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CHAPTER 111

RISK ANALYSIS IN CIVIL ENGINEERING [1]

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[1] Module 1 of Course Notes used for a Short Course on “Risk and Reliability in Civil Engineering” at the Int. Conf. on “Safety, Risk and Reliability – Trends in Engineering”, Malta, March 21, 2001.

1. INTRODUCTION

Until fairly recently there has been a tendency for structural engineering to be dominated by deterministic thinking, characterized in design calculations by the use of specified minimum material properties, specified load intensities and by prescribed procedures for computing stresses and deflections. This deterministic approach has almost certainly been reinforced by the vary large extent to which structural engineering design is codified and the lack of feed-back about the actual performance of structures. For example, the actual stresses are rarely known, deflections are rarely observed or monitored, and since most structures do not collapse the real reserves of strengths are generally not known. In contrast, in the field of hydraulic systems, much more is known about the actual performance of, say, pipe networks, weirs, spillways, etc., as their performance in service can be relatively easily observed or determined.

The lack of information about the actual behaviour of structures combined with the use of codes embodying relatively high safety factors can lead to the view, still held by some engineers as well as by some members of the general public, that absolute safety can be achieved. Absolute safety is of course unobtainable; and such a goal is also undesirable, since absolute safety could be achieved only by deploying infinite resources.

It is now widely recognized, however, that some risk of unacceptable structural performance must be tolerated. The main object of structural design is therefore to ensure, at an acceptable level of probability, which each structure will not become unfit for its intended purpose at any time during its specified design life. Most structures, however, have multiple performance requirements, commonly expressed in terms of a set of serviceability and ultimate limit states, most of which are not independent; and thus the problem is much more complex than the specification of just a single probability.

This chapter is primarily based on Thoft-Christensen & Baker [1], Thoft-Christensen & Thoft-Christensen & Murotsu [2], and Thoft-Christensen [3],[4].

1.1. Current Risk Levels

As a means of assessing the relative importance of structural failures as a cause of death, some comparative statistics for the U.K. are given in table 1 for a number of causes. These figures show that, at least for a typical Western European country, the risk to life from structural failures is negligible. For the 3 year period reported, the average number of deaths per annum directly attributable to structural failure was 14, divided almost equally between failures occurring during construction and the failures of completed structures. Other structural failures occur in which there are no deaths or personal injuries; but data on such failures are more difficult to assemble because in many countries they do not have to be reported.

In comparing the relative risks given in table 1, account should be taken of differences in exposure time typical for the various activities. For example, although air travel is associated with a high risk per hour, a typical passenger may be exposed for between only, say, 10 - 100 hours per year, leading to a risk of death of between 10^{-5} and 10^{-4} per year. In contrast, most people spend at least 70% of their life indoors and are therefore exposed to the possible effects of structural failure; but this leads to an average annual risk per person of only 10^{-7}. Nevertheless, the only fair basis for comparing the risk is comparison with the inescapable minimum risk that has to be accepted by an individual member of society as beyond his control and for which no blame can be attributed to other people - for example, the risk of death by disease. Many people accept voluntary risks many orders of magnitude higher, but these should not be taken into account when considering structural safety.

Activity/Cause	**Number of deaths per hour per 10^8 persons**
Mountaineering (international)	2700
Air travel (international)	120
Deep water trawling	59
Car travel	56
Coal mining	21
Construction sites	7.7
Manufacturing	2.0
Accidents at home (all)	2.1
Accidents at home (able-bodied persons)	0.7
Fire at home	0.1
STRUCTURAL FAILURES	0.002

Table 1. Comparative death risks. [Average 1970-1973 in U.K. based on Central Statistical Office, Abstract 1974]. Taken from Thoft-Christensen & Baker [1].

In assessing the importance of structural failures, account should also be taken of the economic consequences of collapse and unserviceability. In fact the economic aspects may be regarded as dominant since the marginal returns in terms of lives saved for each additional it 1 million invested in improving the safety of structures may be small in comparison with the benefits of investing the same sum in, say, road safety or health care. However, structures should, where possible, be designed in such a way that there is ample warning of impending failure and with brittle failure modes having larger safety margins than ductile modes (i.e. higher reliabilities).

1.2. Structural Codes

Most structural design is undertaken in accordance with codes of practice, which in many countries have legal status. Structural codes typically and properly have a deterministic format and describe what are considered to be the minimum standards for design, construction and workmanship for each type of structure. Most codes can be seen to be evolutionary in nature, with changes being introduced or major revisions made at intervals of 3 - 10 years to allow for new types of structural form (e.g. reinforced masonry), the effects of improved understanding of structural behaviour (e.g. of stiffened plated structures), the effects of changes in manufacturing tolerances or quality control procedures, a better knowledge of loads, etc.

Until recently, structural codes could be considered to be documents in which current good practice was codified; and these documents could be relied upon to produce safe, if not economic, structures. These high standards of safety were achieved for the majority of structures, not from an understanding of all the uncertainties that affect the loading and response, but by codifying practice that was known by experience to be satisfactory. The recent generation of structural codes, including the Euro-codes and the associated model codes for steel and concrete, are however more scientific in nature. They typically cover a wider range of structural elements and incorporate the results of much experimental and theoretical research. They are also more complex documents to assimilate and to use and the associated design costs are consequently higher, as are the risks of errors in interpretation.

The benefits of these new codes must therefore lie in the possibility of:

- increased overall safety for the same construction costs;
- the same or more consistent levels of safety with reduced construction costs;
- or, a combination of these two.

A further aim should be the trend, where appropriate, towards design procedures, which can be applied with confidence to completely new forms of construction without the prior need for prototype testing. The aims and benefits described above can only be achieved by a rational assessment of the various uncertainties associated with each type of structure and a study of their interactions. This is the essence of structural reliability analysis.

2. UNCERTAINTY

2.1. General

Structural reliability analysis is concerned with the rational treatment of uncertainties in structural engineering design and the associated problems of rational decision-making. All quantities (except physical and mathematical constants) that currently enter into engineering calculations are in reality associated with some uncertainty. This fact has been implicitly recognized in current and previous codes. If this were not the case, a "safety factor" only slightly in excess of unity would suffice in all circumstances. The determination of appropriate standards of safety requires the quantification of these uncertainties by some appropriate means and a study of their interaction for the structure under consideration.

It is sometimes argued that the magnitudes of all variables are either bounded or can be restricted within specified limits, and that these bounding values should be used as the basis for design. However, in structural engineering

- upper limits to individual loads and lower limits to material strength are not easily identified in practice (e.g. the cube or cylinder strength of concrete);
- even if such natural limits exist, their direct use in design is likely to be extremely uneconomic;
- limits imposed by quality control and testing can never be completely effective, particularly in the case of properties which can be measured only by destructive tests or in circumstances in which changes in the potential properties take place between sampling and use of the material (e.g. concrete);
- even if recognizable limits do exist, their use may not always be rational.

Example 1. Consider a column supporting n floors of a building on which the loads are known to vary independently with time. Assuming that the load on each floor is physically restricted by some hypothetical fail-safe device so that under no circumstances can it exceed some specified maximum value, and given that each load stays at its maximum value for, say, 1% of the time, the rational design load for the column can generally be shown to be less than the sum of the maximum loads. This design load will, of course, depend on the number of storeys supported and the design life of the structure.

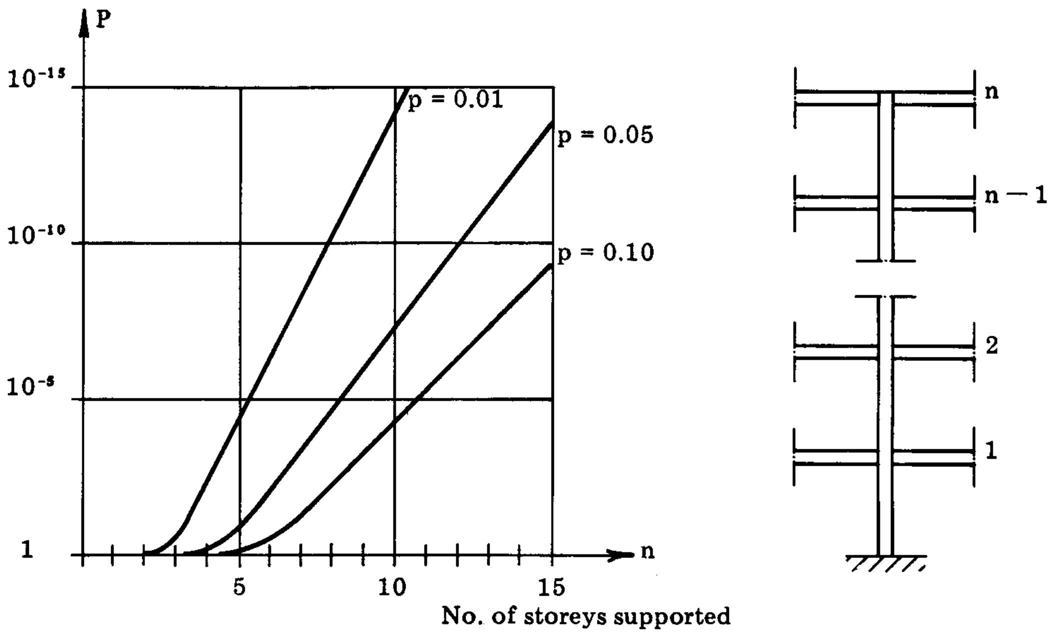

Figure 1.

Figure 1 shows the probabilities P that the maximum column load at ground level will reach the sum of the maxima of the individual floor loads (i.e. the maximum possible column load) at some time during a 50 year period, on the assumption that the floor loads are mutually independent, that they remain constant for an hour and then change to some new random value, and that each has a 1% chance (p = 0.01) of being at its maximum value after each renewal (i.e. each floor is loaded to its maximum value for approximately 1% of the time). The figure shows that even with only six floors, the probability that the maximum possible column load occurs is as small as 10^{-6} in 50 years. Even if each floor is loaded to its maximum value for about 10% of the time (p = 0.1), the probability of the maximum possible column load occurring is still very small if the number of floors supported is 10 or more.

In such cases, it would be irrational and uneconomic to design for this worst possible condition. However, it should be noted that in practice the degree of conservatism depends on whether the individual floor loads are in fact independent; and the acceptable risk level is governed by the consequences of failure and the rapidity with which failure occurs. Some knowledge of the probabilities of occurrence of loads less than the maximum would also be required for each floor for a rational design.

2.2. Basic Variables

For the purposes of quantifying uncertainties in the field of structural engineering and for subsequent reliability analysis it is necessary to define a set of basic variables. These are defined as the set of basic quantities governing the static or dynamic response of the structure.

Basic variables are quantities such as mechanical properties of materials, unit weights, environmental loads, etc. They are basic in the sense that they are the most fundamental quantities normally recognized and used by designers and analysts in structural calculations. Thus, the yield stress of steel can be considered as a basic variable, although this property is itself dependent on chemical composition and various micro-structural parameters.

It should also be mentioned that it is generally impracticable to try to obtain sufficient statistical data to model the variations in the strength of complete structural component directly. Reliance must be placed on the ability of the analyst to synthesize this higher-level information when required.

Ideally, basic variables should be chosen so that they are statistically independent quantities. However, this may not always be possible if the strength of a structure is known to be dependent on, for example, any two mechanical properties that are known to be correlated, e.g. the tensile strength and the compressive strength of a batch of concrete.

2.3. Types of Uncertainty

For the purposes of structural reliability analysis it is necessary to distinguish between at least three types of uncertainty – physical uncertainty, statistical uncertainty and model uncertainty. It should be noted that in the following, random variables will be denoted by upper case letters, e.g. X, and their outcomes by lower case, e.g. x.

Physical uncertainty: Whether or not a structural element fails when loaded depends in part on the actual values of the relevant material properties that govern its strength. The reliability analyst must therefore be concerned with the nature of the actual variability of physical quantities, such as loads, material properties and dimensions. This variability can be described in terms of probability distributions or stochastic processes. However, physical variability can be quantified only by examining sample data; but, since sample sizes are limited by practical and economic considerations, some uncertainty must remain. This practical limit gives rise to so-called statistical uncertainty.

Statistical uncertainty: Statistics, as opposed to probability, is concerned with inference, and in particular with the inferences that can be drawn from sample observations. Data may be collected for the purposes of building a probabilistic model of the physical variability of some quantity, which will involve, firstly the selection of an appropriate probability distribution type, and then determination of numerical values for its parameters. Common probability distributions have between one and four parameters, which immediately place a lower bound on the sample size required, but in practice very large samples are required to establish reliable estimates of the numerical values of parameters. For a given set of data, therefore, the distribution parameters may themselves be considered to be random variables, the uncertainty in which is dependent on the amount of sample data - or, in general, on the amount of data and nay prior knowledge. This uncertainty is termed statistical uncertainty and, unlike physical variability, arises solely as a result of lack of information.

Model uncertainty: Structural design and analysis make use of mathematical models relating desired output quantities (e.g. the deflection at the centre of a reinforced concrete beam) to the values of a set of input quantities or basic variables (e.g. load intensities, modulus of elasticity, duration of loading, etc.). These models are generally deterministic in form (e.g. linear-elastic structural analysis) although they may be probabilistic (e.g. calculation of the peak response of an offshore structure to stochastic wave loading). Furthermore, they may be based on an intimate understanding of the mechanics of the problem (e.g. plastic collapse analysis of a steel portal frame) or they may be highly empirical (e.g. punching shear at tubular joint connections in offshore jacket structures). However, with very few exceptions, it is rarely possible to make highly accurate predictions about the magnitude of the response of typical civil engineering structures to loading even when the governing input quantities are known exactly. In other words, the response of typical structures and structural elements contains a component of uncertainty in addition to those components arising from uncertainties in the values of the basic loading and strength variables. This additional source of uncertainty is termed model uncertainty and occurs as a result of simplifying assumptions, unknown boundary conditions and as a result of the unknown effects of other variables and their interactions, which are not included in the model.

The model uncertainty associated with a particular mathematical model may be expressed in terms of the probability distribution of a variable X_m defined as

$$X_m = \frac{\text{actual strength (response)}}{\text{predicted strength (response) using model}} \tag{1}$$

In many components and structures, model uncertainties have a large effect on structural reliability and should not be neglected.

3. STRUCTURAL RELIABILITY ANALYSIS AND SAFETY CHECKING

3.1 Structural Reliability

The term structural reliability should be considered as having two meanings - a general one and a mathematical one.

- In the most general sense, the reliability of a structure is its ability to fulfill its design purpose for some specified time.
- In a narrow sense it is the probability that a structure will not attain each specified limit state (ultimate or serviceability) during a specified reference period.

In this chapter we shall be concerned with structural reliability in the narrow sense and shall generally be treating each limit state or failure mode separately and explicitly. However, most structures and structural elements have a number of possible failure modes, and in determining the overall reliability of a structural system this must be taken into account, making due allowance for the correlations arising from common sources of loading and common material properties.

However, although the definition above may seem clear, it is necessary to examine with care exactly what is meant by "the probability that a structure will not attain each specified limit state during a specified reference period".

Consider first the need for defining a reference period. Because the majority of structural loads vary with time in an uncertain manner, the probability that any selected load intensity will be exceeded in a fixed interval of time is a function of the length of that interval (and possibly the time at which it begins). Hence, in general, structural reliability is dependent on time of exposure to the loading environment. It is also affected if material properties change with time. Only for the rare cases, when loads and strength are constant, can the reference period be ignored. In such cases the loads are applied once and the structure either does or does not fail (e.g. when the structure or component is loaded entirely by its own self weight).

The second question is more demanding. What is meant by "the probability that …"? This is best explored by a simple example.

Example 2. Assume that an offshore structure is idealized as a uniform vertical cantilever rigidly connected to the seabed. The structure will fail when the moment S induced at the root of the cantilever exceeds the flexural strength R. Assume further that R and S are random variables whose statistical distributions are known very precisely as a result of a very long series of measurements. R is a variable representing the variations in strength between nominally identical structures, whereas S represents the maximum load effects in successive T year periods. The distributions of R and S are both assumed to be stationary with time. Under these assumptions, the probability that the structure will collapse during any reference period of duration T years can then be shown to be given by

$$P_f = P(M \le 0) = \int_{-\infty}^{\infty} F_R(x) f_S(x) dx \tag{2}$$

where $M = R - S$ and where F_R is the probability function of R and f_S the probability density function of S .

Because of the definition of R and S in terms of frequentist probabilities, the probability determined from equation (2) may be interpreted as a long-run failure frequency. Similarly the reliability R, defined as

$$R = 1 - P_f \tag{3}$$

may be interpreted as a long-run survival frequency or long-run reliability. R may therefore be called a frequentist reliability. If, however, we focus our attention on one particular structure (and this is generally the case for "one-off" civil engineering structures), R may also be interpreted as a measure of the reliability of that particular structure.

3.2. Bayesian Reliability

This interpretation of reliability is fundamentally different from that given above, because, although the structure may be sampled at random from the theoretically infinite population described by the random variable R, once the particular structure has been selected (and, in practice, constructed) the reliability becomes the probability that the fixed, but unknown, resistance r will be exceeded by the as yet "un-sampled" reference period extreme load effect S. The numerical value of the failure probability remains the same but is now dependent upon two radically different types of uncertainty - firstly, the physical variability of the extreme load effect, and, secondly, lack of knowledge about the true value of the fixed but unknown resistance. This type of probability does not have a relative frequency interpretation and is commonly called a subjective probability. The associated reliability can be called subjective or *Bayesian reliability*. For a particular structure, the numerical value of this reliability changes as the state of knowledge about the structure changes - for example, if non-destructive tests were to be carried out on the structure to estimate the magnitude of r. In the limit when r becomes known exactly, the probability of failure given by equation (2) changes

$$P_f = P(r - S \leq 0) = 1 - F_S(r) \tag{4}$$

This special case may also be interpreted as a conditional failure probability with a relative frequency interpretation, i.e.

$$P_f = P(R - S \leq 0 \mid R = r) \tag{5}$$

The symbol $\mid$ may be read as "given that".

3.3 Methods of Safety Checking

Methods of structural reliability analysis can be divided into two broad classes. These are:

Level 3: methods in which calculations are made to determine the "exact" probability of failure for a structure or structural component, making use of a full probabilistic description of the joint occurrence of the various quantities which affect the response of the structure and taking into account the true nature of the failure domain.

Level 2: methods involving certain approximate iterative calculation procedures to obtain an approximation to the failure probability of a structure or structural system, generally requiring an idealisation of failure domain and often associated with a simplified representation of the joint probability distribution of the variables.

In theory, both level 3 and level 2 methods can be used for checking the safety of a design or directly in the design process, provided a target reliability or reliability index has been specified.

For the sake of completeness, some mention should also be made of level 1 methods at this stage. These are not methods of reliability analysis, but are methods of design or safety checking.

Level 1: design methods in which appropriate degrees of structural reliability are provided on a structural element basis (occasionally on a structural basis) by the use of a number of partial safety factors, or partial coefficients, related to pre-defined characteristic or nominal values of the major structural and loading variables. A level 1 structural design, with the explicit consideration of a number of separate limit states, is what is now commonly called limit-state design.

The three levels of safety checking should be seen as a hierarchy of methods in which level 2 methods are an approximation to level 3 methods and in which level 1 methods are a discretisation of level 2 methods (i.e. giving identical designs to level 2 methods for only a few discrete sets of values of the structural design parameters).

4. APPLICATION TO STRUCTURAL CODE

4.1. Introduction

Structural codes are documents, which, are subject to periodic revision and amendment, but the 1970 – 2000 was a time of marked activity in code development. This is still continuing. The main features have been

- the replacement of many simple design rules by more scientifically-based calculations derived from experimental and theoretical research,
- the move towards limit state design - whereby the designer and/or code writer specifies the relevant performance requirements (limit states) for each structure explicitly; and where separate sets of calculations are required to check that the structure will not attain each limit state (at a given level of probability),
- the replacement of single safety factors or load factors by sets of partial coefficients,
- the improvement of rules for the treatment of combinations of loads and other actions,
- the use of structural reliability theory in determining rational sets of partial coefficients, and
- the preparation of model codes for different types of structural materials and forms of construction; and steps towards international code harmonization, particularly within the EC.

In comparison with the idealized models used for calculation purposed, the actual behaviour of most structures is extremely complex and there is a tendency, as more research is undertaken and more becomes known, for the design procedures set out in structural codes to become increasingly lengthy and involved. Such changes generally increase design costs and increase the risk of major errors being made. They cannot be classed as improvements unless the new procedures result in improved standards of safety and/or reduced costs of construction and maintenance.

It is therefore clear that the "best" codes are not necessarily those with the most scientifically advanced design clauses. There may often be advantages in using simplified design rules. The effect of this will be to make the overall construction slightly less economic and the reliability of those structures designed to the code marginally more variable, for any specified standard of reliability.

4.2. Level 1 Codes

Level 1 design methods can be described as "design methods in which appropriate degrees of structural reliability are provided on a structural element basis (occasionally on a structural basis) by the use of a number of partial safety factors (partial coefficients) related to pre-defined characteristic or nominal values of the major structural and loading variables". A level 1 code is therefore a conventional deterministic code in which the nominal strengths of the structural members designed to that code are governed by a number of partial coefficients or by equivalent means.

The safety and serviceability of practical structures are achieved by the use of suitable partial coefficients in design, together with appropriate control measures. Both are essential and it is helpful to distinguish their individual roles.

Let us first examine the role of partial coefficients. Consider a structure subjected to a random time-varying load Q having a specified nominal magnitude q_{sp}. The structure is proportioned to carry a design load $Q_d = \gamma_Q q_{sp}$, where γ_Q is a partial coefficient on live load. The effects of increasing γ_Q by, say, 20% will in general be

- an increase in the nominal capacity of the structure to support the load Q
- an increase in the actual capacity of the structure to support the load Q,
- an increase in the sizes of the structural members and the self-weight of the structure
- an increase in the cost of the structural system
- some increase in the actual capacity of the structure to resist any other load Q´
- an increase in the safety of the structure as characterised by a reduction in the probability that it will fail in any given reference period T .

If the design strength of a materials is given by $e_d = e_{sp} / \gamma_m$, where e_{sp} is the specified material strength and γ_m is a partial coefficient, an increase in γ_m will in general have the same effects as an increase in γ_Q.

There are some circumstances, however, when increases in γ_Q or in γ_m may not give rise to these effects. For example, the actual load-carrying capacity of a structural member, as opposed to its nominal capacity, may decrease or may not significantly increase, if, for example, any change in γ_Q or γ_m results in the designer using larger diameter reinforcing bars which, in spite of having the same specified yield stress as the bars they are replacing, may have a lower mean yield stress. Similarly, small changes in γ_Q or γ_m may sometimes have no effect on either the dimensions or the safety of some structural members. This is because of the discrete nature of many structural components (e.g. rolled steel beams) and the need to round up to the next section size above when designing. In such cases the actual strength, and hence the reliability, is not a continuous function of the partial coefficients.

We now consider the reasons for using partial coefficients as opposed to single safety factors or load factors. The main reason is that only by using partial coefficients can reasonably consistent standards of reliability be achieved over a range of different designs within any one code. The most consistent standards can be achieved by associating a partial coefficient or some other safety element with each major source of uncertainty (i.e. with each basic variable). Partial coefficients are also essential for the rational treatment of load combinations, and in particular for situations in which the total load effect in part of a structure is the difference of two load effects of approximately similar magnitude but originating from different load sources - e.g. the effects of gravity loads and wind loads in the up-wind columns of a tall building.

4.3. Control Measures

The safety and serviceability of a structure are influenced as much, if not more, by the nature of the control measures that are in operation as by the magnitude of the partial coefficients that are used in design. Control takes two main forms

- quality control of materials and fabrication, and
- controls to avoid the occurrence of major gross errors in the design and construction processes.

Control of the first type is aimed at reducing variability in the mechanical properties of structural materials and maintaining appropriate mean properties. For example, the variability in the yield stress of steel can be reduced by improved control on chemical composition and rolling conditions. Such control will, in general, reduce the probability of structural failure and thus increase safety. Both the form and the parameters of the probabilistic models for resistance variables are dependent on the standards of quality control and inspection that are in operation.

Control of the second type is clearly more difficult to achieve since the sources of possible errors are almost unlimited. This is the subject of chapter 5.

We continue here with the problem of devising a suitable procedure for evaluating partial coefficients or other safety elements for a level 1 code. A logical sequence of steps is as follows

- set limits on the range of structures and materials for which the code will be applicable,
- specify the deterministic functional relationships to be used as the basis for each design clause,
- select the general form of the probabilistic models for the various load and resistance variables and model uncertainties,
- specify appropriate quality control measures and acceptance criteria for the manufacture and fabrication of basic materials and components,
- determine the parameters of the relevant models from loading data and from materials data obtained under the specified standards of quality control and inspection,
- select a suitable safety format - the number of partial coefficients and their position in the design equations (i.e. the variables associated with partial coefficients), etc.,
- select appropriate representative values of all basic random variables (e.g. nominal, characteristic or mean values) to be used as fixed deterministic quantities in the code,
- determine the magnitude of the partial coefficients to be used in conjunction with the above representative values to achieve the required standards of reliability.

Procedures such as this have already been used in the application of structural reliability theory to practical level 1 codes. Some of these steps have already been considered in some detail, e.g. the modelling of load and resistance variables, and others, e.g. quality control procedures, are beyond the scope of this presentation. In the remainder of this chapter we shall concentrate on the question of choosing suitable safety formats for structural codes and on the calculation of partial coefficients.

4.4. Safety Formats for Level 1 Codes

The safety format of a code is defined as the way in which the various clauses of the code regulate the degree of safety, or more generally the reliability, of structures designed to the code. In particular, it concerns: the number of partial coefficients or other safety elements to be used, their positions in the design equations, and rules for load combinations.

4.4.1. Limit state functions and checking equations

The general conditions for a limit state not to be exceeded may be expressed as

$$f(X_1, X_2, ..., X_n) = f(\bar{X}) > 0 \tag{6}$$

where $\overline{X}$ are the n basic random variables which influence the limit state, and *f* is the limit state function (failure function). The variables $\overline{X}$ may be sub-divided into variable loads and actions $\overline{Q}$, permanent loads $\overline{G}$, material properties $\overline{E}$, geometrical parameters $\overline{D}$, and model uncertainties $\overline{X_m}$. In addition; each limit state function is likely to involve one or more constants $\overline{c}$. Equation (6) may therefore be re-written as

$$f(\overline{Q},\overline{G},\overline{E},\overline{D},\overline{X_m},\overline{c}) > 0 \tag{7}$$

For the purposes of a level 1 code, the equivalent deterministic criterion for safety checking (i.e. checking the sufficiency of a structure or structural member whose design properties are given) is

$$f(\overline{q_d},\overline{g_d},\overline{e_d},\overline{d_d},\overline{x_{md}},\overline{c}) > 0 \tag{8}$$

where *f* is the same limit state function as above, involving n quantities $\overline{x_d}$, and m constants $\overline{c}$, and $\overline{q}_d$ is the deterministic design value of the random variable $\overline{Q}$, etc.

For many structures it is possible to re-write (7) as

$$X_R r(\overline{E},\overline{D_R},\overline{c}) - X_S s(\overline{Q},\overline{G},\overline{D_S},\overline{c}) > 0 \tag{9}$$

where r represents a resistance function and $R = r(\cdot)$, s represents a load effect or action effect function and $S = s(\cdot)$. X_R is a model uncertainty associated with the particular form of the resistance function, X_S a model uncertainty associated with the particular form of the load effect or action effect function, and where $\overline{D_R}$ and $\overline{D_S}$ are sets of different dimensions.

In equation (9) the resistance function r and the load effect function s are shown as uncoupled; and because they share no common variables the two terms are also statistically independent. If such uncoupling is possible, then the deterministic checking equation corresponding to equation (8) may be expressed as

$$\frac{1}{\gamma_R} x_{Rd} r(\overline{e_d},\overline{d_{Rd}},\overline{c}) - \gamma_S x_{Sd} s(\overline{q_d},\overline{g_d},\overline{d_{Sd}},\overline{c}) > 0 \tag{10}$$

where γ_R is a partial coefficient on the computed resistance, γ_S is a partial coefficient on the computed load effect, and where the subscript *d* denotes the design value of the variable.

Equation (10) is the most general form of the checking equation for a structure in which *R* and *S* can be uncoupled. In this case, the safety or serviceability of a structure can clearly be increased or decreased by adjusting any or all of the (*n* - 1) independent design values $\overline{x_d}$ (e.g. $\overline{q_d}$ or $\overline{e_d}$) and the two partial coefficients γ_R and γ_S. Substituting these values into equation (10) gives the required value of the remaining quantity - generally a dimension. Because there is an infinite number of sets of (*n* - 1) values $\overline{x_d}$ which will give the same design, the problem facing the code writer is to select the "best" set of values $\overline{x_d}$.

It should be noted that in practice the quantities *R* and *S* may often be correlated because of common parameters. For example, the self-weight of a reinforced concrete beam and hence the mid-span bending moment *S* will be weakly correlated with the beam's moment-carrying capacity *R*, as both are functions of beam depth.

4.4.2. Characteristic values of basic variables

The term characteristic value was introduced in the late 1950's at the time when probabilistic concepts were first being introduced into structural codes; and when it was recognised that few basic variables have clearly defined upper or lower limits that can sensibly be used in design. Characteristic values of actions and material properties based on a prescribed probability p of not being exceeded were considered to be more rational than arbitrary selected values.

The characteristic value x_k of a basic random variable X is defined as the p^{th} fractile of X given by

$$x_k = F_X^{-1}(p) \tag{11}$$

where F_X^{-1} is the inverse distribution function of X, and p a probability which depends on the type of variable being considered (i.e. a load or a strength).

The selection of the probability p is to a large extent arbitrary but is influenced by the following considerations

- characteristic values of loads and other actions are values which should rarely be exceeded,
- characteristic values of material strength properties should normally be exceeded by actual properties,
- the values of p should neither be so large nor so small that the values x_k are not occasionally encountered,
- it is often sensible to use previously adopted nominal values as specified characteristic values, x_{sp}.

The distinction between characteristic value and specified characteristic value (specified value) should be made clear. The former is a fractile of a random variable, whereas the latter is some specified single value of the same quantity - a constant. For practical reasons it is generally necessary for the user of a level 1 code to work with specified values of all the design variables rather than with actual characteristic values, some of which will not be known at the design stage. For example, the actual characteristic value of the 28-day cube or cylinder strength of concrete is likely to depend on the particular supplier or contractor and is not known in advance. In this case it is necessary for the quality control procedures specified by the code writers to be such that the actual characteristic strength of the material exceeds the specified strength by an appropriate margin or with a stated probability. Similarly, the user of a code should normally work with specified deterministic values of loads and other actions; it is the responsibility of the code writers to relate these values to the distributions of the actual loads and actions, and to recommend associated partial coefficients or other safety elements.

4.4.3. Treatment of geometrical variables

Geometrical variables are of two main types - structural dimensions (e.g. the depth of a beam) and geometrical imperfections (e.g. the out-of-straightness of a column).

Structural dimensions: The uncertainties in most structural dimensions D are generally small and for this reason the mean value μ_D may be taken as the characteristic value (i.e. $d_k = \mu_D$).

Geometrical imperfections: The strength of many structural members, for example most plates, columns and shell structures, depends not only on cross-sectional and overall dimensions but also on the magnitude of relevant geometrical imperfections *I*.

For such structures it is normal to specify an upper limit ε on the imperfection magnitude, i.e. $0 \le I \le \varepsilon$. In this case, ε can be taken as the specified characteristic value of *I*, I_{sp}. The probability that I_{sp} will be exceeded will generally be small and will depend on the standard of inspection. The actual characteristic value of the imperfection i_k can conveniently be chosen as the 95% fractile of I and the acceptance criteria designed so that I_{sp} exceeds i_k by an appropriate margin (or with a stated probability).

Design values of dimensions and imperfections: Typically, the standard deviations of geometrical variables are independent of nominal dimensions (e.g. for given site conditions, the standard deviation in the thickness of a 100 mm slab is likely to be about the same as that of a 200 mm slab; giving a reduction in the coefficient of variation for increasing nominal thickness). For this reason the most uniform standards of reliability can be obtained over a range of different structures by using design values d_d and i_d of the geometrical variables related to the specified values, as follows $d_d = d_{sp} \pm \Delta_d$ and $i_d = i_{sp} \pm \Delta_i$ where Δ_d and Δ_i are additive safety elements.

For many structures, however, the probability of failure is insensitive to small variations in structural dimensions. For these cases, Δ_d and Δ_i should be set to zero and the uncertainties in *D* and *I* should be allowed for by modifications to the partial coefficients on the other design variables.

4.4.4. Treatment of material properties

We shall restrict our attention to the strength properties of structural materials, denoted *E*. For each variable, the characteristic value e_k should be such that it has a reasonably high probability *q* (= 1 - *p*) of being exceeded in any single trial or test. Typically, *q* is taken to be between 0.95 and 0.99, corresponding to the 5% and 1% fractile of the variable *E*. However, the user of a level 1 code may often not know the actual characteristic values for his material properties in advance, and it is generally necessary to design using specified characteristic values, e_{sp}. The acceptance criteria for a material should be devised so that e_k exceeds e_{sp} at a stated level of probability p_e.

The design value e_d of the strength of a material is obtained from the specified or characteristic strength as follows $e_d = e_{sp} / \gamma_m$ where γ_m is a partial coefficient on strength.

4.4.5 Treatment of loads and other actions

Most loads differ from other basic variables in that they vary significantly with time and are generally not amenable to effective control. There are some notable exceptions to both these generalisations. Because of the time-varying nature of most loads, the problem of assessing the combined effect of a number of different loads acting on a structure has arise. As might be expected a rather similar problem arises in treating combined loads within the framework of a deterministic level 1 code.

Characteristic values: The uncertainty in most permanent loads is small and for this reason it is customary to use the mean or nominal values of permanent loads in most design calculations. For the same reason it is appropriate that the characteristic value g_k

of each permanent load *G* is taken as its mean value μ_G. μ_G may be considered by using mean dimensions and mean densities.

For a time-varying load *Q*, the characteristic value q_k is normally defined as that value which has a prescribed probability p of not being exceeded within a given reference period. It is therefore the pth fractile of the extreme value distribution of the load corresponding to that reference period. The nominal loads specified in most loading codes vary rather widely in terms of their probability of exceedance.

Single time-varying loads: If a structure or structural component is subjected to only permanent loads *G* and one time-varying load *Q*, the load-combination problem does not arise. In this case, the values g_d and q_d to be used in the design or safety checking process are obtained from $g_d = \gamma_{fG} g_k$ and $q_d = \gamma_{fQ} q_k$ where γ_{fG} and γ_{fQ} are partial coefficients and g_k and q_k are characteristic values of the random variables *G* and *Q*, respectively.

For failure modes in which part of the permanent load acts in a stabilising or resisting sense and part in a de-stabilising or loading sense, different values of γ_{fG} should be used for the two components; $\gamma_{fG} \leq 1$ when the load is stabilising the structure $\gamma_{fG} \geq 1$ when it is not.

Combinations of time-varying loads: When a structure has to resist a number of stochastically independent time-varying loads, it is clear that the probability of two or more loads exceeding their characteristic values simultaneously is small. If the total load effect in a member were to be determined from the characteristic values of the individual loads multiplied by the corresponding partial coefficients then the resulting load effect *S* would be extremely conservative. For this reason it is necessary to introduce a set of reduction factors ψ_{0i} $(\psi_{0i} \leq 1)$ to be applied to the time-varying loads Q_i to take account of the reduced probability of the design values of the loads being exceeded simultaneously.

In principle, if there are n time-varying loads, it is necessary to undertake *n* design checks (equation (13.6)) on the structure, using a separate set of ψ_0 factors for each check and with ψ_{0j} = 1 for the j^{th} check. The need for a number of design checks using different sets of ψ_0 factors arises from the fact that throughout a structure the contribution of each separate load Q_i to the maximum load-effect in any member varies considerably from member to member. For example, although snow loading may dominate the load effect in the roof beams of a multi-storey building, the same loads have only a small influence on the total load effects in the ground floor columns. In practice, with detailed knowledge of the structure being designed or checked, it is often possible to reduce the number of checks significantly.

4.5. Methods for Evaluation of Partial Coefficients

4.5.1. Introduction

The design clauses given in level 1 codes should be interpreted as a set of decision rules, the outcome of which can be modified by changes to a set of control parameters - the partial coefficients. The process of selecting the set of partial coefficients to be used in a particular code should be seen as a process of optimisation such that the outcome of all designs undertaken to the code is in some sense optimal. Whether or not a formal optimisation is undertaken in practice, it is useful to think of the partial coefficient selection process in this way. It is then clear that the possibility exists for using any simplified set of design clauses together with a modified set of partial coefficients, which on average will achieve the same degree of safety as the more complex set. The penalty to be paid for using the simplified design rules is some increase in materials usage.

In Thoft-Christensen & Baker [3] is detailed shown a general method for evaluation of partial coefficients. Practical codes should have the smallest number of partial coefficients that is consistent with reasonably uniform standards of reliability; moreover, the same partial coefficients should be applicable to a wide range of structural components. This means that they must be applicable without being unsafe or unduly conservative.

4.5.2. A General Method for Code Calibration

A suitable general method for the evaluation of a set of partial coefficients is now presented. The first stage of this process is to decide upon an appropriate standard of reliability or target failure probability for the structures (or more generally, structural components, e.g. beams, columns, slabs) that will be designed using the new code.

Studies of the reliability of structural components designed to traditional codes typically show very wide ranges of reliability. An appropriate choice for the target failure probability P_{ft} for a new code is the weighted average of the failure probabilities exhibited by components designed to existing codes, provided that the least reliable component exhibited satisfactory performance in actual service. The latter is not always easy to verify because existing codes may not have been in use for a sufficiently long period of time and structures may have been subjected to only a fraction of their design loads. The weighting factors ω_i i should be selected to represent the previous frequency of usage of each structural component included in the calibration and should be such that $\sum_i \omega_i = 1$.

Use of the weighted average failure probability rather than, say, the weighted average reliability index means that the target failure probability P_{ft} tends to be governed by the less reliable components in existing codes. This assumed a measure of economy in the new code, but care has to be taken that these reliabilities are not too low. A more direct approach to the choice of target failure probabilities has been recommended by the Nordic Committee on Building Regulations (NKB) In this, the target failure probability depends on the consequences of failure and on the nature of the failure mode, as shown in table 2.

Failure consequences	Failure type I		Failure type II		Failure type III	
Not serious	10^{-3}	3.09	10^{-4}	3.71	10^{-5}	4.26
Serious	10^{-4}	3.71	10^{-5}	4.26	10^{-6}	4.75
Very serious	10^{-5}	4.26	10^{-6}	4.75	10^{-7}	5.20

Table 2. Target failure probabilities and corresponding reliability indices .

The target failure probabilities given in table 2 are for a reference period of 1 year, but should be treated as operational or notional probabilities and not as relative frequencies. The failure types are defined as:

I ductile failure with reserve strength capacity resulting from strain hardening
II ductile failure with no reserve capacity
III brittle failure and instability

Having chosen a target failure probability, the problem of selecting a set of partial coefficients $\bar{\gamma}$ for a code, or part of a code, may now be reduced to the following optimisation problem

$$\min_{\bar{\gamma}} S = \min_{\bar{\gamma}} \sum_{i=1}^{m} \omega_i \Delta(P_{fi}(\bar{\gamma}), P_{fi}) \qquad (12)$$
$$s.t. \ \sum_{i=1}^{m} \omega_i P_{fi}(\bar{\gamma}) = P_{ft} \ , \ \sum_{i=1}^{m} \omega_i = 1.0$$

where $\Delta(P_{fi}(\bar{\gamma}), P_{fi})$ is an agreed function of $P_{fi}(\bar{\gamma})$ (the failure probability of the i^{th} structural component designed using the set of partial coefficients $\bar{\gamma}$) and P_{ft} (the target failure probability), and $\bar{\omega} = (\omega_1, \ldots \omega_m)$ (the set of weighting factors indicating the relative importance of each of the m structural components included in the partial factor evaluation).

In general terms, the aim of this approach is to minimise the deviations of the probabilities P_{fi} from the target probability of failure P_{ft}, whilst maintaining the average probability of failure at the target level. Experience has shown that the values of the partial coefficients are generally very insensitive to the form of the objective function used in equation (12). A suitable function is:

$$S = \sum_{i=1}^{m} \omega_i (\beta_i(\bar{\gamma}) - \beta_t)^2 \qquad (13)$$

where β is the reliability index.

5. GROSS ERRORS AND QUALITY ASSURANCE

5.1. Introduction

In undertaking a reliability analysis, the engineer should take account of all known sources of uncertainty and should use this information to control the probabilities of structural failure and unserviceability within acceptable ranges. This can be done either directly, by modifying some part of the structure, or indirectly by modifying the partial coefficients. Allowance should be made for the possibility of the occurrence of all recognised failure modes, e.g. shear, buckling, plastic collapse, etc., together with various modes of unserviceability.

It is widely recognised, however, that most structural failures occur for unexpected reasons and in ways that have not previously been encountered. No discussion of structural reliability theory is therefore complete without some consideration being given to these additional causes of failure and their possible treatment.

5.2. Gross Errors

In the preceding chapters probabilistic modelling of loads and resistance variables is discussed. These models are selected or devised in such a way as to embody those features of the physical quantity that are essential for the analysis of the practical problems being considered. it should not be thought that the models are intended to be a perfect mirror image of reality, but rather as a "tool" in a decision making process. Depending on the nature of the decision to be taken, the "tool" may need to be changed. For example, in modelling the yield stress of steel, a simple lognormal distribution may often be used (see section 3), but on other occasions a mixed distribution model would be more appropriate.

One important assumption that has been made is that the probabilistic models for loads and resistance variables are representative of events during a particular period of time - the life of the structure in the case of loads and other actions, and the period of construction in the case of material properties. In fact, the models are conditional upon or pre-suppose certain standards of design checking, quality control, inspection and maintenance. If, for example, the standards of quality control used in the manufacture of a structural material change, one would expect to see some change in the probability distribution function of that variable. If the standards of quality control are significantly different between different manufacturers or suppliers (e.g. in the case of steel or concrete), it may be convenient to use a mixed distribution model to allow for these differences.

The problems that need to be considered here, however, are of a different nature. The vast majority of structural failures occur because of gross errors. A gross error is defined as a major or fundamental mistake in some aspect of the processes of planning, design, analysis, construction, use or maintenance of a structure that has the potential for causing failure. Gross errors occur because of inadequacies in the standards of quality assurance - the process by which the various components of complete "building process" (mentioned above) are co-ordinated with the aim of achieving the design objective. It should be noted, however, that not all gross errors make as structure weaker - they can also make it unnecessarily strong. Typical examples of gross errors are mistaken in design calculations, use of the wrong size of reinforcing bars or grade of steel, misinterpretations of geotechnical survey data, subjecting the structure to a class of loading for which it was not intended, etc.

A gross error should not therefore be considered as some extreme value in the tail of the probability distribution used to model a particular random variable, but a discrete event G that radically alters the probability of failure by changing the models that are applicable.

Example 3. Assume, for the sake of simplicity, that a structure has a resistance *R* which is dependent on only one basic variable, the yield stress of steel, and that it is subjected to a single load effect *S*. For simplicity, let *R* and *S* be normally distributed with

$\mu_R = 380 \;\; N/mm^2$; $\sigma_R = 25 \;\; N/mm^2$

$\mu_S = 230 \;\; N/mm^2$; $\sigma_S = 43 \;\; N/mm^2$.

Then it can easily be shown that $\beta = 3.02$ and $P_f = 1.3 \times 10^{-3}$. Assume now that a lower grade of steel is used in place of the correct grade, so that *R* is distributed with parameters $\mu_R' = 280 \;\; N/mm^2$ and $\sigma_R' = 25 \;\; N/mm^2$ giving $\beta' = 1.00$ and $P_f = 0.16$.

The values of μ_R and σ_R given above are typical for the yield stress of grade 50 and 43 weld able structural steels, and it can therefore be seen that a gross error involving the substitution of grade 43 steel for grade 50 in a critical part of a structure is quite likely to cause failure - a chance of about 1 in 6 with the arbitrary assumptions made here.

The preceding example is somewhat simplistic and little attention should be paid to the actual numbers used. However, it illustrates the principle that the models, which are used under normal conditions, without the presence of gross errors, are no longer applicable when a gross error occurs. This does not mean that reliability theory cannot be used under these conditions - it means that the models have to be amended. The problem is to know the various forms, which gross errors can take.

5.3. Classification of gross errors

Table 3 gives a general classification of the nature and sources of gross errors, along with some examples. No such list can, of course, be comprehensive. Indeed, by their very nature, some potential gross errors or hazardous situations must exist which have not yet been recognised. Ignorance of phenomena such as fatigue, brittle fracture and the deterioration of concrete made from high-alumina cement are typical examples from the past of errors in design concept. Early designers cannot be criticised for not knowing about such effects, just as there is no reason to suppose that currently unrecognised failure modes will not cause accidents in the future.

Those gross errors, which cause structural collapse or unserviceability, can also be classified according to the type of failure that occurs, as shown in table 4. The word failure, here, should be interpreted in the general sense of failure to comply with some performance requirement and not just collapse (i.e. it includes unserviceability). Two types are given: those in which the structure fails in a predictable manner by one of a number of foreseen failure modes - here called type A; and those in which unforeseen failure modes occur - called type B.

Source	**Nature**	**Example**
Design concept	- Possible failure mode unrecognised - Incorrect nature of use assumed	- neglect of lateral torsional buckling - room used for storage of heavy equipment in office premises
Design and analysis	- Omission of a load or load combination - Misinterpretation of geotechnical Data - Computational error in analysis - Misinterpretation of units - Error in detailing	- effect of ground-water pressure overlooked - soft stratum not detected - error in computer program - kilogram's interpreted as Newtons - 20 mm bars used instead of 40 mm bars
Construc-tion	- Misinterpretation of drawings - Use of incorrect material - Incorrect farication - Incorrect construction	- 100mm slab instead of 150 mm slab - grade 43 steel used instead of grade 50 steel - omission of heat treatment - error in position of reinforcement
Inspection	- Gross defect not detected	- crack in weld
Use	- Accidental loading - Change of use without structural assessment - Need for specialist maintenance overlooked	- severe impact or explosion - domestic premises used for public library - cathodic protection system becomes inoperative

Table 3. General Classification of the nature and sources of gross errors.

Type of failure	Gross errors
Type A: Failure in a mode of behaviour against which the structure was designed	*Errors affecting:* - load-carrying capacity - ability to remain serviceable - applied load(s)
Type B: Failure in a mode of behaviour against which the structure was not designed	*Errors that relate to the fundamental understanding of structural behaviour, arising from:* - profession’s ignorance - engineers’ ignorance - engineers’ oversight

Table 4. Classification of gross errors according to type of failure.

Failures of type A arise because of gross errors in the calculation of design loads and/or load-carrying capacities, and/or because of weaknesses, which are introduced into the structure during construction. Most of the examples in table 3 are of this type. Failures of type B occur mainly because of lack of knowledge. However, distinction must be made between failures, which are the result of ignorance within the structural engineering profession as a whole, and those that occur because of ignorance or negligence by an individual or design team. In the latter case, the engineer or team is clearly responsible for the failure. In the former, he or they are just unfortunate, unless it can be shown that currently accepted practice has been extrapolated to an unreasonable extent.

The prevention of failures, which arise from lack of knowledge within the profession, as a whole is clearly impossible and occasional failures of this type, will continue to occur. They will then be researched and this will add to the general fund of engineering knowledge. Failures of type A, and failures of type B resulting from an engineer's ignorance or negligence, are in theory preventable, but this requires an appropriate level of expenditure on education, training, design checking, quality control, inspection, maintenance, etc. The planning and coordination of these various tasks is the subject of quality assurance.

Tables 3 and 4 show only two of many possible ways of classifying gross errors. For instance, errors can be classified according to:

- nature of the error (table 3)
- type of failure associated with the error (table 4)
- consequences of failure arising from the error
- those responsible for causing the error
- etc.

Considerable success has been achieved in the analysis of structural failure data using this type of classification. An important conclusion is that many gross errors occur because of lack of experience on the part of those undertaking the work and because the fundamental behaviour of the structure is often not fully understood. Figure 2 illustrates this point in a study of 120 building failures, it was found that over 60% were due to lack of experience by the personnel concerned and that in about 50% of all cases the major cause of failure was a lack of appreciation of the relevant design concepts - for example, ignorance of the need to design against lateral torsional buckling in unsupported compression flanges. Such findings are of considerable value in the planning of quality assurance schemes, but this will not be discussed here.

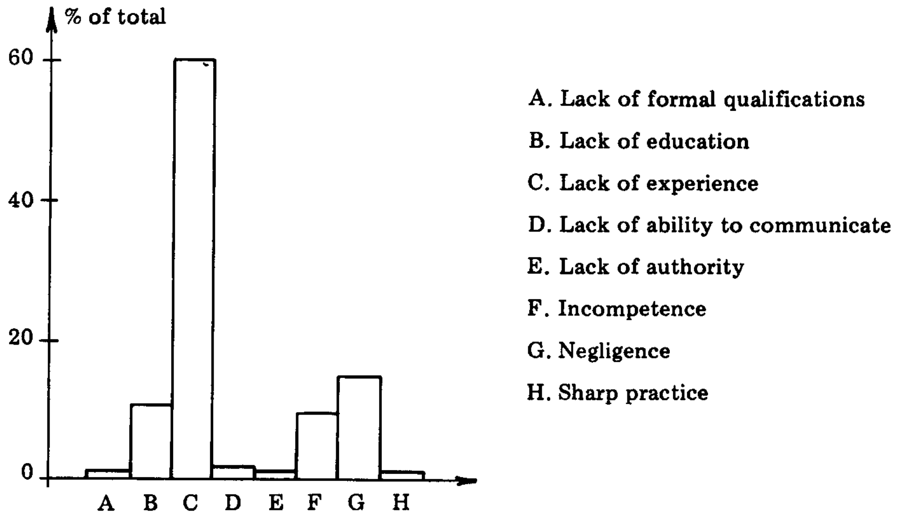

Figure 2. Analysis of underlying causes of structural failures in buildings, from Thoft-Christensen & Baker [3].

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5.4. Quality Assurance

5.4.1. Introduction

The phrase "building process" has been used in the preceding sections as a general term to include planning, design, analysis, construction, maintenance and use of a structure. It will also be used here, to cover all aspects and stages of a structural development, but it should be emphasized that it is applicable to all structural projects, not just buildings.

For a given structure and location, the building process can be divided into two distinct stages:

1) preparing a precise deterministic specification for the structure, and
2) building the structure and checking that the specification is met.

The specification, involving documents and drawings, will typically contain information of the type:

- the column shall contain 16 reinforcing bars of diameter 40 mm
- the nominal thickness of the slab shall be 200 mm
- the structural steel shall have a nominal yield stress of 350 n/mm^2
- etc.

This information, which is passed from the designer to the contractor (or builder), is and needs to be of a deterministic nature. The second part of the building process involves the transformation of the specification into physical reality - the structure – and checking that it is satisfactory.

For new structures, the role played by reliability theory is in the preparation of the structural specification, either directly, by subjecting the proposed design to a reliability analysis or, indirectly, by using a code in which the partial coefficients have been assessed probabilistically. As previously mentioned (for example in chapter 3), the probabilistic models used in reliability analysis are conditional upon the specified standards of quality control and acceptance tests for the materials, and on the standards of inspection for the finished structure. Stages 1) and 2) of the building process are (or should be) intimately related. The role of quality assurance, in its broadest sense, is to coordinate, rationalise and monitor these two stages.

Reliability theory also has a role to play in the assessment of existing structures, particularly when structural damage has occurred as a result of accidental loading, or when a structure is being assessed for a radical change of use. it should be noted that in all cases, the questions to be answered are of the type: Is the structure strong enough? Should the nominal dimensions be increased or reduced, and if so by how much?

5.4.2. The effect of gross errors on the choice of partial coefficients

The problem that must now be considered is whether knowledge that gross errors can occur during the processes of design and construction should affect the rational choice of partial coefficients for use in level 1 codes. This is best explored by means of an example.

Example 4. Take a failure function of the form $f(G,R,K,S) = 0$ and let the safety margin be

$$M = GR - KS \tag{14}$$

where R is a continuously distributed random strength variable, $\mathrm{N}(\mu_R, \sigma_R)$, S is a continuously distributed random load effect, $\mathrm{N}(\mu_S, \sigma_S)$, K is a discrete-valued model uncertainty, and G is a gross error which modifies the strength parameter R, and assume that the quantities R, S, K, and G are statistically independent. This is a reasonable assumption, as least as far as independence of G and the other variables is concerned, since it may be assumed, for example, that the probability of having an incorrect size of reinforcing bars is unrelated to the yield stress of the bars or to the loads that are subsequently applied to the structure.

Let the initial cost of the structure be given by $c_I = \alpha\mu_R$ where alpha is a constant, and let the consequential cost of failure, should it occur, be $c_f = \xi c_I = \xi\alpha\mu_R$.

The probability of failure, given a gross error of magnitude g and a model uncertainty of magnitude k, may be expressed as $P_f \left| g,k = P(gR - kS \le 0) \right.$ and the expected conditional total cost as $E\left[C_T \left| g,k \right.\right] = c_I + c_f P_f \left| g,k \right.$. Assuming that the model uncertainty K can take the following discrete values with probability mass $p(k_i)$

k_i	0.4	0.5	0.6
$p(k_i)$	0.2	0.6	0.2

Table 5.

The expected total cost, given a gross error g, $E\left[C_T \left| g \right.\right]$, is

$$E\left[C_T \left| g \right.\right] = \sum_{i=1}^{3} (c_I + c_f P_f \left| g \right.) p(k) \tag{15}$$

The expected total cost given that there is a gross error g with a probability *p(g)*, and no error (g = 1) with a probability 1 - p(g) is thus

$$E\left[C_T\,|g\right]=\sum_{i=1}^{3}(c_I+c_fP_f\,|g)p(k)p(g)+\sum_{i=1}^{3}(c_I+c_fP_f)p(k)(1-p(g)) \qquad (16)$$

Substituting for c_I and c_f from gives

$$E\left[C_T\,|g\right]=\sum_{i=1}^{3}\alpha\mu_R(1+\xi P_f\,|g)p(k)p(g)+\sum_{i=1}^{3}\alpha\mu_R(1+\xi P_f)p(k)(1-p(g)) \qquad (17)$$

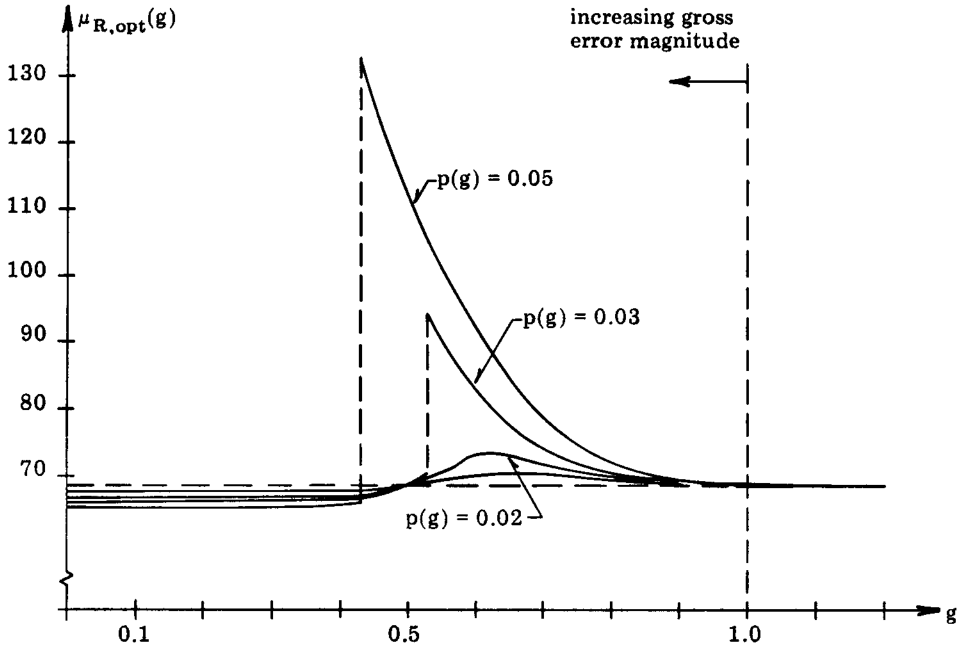

Figure 3.

Let us now undertake an unconstrained minimisation of $E\left[C_T(g)\right]$ with respect to the quantity μ_R and denote the minimum value of μ_R by $\mu_{R,opt}(g)$. This is shown in figure 3 for the set of parameters given in table 6.

Variable	μ	σ/μ
R	-	0.07
S	100	0.10
α =10 cost units		
ξ =20		

Table 6.

An important result for this particular set of assumptions is that the quality assurance process is able to restrict the frequency of gross errors to less than 2%, then $\mu_{R,opt}$ is very insensitive to the occurrence of gross errors of any magnitude. A more important result, however, is that even if the frequency of gross errors rises to say 5%, although $\mu_{R,opt}$ shows a marked increase for gross errors of moderate magnitude, the expected total cost given by

$$E\left[C_T(g)\middle|\mu_{R,opt}\right] = \sum_{i=1}^{3}(c_I + c_f P_f\left|g, \mu_{opt}\right.)\,p(k)\,p(g) + \sum_{i=1}^{3}(c_I + c_f P_f\left|\mu_{R,opt}\right.)\,p(k)(1-p(g)) \tag{18}$$

is insensitive to the decision of whether or not to allow for the possibility of gross errors in calculating $\mu_{R,opt}$ (see figure 4). At the worst, the total expected cost differs by only 15%.

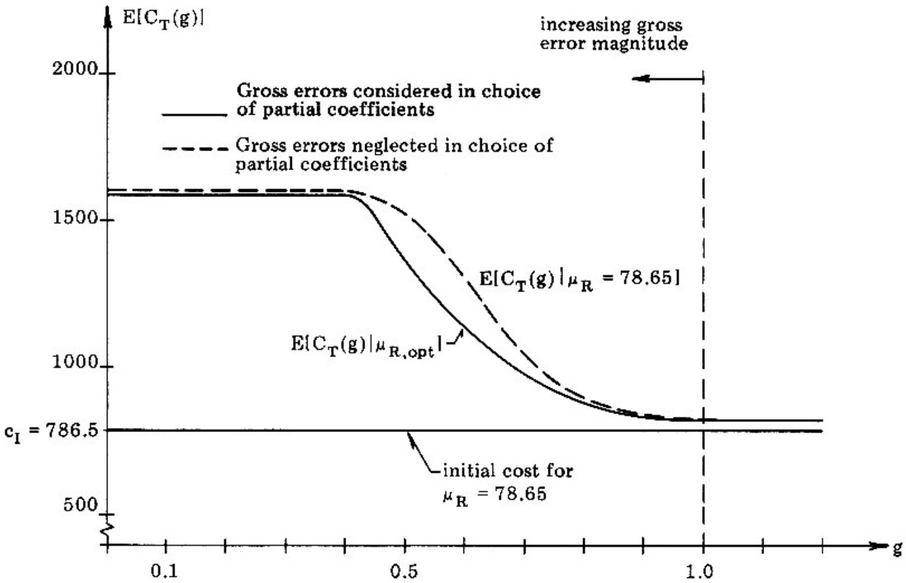

Figure 4. (p(g) = 0.05).

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$$E\left[C_T(g)\middle|\mu_{R,opt}\right] = \sum_{i=1}^{3}(c_I + c_f P_f\left|g, \mu_{opt}\right.)\,p(k)\,p(g) + \sum_{i=1}^{3}(c_I + c_f P_f\left|\mu_{R,opt}\right.)\,p(k)(1-p(g)) \tag{18}$$

is insensitive to the decision of whether or not to allow for the possibility of gross errors in calculating $\mu_{R,opt}$ (see figure 4). At the worst, the total expected cost differs by only 15%.

This example indicates that, at least for the set of models and parameters chosen, the possibility of the occurrence of gross errors should not influence the selection of partial coefficients for use in structural design. The extent to which these results can be generalized depends on circumstances, but it is considered that under many conditions the optimisation of expenditure on the control of gross errors can be undertaken independently of the choice of partial coefficients. Probabilistically based structural integrity analysis has been used for many years in the nuclear industry. It started as early as 1980-81 with the computer code PRAISE (Piping Reliability Analysis Including Seismic Events) developed by the United States Nuclear Regulatory Commission. Later it was expanded to include stress corrosion cracking initiation and growth to failure. Since then more similar codes have been developed.

The basic principle in these codes is to use the deterministic equations but to apply statistical distributions to uncertain quantities. Thereby, instead of giving a single yes or no statement to any given failure criterion (the deterministic approach), the probabilistic approach provides a probability of that criteria being met.

Probability of Failure and Fatigue Life Monitoring for Life Extension: In-service monitoring of transients may in certain cases be used to remove uncertainties related to operation of nuclear plants. From a safety standpoint this will provide a high level of coincidence that no unknown could give rise to possible failure. This knowledge means that a more accurate assessment of the so-called true ASME fatigue usage factor can be made. In-service monitoring combined with service inspection is also of great importance in questions like life extension of a nuclear plant.

The respective values of reliability analysis and quality assurance have been explored earlier in this chapter and have been show to be entirely compatible. The analysis of many structural failures shows that the majority could not have been prevented by minor increases in partial coefficients. This is consistent with the results obtained from example 4 and indicates that relatively more resources should be deployed on control, inspection and checking - i.e. quality assurance. A discussion of this large subject is beyond the scope of the present text.

6. INDUSTRIAL APPLICATIONS OF STRUCTURAL RELIABILITY THEORY

A number of industrial applications of structural reliability theory are given in Thoft-Christensen [3] and Thoft-Christensen [4]. This chapter is based on these two references.

6.1. Nuclear Installations

This section is based on a paper by Chapman, Pitner & Persoz [5] where five applications of structural reliability in the nuclear industry are presented:

- Probability of Failure and Fatigue Life Monitoring for Life Extension
- Maintenance Optimisation and Life Prediction of Steam Generator Tube Bundle
- ISI (In-Service Inspection) as a Confidence Builder
- Integrity Analyses of Reactor Pressure Vessel subjected to Transient Conditions
- Risk Based In-Service Inspection

Probabilistically based structural integrity analysis has been used for many years in the nuclear industry. It started as early as 1980-81 with the computer code PRAISE (Piping Reliability Analysis Including Seismic Events) developed by the United States Nuclear Regulatory Commission. Later it was expanded to include stress corrosion cracking initiation and growth to failure. Since then more similar codes have been developed.

The basic principle in these codes is to use the deterministic equations but to apply statistical distributions to uncertain quantities. Thereby, instead of giving a single yes or no statement to any given failure criterion (the deterministic approach), the probabilistic approach provides a probability of that criteria being met.

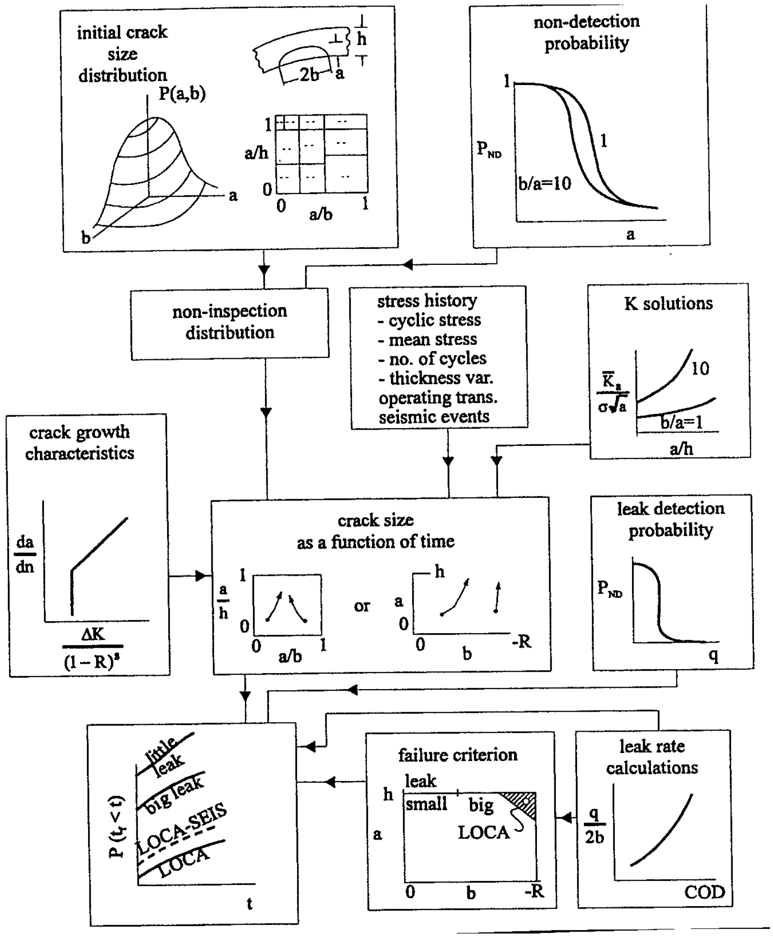

Figure 5. Schematic diagram of steps in analysis of reliability of PRAISE code. Taken from Chapman, Pitner & Persoz [5].

Probability of Failure and Fatigue Life Monitoring for Life Extension: In-service monitoring of transients may in certain cases be used to remove uncertainties related to operation of nuclear plants. From a safety standpoint this will provide a high level of coincidence that no unknown could give rise to possible failure. This knowledge means that a more accurate assessment of the so-called true ASME fatigue usage factor can be made. In-service monitoring combined with service inspection is also of great importance in questions like life extension of a nuclear plant.

Maintenance Optimisation and Life Prediction of Steam Generator Tube Bundle: A serious problem for steam generators (SG) in pressurised water reactors (PWR) is degradation of their tubes. The main causes are corrosion and defect mechanical mechanisms. Therefore, there is a need for a very high degree of reliability. In-service inspections are used as a means of detecting and tracking defects before they reach a critical size

Two main objectives are ascribed to SG tube maintenance:

- From the standpoint of safety, keeping the probability of a tube rupture at a very low level imposed by the design rules.

- From the standpoint of availability, limiting the number of shutdowns caused by primary-to-secondary leakages in excess of specifications.

In Chapman, Pitner & Persoz [5] two significant advantages using probabilistic fracture mechanics to assess the risk of failure of a tube are mentioned:

- All factors of influence can be taken into account as random variables, rather than using pessimistic, conservative, assumptions.
- The influence of maintenance and inspection on risk of failure can be quantified.

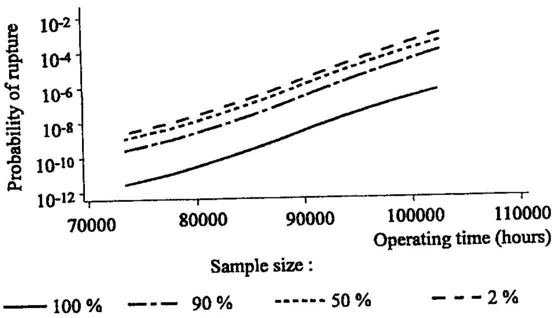

Figure 6. Relative influence of the sample size on the risk of tube rupture. Taken from Chapman, Pitner & Persoz [5].

In Chapman, Pitner & Persoz [5] a number of interesting issues are investigated. Two will be mentioned here. Figure 6 shows the evolution of risks estimated for normal operating conditions with different random sample percentages. Note the small difference between inspection of one tube in eight (12.5%) and in one tube in two (50%). However, a 100% inspection is very effective at detecting and plugging large critical flaws. Figure 7 shows the degree of conservatism in the evaluation of the safety level when worst case values (deterministic approach) are used for the key parameters of the failure analysis.

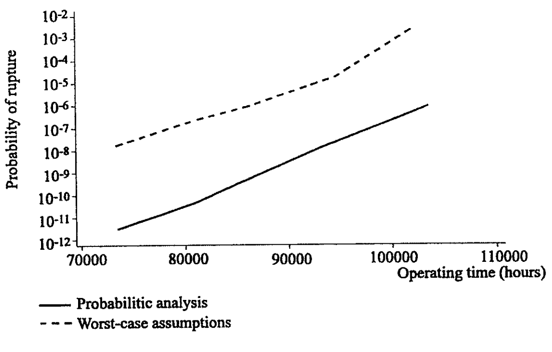

Figure 7. Relative benefit of the probabilistic approach applied to an evaluation of critical crack size for normal operating conditions. Taken from Chapman, Pitner & Persoz [5].

ISI as a Confidence Builder: In-Service Inspection (ISI) is an essential way of demonstrating that an operating plant is safe. Reference Chapman, Pitner & Persoz [5] presents a probability of failure model, which accepts that input parameters are not precisely defined. In the model these dates are considered unknown and the object of the ISI data is to obtain information about them.

Integrity Analyses of Reactor Pressure Vessel subjected to Transient Conditions: The reactor pressure vessel is of major importance for the safety of a PWR nuclear plant. Therefore, the integrity of this structure must be guaranteed, even in the case of the most severe accidents. Further its mechanical state can be decisive for the lifetime of the plant. The vessel wall is one of the most affected zones by embrittlement due to neutron radiation damage. In reference Chapman, Pitner & Persoz [5] a probabilistic fracture mechanics model is described which can be used to:

- estimate the likelihood of vessel failure and perform sensitivity studies,
- identify the relative influence of key input parameters,
- assess the effect of conservatism introduced in deterministic analysis,
- help to define acceptability in-service criteria.

Risk Based In-Service Inspection. In reference Chapman, Pitner & Persoz [5] the application of software for structural reliability risk assessment developed by Rolls-Royce & Associated Ltd., UK, is described. As an example the application on vessel welds is shown in figure 8. The probability of failure per year of a vessel is approximately 2.2×10^{-8} and is dominated by a circumferential weld. However, including the smaller details of the condensate increase this very low probability of failure to 2.5×10^{-6}. The results of these assessments show that the probability of failure depends significantly on the vessel details.

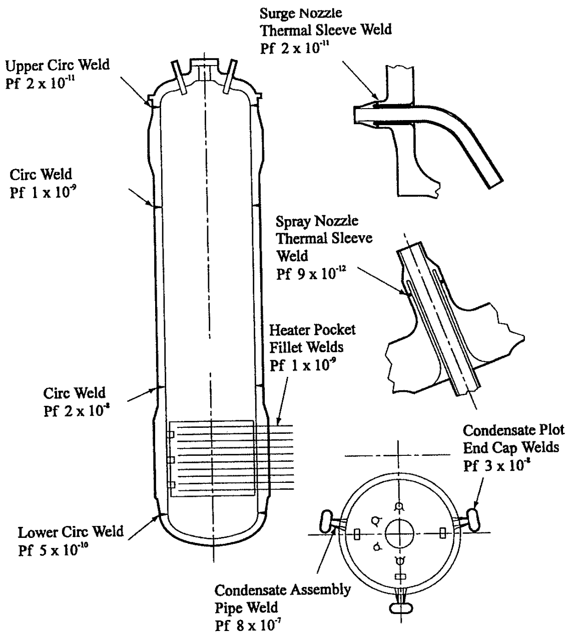

Figure 8. Evaluation of the probability of failure for vessel welds. Taken from Chapman, Pitner & Persoz [5].

6.2. Offshore - Pipeline Free Span Design

This section is based on Hagen & Mørk [6]. Offshore sub sea pipelines are often installed in areas of irregular seabed topography and in deep water. Therefore, free spans in such pipeline systems may occur and submarine currents or wave induced flow velocities may cause significant dynamic excitation of the free span sections. Further, amplified response due to resonant fluid-structure interaction may cause fatigue damage. In 1994 the joint industry R&D MULTISPAN project was established by a number of companies and institutions to provide rational criteria for free spans. Appropriate target safety levels are fundamental to the process of developing new design criteria through the application of reliability methods. In the DNV design rules from 1996 the target safety levels shown in table 7 are recommended.

Limit State Category	Probability Bases	Safety Classes		
		Low	Normal	High
Serviceability	Annual per Pipeline[1)]	10^{-2}	10^{-3}	10^{-3}
Ultimate	Annual per Pipeline[1)]	10^{-3}	10^{-4}	10^{-5}
Fatigue	Lifetime probability per Pipeline[2)]	10^{-3}	10^{-4}	10^{-5}
Accidental	Annual per km[3)]	10^{-4}	10^{-5}	10^{-6}

1 Or the length of the period in the temporary phase
2 No inspection and repair is assumed, temporary and in-service conditions considered together.
3 Refers to the overall allowable probability of severe consequences.

Table 7. Recommended target safety levels in DNV'96. Taken from Hagen & Mørk (1997).

In the DNV'96 rules for submarine pipeline systems new safety classes are defined by:

- *Low Safety Class,* where failure implies no risk of human injury and minor environmental and economic consequences.
- *Normal Safety Class,* i.e. normal classification.
- *High safety Class,* where failure implies risk of human injury, significant environmental pollution or very high economic or political consequences.

A detailed presentation of a reliability based calibration study is given in Hagen & Mørk [6]. One result of the calibration study is shown in figure 9. The five lines in figure 9 correspond to five design cases for the annual extreme current distribution and, the stars indicate proposed safety factors for annual probability of failure for the three safety classes.

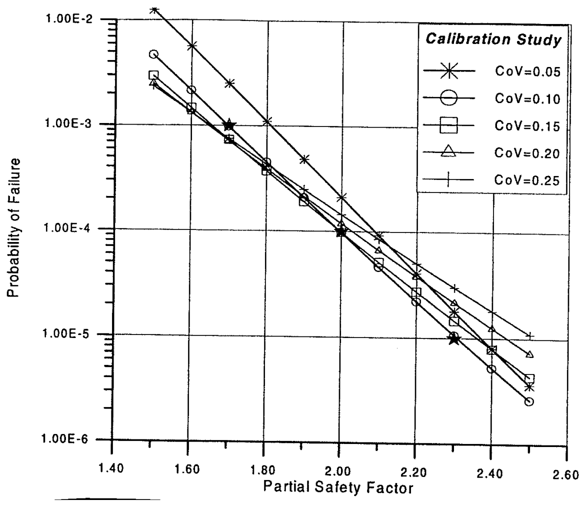

Figure 9. Onset criteria for cross-flow VIV. Calibration study for base case. Taken from Hagen & Mørk (1997).

In reference Hagen & Mørk (1997) significant cross-flow vortex induced vibrations are defined as maximum oscillation amplitudes larger than 0.1D, where D is the diameter of the pipe. For the steady current case the cross-flow no-vibration criterion is given as:

$$f_{0,c} \geq \frac{U_{n,c}}{V_{R,onset} D} \gamma_T \Psi_D \Psi_R \Psi_u$$

Safety Class	**Low** temporary	**Normal** in-service	**High** in-service
γ_T	1.7	2.0	2.3

Table 8. Recommended safety factors. Taken from Hagen & Mørk [6].

where γ_T is a safety factor given in table 8 and where the other quantities are defined in Hagen & Mørk [6].

The work presented above is presently being extended to cover fatigue analysis and onset-criteria for combined wave and current induced loads for unevenness induced and scour induced free spans.

6.3. Requalification of Jacket Structure

This section is based on Sigurdsson, Cramer & Hagen [7]. The objective of a requalification is to establish:

- that the structure is fit for its intended purpose over the residual service life,
- that the consequences in terms of risk to human life, to the environment and to the assets, in the event of structural failure, are acceptable from both a reliability and an economical point of view.

Some typical initiators for requalification are mentioned in reference Sigurdsson, Cramer & Hagen [7]:

- repair of damage or excessive deterioration,
- extension of service life,
- new rules and regulations,
- new load and strength models,
- new analysis and calculation methods,
- significant changes in the consequence of failure,
- discovery of design or construction errors,
- doubts about the safety of the structure/facility for any other reasons,
- detailed planning of I&M.

The requalification procedure presented in reference Sigurdsson, Cramer & Hagen [7] is illustrated for the jacket structure shown in figure 10. The jacket was designed in the sixties on the basis of data, which have been changed later. In reference Sigurdsson, Cramer & Hagen [7] five requalification alternatives are defined. In brief, alternatives 1-4 are different modifications of the structure to reduce the loading and, except for alternative 3, demanning of the platform. Alternative 5 is platform elimination of the structure and replacing it by an underwater connection of the pipelines passing the platform.

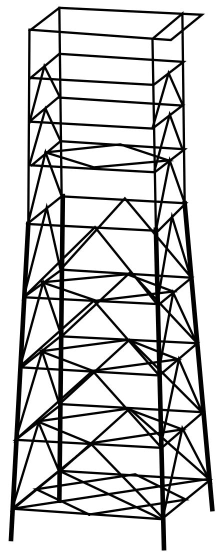

Figure 10. Offshore jacket structure. Taken from Sigurdsson, Cramer & Hagen [7]).

The probabilities of collapse, damage and no-failure are shown in table 3 for all five alternatives.

Name	Decision Alternative				
	1	2	3	4	5
$P_{Collapse}$	0.071	0.010	0.002	0.002	0.000
P_{Damage}	0.270	0.030	0.023	0.023	0.000
$P_{No\text{-}Failure}$	0.659	0.960	0.975	0.975	1.000

Table 9. Annual occurrence probabilities for the five selected alternatives. Taken from Sigurdsson, Cramer & Hagen [7].

The corresponding decision problem is formulated in detail in reference Sigurdsson, Cramer & Hagen [7]. Based on three attributes: Net profit, safety, and reputation and the corresponding utility functions, the conclusion is that alternative 5 should be chosen since this alternative has the largest estimate for its utility.

6.4. Reinforced Concrete Bridges

This section is based on Thoft-Christensen & Middleton [8]. Throughout the world, highway authorities are faced with the task of assessing the strength and safety of their existing bridges. There is a need for increasing the load limit for road bridges and it is a fact that thousands of bridges have deteriorated significantly. It is recognised in many countries that the management of the bridge stock would require knowledge of the overall condition of bridges.

Several highway authorities have identified the potential for applying reliability-based methods for assessing existing bridges. One study is the development of reliability-based codes of practice for the structural analysis of concrete bridges. The goal is to develop a methodology with which a typical short span concrete bridge could be realistically assessed, taking into account the age of the structure and different levels of deterioration. For concrete bridges the primary mechanism of deterioration is corrosion of the steel reinforcement and hence appropriate models were needed to describe this process.

The aim is eventually to develop a risk based assessment procedure for concrete bridges in which satisfactory structures will be defined in terms of a certain (low) probability of failure. By considering the risk of failure at different load levels a simplified assessment code suitable for general use by the profession might then be derived. Clearly such a procedure will need extensive calibration before being adopted but it does hold out the prospect for a rationally based approach to bridge assessment. In particular this methodology should enable bridge managers to allocate resources more rationally on the basis of risk of failure.

Several models can be used to model the deterioration of reinforcement steel in concrete slabs. However, there is a general agreement that the model presented below is acceptable in most cases. Corrosion initiation period refers to the time during which the passivation of steel is destroyed and the reinforcement starts to corrode actively. Practical experience of bridges in wetter countries like UK shows that chloride ingress is far bigger a problem than carbonation.

Fick’s law of diffusion can represent the rate of chloride penetration into concrete, as a function of depth from the concrete surface and time. When corrosion has started the diameter of the reinforcement bars is modelled by a linear function in time. An important aspect of corrosion in addition to the average corrosion is the maximum penetration (pitting of reinforcement). Pitting of reinforcement may have more influence on the reliability than the average deterioration due to localised much higher weakening of the reinforcement.

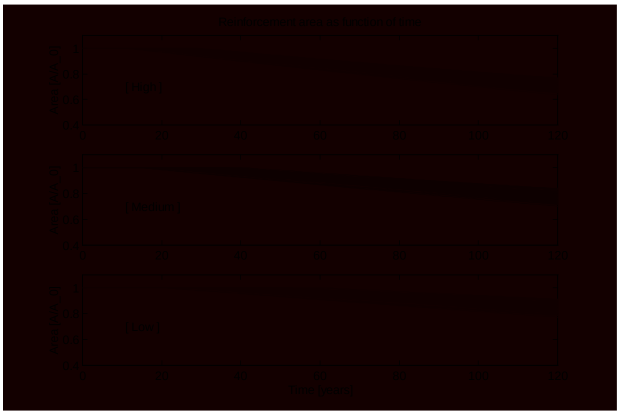

Figure 11. Normalised reinforcement area A/A_0 as a function of time for low, medium, and high deterioration. Taken from Thoft-Christensen & Middleton [8].

In Thoft-Christensen & Middleton [8] three levels of deterioration are proposed: low deterioration, medium deterioration and high deterioration. In figure 11 the sample realisations of the history of the reinforcement area for deterioration models: low, medium, high are shown for a specific case. A collapse (yield line) limit state is used. 7 different failure modes are considered, see figure 12.

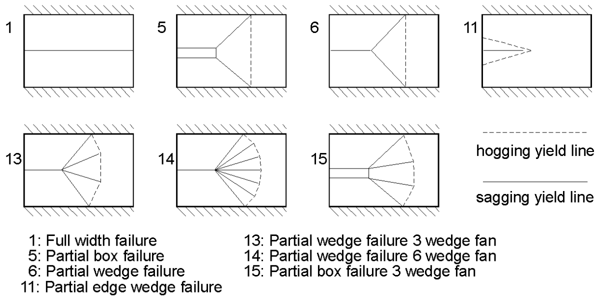

Figure 12. Failure modes for simply supported slab bridges. Taken from Thoft-Christensen & Middleton [8].

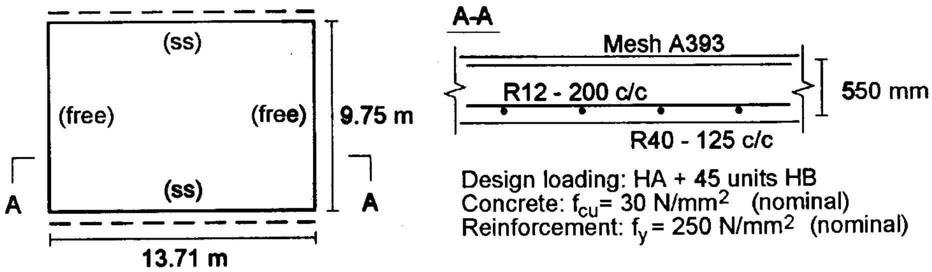

Figure 13. Bridge data. Taken from Thoft-Christensen & Middleton [8].

The following example is used to illustrate the applied methodology, see figure 13. The example is based on an existing UK bridge built in 1975. Based on the corrosion data for the high corrosion model the expected area of the reinforcement as a function of time can be calculated, see figure 14. The normalised reliability profile for the yield line ULS (full width failure) and the corresponding probability of failure profile are shown in figure 15. The reliability index at the time t =0 is β_0 =11.5. Due to the size of the concrete cover (mean value 60 mm) the deterioration does not have any effect until year 70.

Figure 14. Expected relative reinforcement area as a function of time. Taken from Thoft-Christensen & Middleton [8].

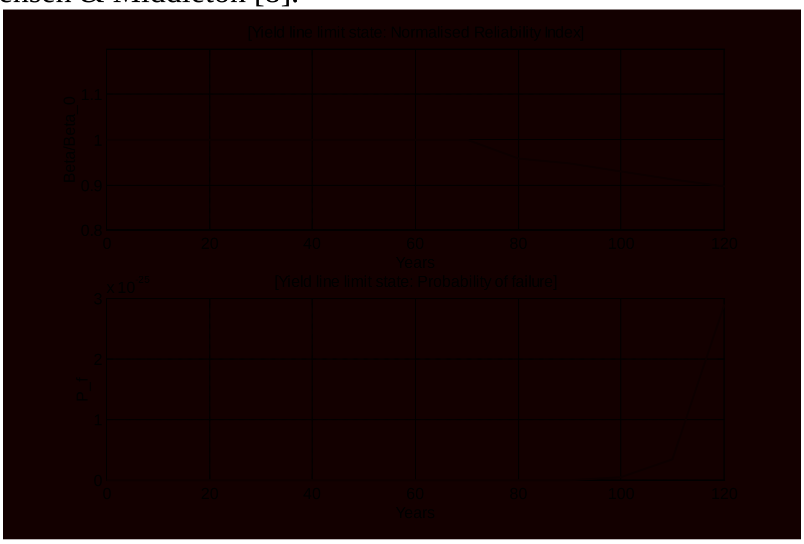

Fig. 15. Reliability profiles using a yield line limit state. Taken from Thoft-Christensen & Middleton [8].

The results from a sensitivity analysis with regard to the mean values are shown for t =0 years and t =120 years in figure 16. The most important variables are, as expected, the thickness of the slab, the yield strength of the reinforcement, and the model uncertainty. Observe that the sensitivity with regard to the cover changes from negative at the time t =0 to positive at the time t =120 due to the corrosion.

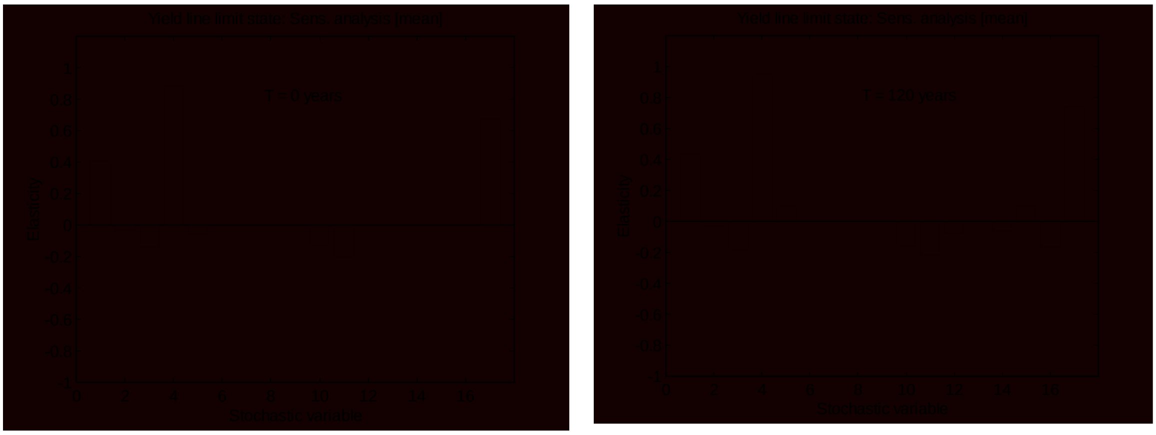

Fig.16**.** Sensitivity analysis for yield line limit state at t =0 and t = 120 years. Taken from Thoft-Christensen & Middleton [8].

6.5. Launch Vehicle Propulsion System

This section is based on Gerez et al. [9]. The overall cost of a launch system consists of several factors such as the expected potential due to catastrophic failure, the manufacturing costs, and the operational costs. The potential for catastrophic failure can be evaluated by a probabilistic risk assessment.

In Gerez et al. [9] the risk evaluation methodologies traditionally used in the aerospace industry are discussed and demonstrated on the US Space Shuttle. Aerospace propulsion has traditionally been based on non-redundant systems. Any failure e.g. in one of the engines or in the support systems would result in either failure of the mission, or catastrophic loss of the vehicle, depending on the severity of the failure.

The traditional propulsion concept changed with the design of the NASA Space Shuttle. The main propulsion in the Space Shuttle is based on three thrust generation systems. However, the main difference between this new concept and earlier designs is that it can tolerate certain failures in the main subsystems and/or their support systems, without endangering mission or crew.

Traditional risk analysis was used in the early stages of the Shuttle project. Traditional risk analysis only gave a qualitative estimate of the risk of loss of mission and catastrophic loss of vehicle, and of the major single point failures in the propulsion system design. However, the Challenger accident made it clear that a quantitative risk analysis is needed. At that time Probabilistic Risk Assessment (PRA) was being extensively applied by the nuclear power industry to evaluate risk levels. The PRA methodology is a scenario-based technique, which characterises accident sequences in an event tree format. Therefore, implementing a similar general strategy, the aerospace industry began applying PRA techniques to manned space vehicles.

The primary objective of the Space Shuttle PRA was to support management and engineering decision-making with respect to the Shuttle programme by producing (see Gerez et al. [9]):

- a quantitative probabilistic risk model of the Space Shuttle during flight,
- a quantitative assessment of in-flight safety risk,
- an identification and prioritisation of the features of design and operations that principally contribute to in-flight safety risk, and
- a mechanism for risk-based evaluation of proposed modifications to the Shuttle system.

In addition PRA analyses are innately tailored for the following applications (see Gerez et al. (1997)):
Estimate the frequency of different undesired consequences (mission failure, catastrophic loss, delay in the program, etc.) for a complex system throughout its lifecycle (design, manufacture, operation, maintenance).

- Determine where the project teams should concentrate their efforts, i.e. what factors generate the overall risk to the system.
- Evaluate which alternatives for design, operation and maintenance represent the least risk to the system.
- Analyse whether a proposed modification reduces the cost-risk ratio sufficiently to justify implementation.
- Identify factors for which uncertainty is high and can be reduced by further tests or analyses.

These capabilities clearly show that PRA has certain advantages over traditional reliability and safety analysis techniques.

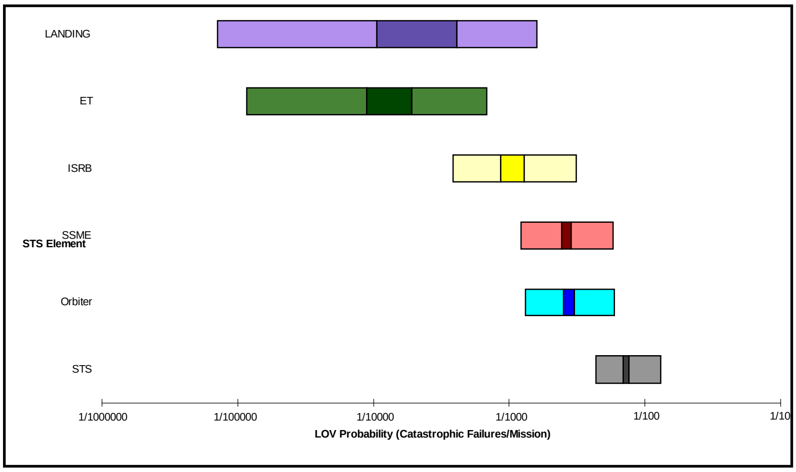

Figure 17**.** LOV risk uncertainty distributions for the total shuttle mission. Taken from Gerez et al. [9].

Using importance analyses, each system component's contribution to the vehicle loss frequency can be estimated. This can also be used to evaluate the risk reduction effectiveness of a proposed modification to components, equipment or subsystem of the Shuttle. Figure 17 shows the LOV (Loss Of Vehicle) risk uncertainty distributions for total Shuttle missions for each of the main subsystems (STS, Orbiter, SSME (Space Shuttle Main Engines), ISRB (Solid Rocket), ET (External Tank), and Landing).

6.6. Hydraulic Steel Structures

This section is based on Manen et al. (1997). Application of structural reliability theory in the design and assessment of hydraulic engineering structures and flood defence works in the Netherlands was enhanced during the damming of the Eastern Scheldt. Advanced techniques to assess the structural reliability were used during the design stage as well as during construction of the Eastern Scheldt Storm Surge Barrier.

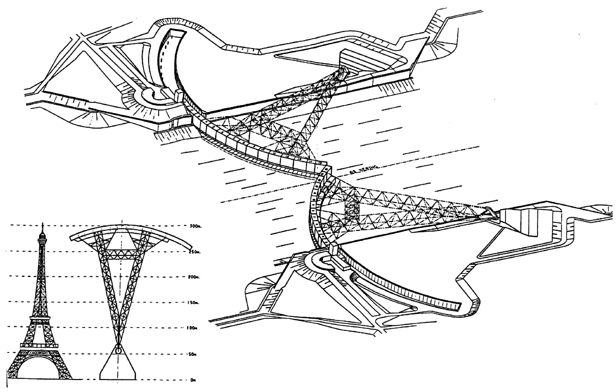

Figure 18**.** Storm surge barrier in the New Waterway near Rotterdam. Taken from Manen et al. [10].

A target failure probability of 10^{-6} per year for the main event "failure of the barrier" was chosen. The target failure probability was then split up into target reliabilities for the different substructures by using the following equation

$$\Phi(-\beta_{\mathrm{ss}}) = \Phi(-\beta_{\mathrm{barrier}})\,\Delta_{\mathrm{ss},i}$$

Correlation between substructures is neglected. The value of $D_{ss,i}$ has been defined by the structural engineer (designer) for each substructure and varies between zero and one. The sum of all $D_{ss,i}$ should be less than or, at most, equal to one. A part of the adopted distribution is shown in Fig. 19.

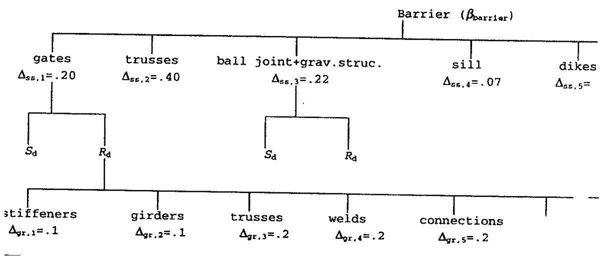

Figure 19. Distribution of failure probability to substructure and the design point approach for some substructures.

For specific elements in the barrier non-linear finite element calculations were performed to investigate the ratio of the actual ultimate load capacity and the capacity according to the design code. The existing overcapacity in relation to the code for a specific limit state can be reduced by a modification of the partial safety factor by:

$$\gamma_{\mathrm{M}} = \frac{\gamma_{\mathrm{code}}}{\gamma_{\mathrm{FEM\text{-}exp}}}$$

For the girders in the steel gate, physical and geometrical non-linear finite element calculations have been performed. In the calculations a load factor of 2.4 is achieved in relation to the basic loads on the model. The girder fails by shear failure of the web below a longitudinal stiffener. The admissible load factor according to the design code in relation to the basic loads on the model is e.g., 1.76. This gives an overcapacity of the structure in relation to the design loads of 2.4 / 1.76 = 1.36. The partial safety factors for the resistance are shown in table 10 for the different groups of gate elements.

Group	γ_m	$\gamma_m{}^d_{gr}$
Girders	1.0	1.35
Stiffeners	1.0	1.36
Trusses	1.0	1.28
Welds	1.25	1.66
Local girders	1.0	1.33
Connections of trusses	1.0	1.26
Supports	1.0	1.25
Locomotive connections	1.0	1.17

Table 10. Safety Factors for the different groups. Taken from Manen et al. [10].

The partial safety factor in the design code amounts to γ_m = 1.0 for limit states governed by yielding and γ_m = 1.25 in case of ultimate strength. The larger reliability index and the series system behaviour of the substructure gives an increase of the partial safety factor by approximately 30 %.

6.7. Dams

This section is based on López-Hernández et al. [11]. Analysis of the reliability of dams is performed to quantify the frequency of possible failures of the dams.

Risk analysis is performed to evaluate the potential consequences of risks. Failures are grouped according to type of dam, height and type of foundation and are analysed in terms of the source of the information, type of degradation, time of failure relative to the age of the dam, methods used to evaluate the damage and the corrective measures taken (see reference López-Hernández et al. {11]).

As mentioned the purpose of the reliability study of a dam is to perform a quantitative evaluation of dam safety in terms of failure probability expressed as failures/year. Therefore, the first step is to identify possible causes of dam failure. The required data include the type of dam, characteristics of the floodplain and terrain, and the existence or not of dams further upstream. The next step is to quantify the possibility of occurrence.

The global failure probability is obtained from the failure probability for each individual cause of failure:

* Failures occurring as a result of hydrological action include dam top overspill, erosion of the spillway and erosion of the dam downstream facing.
* Failures occurring as a result of seismic activity include collapse due to earth movement, high dynamic loads, soil liquefaction causing the structure fall over, displacement of embankments and the consequent water wave.
* Failures occurring as a result of inherent defects include failure of dam foundations, structure or abutments.
* Failure of a dam upstream may occur as a result of any of the aforementioned failures and will have an additional impact on the dam downstream.

The most commonly used approach is based on the collection of experimental data on dam behaviour, e.g. the ICOLD database. The failure is then analysed in terms of the source of the information, type of degradation, time of failure relative to the age of the dam, methods used to evaluate the damage and the corrective measures taken. This database was used to evaluate the failure probability of the dams discussed in this document.

Many factors must be taken into account in a probabilistic study of dams. Firstly, it can be stated that modern dams are safer than old ones. Next, the probability of failure is higher during construction and in the initial years after construction, when they are being filled for the first time. It then decreases and rises slightly at the end of the dams predictable life. Another important factor is the type of dam since. the proportion of failure is higher in embankment dams than in concrete dams. The safety study should also take into account the level of technology of the country in question.

The following annual frequencies of failure were obtained for the three Spanish dams considered in López-Hernández et al. (1997), see table 11.

Dam	Annual Frequency of Failure
Entrepeñas	$p = 2.09 \times 10^{-6}$ incidents/year
Buendía	$p = 2.16 \times 10^{-6}$ incidents/year
Bolarque	$p = 5.79 \times 10^{-6}$ incidents/year

Table 11.

6.8. Fatigue Assessment of Offshore Structure

This section is based on Sigurdsson et al. [12]. In Sigurdsson et al. ([12] an example of the fatigue assessment of an eight-legged North Sea jacket structure located at 107 m water depth is presented. The analysis is a level III reliability analysis, where the joint probability distribution of the uncertainty parameters is applied in the computation of the estimated fatigue failure probabilities.

A *probabilistic S-N fatigue analysis* is performed in Sigurdsson et al. (1997). where the fatigue damage is calculated using the Miners sum and the S-N curve and the following limit state

$$g(D,\Delta) = \Delta - D$$

where D is the damage accumulation and Δ is the fatigue capacity.

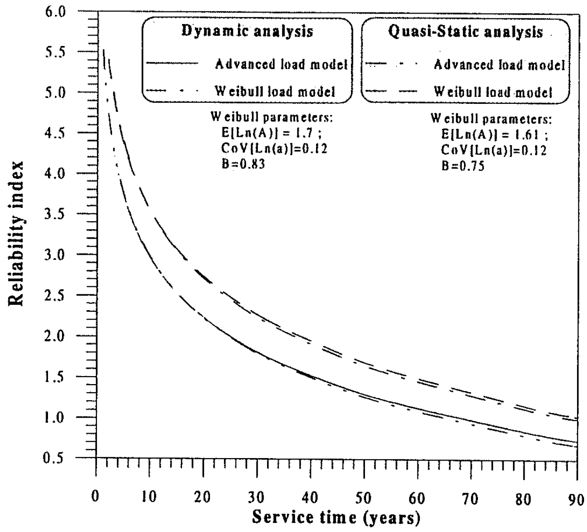

Figure 20. Estimated S-N fatigue reliability of the critical joint as a function of the service time. Taken from Sigurdsson et al. [12].

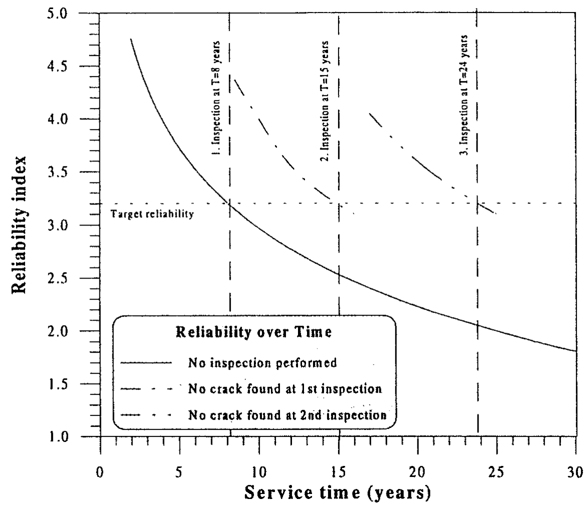

Figure 21. Estimated updated fatigue reliability having inspections with no crack detection. Taken from Sigurdsson et al. [12].

The estimated fatigue reliability over the service life of the structure is shown in figure 20, applying transfer functions derived from both a dynamic and a quasi-static analysis. An importance factor analysis shows that the uncertainties related to the S-N curve and the estimation of the local stress response are the most important uncertainty contributions to the fatigue reliability assessment.

A *probabilistic FM (Fracture Mechanics) fatigue analysis* is also performed. The main advantage of using the FM fatigue model is that structural inspection results can be incorporated for updating the degree of fatigue damage accumulation. A 2-dimensional crack propagation model using Paris equation is used. The parameters describing the crack propagation were fitted on the basis of the application of the S-N approach.

In figure 21 the estimated fatigue reliability is shown as a function of time. For all inspections an MPI inspection technique is applied where no crack was detected.

REFERENCES

[1] Thoft-Christensen, P. & Baker, M.J. Structural Reliability Theory and Its Applications. Springer Verlag, 1982.

[2] Thoft-Christensen, P. & Murotsu, Y. Application of Structural Systems Reliability Theory. Springer Verlag, 1986.

[3] Thoft-Christensen, P. (editor): Industrial Application of Structural Reliability Theory. ESReDA Working Group Report, Aalborg University, Denmark, 1997.

[4] Thoft-Christensen, P. (editor): Industrial Application of Structural Reliability Theory. ESReDA , Det Norske Veritas, Høvik, Norway, 1998.

[5] Chapman, O.J.V. & Pitner, P. & Persoz, M. Probabilistic Structural Integrity in the Nuclear Industry. Chapter 2.1 in Thoft-Christensen [3].

[6] Hagen, Ø. & Mørk, K. Offshore Application Code Calibration - Pipeline Free Span Design. Chapter 3.1 in Thoft-Christensen [3].

[7] Sigurdsson, G., Cramer, E.H. & Hagen, Ø. Offshore Application: Requalification of Offshore Structures. Chapter 3.2 in Thoft-Christensen [3].

[8] Thoft-Christensen, P. & Middleton, C.R. Reliability Assessment of Concrete Bridges. Chapter 4.1 in reference Thoft-Christensen [3].

[9] L. Gerez, L., González, A.Moreno & Maggio, G. Risk-Based Evaluation of Launch Vehicle Propulsion System Designs. Chapter 5.2 in Thoft-Christensen [3].

[10] Manen, S.E. van, Vrouwenvelder, A.C. & van Foeken, R.J. Probabilistic Design of the Steel Structure of the New Waterway Storm Surge Barrier. Chapter 6.1 in Thoft-Christensen [3].

[11] López-Hernández, A., Diaz, E. Salete & González, A. Moreno. Reliability Study of Three Spanish Dams. Chapter 6.2 in Thoft-Christensen [3].

[12] Sigurdsson, G., Cramer, E.H. & Lotsberg, I. Fatigue Assessment of Structures. Chapter 7.2 in Thoft-Christensen [3].