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CHAPTER 129

SERVICE LIFE DEFINITIONS BASED ON CORROSION CRACK WIDTH[1]

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ABSTRACT

In this paper service life assessment of bridges is discussed based on stochastic models and with special emphasis on deterioration of reinforced structures due to reinforcement corrosion. The service life for a reinforced concrete structure has been subject to discussion between engineers for several decades. Definitions related to the reliability of the structure have been proposed in recent years. Several authors have defined the service life as the initiation time for corrosion of the reinforcement. Estimating the initiation time for corrosion by an approach based on diffusion theory seems to have obtained general acceptance.

The service life has later been modified so that the time from corrosion initiation to corrosion crack initiation in the concrete is included. A stochastic model may be developed on the basis of existing deterministic theories for crack initiation. The service life may be further modified so that the time from corrosion crack initiation to formation of a certain (critical) crack width is included. By this modelling it is possible to estimate the reliability of a given structure on the basis of measurements of crack widths on the surface of the concrete structure. In this paper the possibility of using this last-mentioned definition is discussed based on some recent FEM models of the crack width development.

[1] Proceedings of ICOSSAR'05, Rome, Italy, June 19-22, 2005, Millpress, Rotterdam, 2005 (cd). ISBN 90 5966 040 4.

1. INTRODUCTION

The service life $T_{service}$ for a reinforced concrete structure has been subject to discussion for several decades. Several authors have defined the service life as the initiation time for corrosion T_{corr} of the reinforcement. Estimating T_{corr} is a very complicated matter. An approach based on diffusion theory seems to have obtained general acceptance.

The service life $T_{service}$ has later been modified so that the time Δt_{crack} from corrosion initiation to corrosion crack initiation in the concrete is included. The service life is then defined by $T_{service} = T_{crack} = T_{corr} + \Delta t_{crack}$. A stochastic model for Δt_{crack} may be developed on the basis of existing deterministic theories for crack initiation; Thoft-Christensen [1].

The service life $T_{service}$ may further be modified so that the time $\Delta t_{crack\ width}$ from corrosion crack initiation to formation of a certain (critical) crack width is included. By this modelling it is possible to estimate the reliability of a given structure on the basis of measurements of the crack widths on the surface of the concrete structure.

After formation of the initial crack the rebar cross-section is further reduced due to the continued corrosion, and the width of the crack is increased. Experiments show that the function between the reduction of the rebar diameter ΔD_{bar} and the corresponding increase in crack width Δw_{crack} in a given time interval Δt measured on the surface of the concrete specimen can be approximated by a linear function $\Delta w_{crack} = \gamma\, \Delta D_{bar}$, where the factor γ is of the order 1.5 to 5.

This linearization is investigated in some detail in this paper using FEM analysis for simple beam cross-sections. This ongoing research seems to confirm the linearization between the increase of corrosion crack width and the decrease in rebar diameter. Further, the order of the factor γ is also confirmed.

The FEM methodology presented may also be used to identify where new corrosion cracks are developed and in this way perhaps even give an estimation of the time for the first spalling.

2. CORROSION INITIATION

If the rate of chloride penetration into concrete is modelled by Fick´s law of diffusion, then it can be shown that the time T_{corr} to initiation of reinforcement corrosion is

$$T_{corr} = \frac{d^2}{4D}\left(erf^{-1}\left(\frac{C_{cr} - C_0}{C_i - C_0}\right)\right)^{-2} \tag{1}$$

where *d is* the concrete cover, *D* is the diffusion coefficient, C_{cr} is the critical chloride concentration at the site of the corrosion, C_0 is the equilibrium chloride concentration on the concrete surface, C_i is the initial chloride concentration in the concrete, *erf* is the error function. All the parameters mentioned above are modelled by stochastic variables or stochastic processes.

3. CRACK INITIATION

After corrosion initiation the rust products will initially fill the porous zone around the steel/concrete surface caused by the transition from paste to steel and entrapped/entrained air voids and then result in an expansion for the concrete near the reinforcement. As a result of this, tensile stresses are initiated in the concrete. With increasing corrosion the tensile stresses will reach a critical value and cracks will be developed. During this process the volume of the corrosion products at initial cracking of the concrete W_{crit} will occupy three volumes, namely the porous zone W_{porous}, the expansion of the concrete due to rust pressure W_{expan}, and the space of the corroded steel W_{steel}. With this modelling and some minor simplifications it can then be shown that the time from corrosion imitation to crack initiation is; see Liu & Weyers [2]

$$\Delta t_{crack} = \frac{1}{2\times 0.383\times 10^{-3} D_{bar} i_{corr}} \left(\frac{\rho_{steel}}{\rho_{steel} - 0.57\rho_{ust}} \left(W_{porous} + W_{\exp an} \right) \right)^2 \tag{2}$$

where D_{bar} is the diameter of the reinforcement bar, i_{corr} is the annual mean corrosion rate, ρ_{steel} is the density of the steel, and ρ_{rust} is the density of the rust products. It is in the derivation of (2) assumed that the diameter $D_{bar}(t)$ of the reinforcement bar at the time t is modelled by

$$D_{bar}(t) = D_{bar}(T_{corr}) - c_{corr} i_{corr} (t - T_{corr}) \tag{3}$$

where c_{corr} is a corrosion coefficient.

4. CRACK EVOLUTION

After formation of the initial crack the rebar cross-section is further reduced due to the continued corrosion, and the width of the crack is increased. Experiments show that the function between the reduction of the rebar diameter ΔD_{bar} and the corresponding increase in crack width Δw_{crack} in a given time interval Δt measured on the surface of the concrete specimen can be approximated by a linear function

$$\Delta w_{crack} = \gamma\, \Delta D_{bar} \tag{4}$$

where the factor γ is of the order 1.5 to 5. It follows from (3) and (4) that

$$\begin{aligned} w_{crack}(t) &= w_{crack}(T_{crack}) + \gamma \left(D_{bar}(T_{crack}) - D_{bar}(t) \right) \\ &= w_{crack}(T_{crack}) + \gamma c_{corr} i_{corr} \left(t - T_{rack} \right) \end{aligned} \tag{5}$$

Let the critical crack width be $w_{critical}$ occurring at the time $T_{critical}$. By setting $w(T_{critical}) = w_{critical}$ the following expression is obtained for $T_{critical}$

$$T_{critical} = \frac{w_{critical} - w_{crack}(T_{crack})}{\gamma c_{corr} i_{corr}} + T_{crack} \tag{6}$$

$w_{crack}(T_{crack}) \approx 0$ is the initial crack width at the time T_{crack}.

5, EXPERIMENTAL VERIFICATION

The evolution of corrosion cracks in reinforced concrete beams has been experimentally investigated by Andrade, Alonso, & Molina [3]. After formation of the initial crack the rebar cross-section is further reduced due to the continued corrosion, and the width of the crack is increased. In the experiments the function between the reduction of the rebar diameter and the maximum crack width measured in the surface of the concrete specimen can be approximated by a linear function. Let Δw_{crack} be the increase in crack width in the time interval Δt and let the corresponding loss of rebar diameter be ΔD_{bar}. Then

$$\Delta w_{crack} = \gamma \, \Delta D_{bar} \tag{7}$$

where γ is of the order 1.5 to 5 in the experiments reported in Andrade, Alonso, & Molina [3]. The factor γ depends on the cross-sectional data.

6. NUMERICAL VERIFICATION

6.1 Estimation of γ

The coefficient γ in (7) has been estimated using FEM analysis (FEMLAB) by Thoft-Christensen [4]. For illustration, consider the rectangular beam cross-section shown in figure 1. The reinforcement consists of only one reinforcement bar. The diameter of the hole around the rebar at the time of crack initiation is D_{hole} = 20 mm and the cover is c = 10 mm. The initial crack width is 0.1 mm.

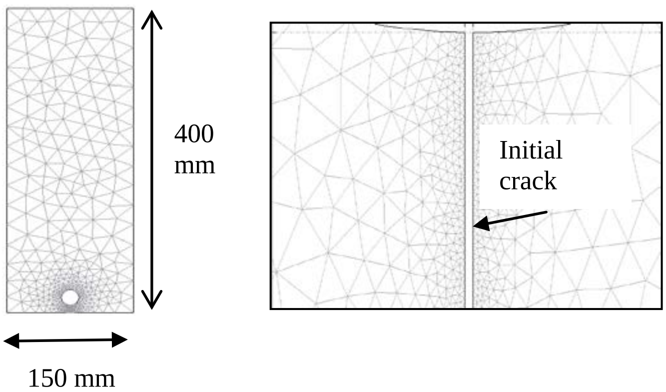

Figure 1. The FEM net. The total net to the left and the local net near the crack to the right.

In the FEM modelling the rectangular cross-section is assumed to have a hole at the site of the reinforcement and a crack from the hole to the boundary as shown in figure 1 to he left. The total triangular FEM net and the very fine local FEM net near the crack are shown in figure 1 to the right. The number of constant strain elements is 5580 and there are 3066 nodes. The FEM analysis is made using FEMLAB/MATLAB. The material is assumed to be linear elastic with the elasticity module E = 25×10^9 Pa and the pressure from the increasing corrosion products is modelled as a uniform loading (pressure) p = 1×10^8 N/m at the boundary of the hole.

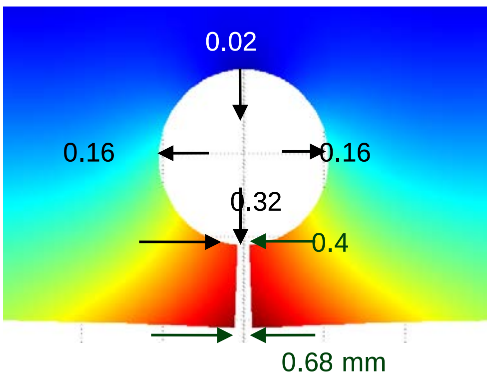

Figure 2. Displacement at four points of the circle and at the ends of the crack.

The result of the analysis is shown in figure 2. To illustrate the procedure an (unrealistically) high pressure is used and the displacements are shown with an enlargement factor equal to 3. The increase in the crack width is $\Delta w_{\mathrm{crack}}$ = 0.67 mm and the average increase in the hole diameter is ΔD_{hole} = 0.31 mm.

Therefore,

$$\eta = \Delta w_{\mathrm{crack}} / \Delta D_{\mathrm{hole}} = 0.67/0.31 = 2.2.$$

From $\Delta w_{crack} = \gamma\, \Delta D_{bar}$ and $\Delta w_{crack} = \eta\, \Delta D_{hole}$ it follows that

$$\Delta D_{bar} = \frac{\eta}{\gamma} \Delta D_{hole} \approx \Delta D_{hole} \quad \text{for small } \Delta w_{crack} \tag{8}$$

Then it follows from (8) that

$$\gamma \approx \eta \quad \text{for small } \Delta w_{crack} \tag{9}$$

so that the finite element estimation of η can be used to estimate the reduction of ΔD_{bar}.

6.2 γ as a function of the rebar diameter and the cover.

The coefficient $\gamma(D, a)$ as a function of the bar diameter D and the concrete cover a has recently been investigated using FEM analyses assuming linear elasticity. The analysis is performed by a procedure similar to the one used in section 6.1. However, the elasticity module is E = 40 GPa and Poisson's ratio is 1/6. The pressure at the boundary of the hole is 1 GPa. The considered cross-section of the reinforced concrete beam is shown in figure 3 and the 30 combinations of the diameter D and the cover a used in the FEM analysis are defined in table 1.

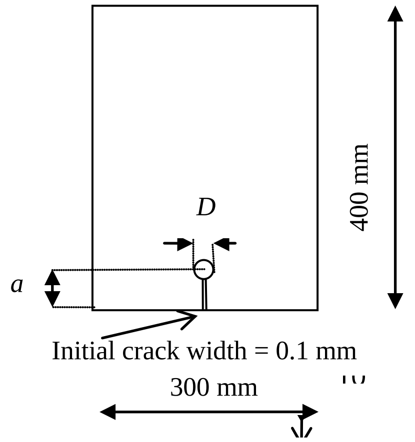

Figure 3. Cross-sectional dimensions.

As an illustration, the FEM mesh is shown in figure 4 for combination 15.

Diameter D, mm	cover a 20	25	30	35	40
10	1	7	13	19	25
~~12~~	~~2~~	~~8~~	~~14~~	~~20~~	~~26~~
14	3	9	15	21	27
16	4	10	16	22	28
18	5	11	17	23	29
20	6	12	18	24	30

Table 1. The 30 combinations of the diameter D and the cover a.

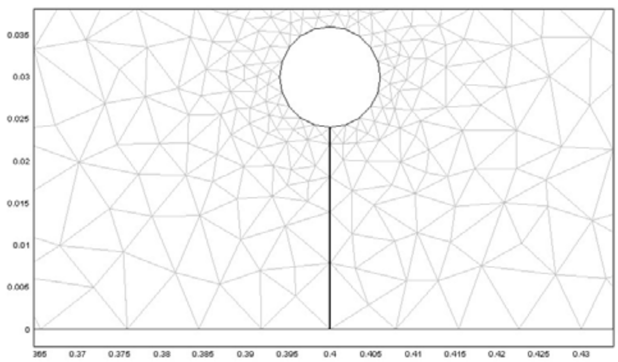

Figure 4. The FEM mesh for combination 15.

For each of the 30 combinations defined in table 1, the displacements of the 8 points shown in figure 5 are calculated by the FEM analysis.
As an example, the displacements for combination 15 are illustrated in figure 6 and the numerical values (mm) of the 8 displacements are shown in table 2. The average change in diameter is

$$\Delta D_{hole} = (-\mathrm{d}1 + \mathrm{d}2 + \mathrm{d}3 - \mathrm{d}4)/4 = 0.375\ \mathrm{mm} \tag{10}$$

and the change in crack width is

$$\Delta w_{crack} = -c3 + c4 = 1.437 \text{ mm} \quad (11)$$

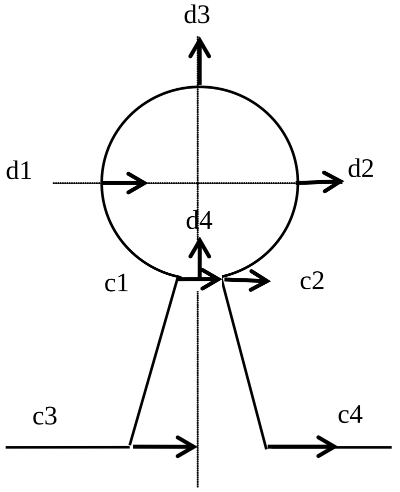

Figure 5. Displacements at the 8 measuring points.

Point	Displacement in mm
d1	-0.468
d2	0.461
d3	-0.111
d4	-0.682
c1	-0.552
c2	0.545
c3	-0.722
c4	0.715

Table 2. Displacements, combination 15.

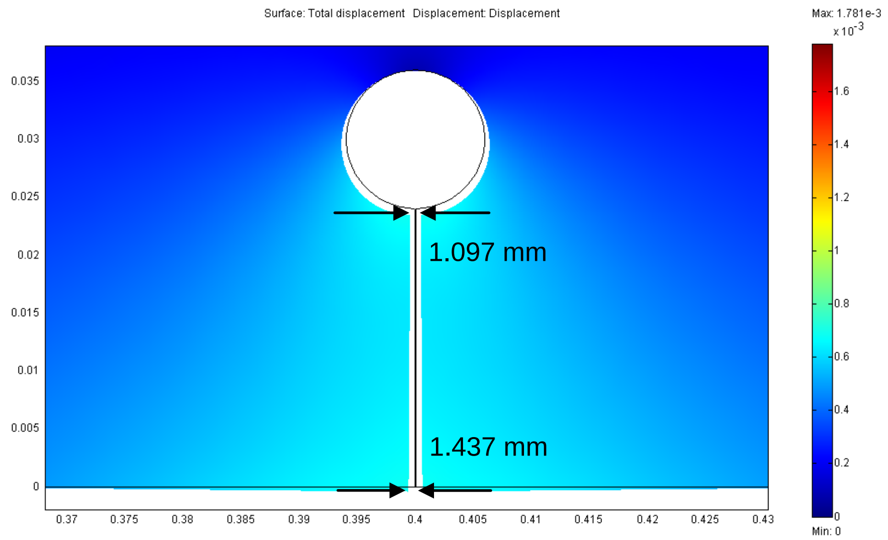

Figure 6. Crack dimensions for combination 15.

Diameter D, mm	Cover a,mm				
	20	25	30	35	40
10	3.72	3.53	3.39	3.30	3.25
12	3.93	3.76	3.63	3.55	3.50
14	4.09	3.94	3.83	3.75	3.71
16	4.20	4.08	3.99	3.92	3.88
18	4.26	4.19	4.11	4.06	4.03
20	4.28	4.27	4.21	4.18	4.15

Table 3. Estimates of $\gamma(D,a)$.

An estimate of the ratio γ for this parameter combination 15 is then

$$\gamma(14,30) \approx \eta = \Delta w_{crack} / \Delta D_{hole} = 3.83 \quad (12)$$

Estimates of the ratio $\gamma(D,a)$ for all 30 combinations of the diameter D and the concrete cover a are shown in table 3 and illustrated in figure 7.

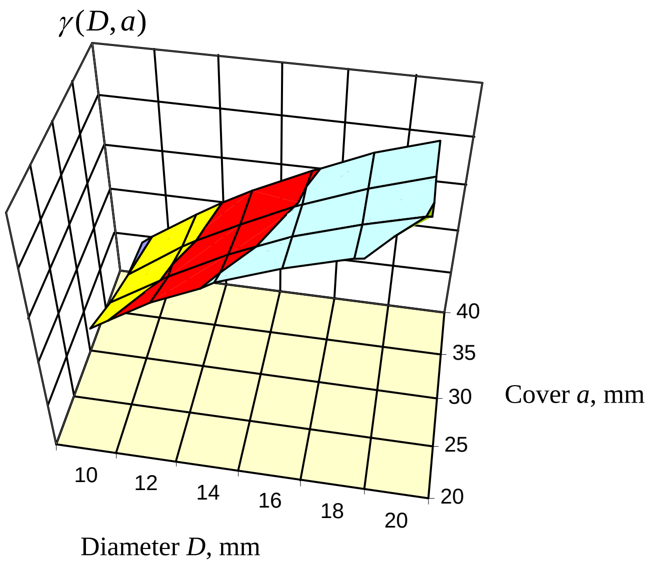

Figure 7. The ratio $\gamma(D,a)$.

It follows from table 2 and figure 7 that the ratio $\gamma(D,a)$ for this example and for the used parameter combinations increases with the diameter D and decreases with the concrete cover a. Based on results like these it may eventually for a given realistic beam be possible on basis of a certain observed corrosion crack to estimate the ratio γ and therefore also the corrosion evolution.

6.3 The ratio γ for different distances from the crack to the vertical side of the beam

In this section it is investigated whether the distance b from the crack to the nearest uncracked vertical side of the bridge is important for the estimate of the ratio γ. For this purpose the same cross-section as in section 6.2 is used, but the hole and crack are placed at different distances from the vertical side of the beam. The parameter combination 15 with rebar diameter D = 14 mm and concrete cover a = 30 mm is chosen for illustration, see figure 8.

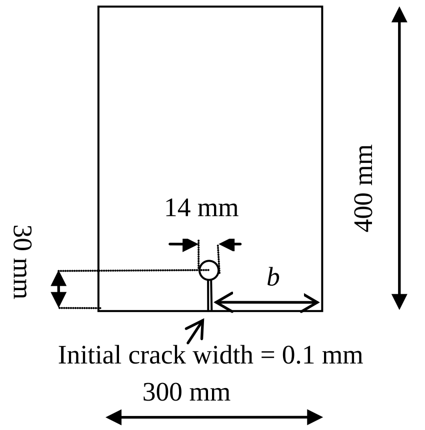

Figure 8. Cross-section dimensions.

FEM analysis is made for the distances b = 30, 60, 90, 120, and 150 mm. The calculated $\gamma \approx \eta$ values are shown in table 4 and figure 9. For b = 30, 90 and 150 mm the displacements are illustrated in figures 10, 11, and 12, respectively.

b, mm	η, mm
30	5.98
60	4.44
90	4.01
120	3.85
150	3.83

Table 4. The ration γ for different distances b to the vertical side of the beam.

It follows from table 4 and figures 9 – 12 that the distance b to the vertical side of the beam is only significant if b is smaller than twice the rebar diameter. Otherwise, $\gamma \approx \eta$ with a good approximation seems to be independent of b.

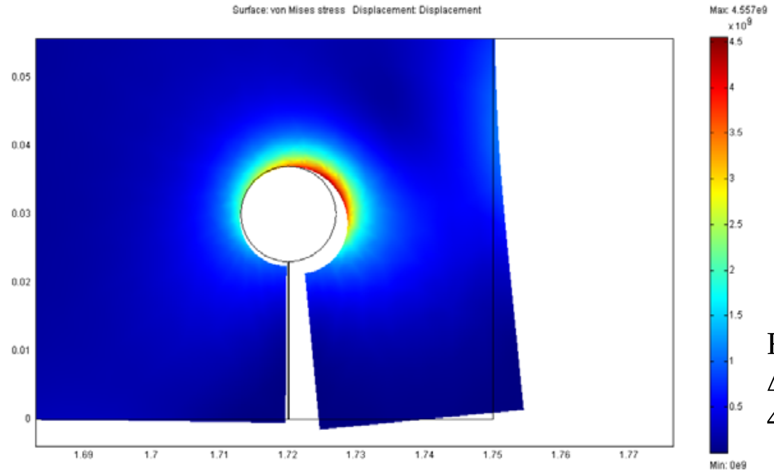

Figure 10. b = 30 mm.
ΔD_{hole}=0.83 mm, Δ w_{crack} = 4.93 mm, $\gamma \approx \eta$ = 5.98.

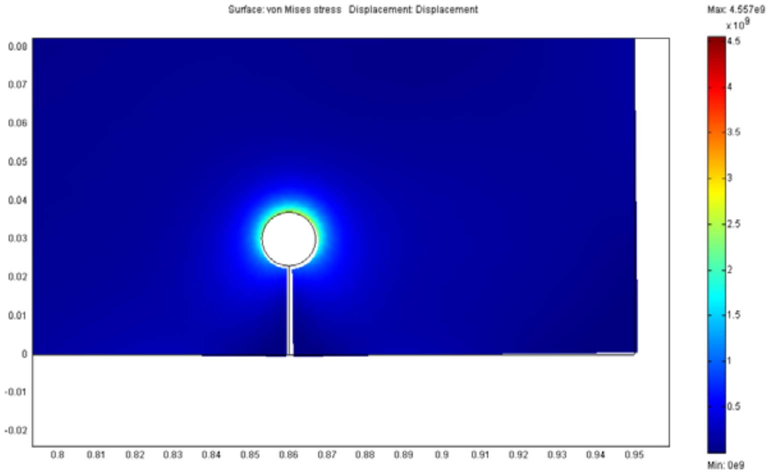

Figure 11. b = 90 mm.
ΔD_{hole}= 0.41 mm, Δ w_{crack} = 1.63 mm

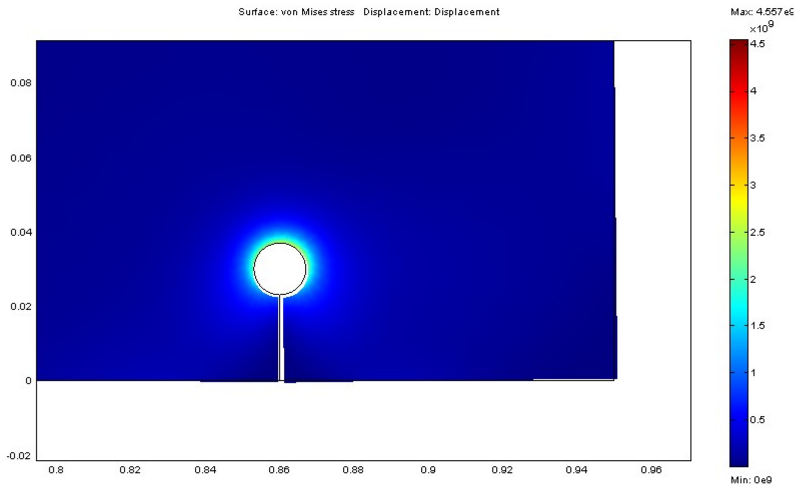

Figure 12. b = 150 mm.
ΔD_{hole} = 0.37, Δ w_{crack} = 1.44 mm
$\gamma \approx \eta$ = 3.83.

7. SPALLING

In this section it is indicated how the FEM methodology used in this paper may be used to identify where the next corrosion crack may occur and perhaps even to identify the first spalling.

As an example consider again the parameter combination 15 with b = 30 mm, see figure 13. The point of maximum first principal stress s_{max} at the surface of the hole is indicated. Its value is 4.1429 MPa. The second corrosion crack may start from that point.

In table 4 is shown the relative values of the maximum first principal stresses for the considered b values.

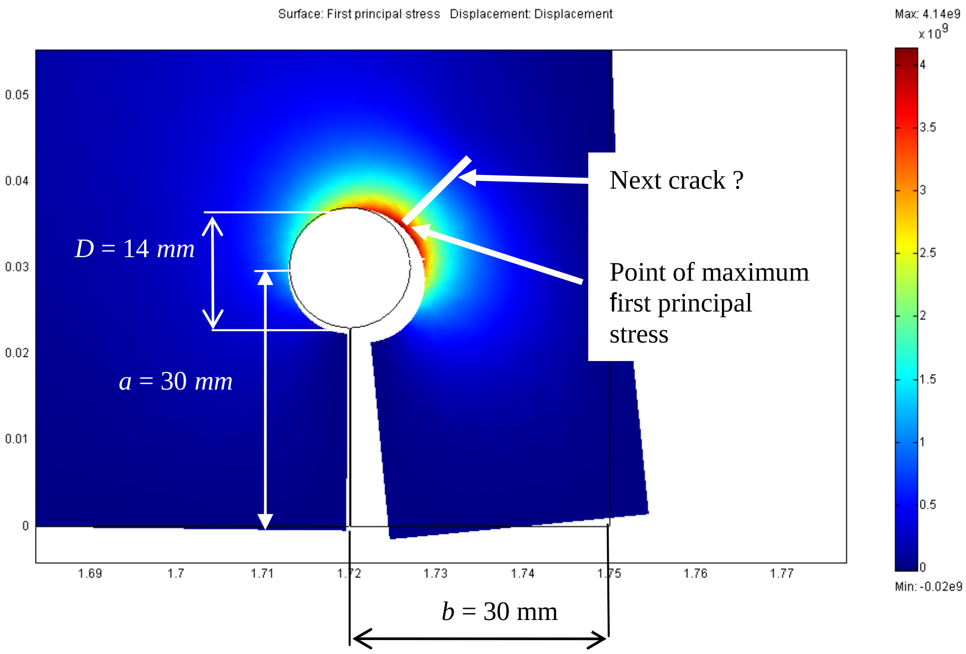

Figure 13. First principal stresses for combination 15 with b = 30.

b, mm	30	60	90	120	150
$s_{max}(B)/ s_{max}(30)$	1.00	0.60	0.51	0.48	0.47

Table 4. Relative maximum first principal stresses.

8. STOCHASTIC MODELLING

The modelling presented in the paper is deterministic, but it may easily be changed to a stochastic modelling simply by modelling the relevant parameters as stochastic variables or processes.

Such a simple procedure has been used by several authors; see e.g. Thoft-Christensen [1], [5]. On the basis of equation (1) outcomes of the corrosion initiation time T_{corr} have been performed on the basis of the following data by simple Monte Carlo simulation: Initial chloride concentration: 0%, Surface chloride concentration: Normal (0.650 ; 0.038), Diffusion coefficient: Normal (30 ; 5), Critical concentration: Normal (0.3 ; 0.05), Cover: Normal (40 ; 8).

A Weibull distribution can be used to approximate the distribution of the simulated data for the corrosion initiation time T_{corr}. The Weibull distribution is

$W(x; \mu, k, \varepsilon)$, where $\mu = 63.67$ years, $k = 1.81$ and $\varepsilon = 4.79$. The corresponding histogram and the density function are shown in figure 14.

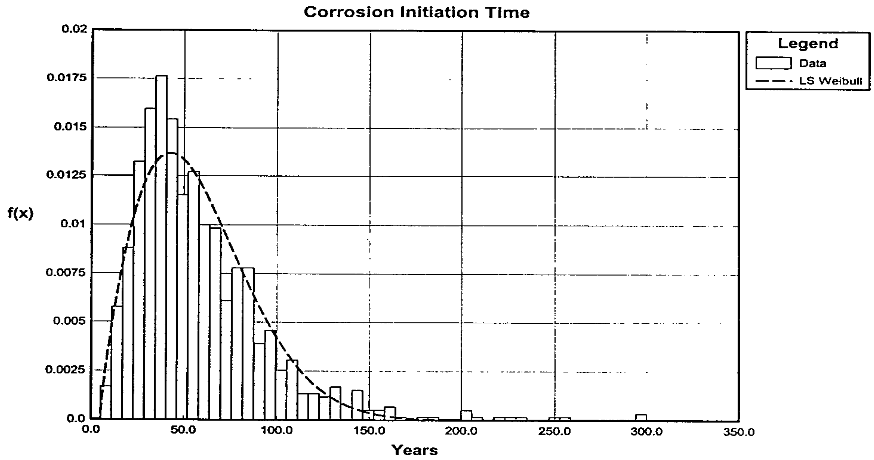

Figure 14. Density function of the corrosion initiation time T_{corr} ;
Thoft-Christensen (2001).

Likewise, for the same example it can be shown by Monte Carlo simulation that Δt_{crack} with a good approximation can be modelled by a Weibull distribution W(3.350; 1.944 ; 0) years, see figure 15. The mean is 2.95 years and the standard deviation 1.58 years.

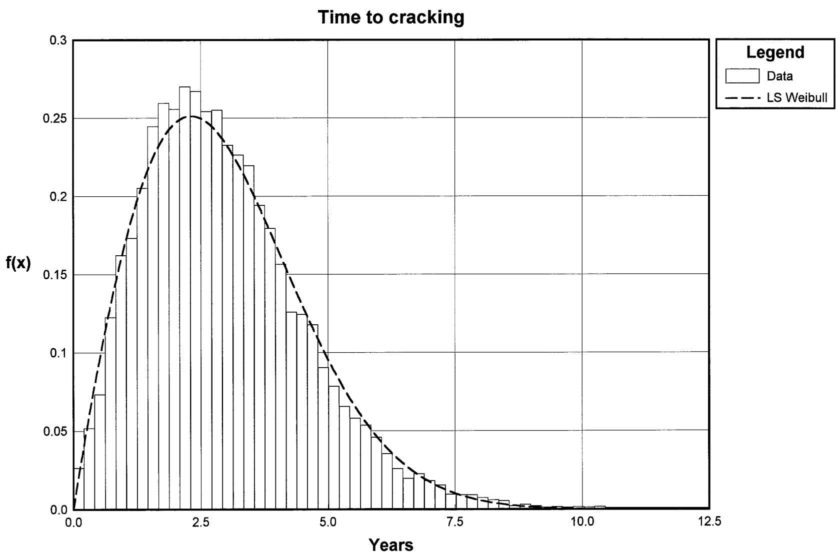

Figure 15. Stochastic modelling of Δt_{crack} ;
Thoft-Christensen (2001).

The last and perhaps the most difficult step is then to model time $\Delta t_{crack\ width}$ from corrosion crack initiation to formation of a certain (critical) crack width. This problem is discussed in this paper where the relation between the increase of the crack with and the corresponding decrease in the rebar diameter is modelled in a deterministic way. A stochastic modelling is also needed for this step since the rate of rust production (decrease in rebar diameter) as well as the relation between the increase of the crack with and the corresponding decrease in the rebar diameter is uncertain quantities. If the last-mentioned relation is linear then a simple Monte Carlo simulation techniques may also be used for this step. However, more research is needed to justify such a procedure, see e.g. Thoft-Christensen [6].

9. CONCLUSIONS

In this paper a first attempt to make a framework for a new methodology to evaluate the safety (reliability) of a reinforced concrete structure is shown. The methodology is based on corrosion crack widths.

The ratio $\gamma \approx \eta = \Delta w_{crack} / \Delta D_{hole}$ is studied for several cross-sections as a function of the rebar diameter *D*, the concrete cover *a*, and the distance between the crack and the nearest side of the beam.

A tentative conclusion is that $\gamma \approx \eta$ increases with the diameter, decreases with the concrete cover, and is almost independent of the distance to the beam side, unless the rebar is within twice the diameter from the beam side. More research is needed to confirm this conclusion.

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