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Thoft-Christensen, Palle

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CHAPTER 118

CORROSION AND CRACKING OF REINFORCED CONCRETE¹

P. Thoft-Christensen
Aalborg University, Aalborg, Denmark

1. INTRODUCTION

Modeling of the deterioration of reinforced concrete has in recent years changed from being a deterministic modeling based on experience to be a stochastic modelling based on sound and consistent physical, chemical, and mechanical principles.

The first step in modelling of the deterioration process is to choose a deterioration profile that is a curve showing how the deterioration develops in time t . Deterioration profiles can be based directly on observations, if such data are available, using curve fitting. It is useful to divide the profile in a number of steps where certain phenomena occur. Often the time T_{corr} to corrosion initiation, the time T_{crack} to crack initiation, and the time $T_{spalling}$ are used for this purpose. In this way the profile is divided in four curves. These curves are normally non-linear, but often a linear approximation may be used, see figure 1.

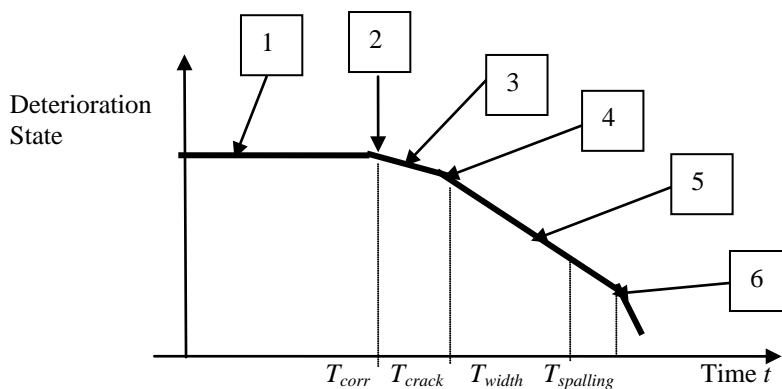


Figure 1. Deterioration profile.

¹ Proceedings IABMAS Workshop, Lausanne, Switzerland, March 2003. In "Life-Cycle Performance of Deteriorating Structures" (eds. Frangopol, Brühwiler, Faber & Adey), ASCE, 2003, pp. 26-36.

In figure 1 the deterioration profile for the case of chloride induced corrosion is described by 6 deterioration steps:

1. Chloride penetration of the concrete
1. Initiation of the corrosion of the reinforcement
2. Evolution of corrosion of the reinforcement
3. Initial cracking of the concrete
4. Evolution of cracks in the concrete.
5. Spalling

A similar deterioration profile for corrosion of the reinforcement due to carbonation may be drawn.

2. DETERIORATION MODELING LEVELS

Modeling of deterioration of reinforced concrete may be divided in 3 levels:

- Level 3 – Scientific Level
- Level 2 – Engineering Level
- Level 1 – Technical Level

Level 3 is the most advanced level. Models on this level are “exact models” in the sense that the modeling of the deterioration profile is based on a sound and consistent scientific basis from a physical, chemical, and mechanical point of view. Advanced information on the structure of the reinforced concrete is used and detailed information on the environmental loading (carbonation and chloride induced corrosion, alkali-aggregate reaction, frost attack, etc) is taken into account. A level 3 modeling is typical used on a new large structure such as a long suspension bridge. It is a very expensive modeling and it is not easy to formulate a level 3 method on basis of a number of measurable physical parameters. An important application of level 3 models is to supply information to be used in a level 2 model.

Level 2 is an average level from a sophistication point of view. Level 2 models are based on semi-physical or average material parameters such as the diffusion coefficient. They are based on average “loading parameters” like the average chloride concentration on the surface of the concrete. They are also based on a number of engineering simplifications regarding the ingress such as Fick’s law. A level 2 model will often limit the deterioration to a single type of deterioration like chloride induced corrosion. Level 2 models may be used for design of new structures and for estimation of deterioration of existing concrete structures. An important application of level 2 models is to supply information to be used in a level 1 model.

Level 1 is the most simplified level of modeling of the deterioration profile. It is based on direct observations on structural elements rather than going into the detailed deterioration mechanisms. A level 1 model is usually based on a limited number of parameters, e.g. obtained from level 2 modeling. Level 1 models may be used on groups of bridges to obtain e.g. optimal maintenance strategies for reinforced concrete bridges. A level 1 model is often used as a first estimation of the deterioration level of existing concrete structures.

In this paper it is shown how information obtained on level 3 approaches may be used to improve level 2 models.

3. APPLICATIONS

Modeling of the deterioration of reinforced concrete structures is of great importance in a number of engineering disciplines. It is outside the scope of this paper to review all applications. Only 2 applications will briefly be presented.

In designing structures the service life plays an important role. The service life $T_{service}$ for a reinforced concrete structure has been defined by several authors as the initiation time for corrosion T_{corr} of the reinforcement

$$T_{service} = T_{corr} \quad (1)$$

The service life $T_{service}$ has later been modified so that the time Δt_{crack} from corrosion initiation to corrosion crack initiation in the concrete is included; Thoft-Christensen [1]. The service life is then defined as

$$T_{service} = T_{crack} = T_{corr} + \Delta t_{crack} \quad (2)$$

In Thoft-Christensen [2] the model is extended to include the situation after initiation of corrosion cracking. The service lifetime $T_{service}$ is then defined as the time T_{width} to a certain corrosion crack width (say 0.3 mm) has developed that is

$$T_{service} = T_{width} = T_{crack} + \Delta t_{width} = T_{corr} + \Delta t_{crack} + \Delta t_{width} \quad (3)$$

In design of optimal Bridge Management Strategies for reinforced concrete structures the deterioration profiles are important to predict as precise as possible since the degree of deterioration control maintenance actions. EU sponsored the first major research on optimal strategies for maintenance of reinforced concretes structures in 1990 to 1993. The results are presented in several reports and papers e.g. Thoft-Christensen [3, [4]. The methodology used in the project is analytic with traditional numerical analysis and rather advanced stochastic modelling.

In modelling reliability profiles for reinforced concrete bridges Monte Carlo simulation seems to be used for the first time in Highways Agency project. Stochastic variables and crude Monte Carlo simulation is used to model reliability profiles model all relevant parameters. The methodology used is presented in detail in the final project report; Thoft-Christensen & Jensen [5].

The simulation approach was extended in another HA Project to include stochastic modelling of rehabilitation distributions and preventive and essential maintenance for reinforced concrete bridges Thoft-Christensen [6], [7]. A similar approach is used in the same project by Frangopol [8] on steel/concrete composite bridges.

In a recent Highways Agency project the simulation technique is extended further to modelling of condition profiles, and the interaction between reliability profiles and condition profiles for reinforced concrete bridges, and the whole life costs. The simulation results are detailed presented by Frangopol [9] and Thoft-Christensen [10].

4. MODELING OF CORROSION

4.1 Initiation of corrosion of the reinforcement.

The corrosion process is very complex and the modelling is often based on observations or speculations rather than a clear understanding of the physical and chemical processes. Corrosion initiation period refers to the time during which the passivation of steel is destroyed and the reinforcement starts corroding actively. In this

paper Fick's law of diffusion is used to model the rate of chloride penetration into concrete as a function of depth x from the concrete surface and as a function of time t

$$\frac{\partial C(x,t)}{\partial t} = D_c \frac{\partial^2 C(x,t)}{\partial x^2} \quad (4)$$

where $C(x,t)$ is the chloride ion concentration, as % by weight of cement, at a distance of x cm from the concrete surface after t seconds of exposure to the chloride source. D_c is the chloride diffusion coefficient expressed in cm^2/sec . The solution of the equation (4) is

$$C(x,t) = C_0 \left\{ 1 - \text{erf} \left(\frac{x}{2\sqrt{D_c \cdot t}} \right) \right\} \quad (5)$$

where C_0 is the equilibrium chloride concentration on the concrete surface, as % by weight of cement, erf is the error function.

More sophisticated models, which e.g. take into account variation of D_c regard to x or the spatial or time variation of C_0 have also been formulated.

Let C_{cr} be a critical chloride corrosion threshold and d the thickness of concrete cover, then the corrosion initiation period T_{corr} can easily be calculated from equation (5)

$$T_{corr} = \frac{d^2}{4D_c} \left(\text{erf}^{-1} \left(\frac{C_{cr} - C_0}{C_i - C_0} \right) \right)^2 \quad (6)$$

It follows from (6) that the time to corrosion imitation is inversely proportional to D_c . It is therefore of great interest to get a good estimate of D_c . The diffusion coefficient D_c is not a real physical constant for a given concrete structure since it depends of a number of factors; Thoft-Christensen [11].

Extensive experimental investigations by Jensen, [12] and Jensen, Hansen, Coats & Glasser [13] have shown that the most important factors affecting D_c are the water/cement ratio w/c , the temperature Φ , and the amount of e.g. silica fume s.f. The diffusion coefficient D_c increases significantly with w/c as well as with the temperature Φ . The influence of w/c and the temperature Φ may be explained by the chloride binding. Only the free chloride is important for the diffusion coefficient D_c . With increased w/c ratio less chloride is bound and D_c therefore increased. The strong influence of the temperature is mainly caused by thermal activation of the diffusion process but may also be due to a reduced chloride binding when the temperature is increased.

Based on the above-mentioned experiments, figure 2 shows the diffusion coefficient D_c as a function of the water-cement ratio w/c and the temperature Φ °C for cement pastes with 0% silica fume. It is clear from figure 2 that the diffusion coefficient D_c increases significantly with w/c as well as the temperature Φ . In the example illustrated in figure 2 the minimum value of D_c is $0.31 \times 10^{-12} \text{ m}^2/\text{s}$ corresponding to $w/c = 0.2$ and the temperature $\Phi = 4^\circ\text{C}$ and the maximum value is $80.00 \times 10^{-12} \text{ m}^2/\text{s}$ corresponding to $w/c = 0.70$ and $\Phi = 35^\circ\text{C}$.

The data above clearly indicate that site information is needed to make e.g. an estimation of the remaining life cycle or any estimation where the diffusion coefficient is involved. This has clearly been confirmed by several authors e.g. in Suda et al. [14], where important information of the distribution of the diffusion coefficient D_c in Japan is shown.

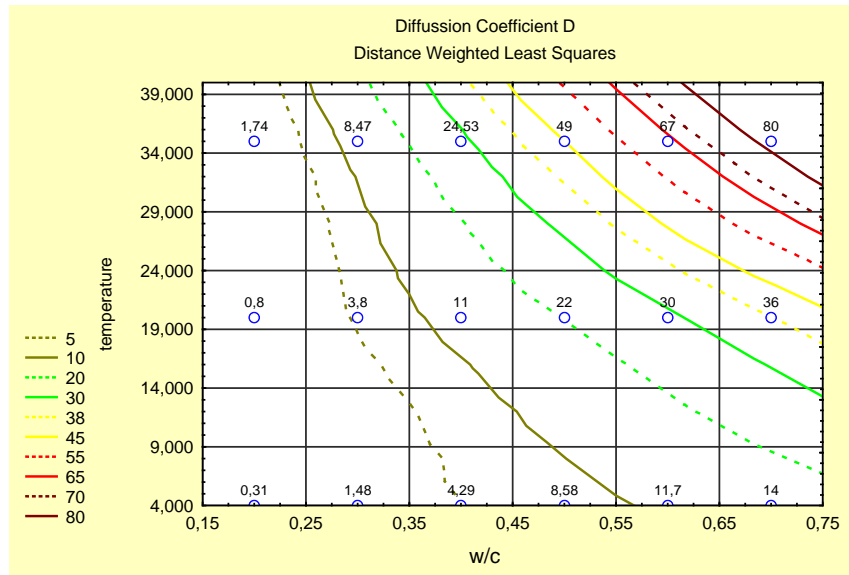


Figure 2. The diffusion coefficient D ($10^{-12} \text{ m}^2/\text{s}$) as a function of the water-cement ratio w/c and the temperature Φ °C (Celsius).

Even in smaller countries it seems essential to take into account the above-mentioned findings using a table like table 1, where D_0 is a characteristic diffusion coefficient based on average yearly temperature and an average w/c value.

Relative D_c	Low temperature	Average temperature	High temperature
Low w/c	$0.1 \times D_0$	$0.2 \times D_0$	$0.1 \times D_0$
Average w/c	$0.5 \times D_0$	D_0	$2 \times D_0$
High w/c	D_0	$2 \times D_0$	$10 \times D_0$

Table 1. Relative diffusion coefficients

4.2 Evolution of Corrosion in the Reinforcement.

It is complicated to model the evolution of corrosion after corrosion initiation at time T_{corr} . A linear relation between the diameter $D(t)$ of the reinforcement bars at the time t is therefore often used

$$D(t) = D_0 - c_{corr} i_{corr} (t - T_{corr}), t \geq T_{corr} \quad (7)$$

D_0 is the initial diameter. c_{corr} is a corrosion coefficient, and i_{corr} is the rate of corrosion.

There is always in the corrosion free state a porous zone around the steel/concrete surface caused by the transition from paste to steel, entrapped/entrained air voids, and corrosion products diffusing into the capillary voids in the cement paste. When the total amount of corrosion products exceeds the amount of corrosion products needed to fill the porous zone around the steel, the corrosion products create expansive pressure on the surrounding concrete. Very close to the bars the porosity is close to one, but the porosity decreases with the distances from the bars. Let t_{pore} be the thickness of an equivalent zone with porosity one around a steel bar. Then the amount of corrosion products necessary to fill the porous zone can be written

$$W_{porous} = t_{pore} \times \pi \rho_{rust} D_0 \quad (8)$$

where ρ_{rust} is the density of the corrosion products.

4.3 Modelling of initiation of cracking

After corrosion initiation $D(t) = t \geq T_{corr}$ the rust products will initially fill the porous zone and then result in an expansion of the concrete near the reinforcement. As a result of this, tensile stresses are initiated in the concrete. With increasing corrosion the tensile stresses will reach a critical value and cracks will be developed.

During this process the corrosion products at initial cracking of the concrete will occupy three volumes, namely the porous zone, the expansion of the concrete due to rust pressure, and the space of the corroded steel. The corresponding total amount of critical rust products W_{crit} to fill these volumes is

$$W_{crit} = W_{porous} + W_{expan} + W_{steel} \quad (9)$$

where W_{expan} is the amount of corrosion products needed to fill in the space due to the expansion of the concrete around the reinforcement, and W_{steel} is the amount of corrosion products at time T_{crack} of cracking.

Let the expansion of the concrete around the reinforcement have the thickness t_{expan} at time T_{crack} , then W_{expan} can be written

$$W_{expan} = t_{crit} \times \rho_{rust} \pi (D + 2t_{por}) \quad (10)$$

where t_{crit} is the thickness of the expansion at crack initiation.

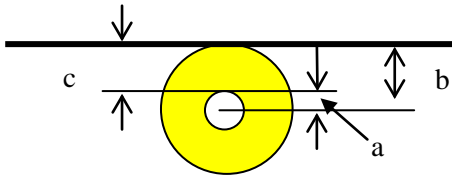


Figure 3. Idealization of the concrete around the reinforcement by a thick-walled cylinder.

Liu & Weyers [15] have estimated t_{crit} by assuming that the concrete is a homogeneous elastic material and can be approximated by a thick-walled concrete cylinder with inner radius $a = (D + 2t_{por}) / 2$ and outer radius $b = c + (D + 2t_{por}) / 2$

where c is the cover depth, see figure 3. Then the approximate value of the critical expansion t_{cr} is

$$t_{crit} = \frac{cf'_t}{E_{ef}} \left(\frac{a^2 + b^2}{b^2 - a^2} + \nu_c \right) \quad (11)$$

where E_{ef} is the effective elastic modulus of the concrete and f'_t is the tensile strength of the concrete. ν_c is Poisson's ratio of the concrete.

Finally

$$W_{steel} = \frac{\rho_{rust}}{\rho_{steel}} M_{steel} \quad (12)$$

where ρ_{steel} is the density of steel, and M_{steel} is the mass of the corroded steel that is proportional to W_{crit} . Liu & Weyers [15] have calculated the factor of proportionality for two kinds of corrosion products as 0.523 and 0.622. For simplicity, it will here be assumed that $M_{steel} = 0.57 \times W_{crit}$. Therefore, equation (12) can be rewritten

$$\begin{aligned}
 W_{crit} &= W_{porous} + W_{expan} + W_{steel} = W_{porous} + W_{expan} + 0.57 \frac{\rho_{rust}}{\rho_{steel}} W_{crit} \\
 &= \frac{\rho_{steel}}{\rho_{steel} - 0.57 \rho_{rust}} (W_{porous} + W_{expan})
 \end{aligned}
 \quad (13)$$

The rate of rust production as a function of time (years) from corrosion initiation can be written; Liu & Weyers [15]

$$\frac{dW_{rust}(t)}{dt} = k_{rust}(t) \frac{1}{W_{rust}(t)} \quad (14)$$

i.e. the rate of corrosion is inversely proportional to the amount of rust products W_{rust} (kg/m). The factor $k_{rust}(t)$ ($\text{kg}^2/\text{m}^2\text{year}$) is assumed to be proportional to the annual mean corrosion rate $i_{cor}(t)$ ($\mu\text{A}/\text{cm}^2$) and the diameter D (m) of the reinforcement.

The proportionality factor depends on the types of rust products, but is here taken as $0.383e-3$.

$$k_{rust}(t) = 0.383 \times 10^{-3} D i_{corr}(t) \quad (15)$$

By integration

$$W_{rust}^2(t) = 2 \int_0^t k_{rust}(t) dt \quad (16)$$

Let $i_{cor}(t)$ be modeled by a time-independent normally distributed stochastic variable $N(3; 0.3)$ ($\mu\text{A}/\text{cm}^2$) then the time from corrosion initiation to cracking Δt_{crack} can be estimated by (16) by setting $W_{rust}(\Delta t_{crack}) = W_{crit}$.

$$\Delta t_{crack} = \frac{W_{crit}^2}{2k_{rust}} = \frac{W_{crit}^2}{2 \times 0.383 \times 10^{-3} D i_{corr}} \quad (17)$$

The time to initial cracking is then given by

$$T_{crack} = T_{corr} + \Delta t_{crack} = \frac{d^2}{4D_c} \left(\text{erf}^{-1} \left(\frac{C_{cr} - C_0}{C_i - C_0} \right) \right)^{-2} + \frac{W_{crit}^2}{2 \times 0.383 \times 10^{-3} D i_{corr}} \quad (18)$$

4.4 Evolution of corrosion cracks

Let the initial crack width be w_0 at time T_{crack} . The crack width will increase for $t > T_{crack}$ when the production of corrosion products is increased. It has not yet been possible to find measurements on real structures, which can indicate how the corrosion crack width increase with time.

A simple modelling of this process is shown in figure 4, where a linear relation

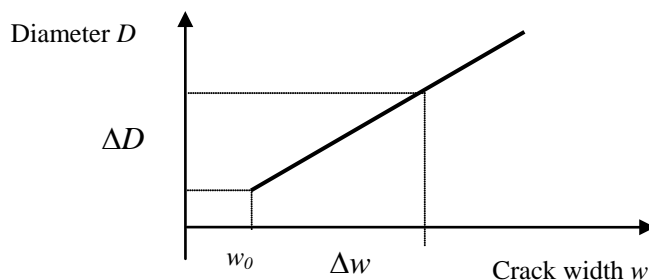


Figure 4. Relation between reduction in diameter and the crack width.

between the crack width $w(t)$ and the decrease of the diameter D .

Andrade, Alonso & Molina [16] have investigated experimentally the evolution of corrosion cracks in reinforced concrete beams. After formation of the initial

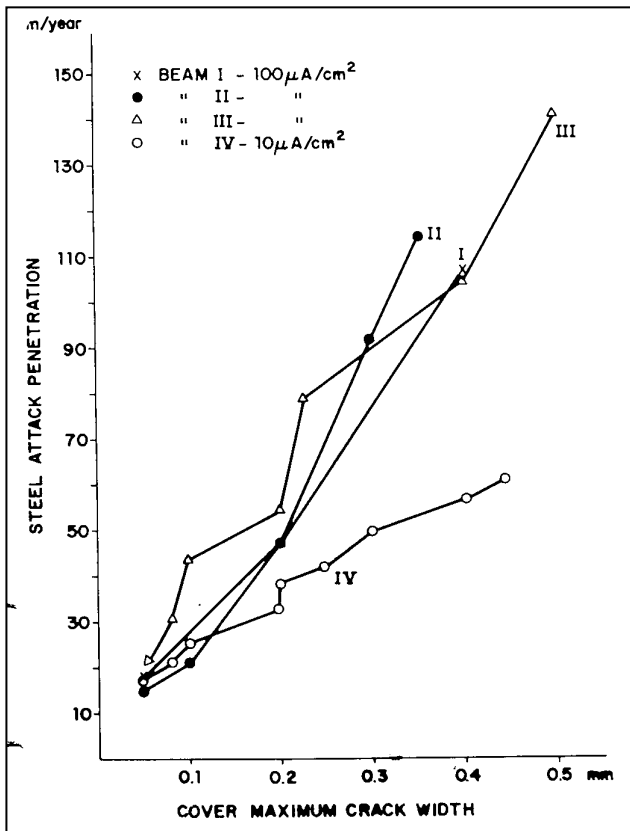


Figure 5. Loss in rebar diameter versus the crack width; Andrade et al. [16].

Andrade et al.[16]. The factor γ depends on the applied current and on the cross-sectional data.

A simple approximate estimation of γ for a given beam cross-section may be performed as shown below Thoft-Christensen [17]. For illustration, assume that the diameter of the rebar at the time of crack initiation is 16 mm and that the cover is 30 mm, see figure 5.

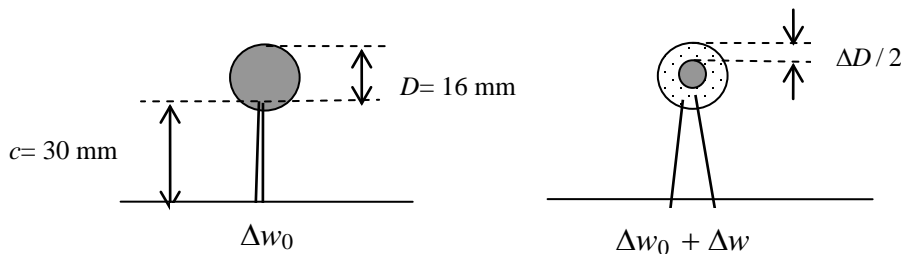


Figure 6. Evolution of cracks.

To the left in figure 6 the crack at the time T_{crack} of the initial crack is shown. The initial crack width is Δw_0 . To the right the assumed crack configuration at the time $t > T_{crack}$ is shown when a further loss ΔD has taken place. The crack width is then $\Delta w_0 + \Delta w$. By assuming that the increase in the volume of the crack is equal to the volume of the corrosion products produced when the diameter is reduced to $D - \Delta D$, a relationship between Δw and D can be obtained approximately by

crack the rebar cross-section is further reduced due to the continued corrosion, and the width of the crack is increased. In the paper four simple test specimens have been investigated. An impressed current artificially corrodes the beams. The loss of bar sections is monitored and the corresponding crack evolution is measured by the use of strain gauges attached to the surface of the beams. In all four experiments the function between the reduction of the rebar diameter and the maximum crack width measured in the surface of the concrete specimen can be approximated by a linear function, see figure 5.

Let Δw be the increase in crack width in the time interval Δt and let the corresponding loss of rebar diameter be ΔD . Then, see figure 4

$$\Delta w = \gamma \Delta D \quad (19)$$

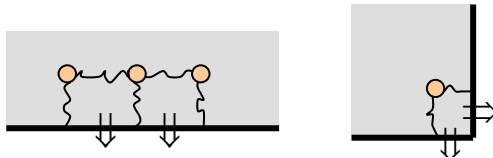
where γ is of the order 1.5 to 5 in the experiments reported by

$$\frac{1}{2} \left(\frac{D/2}{D/2+c} + 1 \right) c \Delta w = (\alpha - 1) \pi D \frac{\Delta D}{2} \quad (20)$$

where $\alpha = \rho_{rust} / \rho_{steel}$ (the relation between the densities of the rust product and the steel) depends on the type of corrosion products. Typical values are 2 - 4. By inserting the above-mentioned data one obtains for this case $\gamma = 1.4 - 4.2$ in good agreement with the experimental results described by Andrade et al. [16]; see figure 5.

4.5 Spalling

Using FEM the procedure presented above can be extended to estimating the time for



corrosion based spalling of concrete for e.g. slabs and beams; see figure 7.

Figure 7. Typical examples of concrete spalling.

5. CONCLUSIONS

In this paper is presented a brief review of modern modelling of the reliability profiles for reinforced concrete structures. Deterministic models for the different steps in the deterioration process are discussed. Several of the parameters used in the modelling are so uncertain that a stochastic modelling is natural. By crude Monte Carlo simulation predictions for time to initial corrosion, time to initial cracking, and time to a given crack width may be obtained.

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