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STOCHASTIC MODELLING AND OPTIMIZATION OF COMPLEX INFRASTRUCTURE SYSTEMS[1]

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ABSTRACT

In this paper it is shown that recent progress in stochastic modelling and optimization in combination with advanced computer systems has now made it possible to improve the design and the maintenance strategies for infrastructure systems. The paper concentrates on highway networks and single large bridges. United States has perhaps the largest highway networks in the world with more than 6 million kilometers of roadway and more than 0.5 million highway bridges; see Chase [1]. About 40% of these bridges are considered deficient and more than $50 billion is estimated needed to correct the deficiencies; see Roberts [2]. The percentage of sub-standard bridges deemed to require urgent actions in other countries such as France (15%) and UK (20%) is also high; see Das [3].

1. INTRODUCTION

Obtaining and maintaining advanced infrastructure systems plays an important role in modern societies. Developed countries have in general well established infrastructure systems but most non-developed countries are characterized by having bad or no effective infrastructure systems. Therefore, in the transition from a non-developed country to a well developed country construction of effective infrastructure systems plays an important role. However, it is a fact that construction of new infrastructure

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systems requires great investments so a careful planning of all details in the system is essential for the effectiveness of the system from an operational but also economical point of view.

Obtaining the resources needed to establish infrastructure systems is only the first step. The next step and perhaps the most expensive step are to maintain the systems. It is recognized in most developed countries that good maintenance of infrastructure systems is in the long run the most economical way to keep the infrastructure in a satisfactory state. Effective maintenance requires however more resources than available in most countries. Therefore, careful planning of maintenance strategies is essential for all types of infrastructures.

2. FORMULATION OF THE COST OPTIMIZATION PROBLEM

An infrastructure system consists of a number of structures. The objective is to minimize the cost of maintaining such a group of structures in the service life of the infrastructure. Estimation of the service life costs is a very uncertain so that a stochastic modelling is clearly needed. This can be expressed mathematically as

$$\min E\left[C\right] = \min\left(E\left[C_M\right] + E\left[C_U\right] + E\left[C_F\right]\right) \tag{1}$$

where

$E\left[C\right]$ is the expected total cost in the service life of the infrastructure

$E\left[C_M\right]$ is the expected maintenance cost in the service life of the infrastructure

$E\left[C_U\right]$ is the expected user costs e.g. traffic disruption costs due to works or restrictions on the structure

$E\left[C_F\right]$ is the expected costs due to failure of structures in the infrastructure.

For a *single* structure *i* in the infrastructure the expected cost $E\left[C_i\right]$ can be written

$$\begin{aligned} E\left[C_i\right] &= E\left[C_{Mi}\right] + E\left[C_{Ui}\right] + E\left[C_{F_i}\right] \\ &= \sum_{t=1}^{T} \{(1+\gamma)^{-1}\left[E[C_{Mi}(t)]P(M_{it}) + E[C_{Ui}(t)]P(U_{it}) + E[C_{Fi}(t)]P(F_i(t))\right]\} \end{aligned} \tag{2}$$

where

γ is the discount rate (factor) e.g. 6 %

$E\left[C_i\right]$ is the expected total cost for structure *i*

$E\left[C_{Mi}(t)\right]$ is the expected maintenance cost for structure *i* in year *t*

$E\left[C_{Di}(t)\right]$ is the expected user costs for structure *i* in year *t*

$E\left[C_{Fi}(t)\right]$ is the expected failure costs for structure *i* in year *t*

$P(M_{it})$ is the probability of the event “maintenance is necessary” for structure *i* in year *t*

$P(D_{it})$ is the probability of the event “maintenance is necessary” for structure *i* in year *t*

$P(F_{it})$ is the probability of the event “maintenance is necessary” for structure *i* in year *t*

T is the remaining service life or reference period (in years).

Let the number of structures in the considered infrastructure be *m*. The expected total cost for the group can then be written

$$\begin{aligned} E\left[C\right] &= \sum_{i=1}^{m} \left\{ E\left(C_{Mi}\right) + E\left(C_{Ui}\right) + E\left(C_{Fi}\right) \right\} \\ &= \sum_{i=1}^{m} \sum_{t=1}^{T} \{(1+\gamma)^{-1} \left[E[C_{Mi}(t)]P(M_{it}) + E[C_{Ui}(t)]P(U_{it}) + E[C_{Fi}(t)]P(F_i(t)) \right] \} \end{aligned} \tag{3}$$

3. BRIDGE NETWORKS

Future advanced bridge management systems will be based on simple models for predicting the residual strength of structural elements. Improved stochastic modelling of the deterioration is needed to be able to formulate optimal strategies for inspection and maintenance of deteriorated reinforced concrete bridges. However, such strategies will only be useful if they are also combined with expert knowledge. It is not possible to formulate all expert experience in mathematical terms. Therefore, it is believed that future management systems will be expert systems or at least knowledge-based systems; see Thoft-Christensen [4].

Methods and computer programs for determining rational inspection and maintenance strategies for concrete bridges must be developed. The optimal decision should be based on the expected benefits and total cost of inspection, repair, maintenance and complete or partial failure of the bridge. Further, the reliability has to be acceptable during the expected lifetime.

The first major research on combining stochastic modelling, expert systems and optimal strategies for maintenance of reinforced concretes structures was sponsored by EU in 1990 to 1993. The research project is entitled *"Assessment of Performance and Optimal Strategies for inspection and Maintenance of Concrete Structures Using Reliability Based Expert systems"*. The results are presented in several reports and papers; see e.g. [4] and [5]. The methodology used in the project is analytic with traditional numerical analysis and rather advanced stochastic modelling.

Monte Carlo simulation has been used in decades to analyze complex engineering structures in many areas, e.g. in nuclear engineering. In modelling reliability profiles for reinforced concrete bridges Monte Carlo simulation seems to be used for the first time in December 1995 in the Highways Agency project *"Revision of the Bridge Assessment Rules based on Whole Life Performance: Concrete"* (1995-1996, Contract: DPU 9/3/44). The project is strongly inspired of the above-mentioned EU-project. The methodology used is presented in detail in the final project report, see Thoft-Cristensen & Jensen [6].

In the Highways Agency project *"Optimum Maintenance Strategies for Different Bridge Types"* (1998-2000, Contract: 3/179) the simulation approach was extended in, Thoft-Christensen [7] and [8] to include stochastic modelling of rehabilitation distributions and preventive and essential maintenance for reinforced concrete bridges. A similar approach is used in the project on steel/concrete composite bridges, see Frangopol [9].

In a recent project *"Preventive Maintenance Strategies for Bridge Groups* (2001-2003, Contact 3/344 (A+B)) the simulation technique is extended further to modelling of condition profiles, and the interaction between reliability profiles and condition profiles for reinforced concrete bridges, and the whole life costs. The simulation results are detailed presented in Frangopol [10], Thoft-Christensen [11], and Thoft-Christensen

& Frier [12].

4. ESTIMATION OF SERVICE LIFE OF INFRASTRUCTURES

In this paper service life assessment of infrastructures is discussed based on stochastic models and with special emphasis on deterioration of reinforced structures due to reinforcement corrosion.

The service life $T_{service}^{(1)}$ for a reinforced concrete structure has been the subject of discussion between engineers for several decades. Several authors; see e.g. Thoft-Christensen [13]; have defined the service life as the initiation time for corrosion T_{corr} of the reinforcement.

The service life $T_{service}^{(1)}$ has later been modified so that the time Δt_{crack} from corrosion initiation to corrosion crack initiation in the concrete is included; see Thoft-Christensen [14]. The service life is then defined by $T_{service}^{(2)} = T_{crack} = T_{corr} + \Delta t_{crack}$. A stochastic model for Δt_{crack} may be developed on the basis of existing deterministic theories for crack initiation; see Liu & Weyers [15].

The service life definition may further be modified so that the time $\Delta t_{crack\ width}$ from corrosion crack initiation to formation of a certain (critical) crack width is included; see Thoft-Christensen [16]. By this modelling it is possible to estimate the reliability of a given structure on the basis of measurements of the crack widths on the surface of the concrete structure.

Corrosion initiation period refers to the time during which the passivation of steel is destroyed and the reinforcement starts corroding actively. If Fick's law of diffusion can represent the rate of chloride penetration into concrete, then it can be shown that the time T_{corr} to initiation of reinforcement corrosion is

$$T_{service}^{(1)} = T_{corr} = \frac{d^2}{4D}\left(erf^{-1}\left(\frac{C_{cr} - C_0}{C_i - C_0}\right)\right)^{-2} \tag{4}$$

where d is the concrete cover, D is the diffusion coefficient, C_{cr} is the critical chloride concentration at the site of the corrosion, C_0 is the equilibrium chloride concentration on the concrete surface, C_i is the initial chloride concentration in the concrete, *erf* is the error function.

After corrosion initiation the rust products will initially fill the porous zone around the steel/concrete surface. As a result of this, tensile stresses are initiated in the concrete. With increasing corrosion the tensile stresses will reach a critical value and cracks will be developed. During this process the volume of the corrosion products at initial cracking of the concrete W_{crit} will occupy three volumes, namely the porous zone W_{porous}, the expansion of the concrete due to rust pressure W_{expan}, and the space of the corroded steel W_{steel}. With this modelling and some minor simplifications it can then be shown that the time from corrosion imitation to crack initiation is; see Liu & Weyers [15]

$$\Delta t_{crack} = \frac{1}{2 \times 0.383 \times 10^{-3} D_{bar} i_{corr}}\left(\frac{\rho_{steel}}{\rho_{steel} - 0.57\rho_{ust}}\left(W_{porous} + W_{expan}\right)\right)^2 \tag{5}$$

where D_{bar} is the diameter of the reinforcement bar, i_{corr} is the annual mean corrosion rate, ρ_{steel} is the density of the steel, and ρ_{rust} is the density of the rust products.

After formation of the initial crack the rebar cross-section is further reduced due to the continued corrosion, and the width of the crack is increased. Experiments (see e.g. Andrade et al. [17]) show that the function between the reduction of the rebar diameter ΔD_{bar} and the corresponding increase in crack width Δw_{crack} in a given time interval Δt measured on the surface of the concrete specimen can be approximated by a linear function

$$\Delta w_{crack} = \gamma\, \Delta D_{bar} \tag{6}$$

where the factor γ is of the order 1.5 to 5. This linearization has been confirmed by FEM analyses; see Thoft-Christensen [18]. Let the critical crack width be $w_{critical}$ corresponding to the service life $T_{service}^{(3)}$. By setting $w(T_{service}^{(3)}) = w_{critical}$ the following expression is obtained for $T_{service}^{(3)}$

$$T_{service}^{(3)} = \frac{w_{critical} - w_{crack}(T_{crack})}{\gamma c_{corr} i_{corr}} + T_{crack} \tag{7}$$

$w_{crack}(T_{crack}) \approx 0$ is the initial crack width at time T_{crack}. Using Monte Carlo simulation, the distribution functions of $T_{service}^{(1)}$, $T_{service}^{(2)}$, and $T_{service}^{(3)}$ can then for a given structure be estimated for any value of the critical crack width when stochastic distributions are known for all parameters.

5. STOCHASTIC MODELLING OF MAINTENANCE STRATEGIES

After a structural assessment of the reliability of a reinforced concrete bridge deck at the time T_0 the problem is to decide if the bridge deck should be repaired and, if so, how and when should it be repaired? Solution of this optimisation problem requires that all future inspections and repairs are taken into account. After each structural assessment the total expected benefits minus expected repair and failure costs in the residual lifetime of the bridge are maximized considering only the repair events in the residual service life of the bridge.

In order to simplify the decision modelling it is assumed that N_R repairs of the same type are performed in the residual service life $T_{service}$ of the bridge. The first repair is performed at time T_{R_1}, and the remaining repairs are performed at equidistant times with the time interval $t_R = (T_{service} - T_{R_1}) / N_R$. This decision model can be used in an adaptive way if the model is updated after an assessment (or repair) and a new optimal repair decision is made with regard to t_R. Therefore, it is mainly the time T_{R_1} of the first repair after an assessment, which is of importance. In order to decide which repair type is optimal after a structural assessment; the following optimisation problem is considered for each repair technique, see Thoft-Christensen [4]

$$\begin{aligned} &\max_{T_R, N_R} W(T_R, N_R) = B(T_R, N_R) - C_R(T_R, N_R) - C_F(T_R, N_R) \\ &s.t. \quad \beta^U(T_{service}, T_R, N_R) \geq \beta^{\min} \quad \text{or/and} \quad T_{service}(T_R, N_R) \geq T_{service}^{\min} \end{aligned} \tag{8}$$

where the optimisation variables are the expected number of repairs N_R in the residual service life and the time T_R of the first repair. W are the total expected benefits minus costs in the residual lifetime of the bridge. B is the benefit. C_R is the repair cost capitalized to the time t = 0 in the residual service life of the bridge. C_F are the expected failure costs capitalized to the time t = 0 in the residual service life of the bridge. $T_{service}$ is the expected service life of the bridge. β^U is the updated reliability index. $\beta^{\min}$ is the minimum reliability index for the bridge (related to a critical element or to the total system). $T_{service}^{\min}$ is the minimum acceptable service life.

The benefits B play a significant role and are modelled by

$$B(T_R, N_R) = \sum_{i=[T_0]+1}^{[T_{service}]} B_i (1+r)^{T_0 - T_{ref}} \frac{1}{(1+r)^{T_i - T_0}} \tag{9}$$

where $[T]$ signifies the integer part of T measured in years and B_i are the benefits in year i (time interval $\left[T_{i-1}, T_i\right]$. T_i is the time from the construction of the bridge. The i^{th} term in (9) represents the benefits from T_{i-1} to T_i. The benefits in year i is modelled by $B_i = k_0 V(T_i)$ where k_0 is a factor modelling the average benefits for one vehicle passing the bridge.

The expected repair costs C_R capitalized to the time t = 0 are modelled by

$$C_R(T_R, N_R) = \sum_{i=1}^{N_R} (1 - P_F^U(T_{R_i})) C_{R_0}(T_{R_i}) \frac{1}{(1+r)^{T_{R_i} - T_0}} \tag{10}$$

$P_F^U(T_R)$ is the updated probability of failure in the time interval $]T_0, T_R]$. The updating is based on a no failure event and the available inspection data at the time T_0. The factor $(1 - P_F^U(T_{R_i}))$ models the probability that the bridge has not failed at the time of repair. r is the discount rate. $C_{R_0}(T_{R_i})$ is the cost of repair.

The capitalized expected costs C_F due to failure are determined by

$$C_F(T_R, N_R) = \sum_{i=1}^{N_R+1} C_F(T_{R_i})(P_F^U(T_{R_i}) - P_F^U(T_{R_{i-1}})) \frac{1}{(1+r)^{T_{R_i}}} \tag{11}$$

where $T_{R_0} = T_0$ is the time of the structural assessment and $T_{R_{N_R+1}} = T_{service}$ is the expected service life. The i^{th} term in (11) represents the expected failure costs in the time interval $]T_{R_{i-1}}, T_{R_i}]$. $C_F(T)$ is the cost of failure at the time T.

6. DESIGN OF LONG BRIDGES

Several short span (< 500 m) suspension bridges collapsed due to the wind. The famous and relatively long (854 m) Tacoma Narrows Bridge failed in 1940. In recent years much longer bridges have been constructed. The longest suspension bridge today is the Akashi Kaikyo Bridge in Japan (main span 1991 m) and the second longest is the Great Belt East Bridge in Denmark (main span 1624 m). Future designs with improved girder forms, lightweight cables, and control devices may be up to 3000-5000 m long. For such extremely long bridges, girder stability to wind action may be a serious problem, especially when the girder depth-to-width ratio is small compared with existing long bridges.

The main dynamic problem with long suspension bridges is the aeroelastic phenomenon called flutter. Flutter oscillations of a bridge girder is a stability problem and the oscillations are perpendicular to the direction of the wind and occur when the bridge is exposed to wind velocity above a critical value called the flutter wind velocity

U_{cr}. U_{cr} decreases with decreasing stiffness and damping. Flutter is therefore a serious problem for bridges with a relatively low stiffness such as long bridges. Installation of passive and active control devices may be a solution to the girder stability problem.

Application of flaps to active control of flutter of long suspension bridges has been proposed in Ostenfeld & Larsen [19] to ensure the aerodynamic stability of slender bridge girders by attaching actively controlled flaps along the girders. The effect of these flaps is that they exert forces on the bridge girder when the flaps are exposed to wind. The Ph.D. thesis by Hansen [20] deals with wind tunnel experiments with a sectional model of a girder where the control flaps are installed as integrated parts of the leading and trailing edges of the girder. Several configurations of the flaps have been tested in a wind tunnel at Instituto Technico in Lisbon, Portugal. An analysis of a full span suspension bridge is performed in the Ph.D. thesis by Huynh [21]. For the used configuration of the flaps it is shown that the flutter wind velocity U_{cr} can be increased by 50% compared with a girder with no flaps.

By assuming potential flow theory, it has shown for thin airfoils in incompressible flow that the motion-induced vertical load $L_{ae}(x,t)$ and the motion-induced moment $M_{ae}(x,t)$ on the airfoil are linear in the theoretical displacement and the torsional angle and their first and second derivatives where x is the coordinate in the direction of the bridge and t the time; see Theodorsen [22]. Let y and z be the coordinates in the direction across the bridge and in the vertical direction. A similar formulation for bridges is introduced in Scanlan & Tomko [23]. The aeroelastic forces L^{deck} and M^{deck} per unit span and for small rotations can then be written, see Simiu & Scanlan [24]:

$$L_{ae}^{deck}(x,t)=\frac{\rho U^2 B}{2}\left[KH_1^*(K)\frac{\dot{v}_z}{U}+KH_2^*(K)\frac{B\dot{r}_x}{U}+K^2H_3^*(K)r_x+K^2H_4^*(K)\frac{v_z}{B}\right] \tag{12}$$

$$M_{ae}^{deck}(x,t)=\frac{\rho U^2 B^2}{2}\left[KA_1^*(K)\frac{\dot{v}_z}{U}+KA_2^*(K)\frac{B\dot{r}_x}{U}+K^2A_3^*(K)r_x+K^2A_4^*(K)\frac{v_z}{B}\right] \tag{13}$$

where $K=B\omega/U$ is the non-dimensional reduced frequency, B is the girder width, U is the mean wind velocity, ω is the bridge oscillating frequency (rad.) at the wind velocity U, and ρ is air density. $H_i^*(K)$ and $A_i^*(K)$ (i =1,2,3,4) are non-dimensional aerodynamic derivatives which can be estimated by wind tunnel experiments. The quantities r_x, $\dot{v}_z/U$ and $B\dot{r}_x/U$ are non-dimensional, effective angles of attack. Two types of actively controlled flaps are shown in figure 1.

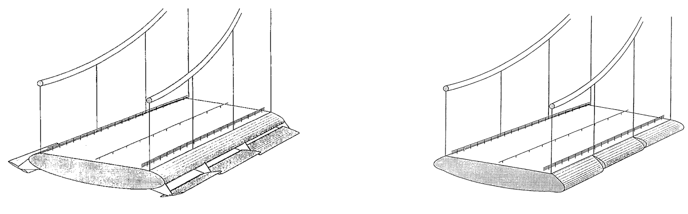

Figure 1: Sections with flaps on pylons and integrated in the section.

By assuming that the angle of a leading flap has no effect on the air circulation it can be shown that the loads due to movement of a leading flap on a thin airfoil are also linear in the angle of the leading flap and in the first and second derivatives. The motion-induced wind loads due to movement of the flaps can therefore be described by additional aerodynamic derivatives.

The total motion-induced wind loads per unit span on the girder and the flaps are, see figure 2

$$L_z^{total} = L_z^{deck} + L_z^{tr}(v_z, r_x^{tr}) + L_z^{le}(v_z, r_x^{le}) \quad (14)$$

$$M_x^{total} = M_x^{deck} + M_x^{tr}(v_z, r_x^{tr}) + M_x^{le}(v_z, r_x^{le}) + \left(L_z^{tr}(v, r_x^{tr}) - L_z^{le}(-v, r_x^{le})\right)\frac{B}{2} \quad (15)$$

where $v_z(x,t)$ and $r_x(x,t)$ are the vertical motion and the rotation of the girder at position x along the bridge girder at the time t. $r_x^{le}(x,t)$ and $r_x^{tr}(x,t)$ are the rotations of the leading and the trailing flaps. Figure 3 shows the calculated flutter velocity U_{cr} for different combinations of flap rotations. α is the rotation of the girder, α_l and α_t are the rotations of the leading and the trailing flaps, φ_l and φ_t are the phase angles between the leading flap, the trailing flap and the girder, respectively.

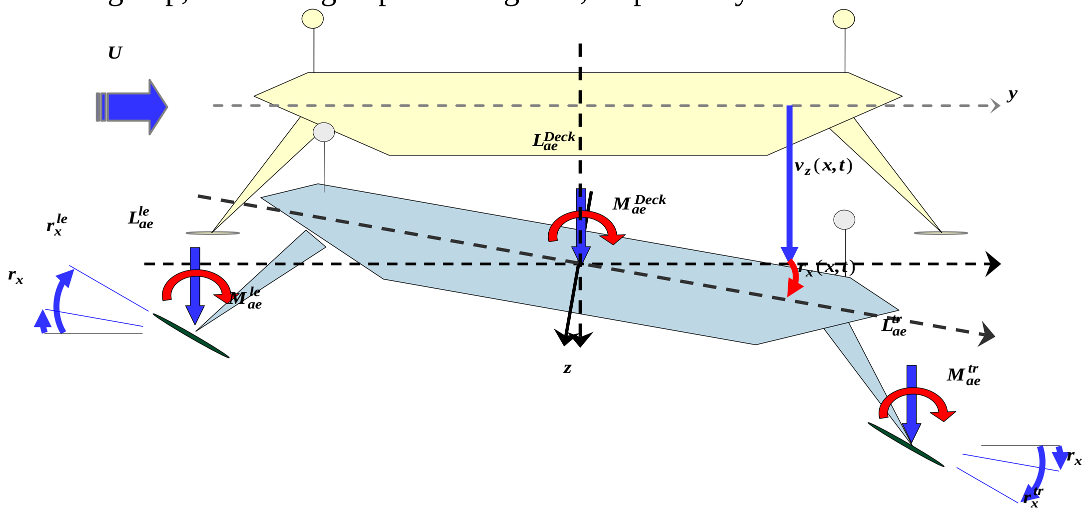

Figure 2: Motion-induced wind loads on the girder and on the flaps.

Figures 4 and 5 show the torsional movement of the model when the flaps are not regulated (configuration 0) and when they are regulated (configuration 2). The wind speed is 6.1 m/s. The conclusion is that configuration 2 is very efficient for controlling the torsional motion of the model. During the first second the torsional motion is reduced from 2.7° to 1.1°, i.e. by 62%.

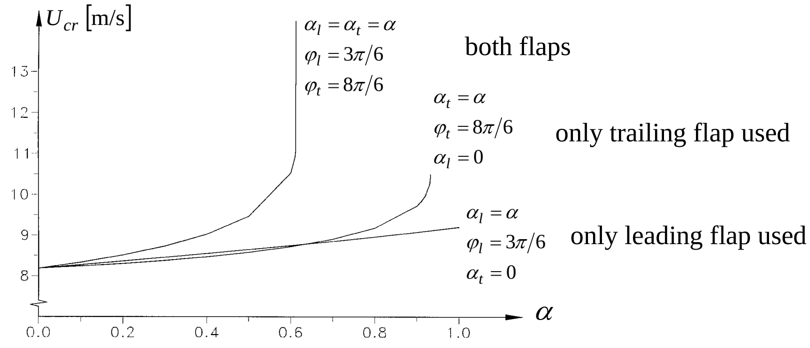

Figure 3: The theoretical effect on the flutter wind velocity of using flaps.

Let $\phi_i(x)$ and $\psi_j(x)$ be the vertical and the torsional mode shapes of the bridge in mode i and mode j which are assumed to be coupled at flutter. The governing modal equations for the two-mode flutter conditions are then

$$M_z\left(\ddot{z}(t) + 2\omega_z\zeta_z\dot{z}(t) + \omega_z^2 z(t)\right) = F_z^{tot}(t) \quad (16)$$

$$M_x\left(\ddot{\alpha}(t)+2\omega_\alpha\zeta_\alpha\dot{\alpha}(t)+\omega_\alpha^2\alpha(t)\right)=F_x^{tot}(t) \tag{17}$$

where $z(t)$ and $\alpha(t)$ are the vertical and the torsional modal coordinates. ω_z, ζ_z and ω_α and ζ_α are the natural frequencies and the damping ratios of the vertical and torsional

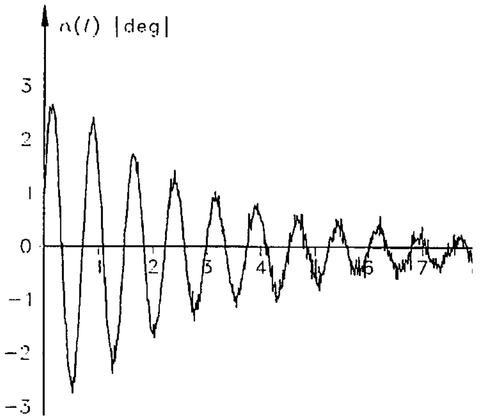

Figure 4: Torsional motion for flap configuration 0 and wind speed 6.1 m/s.

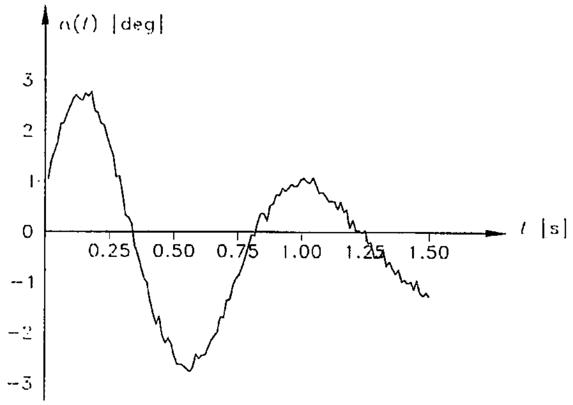

Figure 5: Torsional motion for flap configuration 2 and wind speed 6.1 m/s.

modes. M_z and M_x are the vertical and the torsional modal masses. At the coupled motion, the vertical and the torsional modal responses are both assumed to be proportional to $e^{i\omega t}$, when the critical wind velocity is acting on the bridge, i.e. $z(t) = z_0 e^{i\omega t}$ and $\alpha(t) = \alpha_0 e^{i\omega t}$.

When this is introduced into the above equations the following matrix equation can be derived

$$\boldsymbol{A}\begin{bmatrix} z/B \\ \alpha \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{18}$$

where the system matrix $\boldsymbol{A}$ depends on the natural mode shapes and frequencies, the damping ratios, the derivatives and the wind velocity. This matrix equation has non-trivial solutions when

$$\mathrm{Det}(\boldsymbol{A})=\mathrm{Re}\,\mathrm{Det}(\boldsymbol{A})+i\,\mathrm{Im}\,\mathrm{Det}(\boldsymbol{A})=0 \tag{19}$$

resulting in the following two flutter conditions for a bridge with separate flaps; see Huynh [21]:

$$\begin{aligned}
\mathrm{Re(Det)} = {} & \frac{\omega^4}{\omega_z^4}\left(1+\frac{M3}{J\omega^2\Psi}+\frac{L4}{m\omega^2\Phi}+\frac{1}{mJ\omega^4\Psi\Phi}\left[-\omega^2 L1M2+L4M3-M4L3+\omega^2 M1L2\right]\right)\\
& +\frac{\omega^3}{\omega_z^3}\left(2\zeta_z\frac{M2}{J\omega\Psi}+2\zeta_\alpha\frac{\omega_\alpha}{\omega_z}\frac{L1}{m\omega\Phi}\right)\\
& +\frac{\omega^2}{\omega_z^2}\left(-1-\frac{\omega_\alpha^2}{\omega_z^2}-4\frac{\omega_\alpha}{\omega_z}\zeta_z\zeta_\alpha-\frac{M3}{J\omega^2\Psi}-\frac{\omega_\alpha^2}{\omega_z^2}\frac{L4}{m\omega^2\Phi}\right)+\frac{\omega_\alpha^2}{\omega_z^2}=0
\end{aligned} \tag{20}$$

$$\mathrm{Im(Det)} = \frac{\omega^3}{\omega_z^3}\left(\frac{M2}{J\omega\ \ \Psi} + \frac{L1}{m\omega\ \ \Phi} + \frac{1}{m\omega^3\Phi J\Psi}\left[L1M3 + L4M2 - \ M1L3 - \ M4L2\right]\right)$$
$$+\frac{\omega^2}{\omega_z^2}\left(-2\zeta_z - 2\zeta_\alpha\frac{\omega_\alpha}{\omega_z} - 2\zeta_\alpha\frac{\omega_\alpha}{\omega_z}\frac{L4}{m\omega^2\Phi} - 2\zeta_z\frac{M3}{J\omega^2\Psi}\right)$$
$$+\frac{\omega}{\omega_z}\left(-\frac{M2}{J\omega\ \ \Psi} - \frac{\omega_\alpha^2}{\omega_z^2}\frac{L1}{m\omega\ \ \Phi}\right) + 2\zeta_z\frac{\omega_\alpha^2}{\omega_z^2} + 2\zeta_\alpha\frac{\omega_\alpha}{\omega_z} = 0 \qquad (21)$$

where m is the girder mass per unit span. Φ, Ξ and Ψ are the modal integrals of the girder given by:

$$\Phi = \int_0^L \phi_1^2(x)dx\ ,\ \Xi = \int_0^L \phi_1(x)\psi_1(x)dx\ ,\ \text{and}\ \ \Psi = \int_0^L \psi_1^2(x)dx \qquad (22)$$

and where *L1* to *L4* and *M1* to *M4* contain the modal integrals of the flaps Φ_f, Ξ_f and Ψ_f, the sum of flutter derivatives referred to the girder and the flaps (see Huynh, T. 2000 for full expressions). Finally, note that the flutter mode can be a coupling of more than two modes. In that case, an additional mode gives an additional equation. The determinant condition (19) is still valid, but the calculation of the solution is rather complicated analytically. The obtained critical wind velocity U_{cr} and the critical frequency ω_{cr} will not be varied by more than 5%, if several similar mode shapes with close frequencies are taken into account in the flutter computation, see [9].

7. CONCLUSIONS

It is shown in the paper that recent developed methodologies in stochastic and optimization may be used to solve complex problems related to infrastructure systems. A general formulation of the cost optimization problem is presented with special emphasis on bridge networks. Finally, the difficult (from a formulation and mathematical point of view) problem of estimating the flutter wind velocity for a suspension bridge is solved numerically to show how advanced research can take part in solving infrastructure problems.

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