



AALBORG UNIVERSITY
DENMARK

Aalborg Universitet

Papers, volume 5 - 1997-2000

Thoft-Christensen, Palle

Publication date:
2006

Document Version
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

Citation for published version (APA):
Thoft-Christensen, P. (2006). *Papers, volume 5 - 1997-2000*. Aalborg Universitetsforlag.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal -

Take down policy

If you believe that this document breaches copyright please contact us at vbn@aub.aau.dk providing details, and we will remove access to the work immediately and investigate your claim.

CHAPTER 101

ON RELIABILITY BASED OPTIMAL DESIGN OF CONCRETE BRIDGES¹

P. Thoft-Christensen
Aalborg University, Aalborg, Denmark

ABSTRACT

In recent years important progress has been made in the assessment of the lifetime behaviour of concrete bridges. Due to the large uncertainties related to the loading and the deterioration of such bridges, an assessment based on stochastic modelling of the significant parameters seems to be the only relevant modelling. However, a great number of difficulties are involved in the modelling. The main purpose of this paper is to give an overview of areas where more research is needed and in some cases propose solutions.

There seems to be a general agreement that optimal design of concrete bridges should include assessment of not only the initial costs but also expected costs related to preventive and essential maintenance, and expected failure costs. The total lifetime costs of a system of bridges or a single bridge will therefore not only depend on load and strength parameters but also on inspection and repair costs and also on design criteria (limit states). Further, it is of interest to study the reliability of a single bridge from a systems point of view but also to study whether bridges can be considered from an individual basis or whether a system (group) approach should be used.

The conclusion of this study is that reliability based optimal design of an infrastructure system of concrete bridges can now be performed based on recent research results obtained worldwide. However, much more research in this area is needed before a general procedure can be established. A number of important factors are only partly investigated or not addressed at all. It is believed that optimal design in this context will reduce drastically the lifetime costs of operating this type of infrastructure systems.

¹ Proceedings 2000 Structures Congress, Philadelphia, USA, May 8-10, 2000.

1. INTRODUCTION

Reliability based structural design is now a well-established concept at least from a theoretical point of view. In a review paper from 1991 by Thoft-Christensen [1] 125 references on reliability based structural optimization were included. From a very slow start in the sixties a drastic increase in the number of papers took place in the years 1985-1989. In the nineties the number of papers has almost exploded. In the textbook by Thoft-Christensen & Murotsu, [2], from 1986 there is a chapter on reliability based optimum design where element structural reliability as well as structural systems reliability is considered. Since then a large number of papers in this area have been published in journals like *Structural Safety* or in proceedings from conferences like the IFIP WG 7.5 conferences [3], [4], [5], [6], [7], [8], [9], [10], other conferences [11], [12], [13], [14], [15], [16], [17] and several others.

Modern reliability based optimal design of structures is based on the life-cycle costs i.e. not only the initial costs $C_{initial}$ but also the expected inspection costs $C_{inspect}$, the expected repair costs C_{repair} , and the expected failure costs $C_{failure}$ are taken into consideration. Accepting this then the design problem is an optimization problem based on expected costs. The optimization problem can be formulated in different ways. However, the two most common are based on a minimization of the expected lifecycle costs $C_{lifecycle}$ and a maximization of the expected life-cycle benefits $B_{lifecycle}$ minus the expected life-cycle costs $C_{lifecycle}$, respectively. These two quantities can be formulated in the following way:

$$C_{lifecycle} = C_{initial} + C_{inspect} + C_{repair} + C_{failure} \quad (1)$$

and

$$\begin{aligned} W &= B_{lifecycle} - C_{lifecycle} \\ &= B_{lifecycle} - C_{initial} - C_{inspect} - C_{repair} - C_{failure} \end{aligned} \quad (2)$$

In this paper the optimization problem is based on W as defined in (2). The optimization variables \bar{x} can be the dimensions of the structure, the reinforcements etc., but also parameters related to the inspection and the repair. The constraints are related to the reliability; however, they may also be deterministic constraints (e.g. code requirements).

$$\begin{aligned} \max_{\bar{x}} W[\bar{x}] \\ \text{s.t.} \quad \text{reliability and other constraints} \end{aligned} \quad (3)$$

It has been proposed by Thoft-Christensen [18], to use a risk based structural optimization rather than reliability based optimization. The risk for a failure mode is defined as the product of the failure cost and the failure probability.

In the following the different terms included in (2) and (3) are briefly discussed from a stochastic modelling viewpoint.

2. RELIABILITY PROFILES

Estimation of the reliability of a reinforced concrete bridge is a fundamental problem in reliability based optimal design since the reliability as a function of time (the reliability profile) to some degree controls the bridge rehabilitation. Further, the reliability is also included in the constraints in (3). In Thoft-Christensen [19] the reliability profiles for

reinforced concrete slab bridges are estimated. The deterioration mechanism is corrosion of the reinforcement due to chloride penetration. Using Fick's law of diffusion the corrosion initiation time is estimated. When corrosion has started, then the diameter of the reinforcement bar in question is supposed to decrease linearly with the time t . The normalized reliability profile for a given bridge (yield line limit state) as a function of the time t is shown in figure 1. The normalized reliability index $\beta(t)/\beta(0)$ is seen for this case to be constant until corrosion initiation at $t = 65$ years and then decrease in an approximate linear way with a deterioration rate α . The same modelling of the reliability profile by two straight lines was found to be a good approximation for several other concrete slab bridges.

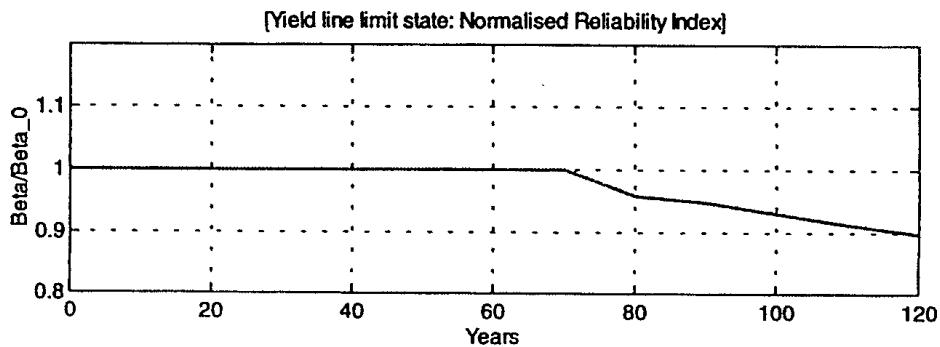


Figure 1. Reliability profile for reinforced concrete slab bridge.

In the next section it will be shown how the above-mentioned modelling of reliability profiles makes it possible to estimate the so-called reliability profiles for concrete bridges.

3. RELIABILITY DISTRIBUTIONS

This section is based on Thoft-Christensen [20] where a group of 15 “good” UK bridges are analysed. A number of distributions for groups of bridges are important for reliability based optimal design of bridges when maintenance is included. The *initial reliability distribution* $\beta(0)$

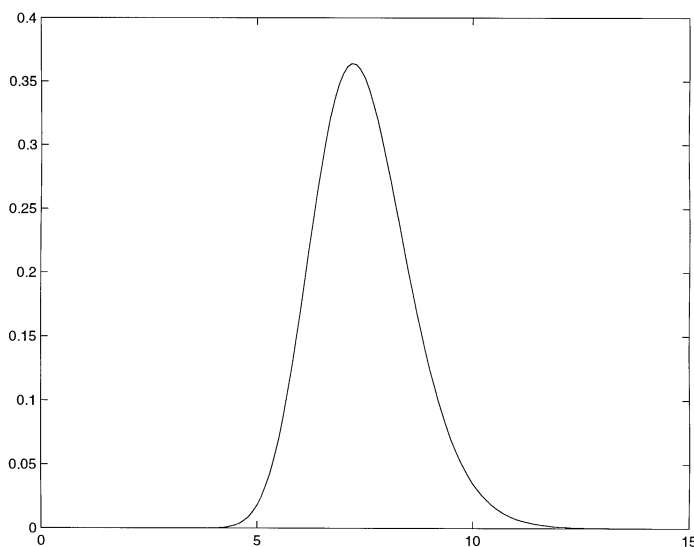


Figure 2. Initial reliability distribution.

is the distribution of reliability indices for the group of bridges at the time 0, i.e. in the year when each of the bridges was constructed. For the above-mentioned bridges a lognormal distribution was chosen, see figure 2.

The *corrosion initiation time distribution* is obtained by crude Monte Carlo simulation and the corrosion modelling mentioned above. For the group of bridges in question the corrosion initiation time

can be approximated by a Weibull distribution, see figure 3. The *deterioration rate distribution*, i.e. the distribution of α , is proposed by Thoft-Christensen [20] to be modelled by a uniform distribution.

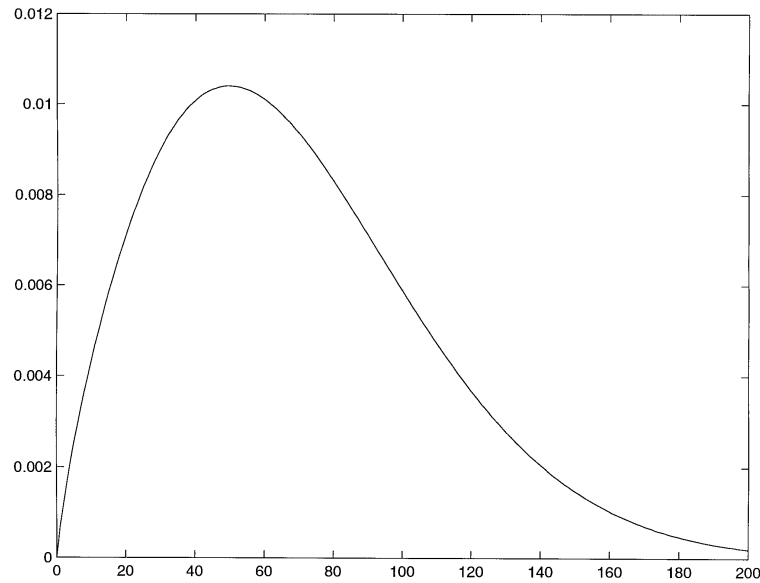


Figure 3. Corrosion initiation time distribution.

Using Monte Carlo simulation a number of important distributions can be estimated on basis of the above-mentioned three distributions. For maintenance the so-called *first rehabilitation distribution* is of great interest. It shows the time to the crossing of a critical level when no maintenance has taken place. For a group of 970 reinforced concrete bridges in UK the first rehabilitation time distribution is shown in figure 4, where the critical level is $\beta = 4.6$.

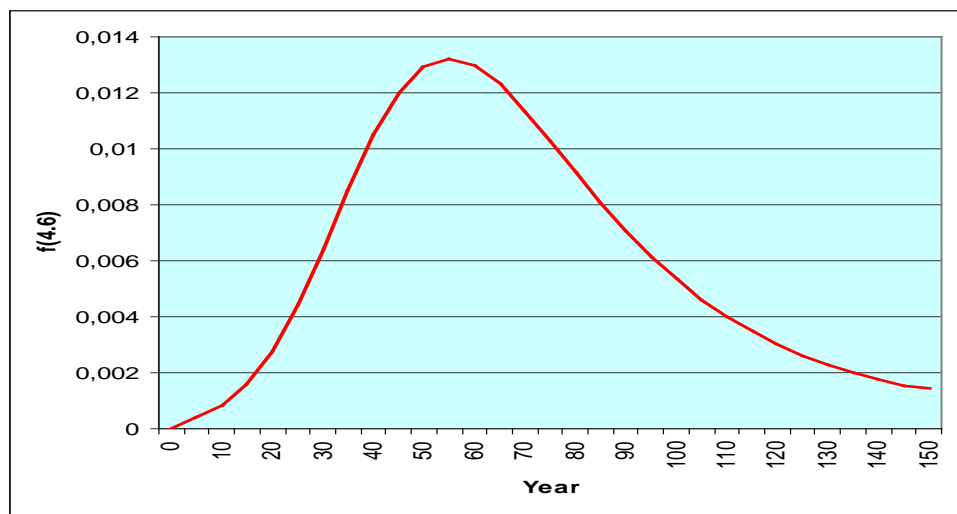


Figure 4. First rehabilitation time distribution.

Similar distributions can be estimated when preventive and essential maintenance is taken into account. Optimal maintenance strategies can be designed on the basis of such distributions. When a maintenance strategy has been chosen then the optimal design of new bridges can be performed on the basis of (2) and (3).

4. MODELLING OF THE BENEFITS

The benefits in the expected lifetime t_L (in years) are modelled by, see Thoft-Christensen [21]

$$B_{lifecycle} = \sum_{i=1}^{t_L} B_i \frac{1}{(1+r)^{t_i}} \quad (4)$$

where B_i are the benefits in year i (time interval $[t_{i-1}, t_i]$). r is the discount rate. t_i is the time from the construction of the bridge. The i th term in (4) represents the benefits from t_{i-1} to t_i .

The benefits in year i are modelled by

$$B_i = k_0 V(t_i) \quad (5)$$

k_0 is a factor modelling the average benefits for one vehicle passing the bridge. It can be estimated as the price of rental of an average vehicle/km times the average detour length. It is assumed that bridges are considered in isolation. Therefore, the benefits are considered marginal benefits by having a bridge (with the alternative that there is no bridge but other nearby routes for traffic). V is the traffic volume per year which is estimated by

$$V(t) = V(0) + V_1 t \quad (6)$$

where $V(0)$ is the traffic volume per year at the time of construction, V_1 is the increase in traffic volume per year.

5. MODELLING OF THE INSPECTION COSTS

Inspection costs are a part of the lifecycle costs for any bridge. However, if a certain inspection strategy has been chosen, then the inspection costs will not influence the optimal design significantly. The inspection costs $C_{inspect}$ can therefore be eliminated from the optimization problem. Anyway, the inspection strategy will often be chosen on the basis of other factors than the reliability of the bridge. The reliability based optimal design can therefore be performed on the basis of the following objective function

$$W^* = B_{lifecycle} - C_{initial} - C_{repair} - C_{failure} \quad (7)$$

The optimization variables are dimensions defining the bridge but also e.g. repair parameters. The result from the optimization is the design of the structure and an optimal repair strategy.

6. MODELLING OF THE REPAIR COSTS

Modelling of the expected repair costs is complicated. It is necessary to simplify the modelling as much as possible but still keep the modelling reasonable. Consider a repair at the time $t_{r,i}$. The corresponding repair costs $C_{repair}(t_{r,i})$ can then e.g. be divided into three terms

$$C_{repair}(t_{r,i}) = C_1(t_{r,i}) + C_2(t_{r,i}) + C_3(t_{r,i}) \quad (8)$$

where the three terms are the functional repair costs, the fixed repair costs, and the unit dependent repair costs, respectively. The first term in (8) represents the functional costs

and the last two terms represent the direct repair costs. The functional costs $C_1(t_{r,i})$ can e.g. be modelled by the duration of the repair (in days), the number of lanes closed, the total number of lanes, the marginal functional repair costs for one vehicle. The second term in (8) represents the fixed costs. The fixed costs $C_2(t_{r,i})$ can e.g. be modelled by the costs due to the distance from the headquarter (Euro/km), the roadblock costs per number of hours and lanes, and the number of hours needed to perform the repair of the bridge. The third term in (8) represents the unit costs. The unit costs $C_3(t_{r,i})$ can e.g. be modelled by how easy it is to perform the repair, time needed to perform the repair, the extent of the repair for a given relevant repair technique, man hour cost (Euro/h), and the material/equipment costs. In Thoft-Christensen [21] models for all three terms are proposed.

The expected repair cost capitalized to the time $t = 0$ can then be modelled by adding the single repair costs.

$$C_{repair} = \sum_{i=1}^{n_r} (1 - P_f(t_{r,i})) C_{repair}(t_{r,i}) \frac{1}{(1+r)^{t_{r,i}}} \quad (9)$$

where P_f is the updated failure probability, and n_r are the number of failures in the lifetime of the bridge. r is the discount rate.

7. MODELLING OF THE FAILURE COSTS

The expected failure costs in the time interval $[t_{i-1}, t_i]$, i.e. in the one year time interval from year $i-1$ to year i , can be written

$$C_{failure,i} = C_{failure}(t_i) (P_f(t_i) - P_f(t_{i-1})) \frac{1}{(1+r)^{t_i}} \quad (10)$$

where the cost of failure $C_{failure}(t_i)$ at the time t_i can be modelled by the sum of the direct failure costs the functional costs

$$C_{failure}(t_i) = C_{failure}^{direct}(t_i) + C_{failure}^{functional}(t_i) \quad (11)$$

The functional cost can be modelled by the loss of benefit. The capitalized expected lifecycle failure costs can then be written

$$C_{failure} = \sum_{i=1}^T C_{failure}(t_i) \quad (12)$$

where T is the expected lifetime of the bridge.

8. DISCUSSION AND CONCLUSIONS

In this paper a brief presentation of what is believed to be the state of art in reliability based optimal design of concrete bridges is given. The importance of using stochastic modelling and a lifecycle approach is stressed although much more research is needed to make a fully satisfactory reliability based optimal design of a concrete bridge. In the last three decades a lot of progress is made. The general acceptance of the presented approach has initiated extensive research all over the world and in the near future Highways Agencies in many countries are or plan to include in their design codes elements from the presented approach. In the next few years we will observe a growing interest and part of the general approach will be implemented for the benefit of the

society.

A number of areas, where more research is needed, are discussed in this section.

- Identification of significant failure modes is essential for the estimation of the reliability of a given bridge. Ultimate as well as serviceability limit states must be included in the reliability assessment. Likewise system behaviour must be taken into consideration since the hidden reliability due to redundancy will often make the bridge safer than assessed.
- The stochastic modelling should be improved in different ways. First of all there is a need for more data concerning loading and strength parameters. As an example it can be mentioned that one of the most significant parameters for concrete bridges, namely the yield strength of the reinforcement, is not well documented. It is also necessary to model all factors affecting the deterioration carefully. Several authors have treated corrosion due to chloride penetration but the effect of cracks etc. is not well understood. Chloride penetration is not the only important factor. All factors should be investigated carefully.
- Stochastic modelling of inspection and repair techniques should be improved. More experimental work on structural elements is needed. Experience from other research areas and practice should be fully utilized e.g. regarding p.o.d. curves. The use of new materials should be investigated not only from a strength viewpoint but also from a reliability viewpoint.
- The target reliability level is important for inspection and repair strategies. New target levels will probably be based on economic considerations so classical optimal decision theory is relevant to use. The risk of loss of human life must be evaluated and minimized, e.g. by making sure that failure modes are ductile by introducing high degrees of redundancy in the structure.
- Consulting companies, Highways Agencies, Universities etc. must cooperate to make sure that the theoretical work is suited for practical implementations. International cooperation is needed in this area to obtain an optimal utilization of the sparse resources allocated to this area.

REFERENCES

- [1] Thoft-Christensen, P. *On Reliability-Based Design of Structures*. Reliability and Optimization of Structural Systems '90 (eds. Der Kiureghian & Thoft-Christensen). Lecture Notes in Engineering, Vol.61, Springer Verlag, 1991, pp. 387-402.
- [2] Thoft-Christensen, P. & Y Murotsu. *Application of Structural Systems Reliability Theory*. Springer Verlag, 1986.
- [3] *Reliability and Optimization of Structural Systems* (ed. P. Thoft-Christensen). Proc. IFIP WG 7.5 Working Conference, Aalborg, Denmark. Springer Verlag, 1987.
- [4] *Reliability and Optimization of Structural Systems '88* (ed. P. Thoft-Christensen). Proc. IFIP WG 7.5 Working Conference, London, UK. Springer Verlag, 1989.
- [5] *Reliability and Optimization of Structural Systems '90* (eds. A. Der Kiureghian, & P. Thoft-Christensen). Proc. IFIP WG 7.5 Working Conference, Berkeley, California, USA. Springer Verlag, 1990.
- [6] *Reliability and Optimization of Structural Systems '91* (eds. R. Rackwitz & P. Thoft-Christensen). Proc. IFIP WG 7.5 Working Conference, Munich, Germany.

- Springer Verlag, 1992.
- [7] *Reliability and Optimization of Structural Systems V* (eds. P. Thoft-Christensen, H. Ishikawa). Proc. IFIP WG 7.5 Working Conference, Takamatsu, Kagawa, Japan. North-Holland, 1991.
 - [8] *Reliability and Optimization of Structural Systems* (eds. R.Rackwitz, G. Augusti & A. Borri). Proc. IFIP WG 7.5 Working Conference, Assisi, Italy. Chapman & Hall, 1995.
 - [9] *Reliability and Optimization of Structural Systems* (eds. D.M. Frangopol, R.B. Corotis & R. Rackwitz). Proc. IFIP WG 7.5 Working Conference, Boulder, Colorado, USA. Pergamon, 1997.
 - [10] *Reliability and Optimization of Structural Systems* (ed. A.S. Nowak). Proc. IFIP WG 7.5 Working Conference, Crakow, Poland. The University of Michigan, Ann Arbor, Michigan, USA, 1998.
 - [11] *New Directions in Structural System Reliability* (ed. D.A. Frangopol). University of Colorado, Boulder, 1989.
 - [12] *Optimal Performance of Civil Infrastructure Systems* (ed. D.A. Frangopol). SEI, ASCE, 1997
 - [13] *Case Studies in Optimal Design and Maintenance Planning of Civil Infrastructure Systems* (ed. D.A.Frangopol). SEI, ASCE, 1999.
 - [14] *Safety of Bridges* (ed. P.C. Das). Proc. Symposium on Safety of Bridges, London, UK. Thomas Telford, 1997.
 - [15] *Current and Future Trends in Bridge Design Construction and Maintenance* (eds. P.C. Das, D.M. Frangopol & A.S. Nowak). Proc. ICE Conference, Singapore. Thomas Telford, 1999.
 - [16] *Recent Advances in Bridge Engineering* (eds. J.R. Casas, F.W. Klaiber & A.R. Mari). Proc. US-Europe Workshop on Bridge Engineering, Barcelona, Spain. CIMNE 1996.
 - [17] *Bridge Rehabilitation* (eds. G. König & A.S. Nowak). Proc. Workshop on Bridge Rehabilitation, Darmstadt, Germany. Ernest & Sohn, 1992.
 - [18] Thoft-Christensen, P. *Risk Based Structural Optimization*. Proc. Sixth ASCE Specialty Conference, Denver, Colorado, USA, July 1992, pp. 535-538.
 - [19] Thoft-Christensen, P. *Assessment of the Reliability Profiles for Concrete Bridges*. Engineering Structures, Vol. 20, 1998, pp. 1004-1009.
 - [20] Thoft-Christensen, P. *Estimation of Bridge Reliability Distributions*. In [15] pp.15-25.
 - [21] Thoft-Christensen, P. *Life-Cycle Cost Evaluation of Concrete Bridges*. Proceedings Structural Engineers World Congress, San Francisco, July 1998. Elsevier Sciences, CD Rom, Paper T132-6.