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CHAPTER 131

RECENT PROGRESS IN MODELLING DETERIORATION OF REINFORCED CONCRETE STRUCTURES ¹

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ABSTRACT

The main purpose of this paper is to give an overview of some recent progress made in modelling of deterioration of reinforced concrete structures. The review is not claimed to be complete. It is based on some of the author's contributions in this research area especially the papers Thoft-Christensen [1] and Thoft-Christensen, Svensson & Frandsen [2].

In the paper, corrosion of reinforced concrete structures is discussed from the point of view of corrosion products. The different types of corrosion products are presented and the important diffusion coefficient is discussed. Modelling of corrosion initiation, corrosion propagation, corrosion cracking and corrosion crack opening is presented.

1. INTRODUCTION

The classical modelling of deterioration of reinforced concrete structures is based on observations of the deterioration of existing structures and on comprehensive experiments in laboratories all over world. By such observations the so-called deterioration profile (deterioration, reliability, or capacity as a function of time) may be estimated. A large number of such observation profiles have been estimated by curve fitting. The advantage of this approach is that the deterioration profile is described by simple curves which are easy to use in analysis and design of reinforced concrete structures; see e.g. Ellingwood [3]. The disadvantage is that it is not easy to estimate

¹ In Proceedings of the Luis Esteva Symposium on "Earthquake Engineering, Challenges and Tendencies", Mexico City, Mexico, September 7-11, 2005, pp. 293-305.

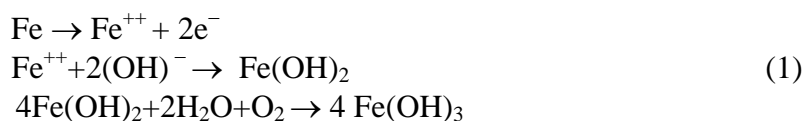
the deterioration profiles for new structures and new materials, since the deterioration curves are not directly related to physical, mechanical, or chemical parameters.

In a recent more modern approach on the estimation of the deterioration is based on a detailed understanding of the chemical and physical processes that take place during deterioration. This approach is used in this paper. In the paper, only deterioration related to corrosion of the reinforcement is considered, since corrosion of the reinforcement is one of the major reasons for deterioration of reinforced concrete structures. Initially, the main effect of corrosion is reduced strength of the structural element in question due to a reduced cross-section of the reinforcement, and therefore, also a reduction of the structural reliability. However, there is a close interaction between corrosion and the bond between the concrete and the reinforcement. A reduced bond will also influence the structural reliability of the structure and must therefore be included in a complete estimation of the deterioration process. Important research on bonding is taking place in several research institutions; see e.g. Lundgren [4] and Maurel, Dekoster and Buyle-Bodin [5].

The corrosion is a serious problem due to the reduction of the reinforcement and since the volume of the rust products is higher than the volume of the corroded steel. The porous zone around the steel/concrete surface can to some extent absorb the higher volume of the rust products. However, at a certain time the total amount of corrosion products exceeds the amount of corrosion products needed to fill the porous zone around the reinforcement. The rust products will then create expansive pressure on the surrounding concrete. The expansion of the concrete near the reinforcement will initiate tensile stresses in the concrete. After some time with increasing corrosion the tensile stresses will reach a critical value and corrosion cracks may develop. With further production of rust, the crack opening will increase and eventually result in spalling. This last part of the corrosion process is still not well understood. More research is certainly needed to clarify these important problems for a corroded reinforced concrete structure.

2. CHLORIDE-INDUCED CORROSION

Chloride-induced corrosion has been investigated by e.g. Neville [6]. Reinforced concrete is an excellent type of structure from a corrosion point of view, since the alkaline environment in the concrete maintains a passive film on the surface of the reinforcement, and this film protects the reinforcement against corrosion. However, if the concrete is penetrated by e.g. water or carbon dioxide, then this passive film breaks down and the reinforcement is open to corrosion. An anodic region is established, where the passive film is broken down, whereby an electrochemical cell is formed. The passive surface is the cathode, and the electrolyte is the pore water in the concrete. At the anode the following reactions take place:



Chloride ions Cl^{-} activate the unprotected surface and form an anode. The chemical reactions are



It follows from Eqs. (1) and (2) that two rust products $\text{Fe}(\text{OH})_2$ and $\text{Fe}(\text{OH})_3$ are produced in this case.

Corrosion product	Colour	Volume, cm ³
Fe ₃ O ₄	Black	2.1
Fe(OH) ₂	White	3.8
Fe(OH) ₃	Brown	4.2
Fe(OH) ₃ ·3H ₂ O	Yellow	6.4

Table 1. Volume of corrosion products corresponding to corrosion of 1 cm³ Fe.

The different types of rust products are interesting to study because they have great influence on corrosion cracking, since the volume of the rust products corresponding to a given volume of the steel varies a lot. This problem has been studied for several corrosion products by Nielsen [7]. He has obtained the rust volumes corresponding to corrosion of 1 cm³ Fe, see Table 1.

3. THE CHLORIDE DIFFUSION PROCESS

The penetration of chloride ions into the concrete is difficult to model. Some simplifying assumptions are needed. There seems to be a general agreement that a model based on diffusion theory is a reasonably good approximation. If the chloride concentration C_0 on the surface of the concrete and the diffusion coefficient D for concrete are independent of space (location) and time, then Fick's law of diffusion can represent the rate of chloride penetration into concrete, as a function of depth from the concrete surface and as a function of time

$$\frac{dC(x,t)}{dt} = D \frac{d^2C(x,t)}{dx^2} \quad (3)$$

where $C(x,t)$ is the chloride ion concentration, as % by weight of cement, at a distance of x cm from the concrete surface after t seconds of exposure to the chloride source. D is the chloride diffusion coefficient expressed in cm²/sec. The solution of the differential Eq. 3 is

$$C(x,t) = C_0 \left\{ 1 - \operatorname{erf} \left(\frac{x}{2\sqrt{D \cdot t}} \right) \right\} \quad (4)$$

where C_0 is the equilibrium chloride concentration on the concrete surface, as % by weight of cement, erf is the error function.

The chloride ingress with a time-dependent chloride concentration at the concrete surface has been investigated by Frederiksen, Mejlbro and Poulsen [8] and by Mejlbro [9] on the basis of a solution of the diffusion law with time-dependent diffusion coefficient and time-dependent surface concentration. Mejlbro and Poulsen [10] have considered the special case where the time dependent chloride concentration on the surface is approximated by a piecewise set of linear functions versus time. Such an approach is relevant for e.g. bridges where de-icing salt containing chloride is used during the winter period.

It is further assumed that a corrosion process is initiated when the chloride concentration at the site of the reinforcement reaches a certain critical chloride corrosion threshold value C_{cr} . The critical chloride threshold depends on the type of

concrete and several other factors; see e.g. Frederiksen [11]. If C_{cr} is assumed to be the chloride corrosion threshold and d is the thickness of the concrete cover, then the corrosion initiation period T_{corr} can be calculated. The time T_{corr} to initiation of reinforcement corrosion is

$$T_{corr} = \frac{d^2}{4D} \left(\operatorname{erf}^{-1} \left(\frac{C_{cr} - C_0}{C_i - C_0} \right) \right)^{-2} \quad (5)$$

On the basis of Eq. 5 outcomes of the corrosion initiation time T_{corr} have been performed on the basis of the following data by simple Monte Carlo simulation; Thoft-Christensen [12]

Initial chloride concentration:	0 %,
Surface chloride concentration:	Normal (0.650 ; 0.038),
Diffusion coefficient:	Normal (30 ; 5),
Critical concentration:	Normal (0.3 ; 0.05),
Cover:	Normal (40 ; 8).

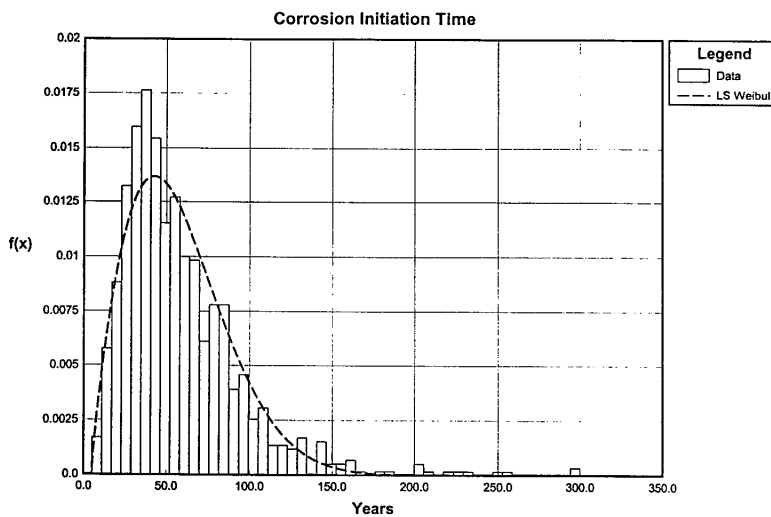


Figure 1. Density function of the corrosion initiation time T_{corr} ; Thoft-Christensen [12].

A Weibull distribution can be used to approximate the distribution of the simulated data. The Weibull distribution is $W(x; \mu, k, \varepsilon)$, where $\mu = 63.67$, $k = 1.81$ and $\varepsilon = 4.79$. The corresponding histogram and the density function are shown in Fig. 1.

4. THE DIFFUSION COEFFICIENT D

It follows from Eq. 5 that the time to corrosion initiation T_{corr} is inversely proportional to the diffusion coefficient D . It is therefore of great interest to get a good estimate of D . The diffusion coefficient D is not a real physical constant for a given concrete structure, since it depends on a number of physical factors. The most important factors are the water/cement ratio w/c , the temperature Φ , and the amounts of e.g. silica fume (s.f.); see Jensen [13] and Jensen, Hansen, Coats & Glasser [14]. The data presented in this section are all based on Jensen [13] and Jensen, Hansen, Coats & Glasser [14].

Fig. 2 shows the diffusion coefficient D as a function of the water-cement ratio w/c and the temperature Φ °C for cement pastes with 0 % silica fume. It is clear from Fig. 2 that the diffusion coefficient D increases significantly with w/c as well as the temperature Φ . It is clearly of great importance to get good estimates of w/c and Φ . The w/c value to be used is the original w/c value when the concrete was produced. If the original value of w/c is not available, then it can be estimated by testing of the concrete. The temperature Φ is more complicated to estimate, since the temperature usually varies a lot. As a first estimate, it is suggested to use an equivalent value based on

information of the variation of the temperature during the year at the site of the structure. The addition of silica fume is of great importance for the chloride ingress. Silica fume additions reduce the chloride ingress because of changes in the pore structure; Jensen [13].

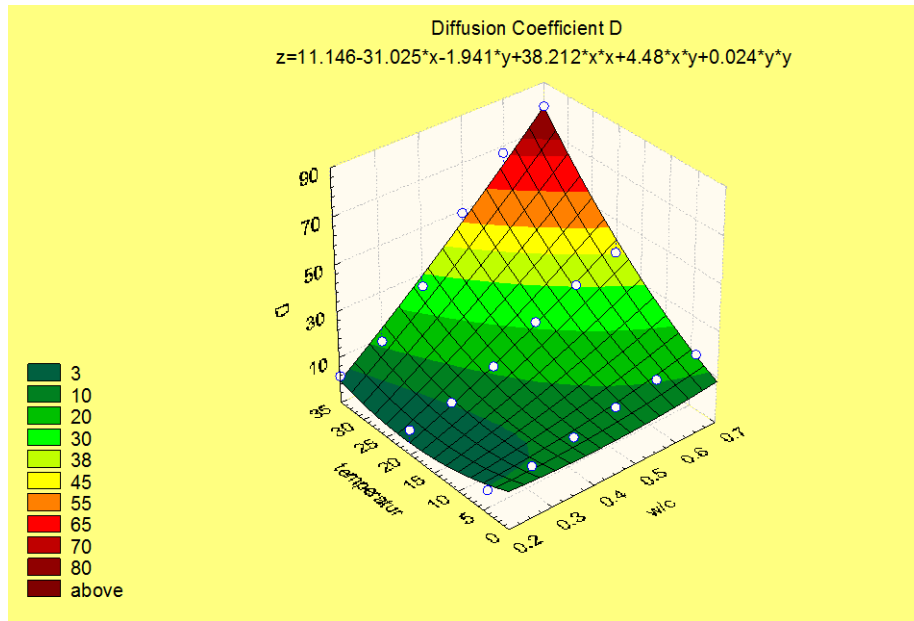


Figure 2. The diffusion coefficient D ($10^{-12} \text{ m}^2/\text{s}$) as a function of w/c and of the temperature Φ ; Thoft-Christensen [15].

5. INITIATION OF CORROSION CRACKS

The corrosion process after corrosion initiation is very difficult to model, since a large number of changes in the rebars as functions of the time have been observed and reported in the literature. The simplest model is to assume that the diameter $d(t)$ of the reinforcement bars at the time $t > T_{corr}$ is modelled by

$$d(t) = d_0 - c_{corr} i_{corr} (t - T_{corr}) \quad (6)$$

where d_0 is the initial diameter, c_{corr} is a corrosion coefficient, and i_{corr} is the rate of corrosion. Based on a survey, three models for chloride penetration have been proposed (the initial chloride concentration in the concrete is assumed to be zero): low, medium, and high deterioration; Thoft-Christensen & Jensen [16]. There is a porous zone around the steel/concrete surface caused by the transition from paste to steel, entrapped/entrained air voids, and corrosion products diffusing into the capillary voids in the cement paste. When the total amount of corrosion products exceeds the amount of corrosion products needed to fill the porous zone around the steel, the corrosion products create expansive pressure on the surrounding concrete. Close to reinforcement bars the concrete has some porosity. Very close to the bars the porosity is close to 1, but the porosity decreases with the distances from the bars. The porosity is typically of the order of 0.5 about 10-20 μm from the bars. Let t_{por} be the thickness of an equivalent zone with porosity 1 around a steel bar, and let t_{por} be modelled by a lognormal distribution with the mean 12.5 μm and a standard deviation of 2.54 μm . Further, let the density ρ_{rust} of the rust product and the initial diameter d_0 be modelled

by normal distributions $N(3600,360)$ kg/m^3 and $N(16,1.6)$ mm, respectively. Then it can be shown by Monte Carlo simulation that the volume of the porous zone W_{porous} with a fairly good approximation can be modelled by a shifted lognormal distribution with a mean $2.14e-03$ kg/m , a standard deviation $0.60e-03$ kg/m and a shift of $0.82e-03$ kg/m ; see Thoft-Christensen [12].

After a certain time the rust products will fill the porous zone completely and then result in an expansion of the concrete near the reinforcement. As a result of this, tensile stresses are initiated in the concrete. With increasing corrosion the tensile stresses will reach a critical value and cracks will be developed. During this process the corrosion products at initial cracking of the concrete will occupy three volumes, namely the porous zone W_{porous} , the expansion of the concrete due to rust pressure W_{expan} , and the space of the corroded steel W_{steel} . The corresponding total amount of critical rust products W_{crit} needed to fill these volumes is

$$W_{crit} = W_{porous} + W_{expan} + W_{steel} \quad (7)$$

Using Monte Carlo simulation it can be shown that W_{expan} with a good approximation can be modelled by a normal distribution $N(0.0047, 0.0011)$ kg/m when the data shown above are used; see Thoft-Christensen [12]. Finally, W_{steel} can be written

$$W_{steel} = \frac{r_{rust}}{r_{steel}} M_{steel} \quad (8)$$

where ρ_{steel} is the density of steel and M_{steel} is the mass of the corroded steel. Clearly, M_{steel} is proportional to W_{crit} . Liu & Weyers [17] have calculated the factor of proportionality for two kinds of corrosion products as 0.523 and 0.622. For simplicity, it will be assumed that $M_{steel} = 0.57W_{crit}$. Therefore, Eq. 7 can be rewritten

$$W_{crit} = \frac{\rho_{steel}}{\rho_{steel} - 0.57\rho_{rust}} (W_{porous} + W_{expan}) \quad (9)$$

Let ρ_{steel} be modelled by a normal distribution $N(8000; 800)$ kg/m^3 . Then by Monte Carlo simulation it can be shown that W_{crit} with a good approximation can be modelled by a normal distribution $N(0.010; 0.0027)$ kg/m ; see Thoft-Christensen [12]. The rate of rust production as a function of time t (years) from corrosion initiation Liu & Weyers [17] can be written

$$\frac{dW_{rust}(t)}{dt} = k_{rust}(t) \frac{1}{W_{rust}(t)} \quad (10)$$

i.e. the rate of corrosion is inversely proportional to the amount of rust products W_{rust} (kg/m). The factor $k_{rust}(t)$ ($\text{kg}^2/\text{m}^2/\text{year}$) is assumed to be proportional to the annual mean corrosion rate $i_{cor}(t)$ ($\mu\text{A}/\text{cm}^2$) and the diameter d (m) of the reinforcement. The proportionality factor depends on the types of rust products, but is here taken as $0.383e-3$.

$$k_{rust}(t) = 0,383 \times 10^{-3} di_{corr}(t) \quad (11)$$

By integration

$$W_{rust}^2(t) = 2 \int_0^t k_{rust}(t) dt \quad (12)$$

Let $i_{cor}(t)$ be modelled by a time-independent normally distributed stochastic variable $N(3 ; 0.3)$ ($\mu A/cm^2$) then the time from corrosion initiation to cracking Δt_{crack} can be estimated by (16) by setting $W_{rust}(\Delta t_{crack}) = W_{crit}$.

$$\Delta t_{crack} = \frac{W_{crit}^2}{2k_{rust}} = \frac{W_{crit}^2}{2 \times 0.383 \times 10^{-3} di_{corr}} \quad (13)$$

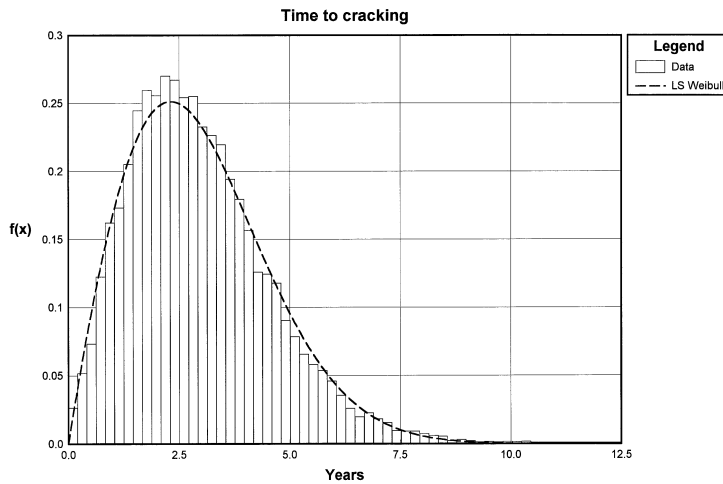


Figure 3. Density function of Δt_{crack} ; Thoft-Christensen [12].

Then it can be shown by Monte Carlo simulation that Δt_{crack} with a good approximation can be modelled by a Weibull distribution $W(3.350 ; 1.944 ; 0)$ years, see Fig. 3. The mean is 2.95 years and the standard deviation is 1.58 years. The mean value of T_{crack} is of the same order as the experimental values (and the deterministic values) obtained by Liu & Weyers [17].

6. THE CRACK - CORROSION INDEX

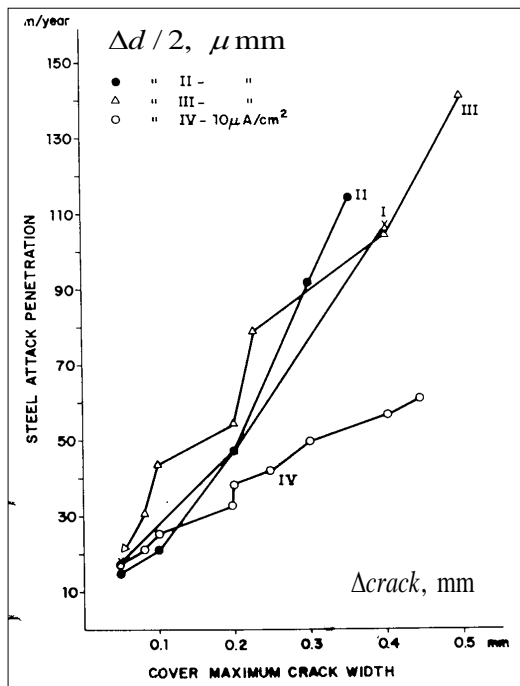


Figure 4. Loss in rebar diameter Δd versus the increase in crack opening $\Delta crack$; Andrade, Alonso & Molina [18].

After formation of the initial crack the rebar cross-section is further reduced due to the continued corrosion, and the opening of the crack is increased. Experiments by Andrade, Alonso & Molina [18] show that the function between the reduction of the rebar diameter Δd and the corresponding increase in crack opening $\Delta crack$ measured on the surface of the concrete specimen can be approximated by a linear function $\Delta crack = \gamma \Delta d$, where the crack-corrosion factor γ is of the order 1.5 to 5, see Fig. 4.

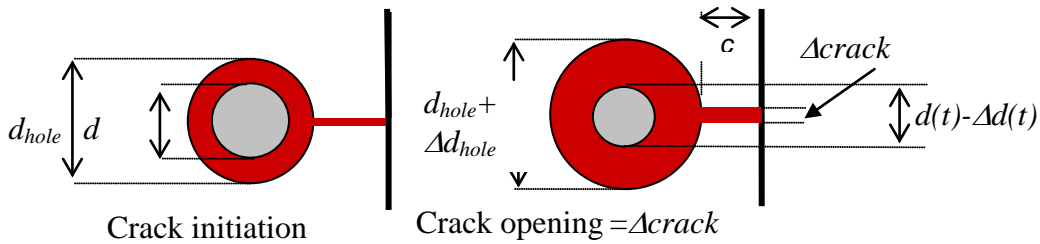


Figure 5. Crack opening increase from crack initiation (opening ~ 0 mm) to $\Delta crack$.

An approximate relation between the decrease in the steel bar diameter Δd_r and the increase in the hole Δd_{hole} can easily be obtained by considering the volume of the produced rust products $\pi \Delta d d \alpha$ ($\alpha = \rho_{steel} / \rho_{rust}$ is the relation between the densities of the steel and the rust product; see Table 1) and the volume due to the expansion of the hole $\pi \Delta d d + \pi \Delta d_{hole} d_{hole} + c \Delta crack$; see Fig. 5. By equalizing these two volumes and assuming that $d \approx d_{hole}$ one gets

$$(\alpha - 1) \Delta d d \approx \pi \Delta d_{hole} d_{hole} + \text{crack volume} = \pi \Delta d_{hole} d_{hole} + c \Delta crack \quad (14)$$

where c is the cover. Therefore,

$$\Delta d \approx (\Delta d_{hole} + c \Delta crack / \pi d) / (\alpha - 1) \quad (15)$$

7. FEM-Modelling of Corrosion Cracking in a Beam Cross Section

The first FEM estimation of the crack-corrosion index $\gamma = \Delta w_{crack} / \Delta D_{bar}$ was presented at the IFIP TC7 Conference on “System Modelling and Optimization” in Sophia Antipolis, France, July 2003; see Thoft-Christensen [19], using FEMLAB/MATLAB. A rectangular beam cross-section with only one reinforcement bar was considered, see figure 6. The diameter of the hole around the rebar at the time of crack initiation is $d_{hole} = 20$ mm and that the cover is $c = 10$ mm.

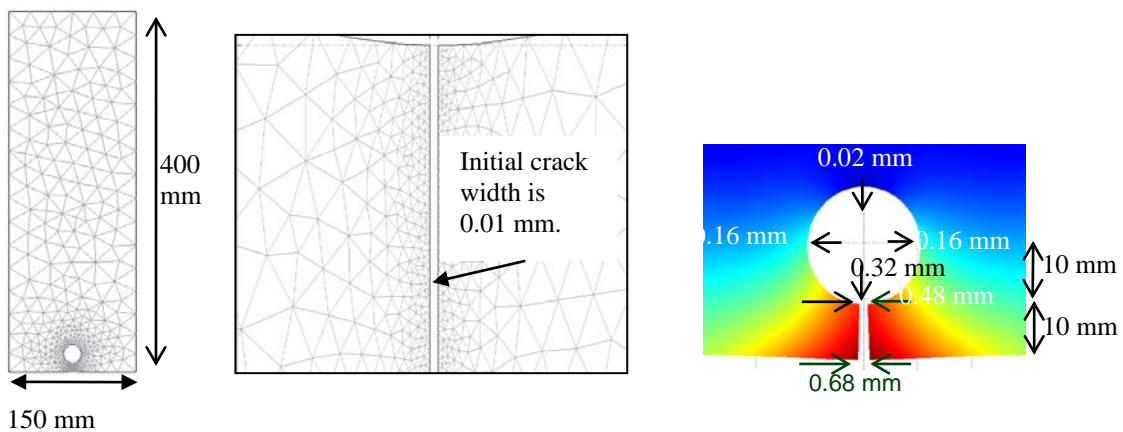


Figure 6. FEM net. The total net to the left, and the local net near the crack in the middle, and the displacement at the right, Thoft-Christensen [19].

In the FEM modelling the rectangular cross-section in a long beam (plain strain) is assumed to have a hole at the location of the reinforcement and a crack (0.01 mm) from the hole to the boundary. The material is assumed to be linear elastic with the

modulus of elasticity $E = 25 \times 10^9$ Pa. It is assumed that the pressure on the boundary of the hole from the increasing corrosion products can be modelled as a uniform loading (pressure) $p = 1 \times 10^8$ N/m at the boundary of the hole. The increase in the crack opening is $\Delta w_{crack} = 0.67$ mm and the average increase in the hole diameter is $\Delta D_{hole} = 0.31$ mm.

For the example shown in Fig. 6 one gets by Eq. 15

$$\Delta d \approx (0.31 + 0.67 \times 10 / (20 \times \pi)) / (\alpha - 1) = 0.42 \times (\alpha - 1)^{-1} \quad (16)$$

and

$$\gamma \approx \Delta crack / \Delta d = 1.60 \times (\alpha - 1) \quad (17)$$

or γ equal to 1.8 and 5.1 for black and brown rust, respectively, in good agreement with the values obtained by the experiments in Fig. 5.

The results of a similar FEM analysis of a cross-section like the one illustrated in figure 6 are presented for 10 different combinations of the cover c and the diameter d in Thoft-Christensen [20]. The conclusion is that γ increases with the cover c for a fixed rebar diameter d , and that γ also increases with the diameter d for a fixed cover c . A similar analysis with 30 combinations of d and c , but with a different cross-section partly confirms this conclusion; see Thoft-Christensen [21]. In the same paper it is also investigated whether the distance b from the crack to the nearest uncracked vertical side of the beam is important for the estimate of crack-corrosion index γ . The same cross-section is used, but the hole and the crack are placed at different distances from the vertical side of the beam. The conclusion is that the distance b to the vertical side of the beam is only significant if b is smaller than twice the rebar diameter. Otherwise, with a fairly good approximation, γ seems to be independent of b .

8. FEM MODELLING OF CORROSION CRACKING IN A BEAM

In this section the procedure used in 2D-modelling of corrosion crack propagation is extended to a 3D-modelling of a reinforced concrete beam; see Thoft-Christensen, Svensson & Frandsen [2]). The 2D-modelling in section 2 is based on several assumptions and simplifications, e.g. that the movement of corrosion products is restricted to the considered cross-section. In a 3D-modelling the corrosion products may also move in the direction of the reinforcement. Further, it is possible in a 3-D

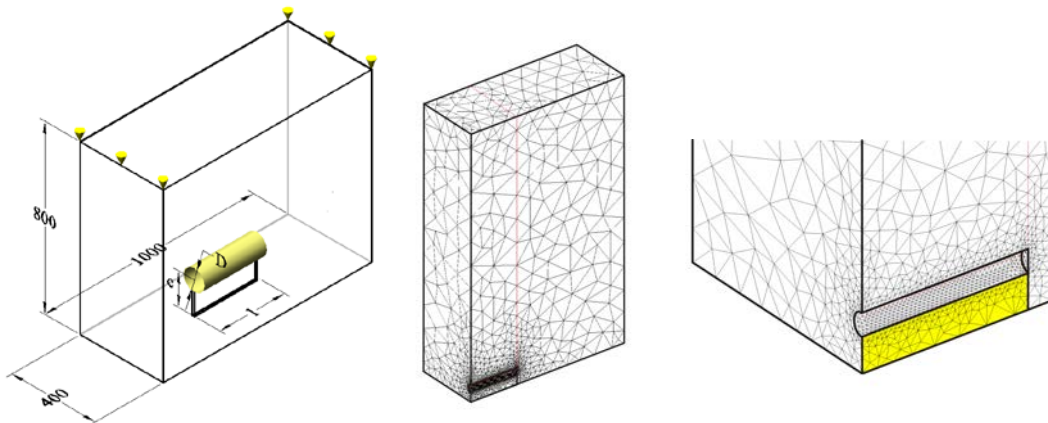


Figure 7. Beam element with corroded part of the reinforcement and the initial crack from the rebar to the surface of the beam is shown to the left. In the middle the FEM-net is shown for one quarter of the beam, and to the right the FEM-net near the reinforcement and the crack is shown.

modelling to consider situations where only parts of the reinforcement are corroded. Eventually, this modelling should be able also to handle pit corrosion.

A beam element with the dimensions $400 \times 800 \times 1000$ mm is considered. It contains only one rebar with the diameter $d = 20$ mm, and the cover c , see Fig. 7. Corrosion is supposed to take place in the central part with the length l ($0 < l < 1000$ mm) of the reinforcement. An initial thin crack (crack width ≈ 0 mm) connects the corroded part of the reinforcement with the surface of the beam as shown in Fig. 7.

The FEM model of the beam element is assumed to have a cylindrical hole at the location of the reinforcement and a crack (≈ 0 mm thick) from the hole to the lower boundary of the beam element. The beam element material is assumed to be linear elastic with the modulus of elasticity module $E = 25 \times 10^9$ Pa. It is assumed that the pressure on the boundary of the hole due to increasing corrosion products can be modelled as a uniform loading (pressure) $p = 1 \times 10^8$ N/m² normal to the boundary of the hole. Due to symmetries, the FEM analysis is based on only one quarter of the considered beam element. The FEM analysis is performed with 5 covers $c = 20, 30, 40, 50,$ and 60 for the corrosion length $l = 10, 20, 30, 40, 50, 100, 200, 300, 400, 500, 600, 700, 800, 900,$ and 1000 mm. The crack-corrosion index $\gamma = \Delta crack / \Delta d$ is calculated for each combination of the above-mentioned values of the cover c and the corrosion (crack) length l . It follows from Fig. 8 that the crack-corrosion index γ decreases with decreased length l especially for small lengths of l . $\gamma \rightarrow 0$ in the case of pit corrosion ($l \rightarrow 0$).

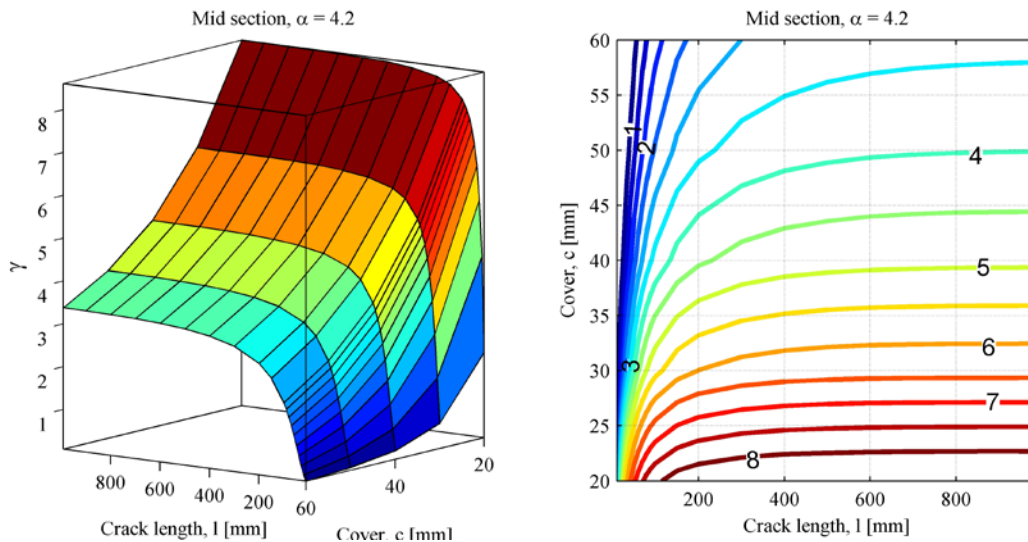


Figure 8. The crack-corrosion index $\gamma(l, c)$ estimated on the basis of the mid section of the corroded part of the rebar.

9. CONCLUSIONS

During the last 20-30 years, the interest in improving the modelling of deterioration of structural materials has been growing in all countries. Hundreds of papers have been published in material science journals and in all kind of engineering journals. A great number of conferences and meetings have been organized. New materials with interesting properties have been developed and new types of exciting structures have been designed. Very tall buildings, offshore structures, long bridges etc. are new

challenges for architects and engineers. All this combined with a need to maintain the infrastructures everywhere have shown a need for a more precise and reliable description of strength and deterioration of structural materials.

In this paper the deterioration of reinforced concrete structures is presented with special emphasis on the corrosion crack opening during corrosion of the reinforcement. The first sign that something is wrong with a reinforced structure is often corrosion cracks leading to spalling and eventually to failure of the structure. However, corrosion-based failure is only one of several failure modes, but in many countries it is one of the most important types of failure.

It is therefore of great interest to design a reinforced concrete structure so that corrosion is not likely to take place in the planned lifetime of the structure, say 100 years. Especially structures near or in the sea and bridges, where de-icing is used, have a high risk of failure. It is therefore of great importance to understand the chemical, physical, and mechanical properties of reinforced materials. To this purpose a great number of deterministic and stochastic models for deterioration of reinforced structures have been developed. Some models are very complicated, others are simpler.

In this paper a framework for a complete model of the total deterioration process of reinforced concrete structures resulting from corrosion of the reinforcement is presented. The model includes chloride ingress, corrosion initiation, corrosion propagation, crack initiation, and crack opening models. The model is made as simple as possible, but may easily be extended to include space dependence, time dependence, and inhomogeneous and anisotropic materials.

It is necessary to make all models presented in this paper stochastic, since a number of uncertain parameters are included in the models. This includes the diffusion coefficient, the corrosion parameters, the rebar diameter, and the concrete cover etc., but also model uncertainties. The deterministic models presented are easily made stochastic by assuming that the involved parameters are stochastic variables or stochastic processes.

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