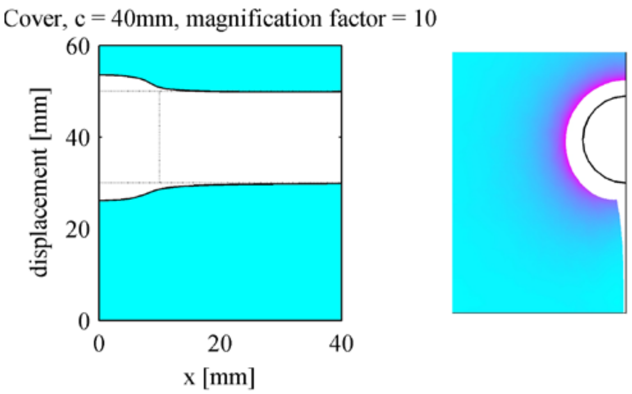


Figure 9. The displacements for a short crack (pit corrosion).

4. SAFETY ESTIMATION

The reduction of the rebar diameter ΔD_{bar} corresponding a given corrosion crack width may be estimated by adding the estimation of the rebar reduction at crack initiation $\Delta D_{bar,ci}$ and the increased reduction $\Delta D_{bar,cw}$ from crack initiation to the crack width Δw_{crack}

$$\Delta D_{bar} = \Delta D_{bar,ci} + \Delta D_{bar,\,cw} \tag{4}$$

Liu & Weyers [9] have estimated the increase in the hole diameter $\Delta D_{hole,ci}$ at crack initiation by assuming that the concrete is a homogeneous elastic material and can be approximated by a thick-walled concrete cylinder around the rebar. The approximate value of the critical expansion $\Delta D_{hole,ci}$ is

$$\Delta D_{hole,ci} = \frac{c f_t'}{E_{ef}}\left(\frac{a^2+b^2}{b^2-a^2}+\nu_c\right) \tag{5}$$

where E_{ef} is the effective elastic modulus of the concrete and f_t' is the tensile strength of the concrete. ν_c is Poisson's ratio of the concrete. *a* and *b* are geometrical constants. An approximate relation between $\Delta D_{bar,ci}$ and $\Delta D_{hole,ci}$ may be obtained by equalizing the volume $\Delta\pi D_{bar,ci}\, D_{bar}\alpha$ ($\alpha = \rho_{steel}/\rho_{rust}$) of the produced rust products with the volume $\Delta\pi D_{bar,ci}\, D_{bar} + \Delta\pi D_{hole,ci}\, D_{bar}$ due to the expansion of the hole. One gets

$$\Delta D_{bar,ci} = \Delta D_{hole,ci} / (\alpha - 1) \tag{6}$$

By (4) - (6) and $\Delta D_{bar,cw} = \gamma\, w_{crack}$ the reduction of the rebar diameter ΔD_{bar} when the corrosion crack width is Δw_{crack} is then estimated by

$$\Delta D_{bar} = \frac{c f_t'}{E_{ef}(1-\alpha)}\left(\frac{a^2+b^2}{b^2-a^2} + \nu_c\right) + \gamma\, \Delta w_{crack} \tag{7}$$

The reductions of the rebar cross section area and thereby the safety of the beam may then be estimated by (7).

5. CONCLUSIONS

Modelling of the deterioration of reinforced concrete structures is presented with special emphasis on the corrosion crack opening during corrosion of the reinforcement. Experiments and 2-dimensional FEM analysis seem to support that the relation between the reduction of the reinforcement diameter ΔD_{bar} and the corresponding increase in crack width Δw_{crack} measured on the surface of the concrete specimen may be approximated by a linear function.

In this paper, recent work on the crack-corrosion index $\gamma = \Delta w_{crack} / \Delta D_{bar}$ is extended to more realistic beam configurations when a 3-dimensional FEM modelling is used. The crack corrosion index may then be evaluated for a 3-dimensional beam by assuming a concentrated corrosion area of the reinforcement. In this way it is possible not only to investigate homogeneous corrosion, but also pit corrosion.

It is shown by equalizing the volume of the produced rust products with the volume due to the expansion of the hole and the increased crack volume that the reductions of the rebar cross section area and thereby the safety of the beam may then be estimated. The results presented in this paper are preliminary outputs of an ongoing research.

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