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CHAPTER 135

STOCHASTIC MODELLING OF CHLORIDE PENETRATION IN CONCRETE STRUCTURES¹

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1. INTRODUCTION

Modelling of deterioration of reinforced concrete structures is usually based on observations of the deterioration of existing structures and on comprehensive experiments in laboratories. By such observations the so-called deterioration profile (deterioration, reliability, or capacity as a function of time) may be estimated. In the past a large number of such observation based profiles have been estimated by curve fitting.

The deterioration profiles obtained by this approach have the advantage that they are easy to use in analysis and design of reinforced concrete structures; see e.g. Ellingwood [1]. The disadvantage is that it is not easy to estimate the deterioration profiles for new structures and new materials, since the deterioration curves are not directly related to physical, mechanical, or chemical parameters.

In a more modern approach, the estimation of the deterioration is based on a detailed understanding of the chemical and physical processes that take place during deterioration. This approach is used in this paper but only deterioration related to corrosion of the reinforcement is considered. Other types of deterioration may be handled in a similar way. Corrosion of the reinforcement is one of the major reasons for deterioration of reinforced concrete structures.

Initially, the main effect of corrosion is a reduced strength of the structural element in question due to a reduced cross-section of the reinforcement. Another effect of the corrosion is the close interaction between corrosion and the bond between the concrete and the reinforcement. A reduced bond will also influence the structural

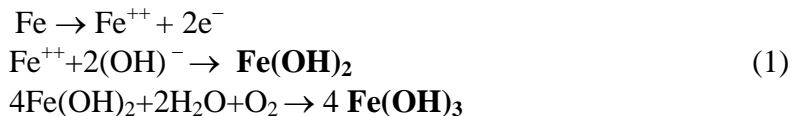
¹ Proceedings Int. Conf. on “Modelling and Optimization of Structures, Processes and Systems”, ICMOSPS 07, Durban, South Africa, Jan. 22-24, 2007.

strength of the structure and must therefore be included in a complete estimation of the deterioration process. Important research on bonding is taking place in several research institutions; see e.g. Lundgren [2] and Maurel, Dekoster & Buyle-Bodin [3].

It is a serious problem that the volume of the rust products is higher than the volume of the corroded steel. The porous zone around the steel/concrete surface can to some extent absorb the higher volume of the rust products. However, at a certain time the total amount of corrosion products exceeds the amount of corrosion products needed to fill the porous zone around the reinforcement. The rust products will then create expansive pressure on the surrounding concrete. The expansion of the concrete near the reinforcement will initiate tensile stresses in the concrete. After some time with increasing corrosion the tensile stresses will reach a critical value and corrosion cracks may develop. With further production of rust, the crack opening will increase and eventually result in spalling. This last part of the corrosion process is still not well understood. More research is certainly needed to clarify these important problems for a corroded reinforced concrete structure.

2. CHLORIDE INGRESS

Chloride-induced corrosion has been investigated by e.g. Neville [4]. Reinforced concrete is an excellent type of material from a corrosion point of view, since the alkaline environment in the concrete maintains a passive film on the surface of the reinforcement, and this film protects the reinforcement against corrosion. However, if the concrete is penetrated by e.g. water or carbon dioxide, then this passive film breaks down and the reinforcement is open to corrosion. An anodic region is established, where the passive film is broken down, whereby an electrochemical cell is formed. The passive surface is the cathode, and the electrolyte is the pore water in the concrete. At the anode the following reactions take place:



Chloride ions Cl^{-} activate the unprotected surface and form an anode. The chemical reactions are



It follows from Eqs. (1) and (2) that two rust products $\text{Fe}(\text{OH})_2$ and $\text{Fe}(\text{OH})_3$ are produced in this case.

<i>Corrosion product</i>	<i>Colour</i>	<i>Volume, cm³</i>
Fe_3O_4	Black	2.1
$\text{Fe}(\text{OH})_2$	White	3.8
$\text{Fe}(\text{OH})_3$	Brown	4.2
$\text{Fe}(\text{OH})_3, 3\text{H}_2\text{O}$	Yellow	6.4

Table 1. Volume of corrosion products corresponding to corrosion of 1 cm³ Fe.

The different types of rust products are interesting to study because they have great influence on corrosion cracking, since the volume of the rust products corresponding to a given volume of the steel varies a lot. This problem has been studied

for several corrosion products by Nielsen [5]. Nielsen [5] has obtained the rust volumes corresponding to corrosion of 1 cm³ Fe, see Table 1.

3. THE CHLORIDE DIFFUSION PROCESS

The penetration of chloride ions into the concrete is difficult to model. Some simplifying assumptions are needed. There seems to be a general agreement that a model based on diffusion theory is a reasonably good approximation. If the chloride concentration C_0 on the surface of the concrete and the diffusion coefficient D for concrete are independent of space (location) and time, then Fick's law of diffusion can represent the rate of chloride penetration into concrete, as a function of depth from the concrete surface and as a function of time

$$\frac{\partial C(x,t)}{\partial t} = D \frac{\partial^2 C(x,t)}{\partial x^2} \quad (3)$$

where $C(x,t)$ is the chloride ion concentration, as % by weight of cement, at a distance of x cm from the concrete surface after t seconds of exposure to the chloride source. D is the chloride diffusion coefficient expressed in cm²/sec. The solution of the differential equation (3) is

$$C(x,t) = C_0 \left\{ 1 - \operatorname{erf} \left(\frac{x}{2\sqrt{Dt}} \right) \right\} \quad (4)$$

where C_0 is the equilibrium chloride concentration on the concrete surface, as % by weight of cement, erf is the error function.

The chloride ingress with a time-dependent chloride concentration at the concrete surface has been investigated by Frederiksen, Mejlbro & Poulsen [6] and by Mejlbro [7] on the basis of a solution of the diffusion law with time-dependent diffusion coefficient and time-dependent surface concentration. Mejlbro & Poulsen [8] have considered the special case where the time dependent chloride concentration on the surface is approximated by a piecewise set of linear functions versus time. Such an approach is relevant for e.g. bridges where de-icing salt containing chloride is used during the winter period.

It is further assumed that a corrosion process is initiated when the chloride concentration at the site of the reinforcement reaches a certain critical chloride corrosion threshold value C_{cr} . The critical chloride threshold depends on the type of concrete and several factors; see e.g. Frederiksen [9]. If C_{cr} is assumed to be the chloride corrosion threshold and d is the thickness of the concrete cover, then the corrosion initiation period T_{corr} can be calculated. The time T_{corr} to initiation of reinforcement corrosion is

$$T_I = \frac{(d_1 - D_1/2)^2}{4D_C} \left(\operatorname{erf}^{-1} \left(\frac{C_{cr} - C_0}{C_i - C_0} \right) \right)^2 \quad (5)$$

On the basis of equation (5) outcomes of the corrosion initiation time T_{corr} have been performed by simple Monte Carlo simulation on the basis of the data in table 2; see Thoft-Christensen [10].

A Weibull distribution can be used to approximate the distribution of the simulated data. The Weibull distribution is $W(x; \mu, k, \varepsilon)$, where $\mu = 63.67$, $k = 1.81$ and $\varepsilon = 4.79$. The corresponding histogram and the density function are shown in figure 1.

Initial chloride concentration	0 %,
Surface chloride concentration	Normal (0.650 ; 0.038)
Diffusion coefficient	Normal (30 ; 5)
Critical chloride concentration	Normal (0.3 ; 0.05)
Cover	Normal (40 ; 8)

Table 2. Parameters used in the simulation study of T_{corr} .

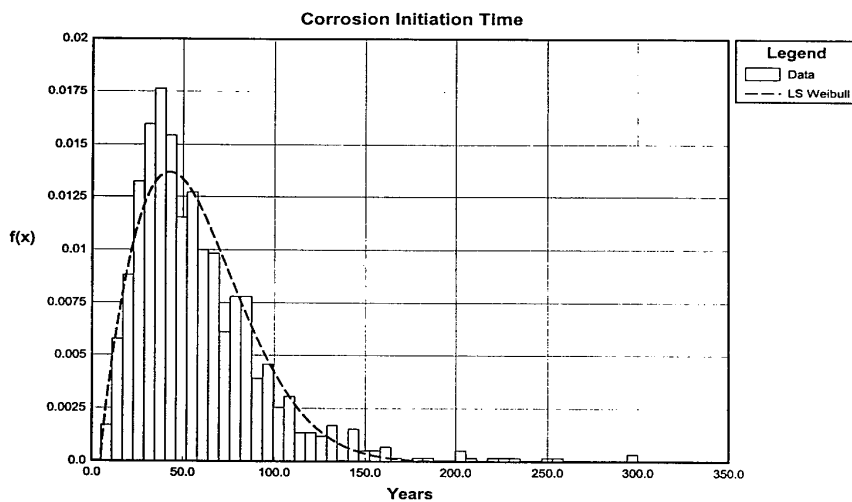


Figure 1. Density function of the corrosion initiation time T_{corr} , Theft Christensen [10].

4. THE DIFFUSION COEFFICIENT D

It follows from Eq. 5 that the time to corrosion initiation T_{corr} is inversely proportional to the diffusion coefficient D . It is therefore of great interest to get a good estimate of D . The diffusion coefficient D is not a real physical constant for a given concrete structure, since it depends on a number of physical factors. The most important factors are the water/cement ratio w/c , the temperature Φ , and the amount of e.g. silica fumes (s.f.); see Jensen [11] and Jensen, Hansen, Coats & Glasser [12]. The data presented in this section are all based on Jensen [11] and Jensen, Hansen, Coats & Glasser [12].

Figure 2 shows the diffusion coefficient D as a function of the water-cement ratio w/c and the temperature Φ °C for cement pastes with 0% silica fume. It is clear from figure 2 that the diffusion coefficient D increases significantly with w/c as well as the temperature Φ . In this example the minimum value of D is 0.31×10^{-12} m²/s corresponding to $w/c = 0.2$ and the temperature $\Phi = 4$ °C. The maximum value of D is 80.00×10^{-12} m²/s corresponding to $w/c = 0.70$ and $\Phi = 35$ °C. In figure 3 the contour lines for the same data are shown.

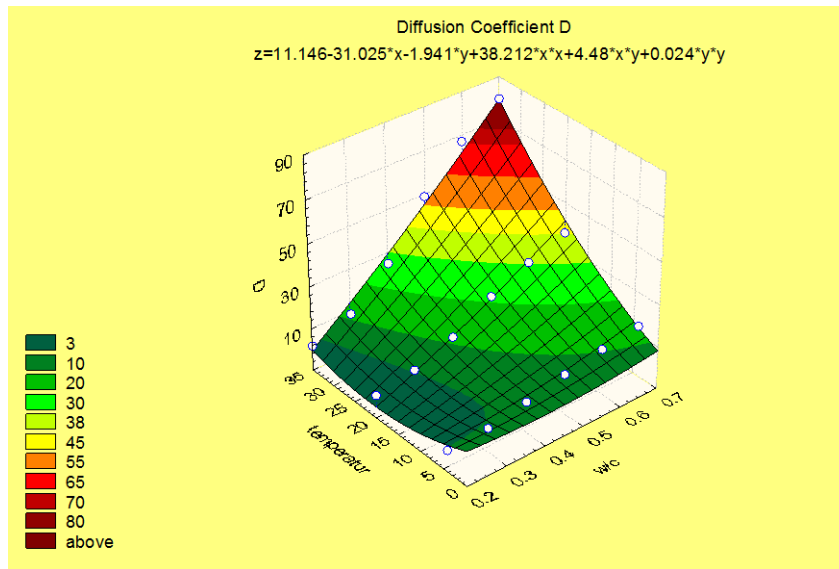


Figure 2. The diffusion coefficient D (10^{-12} m²/s) as a function of w/c and of the temperature Φ , Thoft-Christensen [13].

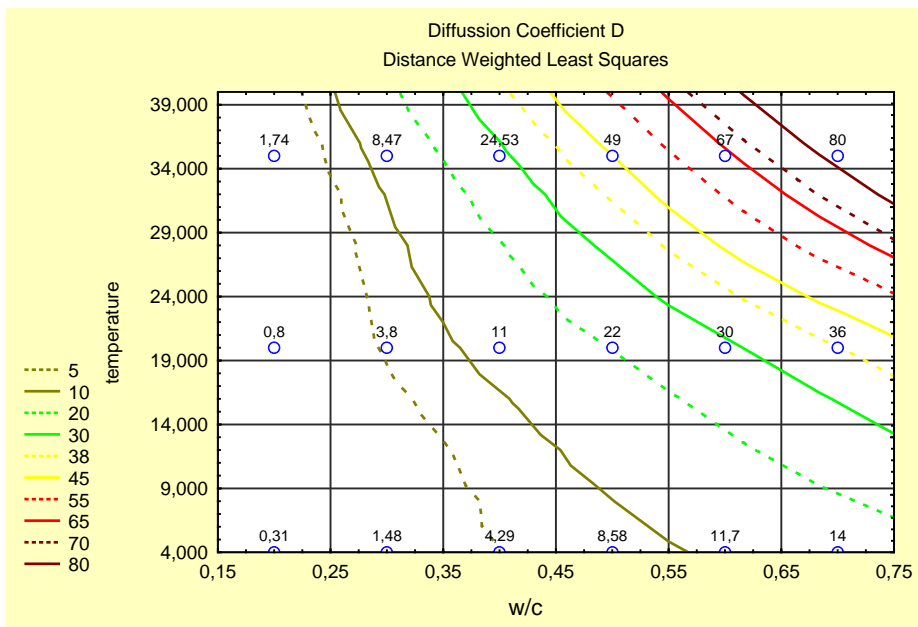


Figure 3. The diffusion coefficient D (10^{-12} m²/s) as a function of the water-cement ratio w/c and the temperature Φ °C, Thoft-Christensen [13].

The addition of silica fume (s.f.) is of great importance for the chloride ingress. Silica fume additions reduce the chloride ingress because of changes in the pore structure; see table 3 and Jensen [11].

w/c	0.3					0.5				
% s.f.	0	3	6	10	20	0	3	6	10	20
D (10^{-12} m/s)	3.8	2.0	0.42	0.12	0.05	22	13	4.2	1.6	

Table 3. The diffusion coefficient D as a function of % s.f. for two values of w/c and $\Phi = 20$ °C.

5. SITE DEPENDENCY OF THE DIFFUSION COEFFICIENT D

The data in section 4 clearly indicate that site information is needed to make e.g. an estimation of the remaining life cycle of a bridge or any estimation where the diffusion coefficient D is involved.

This has been confirmed by several authors see e.g. Suda et al. [14], where important information of the distribution of the diffusion coefficient D in Japan is shown. Figure 4 shows the mean air temperature Φ and the w/c ratio in Japan. As expected, the temperature is much higher (21–25°C) in the southern part of Japan than in the northern part (4–9°C). The w/c ratio has a smaller variation, but the highest ratios are in the Kanto area where also the temperature Φ is relatively high.

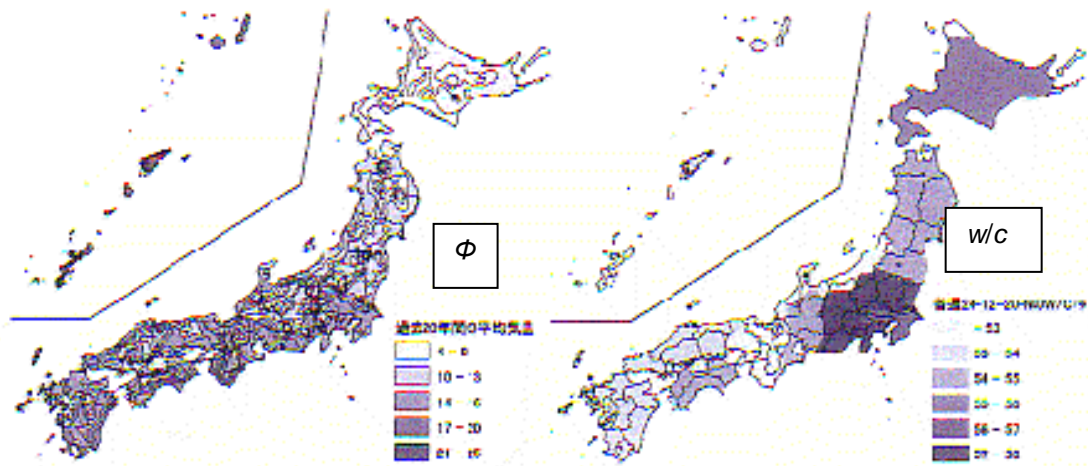


Figure 4. Mean air temperature Φ and w/c ratio in Japan (figures 3 and 5 in Suda et al. [14]).

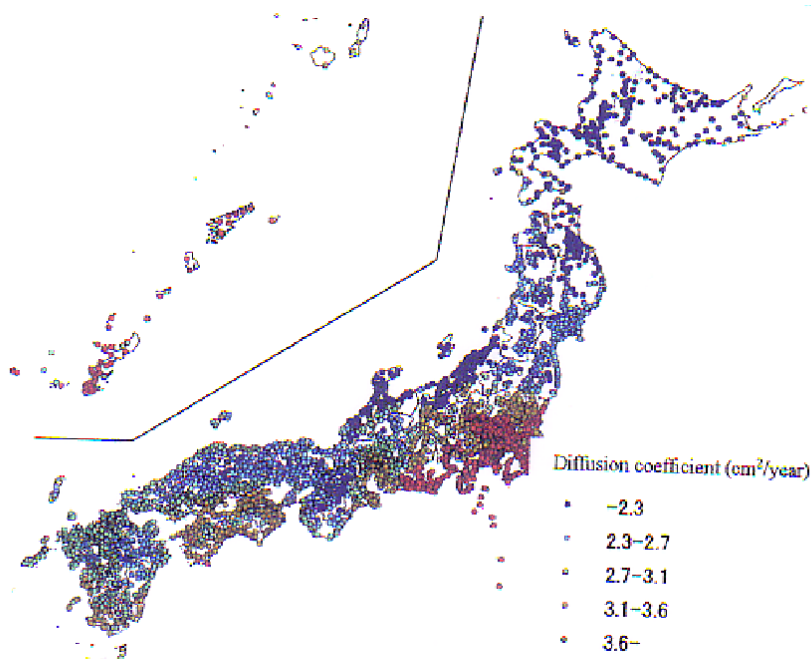


Figure 5. Distribution of the diffusion coefficient D (cm^2/year) in Japan (Figure 15 in Suda et al. [14]).

Figure 5 shows measured values of the diffusion coefficient D in Japan. The D nicely confirms the theoretical results in section 4 namely that D increases with w/c and with the temperature Φ .

6. BINDING MODIFIED DIFFUSION LAW

Fick's law for diffusion was used as early as 1970 for calculation of the diffusion coefficients for various concrete compositions; see Collepardi et al. [15]. Since then, Fick's law has been the basis for chloride ingress modelling, but a large number of modifications have been introduced, see Jensen [11]. Fick's diffusion law has e.g. been modified taking into account binding of chloride by assuming that the free chloride follows Fick's law; see Jensen [11] and Jensen et al. [12]. It is also assumed that the chloride binding follows a Freundlich isothermal equation; Luping [16].

$$c_b = \alpha \times c_f^\beta \quad (6)$$

where c_b [mg/n-gel] is the bound chloride, c_f [mol Cl/l solution] and α and β are empirical constants. The chloride binding significantly modifies the shape of the chloride ingress profiles as well as the calculated chloride diffusion coefficient D . In figure 6 the principal difference between chloride ingress profiles with and without binding is shown. Without bonding the profile is strongly concave and with binding the profile is almost linear. Generally, measured profiles are almost linear so a modelling with binding seems to be a great improvement.

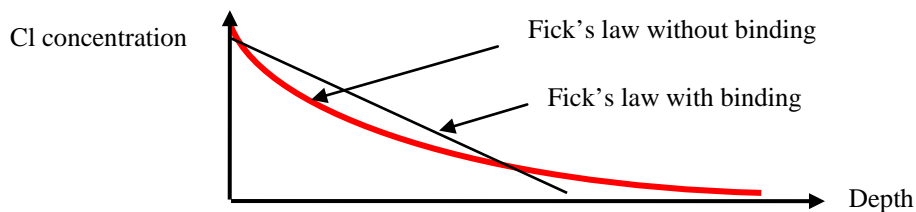


Figure 6. Chloride ingress profiles.

In Jensen [11] is presented an example that strongly supports this conclusion. A cement paste with $w/c = 0.3$ and with no additives was exposed to chlorides for 30 days at 35° . Without binding the diffusion coefficient is $1.5 \times 10^{-12} \text{m}^2/\text{s}$ and with binding included the diffusion coefficient for the free chloride is $3.7 \times 10^{-12} \text{m}^2/\text{s}$. If binding is included the description is substantially improved.

7. THE w/c RATIO

The w/c ratio for an existing concrete element may be estimated using Optical Fluorescence Microscopy; see Jakobsen [17] and Concrete-experts [18]. Thin sections of the concrete are fluorescent impregnated and analysed under an optical microscope using a combination of a blue excitation filter and a yellow blocking filter. In fluorescent light the epoxy filled air voids and cracks then appear yellow. Cement paste appears as shades of green and aggregates black. The shade of green of the cement paste depends on the capillary porosity. A sample with a low w/c ratio appears dark green and a sample with high w/c ratio appears light green. These shades of green are used to estimate the w/c ratio by comparing with the colours of a standard cement pastes

where the w/c ratios are known.

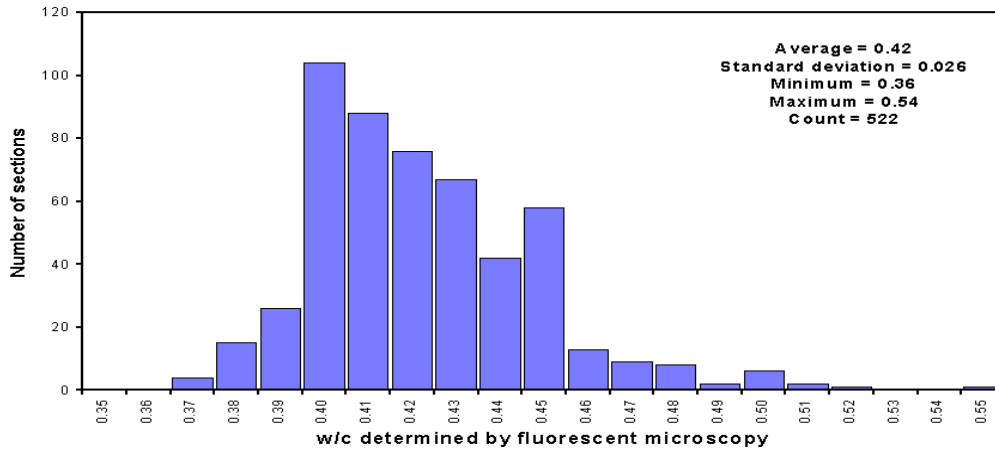


Figure 7. Distribution of w/c measure in 522 thin sections by fluorescence microscopy (Concrete-experts).

In figure 7 is shown the determination of the w/c ratio of precast railroad ties performed on 522 thin sections representing 127 ties. The average value is 0.42 and the standard deviation is 0.026 (the coefficient of variation is 6%).

8. THE TEMPERATURE Φ

The temperature Φ is modelled as a stochastic variable based on the temperature at the site of the structure. The modelling should take into account that the yearly mean temperature varies significantly from year to year. The data needed for the stochastic modelling are in most cases available from national meteorological institutions.

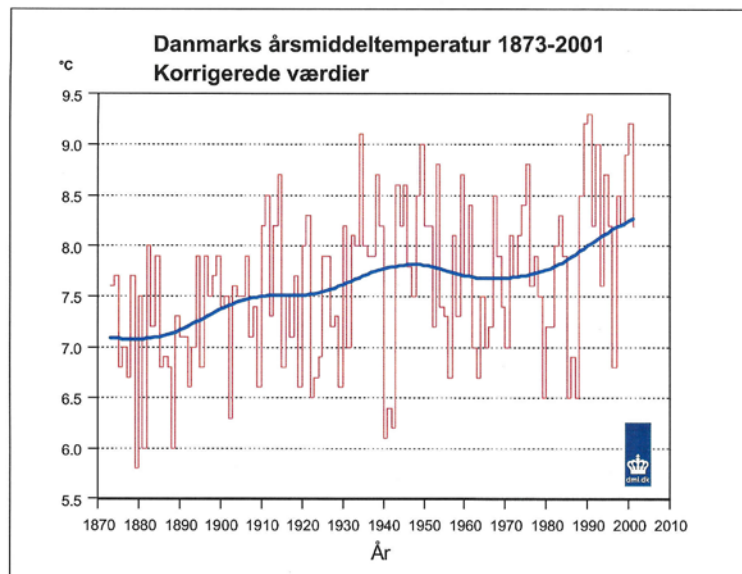


Figure 8. Yearly mean temperature for Denmark 1873-2001.

In figure 8 is as example shown the yearly mean temperature in Denmark for the years 1873-2001, see Cappelen [19]. Notice that the mean yearly temperature in this

time period varies between below 6°C to over 9°C and that there has been a significant increase in the mean yearly temperature. These facts can be taken into consideration by weighting the estimated diffusion coefficient.

9. STOCHASTIC MODELLING OF THE DIFFUSION COEFFICIENT D

Based on the experimental results presented in section 4, the following formula may be used to approximately describe the diffusion coefficient D as a function of the w/c ratio and the temperature Φ :

$$D = 11.146 - 31.025 \times w/c - 1.941 \times \Phi + 38.212 \times (w/c)^2 + 4.48 \times w/c \times \Phi + 0.024 \times \Phi^2 \quad (7)$$

As an example assume that w/c is log-normally distributed LN(0.45,0.02) and that the temperature Φ is normally distributed N(10.0°C , 1.0°C). Then by crude Monte Carlo simulation (10.000 samples) it may be shown that the diffusion coefficient may be modelled by a normally distributed stochastic variable N(8.11×10^{-12} m²/s, 1.11×10^{-12} m²/s), that is with a coefficient of variation equal to 14%.

The diffusion coefficient D is, as expected strongly sensitive to the mean values of w/c and Φ as shown in figure 9. In the diagram to the left, w/c is log-normally distributed LN(E[w/c],0.02) and the temperature Φ is normally distributed N(10.0°C , 1.0°C) and E[D] is the expected value of D . In the diagram to the right, w/c is log-normally distributed LN(0.45,0.02) and the temperature Φ is normally distributed N(E[Temperature] , 1.0°C).

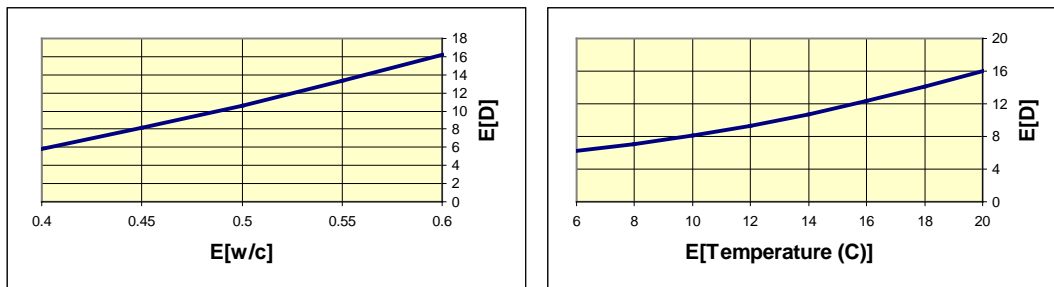


Figure 9. Sensitivity analysis with regard to the mean values of w/c and of the temperature Φ .

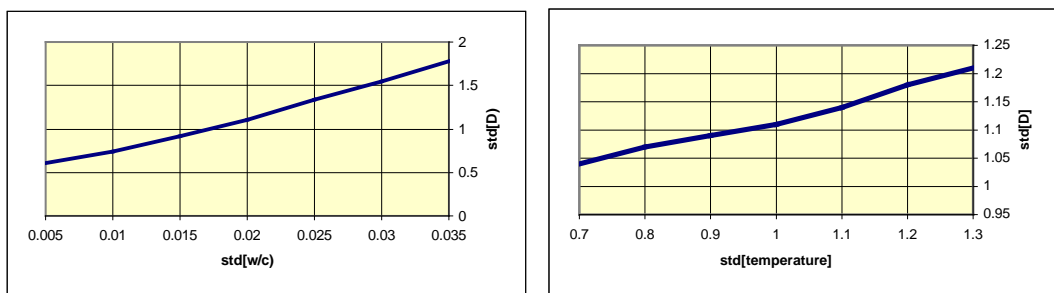


Figure 10. Sensitivity analysis with regard to the standard deviations of w/c and of the temperature.

The standard deviation $\text{std}[D]$ of the diffusion coefficient D (but not the expected value $E[D]$) is sensitive to the standard deviations of w/c and Φ as shown in figure 10.

In the diagram to the left, w/c is log-normally distributed $\text{LN}(0.45, \text{std}[\text{w/c}])$ and the temperature Φ is normally distributed $\text{N}(10.0^\circ\text{C}, 1.0^\circ\text{C})$. In the diagram to the right, w/c is log-normally distributed $\text{LN}(0.45, 0.02)$ and the temperature Φ is normally distributed $\text{N}(10^\circ\text{C}, \text{std}[\text{temperature}])$.

10. INITIATION OF CORROSION CRACKS

The corrosion process after corrosion initiation is very difficult to model, since a large number of changes in the rebars as functions of the time have been observed and reported in the literature. The simplest model is to assume that the diameter $d(t)$ of the reinforcement bars at the time $t > T_{corr}$ is modelled by

$$d(t) = d_0 - c_{corr} i_{corr} (t - T_{corr}) \quad (8)$$

where d_0 is the initial diameter, c_{corr} is a corrosion coefficient, and i_{corr} is the rate of corrosion. Based on a survey, three models for chloride penetration have been proposed (the initial chloride concentration in the concrete is assumed to be zero): low, medium, and high deterioration; see Thoft-Christensen & Jensen [20].

There is a porous zone around the steel/concrete surface caused by the transition from paste to steel, entrapped/entrained air voids, and corrosion products diffusing into the capillary voids in the cement paste. When the total amount of corrosion products exceeds the amount of corrosion products needed to fill the porous zone around the steel, the corrosion products create expansive pressure on the surrounding concrete. Close to reinforcement bars the concrete has some porosity. Very close to the bars the porosity is close to 1, but the porosity decreases with the distances from the bars. The porosity is typically of the order of 0.5 about 10 - 20 μm from the bars.

Let t_{por} be the thickness of an equivalent zone with porosity 1 around a steel bar, and let t_{por} be modelled by a lognormal distribution with the mean 12.5 μm and a standard deviation of 2.54 μm . Further, let the density ρ_{rust} of the rust product and the initial diameter d_0 be modelled by normal distributions $\text{N}(3600, 360)$ kg/m^3 and $\text{N}(16, 1.6)$ mm, respectively.

Then it can be shown by Monte Carlo simulation that the volume of the porous zone W_{porous} with a fairly good approximation can be modelled by a shifted lognormal distribution with a mean $2.14\text{e}-03$ kg/m , a standard deviation $0.60\text{e}-03$ kg/m and a shift of $0.82\text{e}-03$ kg/m ; see Thoft-Christensen [10].

After a certain time the rust products will fill the porous zone completely and then result in an expansion of the concrete near the reinforcement. As a result of this, tensile stresses are initiated in the concrete. With increasing corrosion the tensile stresses will reach a critical value and cracks will be developed. During this process the corrosion products at initial cracking of the concrete will occupy three volumes, namely the porous zone W_{porous} , the expansion of the concrete due to rust pressure W_{expan} , and the space of the corroded steel W_{steel} . The corresponding total amount of critical rust products W_{crit} needed to fill these volumes is

$$W_{crit} = W_{porous} + W_{expan} + W_{steel} \quad (9)$$

Using Monte Carlo simulation it can be shown that W_{expan} with a good approximation can be modelled by a normal distribution $\text{N}(0.0047, 0.0011)$ kg/m when

the data shown above are used; see Thoft-Christensen [10]. Finally, W_{steel} can be written

$$W_{steel} = \frac{\rho_{rust}}{\rho_{steel}} M_{steel} \quad (10)$$

where ρ_{steel} is the density of steel and M_{steel} is the mass of the corroded steel. Clearly, M_{steel} is proportional to W_{crit} .

Liu & Weyers [21] have calculated the factor of proportionality for two kinds of corrosion products as 0.523 and 0.622. For simplicity, it will be assumed that $M_{steel} = 0.57W_{crit}$. Therefore, Eq. 8 can be rewritten

$$W_{crit} = \frac{\rho_{steel}}{\rho_{steel} - 0.57\rho_{rust}} (W_{porous} + W_{expan}) \quad (11)$$

Let ρ_{steel} be modelled by a normal distribution $N(8000 ; 800) \text{ kg/m}^3$. Then by Monte Carlo simulation it can be shown that W_{crit} with a good approximation can be modelled by a normal distribution $N(0.010 ; 0.0027) \text{ kg/m}$; see Thoft-Christensen [10]. The rate of rust production as a function of time t (years) from corrosion initiation can be written, Liu & Weyers [21]

$$\frac{dW_{rust}(t)}{dt} = k_{rust}(t) \frac{1}{W_{rust}(t)} \quad (12)$$

i.e. the rate of corrosion is inversely proportional to the amount of rust products W_{rust} (kg/m). The factor $k_{rust}(t)$ ($\text{kg}^2/\text{m}^2\text{year}$) is assumed to be proportional to the annual mean corrosion rate $i_{cor}(t)$ ($\mu\text{A}/\text{cm}^2$) and the diameter d (m) of the reinforcement. The proportionality factor depends on the types of rust products, but is here taken as $0.383e-3$.

$$k_{rust}(t) = 0,383 \times 10^{-3} di_{corr}(t) \quad (13)$$

By integration

$$W_{rust}^2(t) = 2 \int_0^t k_{rust}(t) dt \quad (14)$$

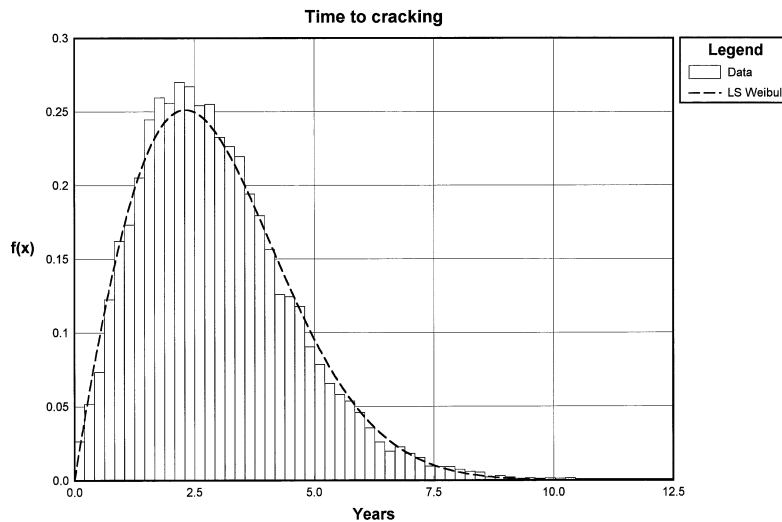


Figure 11. Density function of Δt_{crack} , Thoft-Christensen [10].

Let $i_{cor}(t)$ be modelled by a time-independent normally distributed stochastic variable $N(3 ; 0.3)$ ($\mu A/cm^2$) then the time from corrosion initiation to cracking Δt_{crack} can be estimated by (16) by setting $W_{rust}(\Delta t_{crack}) = W_{crit}$.

$$T_{crack} = \frac{W_{crit}^2}{2k_{rust}} = \frac{W_{crit}^2}{2 \times 0.383 \times 10^{-3} D i_{corr}} \quad (15)$$

Then it can be shown by Monte Carlo simulation that Δt_{crack} with a good approximation can be modelled by a Weibull distribution $W(3.350 ; 1.944 ; 0)$ years, see figure 11. The mean is 2.95 years and the standard deviation is 1.58 years. The mean value of T_{crack} is of the same order as the experimental values (and the deterministic values) obtained by Liu & Weyers [21].

11. THE CRACK - CORROSION INDEX

After formation of the initial crack the rebar cross-section is further reduced due to the continued corrosion, and the opening of the crack is increased. Experiments by Andrade, Alonso & Molina [22] show that the function between the reduction of the rebar diameter Δd and the corresponding increase in crack opening $\Delta crack$ measured on the surface of the concrete specimen can be approximated by a linear function $\Delta crack = \gamma \Delta d$, where the crack-corrosion factor γ is of the order 1.5 to 5.

An approximate relation between the decrease in the steel bar diameter Δd and the increase in the hole Δd_{hole} can easily be obtained by considering the volume of the produced rust products $\pi \Delta d d \alpha$ ($\alpha = \rho_{steel}/\rho_{rust}$) is the relation between the densities of the steel and the rust product; see Table 1) and the volume due to the expansion of the hole $\pi \Delta d d + \pi \Delta d_{hole} d_{hole} + c \Delta crack$. By equalizing these two volumes and assuming that $d \approx d_{hole}$ one gets

$$(\alpha - 1) \Delta d d \approx \pi \Delta d_{hole} d_{hole} + \text{crack volume} = \pi \Delta d_{hole} d_{hole} + c \Delta crack \quad (16)$$

where c is the cover. Therefore,

$$\Delta d \approx (\Delta d_{hole} + c \Delta crack / \pi d) / (\alpha - 1) \quad (17)$$

FEM estimation of the crack-corrosion index $\gamma = \Delta w_{crack} / \Delta D_{bar}$ has been done using FEMLAB/MATLAB; see Thoft-Christensen [23]. A rectangular beam cross-section with only one reinforcement bar was considered as an example.

In the FEM modelling the rectangular cross-section in a long beam (plain strain) is assumed to have a hole at the location of the reinforcement and a crack (0.01 mm) from the hole to the boundary. The material is assumed to be linear elastic with the modulus of elasticity $E = 25 \times 10^9$ Pa. It is assumed that the pressure on the boundary of the hole from the increasing corrosion products can be modelled as a uniform loading (pressure) $p = 1 \times 10^8$ N/m at the boundary of the hole. The increase in the crack opening is $\Delta w_{crack} = 0.67$ mm and the average increase in the hole diameter is $\Delta D_{hole} = 0.31$ mm.

For this example one gets γ equal to 1.8 and 5.1 for black and brown rust, respectively, in good agreement with the values obtained by the experiments by Andrade, Alonso & Molina [22].

The above mentioned procedure used in a 2D-modelling of corrosion crack propagation is extended to a 3D-modelling of a reinforced concrete beam; see Thoft-

Christensen, Svensson & Frandsen [24]. The 2D-modelling in section 2 is based on several assumptions and simplifications, e.g. that the movement of corrosion products is restricted to the considered cross-section. In a 3D-modelling the corrosion products may also move in the direction of the reinforcement. Further, it is possible in a 3-D modelling to consider situations where only parts of the reinforcement are corroded. Eventually, this modelling should be able also to handle pit corrosion.

12. CONCLUSIONS

During the last 20-30 years, the interest in improving the modelling of deterioration of structural materials has been growing in all countries. Hundreds of papers have been published in material science journals and in all kind of engineering journals. A great number of conferences and meetings have been organized. New materials with interesting properties have been developed and new types of exciting structures have been designed. Very tall buildings, offshore structures, long bridges etc. are new challenges for architects and engineers. All this combined with a need to maintain the infrastructures everywhere have shown a need for a more precise and reliable description of strength and deterioration of structural materials.

In this paper the deterioration of reinforced concrete structures is presented with special emphasis on chloride penetration of the concrete. The first sign that something is wrong with a reinforced structure is often corrosion cracks leading to spalling and eventually to failure of the structure. However, corrosion-based failure is only one of several failure modes, but in many countries it is one of the most important types of failure.

It is therefore of great interest to design a reinforced concrete structure so that corrosion is not likely to take place in the planned lifetime of the structure, say 100 years. Especially structures near or in the sea and bridges, where de-icing is used, have a high risk of failure. It is therefore of great importance to understand the chemical, physical, and mechanical properties of reinforced materials. To this purpose a great number of deterministic and stochastic models for deterioration of reinforced structures have been developed. Some models are very complicated, others are simpler.

In this paper a framework for a complete model of the total deterioration process of reinforced concrete structures resulting from corrosion of the reinforcement is presented. The model includes chloride ingress, corrosion initiation, corrosion propagation, crack initiation, and crack opening models. The model is made as simple as possible, but may easily be extended to include space dependence, time dependence, and inhomogeneous and anisotropic materials.

It is necessary to make all models presented in this paper stochastic, since a number of uncertain parameters are included in the models. This includes the diffusion coefficient, the corrosion parameters, the rebar diameter, and the concrete cover etc., but also model uncertainties. The deterministic models presented are easily made stochastic by assuming that the involved parameters are stochastic variables or stochastic processes.

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