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Published in:
ICWMC 2015, The Eleventh International Conference on Wireless and Mobile Communications

Publication date:
2015

Document Version
Accepted author manuscript, peer reviewed version

Throughput Analysis of Full Duplex Communication with Asymmetric Traffic in Small Cell Systems

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***Abstract*—Full duplex communication promises a 100% throughput gain by enabling simultaneous transmission and reception. However, such simultaneous communication leads to a corresponding increase in the network interference. In addition, full duplex communication can only be exploited when traffic is available in both uplink and downlink directions; while, cellular network traffic tend to be downlink heavy in practice. The potential throughput gains of full duplex communication over conventional half duplex transmission in a small cell network with asymmetric traffic conditions are investigated in this contribution using network analysis tools from stochastic geometry. The analytical findings are further confirmed through computer-based Monte-Carlo simulations. Asymmetric downlink/uplink traffic pattern and the increased network interference stemming from full duplex transmissions are found to limit its potential performance to well below the promised 100% throughput gain.**

***Index Terms*—Full duplex communication; small cells; stochastic geometry; 5G.**

I. INTRODUCTION

Full duplex communication, i.e. simultaneous transmission and reception over the same frequency band, promises a 100% throughput gain over conventional half duplex (HD) transmissions. Historically, full duplex (FD) communication had been considered impractical due to the overwhelming loopback interference from the transmission-end. Recent advances in self-interference cancellation (SIC) in both analog and digital domain allow suppressing this loopback interference to within tolerable limits, thereby making FD communication appealing with viable costs [1]. In that respect, FD has the potential of becoming a significant breakthrough in the design of a novel 5th Generation (5G) radio access technology.

Alongside the self interference, the promise of doubling the network throughput (TP) through FD communication with respect to HD transmission may be jeopardized by a number of other factors. Simultaneous transmissions from both ends of a communication link inevitably results in extra interference to the network compared to conventional HD transmission [2]. Furthermore, FD transmissions can only be exploited with traffic available at both downlink (DL) and uplink (UL) directions; whereas in practice, networks have a traffic profile skewed in favor of the DL direction.

The TP performance of wireless networks with FD capable radios have been investigated in [3]–[5] among others. The authors in [3] show that FD capabilities can significantly increase the aggregate throughput of current cellular systems with FD enabled access points (AP) and HD user equipments (UE) under symmetric traffic conditions and relatively isolated cells. On a different note, reference [5] considers a large wireless network and analytically investigates the TP gain of FD communication using stochastic geometry tools.

Building on our earlier system level simulations based exercise investigating the performance of local area network with FD-capable radios [4], the TP performance of FD communication in a small cell system is analytically derived in this contribution. We consider a fixed number of small cells, where the APs and UEs in each cell transmit randomly with independent transmission probabilities. Statistics of the random interference power at a generic receiver are thereby obtained analytically, and applied to evaluate the corresponding ergodic TP for the equivalent FD/HD system. Analytical findings are further validated through simulations.

Similar to [5], stochastic geometry based tools are used in this study to model the wireless network. However, reference [5] models the wireless network as a Poisson point process (PPP), which better reflects an ad-hoc network with a large number of nodes [6]. Here, the network is modelled as a binomial point process (BPP), which closely reflects a local area network with an arbitrary number of small cells [6]. Furthermore, we derive the TP gain of FD while in [5] only upper and lower bounds are found. Finally, we address asymmetric UL/DL traffic profiles which are typically disregarded in analytical studies.

Organization: Section II introduces the system model, followed by statistical representation of the sum interference power in Section III. Numerical results and concluding remarks are then presented in Sections IV and V respectively.

II. SYSTEM MODEL

We consider a local area system with a number of small cells distributed in a circular area $\mathcal{R}$ of radius R in the two-dimensional plane $\mathbb{R}^2$. Each small cell consists of an AP and a single active UE. Our analysis focuses on the performance of a generic reference cell with the desired receiver located at the origin. K interfering cells are assumed to be uniformly distributed around the reference cell, as shown in Figure 1.

The locations of the interfering nodes (i.e. APs and UEs) can be modelled as realizations of random spatial point processes. Such an assumption allows us to analyze the problem in hand using tools from stochastic geometry [6]. Each interfering cell can be modelled as two independent and identically distributed (iid) points in $\mathcal{R}$ representing the AP and UE respectively. The resulting wireless network with a fixed number of nodes can be modelled as the BPP, which closely represents a small cell system [6].

A slotted ALOHA access protocol is considered, where at any given time, the AP and UE in cell k transmits data with independent access probabilities $\rho_{AP,k}$ and $\rho_{UE,k}$ respectively. The desired transmitter receiver separation distance is fixed at d meters. Assuming $R \gg d$, the UL and the DL transmissions in the reference cell can be considered to experience similar interference conditions.

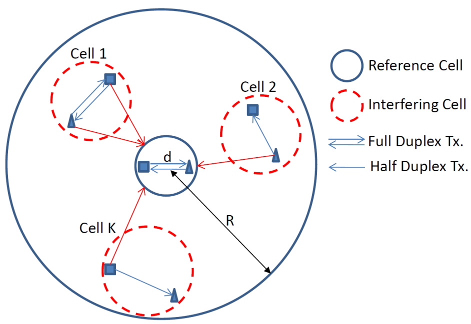

Figure 1. System Model depicting the Reference cell at the center of $\mathcal{R}$ with a random number of interfering cells in FD or HD transmission mode.

Signal Model: The interference power at the desired receiver from a random interferer k located r_k meters away is given by $\zeta_k = \eta g \beta(r_k)$, where $\eta = \eta_0 p \nu$ is a constant path loss factor accounting for the transmit power p, the interference isolation among neighbouring cells ν (commonly known as wall loss) and η_0 : the path loss at reference distance.

A. Sum Interference Power from Multiple Interferers

Let $\Omega \in \{FD(AP), FD(UE), HD\}$ denote the index of set of interferers with the respective transmission mode. Note that, the APs and UEs can both transmit simultaneously with FD transmission, whereas either the AP or the UE of a particular cell transmits in the conventional HD case. The sum interference power for the various transmission modes is modelled in this subsection.

1) Distribution of the Number of Interfering Cells: Due to the assumed slotted ALOHA random access mode, the number of active interfering cells is a random variable (r.v.). Assuming each cell transmits independently with probability ρ_Ω; let $\Lambda_\Omega(\rho_\Omega)$ represent the number of active interferers for the transmission mode Ω with $\lambda \in \{0, 1, \ldots, K\}$ being its realization. By virtue of the assumed BPP network model, the probability mass function (PMF) of $\Lambda_\Omega(\rho_\Omega)$ is given as [8]

$$f_{\Lambda_\Omega(\rho_\Omega)}(\lambda; \rho_\Omega) = \binom{K}{\lambda} \rho_\Omega^\lambda \left(1 - \rho_\Omega\right)^{K-\lambda}. \tag{1}$$

2) Sum Interference Power with HD Transmission: In the conventional HD case, a particular cell can only transmit in either the UL or the DL direction. Considering independent UL and DL traffic, the transmission probability of a particular cell is given by $\rho_{HD} = \rho_{AP} + \rho_{UE} - \rho_{AP}\rho_{UE}$. Note, henceforth we assume $\rho_{AP,k} = \rho_{AP}$ and $\rho_{UE,k} = \rho_{UE}\, \forall k$. The sum interference power with HD transmission is then readily given by $\zeta_{HD} = \sum_{k \in \Lambda_{HD}(\rho_{HD})} \zeta_k$, where ζ_k is the interference from a single random interferer.

3) Sum Interference Power with FD Transmission: With FD transmissions, the AP and the UE can both transmit simultaneously as long as there are available packets to transmit. Therefore, the APs and the UEs can be treated as two different sets of interferers with the respective transmission probabilities ρ_{AP} and ρ_{UE}. Correspondingly, the total interference with FD transmission is the sum of the interference contributions from the set of APs and the UEs, i.e. $\zeta_{FD} = \zeta_{FD(AP)} + \zeta_{FD(UE)}$, where $\zeta_{FD(AP)} = \sum_{k \in \Lambda_{FD(AP)}(\rho_{AP})} \zeta_k$ and $\zeta_{FD(UE)} = \sum_{k \in \Lambda_{FD(UE)}(\rho_{UE})} \zeta_k$.

B. Ergodic Throughput Calculation

In this contribution, we consider the Shannon rate $R(\gamma) = \log_2(1+\gamma)$ as a measure of the instantaneous throughput, where γ is the instantaneous signal to interference plus noise ratio (SINR). Let ϕ denote the desired signal power at the considered receiver. The instantaneous SINR for the transmission mode $\Omega \in \{FD, HD\}$ can be expressed as $\gamma_\Omega = \frac{\phi}{\zeta_\Omega + N_0}$, where N_0 is the additive white Gaussian noise power. The ergodic TP can be obtained by averaging the instantaneous TP over the distribution of the SINR i.e. $R_\Omega = \mathbb{E}_{\gamma_\Omega}\left[\log_2(1+\gamma_\Omega)\right]$, where $\mathbb{E}_X\left[\cdot\right]$ is the expectation operator over the distribution of the r.v. X. This expectation requires a two-fold integration over the distributions of ϕ and ζ_Ω, and is not easy to evaluate directly. A simpler expression for the ergodic TP involving a single integration can instead be obtained using the moment generating functions (MGF) of the ϕ and ζ_Ω as follows [9]

$$R_\Omega = \frac{1}{\ln(2)} \int_0^\infty \frac{\mathcal{M}_{\zeta_\Omega}({}^{s}/_{N_0})\left(1 - \mathcal{M}_\phi({}^{s}/_{N_0})\right)}{s} \exp(-s)\ \mathrm{d}s, \tag{2}$$

where $\mathcal{M}_{\zeta_\Omega}(s)$ and $\mathcal{M}_\phi(s)$ are the MGFs of ζ_Ω and ϕ respectively, which are derived in Section III below. Eq. (2) can be easily evaluated using mathematical software or suitable numerical integration techniques.

Throughput Gain of Full duplex over Half duplex

The TP gain of FD over HD transmission is given as

$$\xi = \frac{TP_{FD} - TP_{HD}}{TP_{HD}}, \tag{3}$$

where $TP_{HD} = \rho_{HD} R_{HD}$ and $TP_{FD} = (\rho_{AP} + \rho_{UE}) R_{FD}$ are the average TP at the reference cell with HD and FD transmission mode, respectively.

III. Statistical Representation of the Signal Powers

Following [10], we define the MGF of a r.v. x to be a function of the complex variable s with a negative argument as follows: $\mathcal{M}_x(s) = \mathbb{E}\left[\exp(-sx)\right]$. The MGFs $\mathcal{M}_{\zeta_\Omega}(s)$ and $\mathcal{M}_\phi(s)$ of the r.vs ζ_Ω and ϕ are derived in this section.

A. MGF of the Desired Signal Power ϕ

Assuming the transmit power at the desired transmitter, and the desired channel fading power gain are given by p_0 and g_0 respectively, the power of the received signal of interest is $\phi = \eta_0 p_0 g_0 \beta(d)$. Considering g_0, which is the only r.v. in ϕ, to be Gamma distributed with parameter m, the MGF of ϕ readily evaluates to $\mathcal{M}_\phi(s) = \left(1 + \frac{s\eta_0 p_0 \beta(d)}{m}\right)^{-m}$ [10].

B. MGF of the Single Cell Interference Power ζ_k

The MGF of the interference power from a random interferer k located r_k meters away is given by $\mathcal{M}_{\zeta_k}(s) = \mathbb{E}\Big[\exp\left(-s\zeta_k\right)\Big] = \mathbb{E}_{g,\beta(r_k)}\Big[\exp\left(-s\eta g\beta\left(r_k\right)\right)\Big]$.

Corollary III.1. *The distribution of the distance dependent path loss $\beta(r_k)$ is given by*

$$f_{\beta(r_k)}(t) = \frac{2}{\alpha R^2}\left(t^{-\frac{2}{\alpha}-1} - t^{-\frac{1}{\alpha}-1}\right), \quad (1+R)^{-\alpha} \le t \le 1. \quad (4)$$

Proof. Under uniform distribution of a user, the distribution of the random distance r_k between a generic receiver located at the origin and a random transmitter k in $\mathcal{R}$ is given by $f_{r_k}(r) = \frac{2r}{R^2}$ for $0 \le r \le R$ [8]. The distribution of $\beta(r_k)$ in Eq. (4) follows directly from $f_{r_k}(r)$ through a change of variable involving $t \triangleq \beta(r_k) = (1+r_k)^{-\alpha}$. □

Using conditional expectations, the MGF of ζ_k can be expanded as $\mathcal{M}_{\zeta_k}(s) = \mathbb{E}_t\Big[\mathbb{E}_g\big[\exp\left(-s\eta g t\right)|t\big]\Big]$. Considering g to be Gamma distributed with parameter m, the inner expectation over the r.v. g conditioned on t evaluates to $\left(1+\frac{s\eta t}{m}\right)^{-m}$ [10]. Elaborating the outer expectation over the r.v. t, $\mathcal{M}_{\zeta_k}(s)$ can be expanded as $\mathcal{M}_{\zeta_k}(s) = \int \left(1+\frac{s\eta t}{m}\right)^{-m} f_{\beta(r_k)}(t)\ \mathrm{d}t$. Using Eq. (4), the above integral can then be expressed as

$$\mathcal{M}_{\zeta_k}(s) = \frac{2}{\alpha R^2}\int_{(1+R)^{-\alpha}}^{1} \frac{t^{-\frac{2}{\alpha}-1} - t^{-\frac{1}{\alpha}-1}}{\left(1+\frac{s\eta t}{m}\right)^m}\ \mathrm{d}t. \quad (5)$$

Considering a change of variable involving $u = \frac{-s\eta t}{m}$ followed by some algebraic manipulations, Eq. (5) can be reduced to the following closed form expression [7]

$$\begin{aligned}\mathcal{M}_{\zeta_k}(s) = &\frac{2}{\alpha R^2}\times \\ &\left[\left(-\frac{s\eta}{m}\right)^{\frac{2}{\alpha}} \mathrm{B}\left(\frac{-s\eta}{m}, \frac{-s\eta(1+R)^{-\alpha}}{m}; \frac{-2}{\alpha}, 1-m\right) - \right.\\ &\left.\left(-\frac{s\eta}{m}\right)^{\frac{1}{\alpha}} \mathrm{B}\left(\frac{-s\eta}{m}, \frac{-s\eta(1+R)^{-\alpha}}{m}; \frac{-1}{\alpha}, 1-m\right)\right], \quad (6)\end{aligned}$$

where $\mathrm{B}\left(z_1, z_2; a, b\right)$ is the difference of two incomplete beta functions known as the generalized incomplete beta function: $\mathrm{B}\left(z_1, z_2; a, b\right) \triangleq \mathrm{B}\left(z_2; a, b\right) - \mathrm{B}\left(z_1; a, b\right)$; with the incomplete beta function defined as $\mathrm{B}\left(z; a, b\right) = \int_0^z u^{a-1}(1-u)^{b-1}$ [11, Eq. 6.6.1].

C. MGF of the Sum Interference Power ζ_Ω

With multiple interfering cells, the total interference power ζ_Ω is the sum of the interference powers from the individual interfering cells; i.e. $\zeta_\Omega = \sum_{k\in\Lambda_{\Omega(\rho_\Omega)}} \zeta_k$, where ζ_k is the interference from a single random interferer.

Theorem III.2. *The MGF of the sum interference power ζ_Ω from K cells transmitting randomly with independent transmission probability ρ_Ω is*

$$\mathcal{M}_{\zeta_\Omega}(s;\rho_\Omega) = \Big(1 - \rho_\Omega\big[1 - \mathcal{M}_{\zeta_k}(s)\big]\Big)^K, \quad (7)$$

where $\mathcal{M}_{\zeta_k}(s)$ is given by Eq. (6).

Proof. Using conditional expectations, the MGF of ζ_Ω can be written as $\mathcal{M}_{\zeta_\Omega}(s;\rho_\Omega) = \mathbb{E}_{\Lambda_{\Omega(\rho_\Omega)}}\left[\mathcal{M}_{\zeta_\Omega}(s)|\lambda\right]$, where $\mathcal{M}_{\zeta_\Omega}(s)|\lambda$ is the MGF of the sum interference power from λ iid interferers. Utilizing the fact that $\mathcal{M}_{(X+Y)}(s) = \mathcal{M}_X(s)\mathcal{M}_Y(s)$ for independent r.vs X and Y, we obtain $\mathcal{M}_{\zeta_\Omega}(s)|\lambda = \left(\mathcal{M}_{\zeta_k}(s)\right)^\lambda$. Using Eq. (1) and the law of total probability, $\mathcal{M}_{\zeta_\Omega}(s;\rho_\Omega) = \mathbb{E}_{\Lambda_{\Omega(\rho_\Omega)}}\left[\mathcal{M}_{\zeta|\lambda}(s)\right]$ can thereby be expanded as

$$\mathcal{M}_{\zeta_\Omega}(s;\rho_\Omega) = \sum_{\lambda=0}^{K}\binom{K}{\lambda}\left(\rho_\Omega\mathcal{M}_{\zeta_k}(s)\right)^\lambda\left(1-\rho_\Omega\right)^{K-\lambda},$$

which reduces to Eq. (7) by virtue of the Binomial Theorem. □

D. MGF of the Sum Interference Power with FD and HD transmissions

Having derived the expression of the MGF for the sum interference power, we now specifically address the MGF of the sum interference power with FD and HD transmission as a function of the respective channel access probabilities discussed in Section II-A. In the conventional HD case, a particular cell transmits with channel access probability ρ_{HD}. The ensuing sum interference power MGF $\mathcal{M}_{\zeta_{HD}}(s;\rho_{HD})$ is then readily given by Th. III.2. With FD communication, the APs and the UEs are treated as two independent sets of interferers having transmission probabilities ρ_{AP} and ρ_{UE} respectively. The resulting sum interference power is $\zeta_{FD} = \zeta_{FD,AP} + \zeta_{FD,UE}$. Applying Th. III.2, the corresponding MGF is thereby obtained as $\mathcal{M}_{\zeta_{FD}}(s) = \mathcal{M}_{\zeta_{FD,AP}}(s;\rho_{AP})\mathcal{M}_{\zeta_{FD,UE}}(s;\rho_{UE})$.

The ergodic TP with FD or HD transmission can now be readily computed by inserting the respective sum interference power MGF into Eq. (2).

IV. Numerical Results

Matlab® based Monte Carlo simulation results validating the derived analytical findings are presented in this Section. At least $100,000$ independent snapshots of each scenario are simulated to ensure statistical reliability. The following general simulation parameters are assumed to reflect a typical dense small cell system: path loss exponent $\alpha = 3$, cell radius $R = 100$, transmit power $p_0 = 13$ dBm, reference path loss $\eta_0 = -38$ dB, gamma parameter $m = 2$ and desired AP-UE distance $d = 10$ m.

A. Impact of Traffic Asymmetry in Isolated Cell

The impact of traffic asymmetry on the TP gain with FD transmission is investigated first. To this end, a single cell scenario (i.e. $\zeta_{FD} = \zeta_{HD} = 0$) is considered. The ensuing TP gain for this special case is $\xi = \frac{\rho_{AP}\rho_{UE}}{\rho_{AP}+\rho_{UE}-\rho_{AP}\rho_{UE}}$, as presented in Figure 2. The analytical TP gains are found to match closely with the simulation results, thereby validating the derived findings.

Interestingly, the promised 100% TP gain is only achieved with a full buffer traffic at both transmission directions (i.e. $\rho_{AP} = \rho_{UE} = 1$). In general, the TP gain is low when either of the two transmission probabilities ρ_{AP} and ρ_{UE} are low.

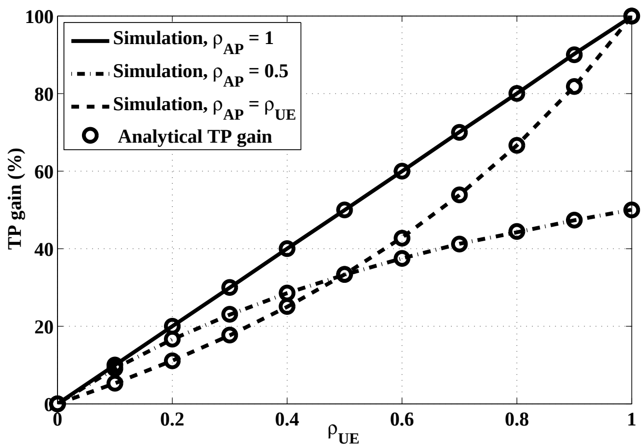

Figure 2. TP gain of FD over HD transmission with asymmetric traffic.

B. Impact of Traffic Asymmetry in FD System

The FD TP gain is next investigated in a system level setting with multiple interferers and with variable transmission probabilities ρ_{AP} and ρ_{UE}. Figure 3 presents the TP gain of FD over HD vs ρ_{UE} with $K = 10$ potential interferers for different values of ρ_{AP}. The analytical TP gains are found to closely match the simulation results, as in Figure 2. The maximum TP gain with FD is more than halved in a system level scenario with wall loss of up to 5 dB due to the resulting increase in inter-cell interference (ICI) with FD transmission. The TP gain is expectedly found to improve with increasing isolation among the cells (i.e. the wall loss). This reinforces the fact that the increased network interference due to FD transmissions has an adverse impact on the promised TP gains.

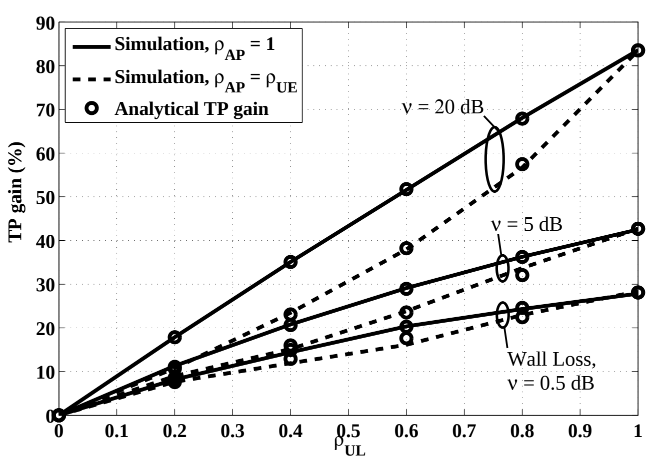

Figure 3. Throughput gain of FD over HD communication in a system level setting with 10 potential interferers with various transmission probabilities ρ_{AP} and different wall loss values ν.

An interesting paradox is observed for the $\rho_{AP} = \rho_{UE}$ scenario. The ICI is reduced at low transmission probabilities (i.e. low ρ_{AP}). However, opportunities to exploit FD communication, which is enabled by overlapping channel access at both ends is also reduced; thereby resulting in the observed low TP gain. Although detailed results had to be omitted due to space constraint, our study further reveals that the TP gain of FD also reduces with increasing the network size K and decreasing the parameters R and α.

V. Conclusion

The potential TP gain of FD communication over conventional HD transmissions in a dense small cell scenario, as targeted by the upcoming 5G radio access technology, is analytically derived and cross validated through extensive Monte Carlo simulations in this contribution. An ideal SIC transceiver model is assumed in order to isolate the impact of the interference coupling and the traffic asymmetry. TP gains obtained numerically are found to closely match the simulation results. The derived analytical results provide with a rather simple model to evaluate the potential performance of FD communication in a system level setting without invoking lengthy system level simulations.

The results reveal that the mean TP gain of FD over HD is only 100% for an isolated cell. In a system level setting, the TP gains depend on two critical factors; namely the interference coupling among the cells, and the availability of traffic at both ends in order to exploit the FD potential. The interference coupling among the cells is in turn a function of the wall loss, the path loss exponent, fading parameters and the user density. Contrary to the promised 100% gain, modest gains of around 40% was observed considering realistic parameter values. As part of the future work, we plan to extend our study of FD by considering more practical constraints on the transceiver operations; and investigate algorithms that can better exploit the potential of FD communication.

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