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CHAPTER 102

STOCHASTIC MODELLING OF THE CRACK INITIATION TIME FOR REINFORCED CONCRETE STRUCTURES[1]

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ABSTRACT

According to EUROCODE 2, limit states are states beyond which the structure no longer satisfies the design performance requirements. Ultimate limit states (ULS) are those associated with collapse or with other forms of structural failure which may

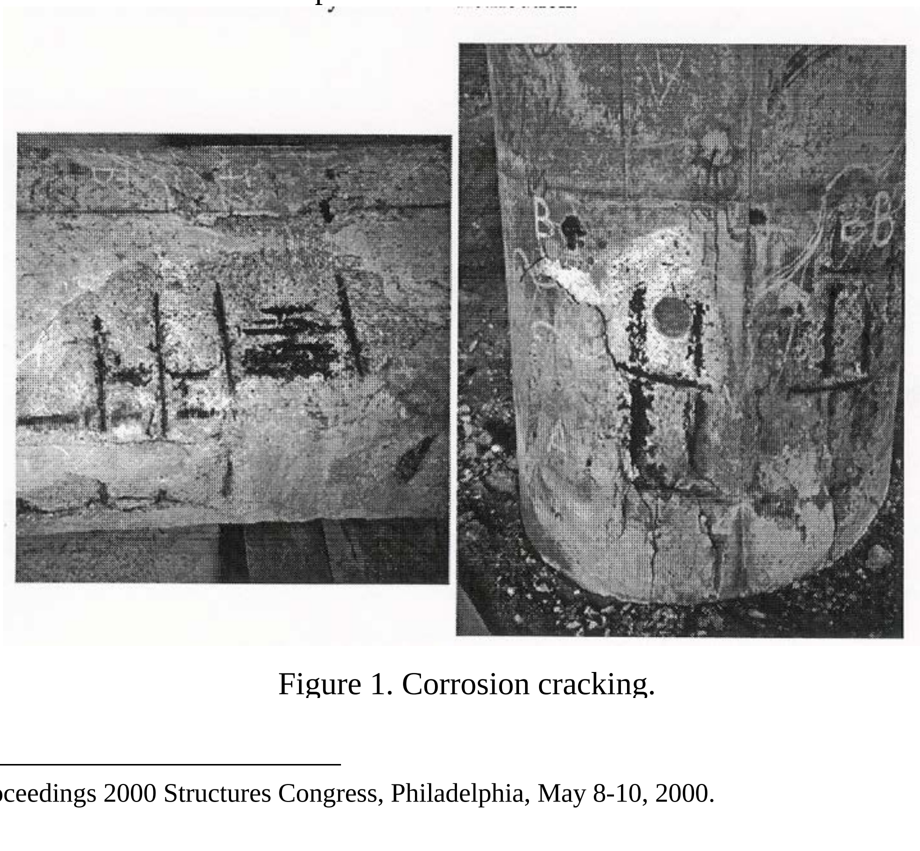

Figure 1. Corrosion cracking.

[1]Proceedings 2000 Structures Congress, Philadelphia, May 8-10, 2000.

endanger the safety of people. Examples are loss of equilibrium of the structure or failure due to excessive deformation, rupture or loss of stability. Serviceability limit states (SLS) correspond to states beyond which specified service requirements are no longer met. Examples are deformations or deflections, vibrations, cracking of concrete, or damaging of concrete.

In an earlier paper by Thoft-Christensen [1] the service life for a reinforced concrete structure is defined as the initiation time for corrosion T_{corr} of the reinforcement. This is a rational definition viewed from a life-cycle cost angle since repair of corroded elements is a major contributor to the life-cycle cost. Using the diffusion modelling of corrosion it is shown on the basis of simulation data that applying this definition the service life can be modelled by a Weibull distribution.

In this paper a similar analysis of the time to corrosion cracking is performed. It is shown on the basis of an example that also this quantity can be modelled by a Weibull distribution. Further, it is shown that the time to corrosion cracking is very short and also very uncertain.

Two examples of severe corrosion cracking are shown in figure 1.

1. INTRODUCTION

In this paper the definition of service life has been modified so that the time T_{crack} from corrosion initiation to crack initiation in the concrete due to corrosion is included. The service life is then defined as

$$T_{service} = T_{corr} + T_{crack}$$

In the paper a stochastic model for ~rack is developed on the basis of existing deterministic theories, see Y. Liu & R.E. Weyers [2]. The corrosion-cracking model is restricted to the stresses resulting from the expansion of corrosion products. Three stages are considered in the model:

1. Free Expansion. It is assumed that there is a porous zone around the steel/concrete surface caused by the transition from paste to steel, entrapped/entrained air voids, and corrosion products diffusing into the capillary voids in the cement paste.
2. Stress Initiation. When the total amount of corrosion products exceeds the amount of corrosion products needed to fill the porous zone around the steel, the corrosion products create expansive pressure on the surrounding concrete.
3. With increasing corrosion the internal stresses will exceed the tensile strength of the concrete and the cover concrete will crack.

Using crude Monte Carlo simulation the distribution of crack is estimated.

2. MODELLING OF THE AMOUNT OF CORROSION PRODUCT NEEDED TO FILL THE POROUS ZONE

Close to reinforcement bars the concrete has some porosity. Very close to the bars the porosity is close to one but the porosity decreases with the distances from the bars. The porosity is typically of the order of 0.5 about 10-20 μm from the bars so the porous zone is very narrow. Let t_{por} be the thickness of an equivalent zone with porosity one around a steel bar. Then the amount of corrosion products necessary to fill the porous zone can be written

$$W_{porous} = \pi \rho_{rust} t_{por} D \qquad (1)$$

where D is the diameter of the reinforcement bar and ρ_{rust} the density of the corrosion products.

For illustration, let t_{por} be modelled by a log-normal distribution with the mean 12.5 μm and a standard deviation of 2.54 μm. Further, let ρ_{rust} and D be modelled by normal distributions N(3600,360) kg/m^3 and N(16,1.6) mm, respectively.

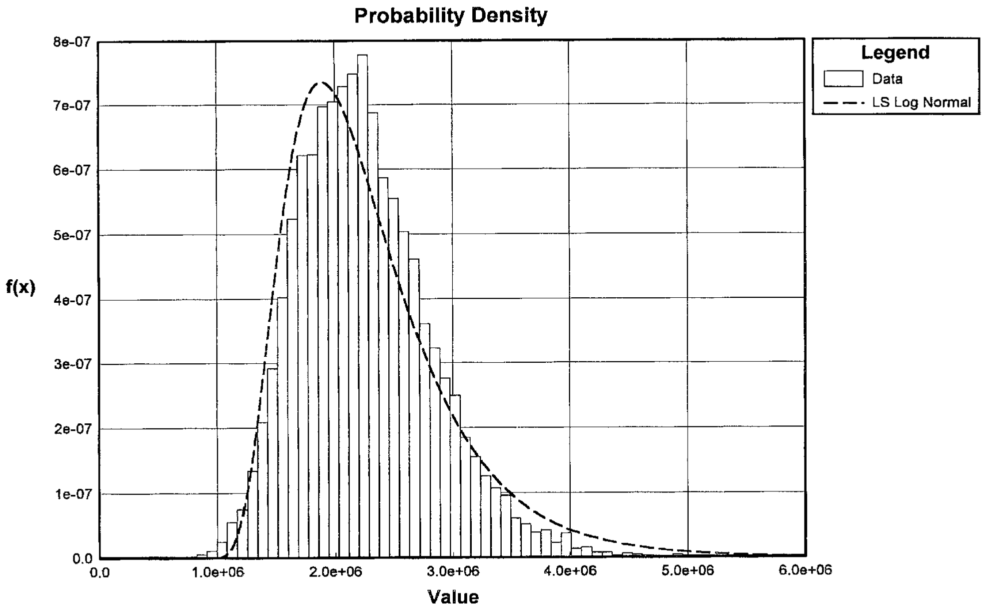

Figure 2. Stochastic modelling of W_{porous}.

Then by Monte Carlo simulation it can be shown that W_{porous} with a good approximation can be modelled by a shifted log-normal distribution with a mean 2.14 e-03 kg/m, a standard deviation 0.60 e-03 kg/m and a shift of 0.82 e-03 kg/m, see figure 2.

3. MODELLING OF THE CRITICAL AMOUNT OF CORROSION PRODUCTS

After corrosion initiation the rust products will initially fill the porous zone and then result in an expansion of the concrete near the reinforcement. As a result of this, tensile stresses are initiated in the concrete. With increasing corrosion the tensile stresses will reach a critical value and cracks will be developed.

During this process the corrosion products at cracking of the concrete will occupy three volumes, namely the porous zone, the expansion of the concrete due to rust pressure, and the space of the corroded steel. The corresponding total amount of critical rust products W_{rust} to fill these volumes is

$$W_{crit} = W_{porous} + W_{expan} + W_{steel} \qquad (2)$$

where W_{expan} is the amount of corrosion Products needed to fill in the space due to the expansion of the concrete around the reinforcement, and W_{steel} is the amount of corrosion products that generate the cracking.

Let the expansion of the concrete around the reinforcement have the thickness t_{expan}, then W_{expan} be written

$$W_{expan} = \rho_{rust}\pi\left(D + 2t_{por}\right)t_{crit} \tag{3}$$

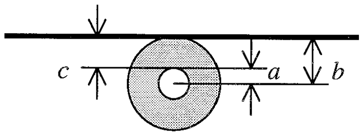

Figure 3. Idealization of the concrete around the reinforcement by a thick-walled cylinder.

where t_{crit} is the thickness of the expansion at crack initiation.

Y. Liu & R.E. Weyers [2] have estimated t_{crit} by assuming that the concrete is a homogeneous elastic material and can be approximated by a thick-walled concrete cylinder with inner radius $a = (D + 2t_{por}) / 2$ and outer radius $b = c + (D + 2t_{por}) /2$ where c is the cover depth, see figure 3.

Then the approximate value of the critical expansion t_{crit} is

$$t_{crit} = \frac{cf_t'}{E_{ef}}\left(\frac{a^2 + b^2}{b^2 - a^2} + \nu_c\right) \tag{4}$$

where E_{ef} is the effective elastic modulus of the concrete and f_t' is the tensile strength of the concrete. ν_c is Poisson's ratio of the concrete.

In this paper E_{ef} , ν_c and c are considered deterministic with values 10 GPa, 0.25, and 60 mm, respectively.

The tensile strength f_t' is modelled as a normally distributed variable with the mean value 4MPa and the standard deviation 0.6 MPa.

Then by Monte Carlo simulation it can be shown that W_{expan} with a good approximation can be modelled by a normal distribution N(0.0047 , 0.0011) kg/m, see figure 4.

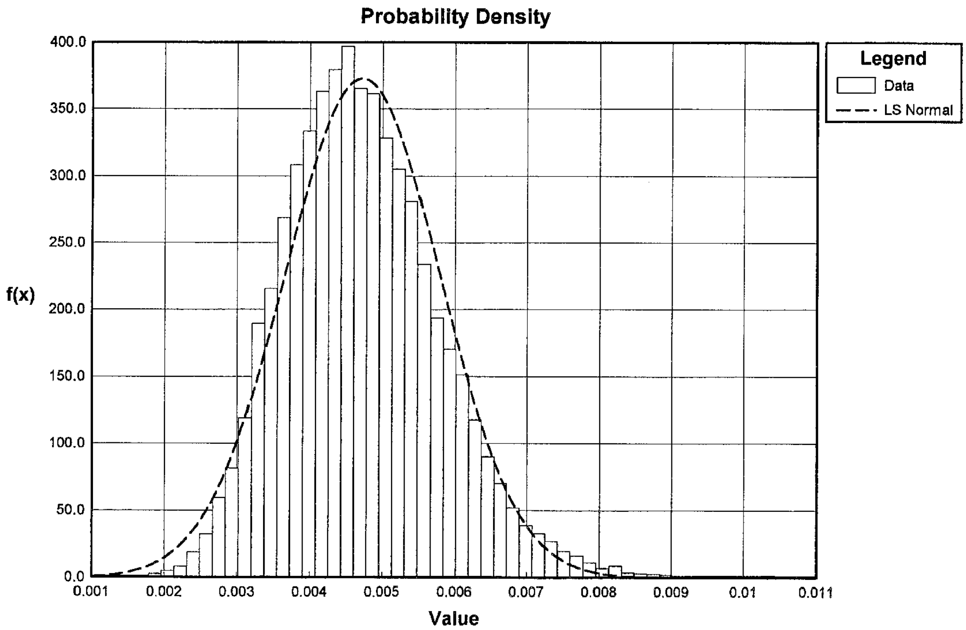

Figure 4. Stochastic modelling of W_{expan}.

Finally, W_{steel} can be written

$$W_{steel} = \frac{\rho_{rust}}{\rho_{steel}} M_{steel} \tag{5}$$

where ρ_{steel} is the density of steel and M_{steel} is the mass of the corroded steel. Clearly, M_{steel} is proportional to W_{crit}. Y. Liu & R.E. Weyers [2] have calculated the factor of proportionality for two kinds of corrosion products to be 0.523 and 0.622. For simplicity, it will be assumed that M_{steel}= 0.57W_{crit}.

Therefore, equation (2) can be rewritten

$$W_{crit} = \frac{\rho_{steel}}{\rho_{steel} - 0.57\rho_{rust}} \left(W_{porous} + W_{\exp an} \right) \tag{6}$$

Let ρ_{steel} be modelled by a normal distribution N(8000,800) kg/m^3. Then by Monte Carlo simulation it can be shown that W_{crit} with a good approximation can be modelled by a normal distribution N(0.010, 0.0027) kg/m, see figure 5.

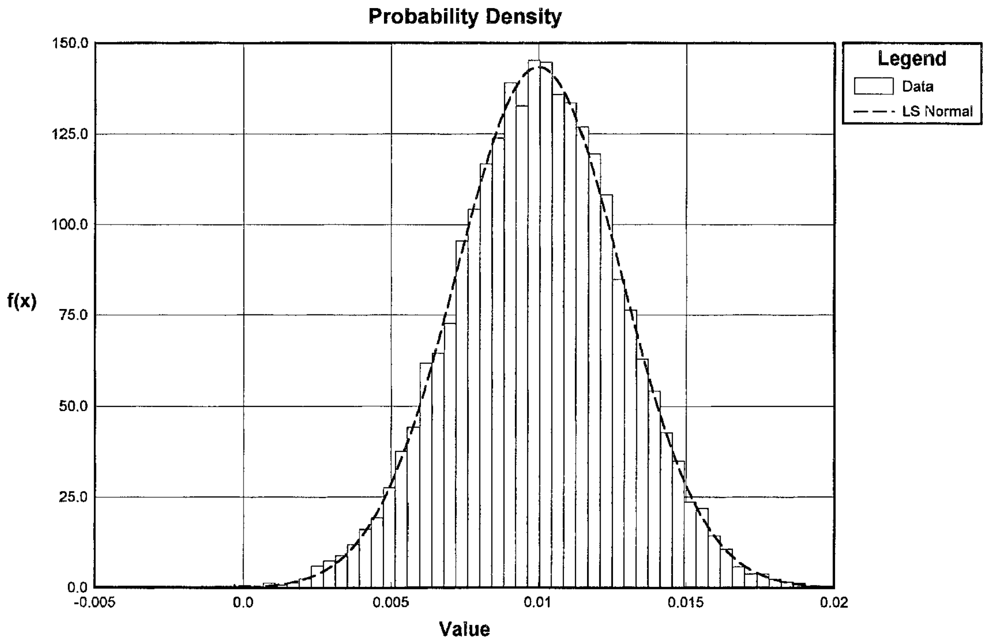

Figure 5. Stochastic modelling of W_{crit} .

4. ESTIMATION OF GROWTH OF RUST PRODUCTS

The rate of rust production as a function of time *t* (years) from corrosion initiation can (see Y. Liu & R.E. Weyers [1]) be written

$$\frac{dW_{rust}(t)}{dt} = k_{rust}(t) \frac{1}{W_{rust}(t)} \tag{7}$$

i.e. the rate of corrosion is inversely proportional to the amount of rust products W_{rust} (kg/m). The factor $k_{rust}(t)$ (kg^2/m^{2}t) is assumed to be proportional to the annual mean corrosion rate $i_{corr}(t)$ (μA/cm^2) and the diameter *D*(*m*) of the reinforcement. The proportionality factor depends of the types of rust products but is here taken as 0.383 e-3.

$$k_{rust}(t) = 0.383 \times 10^{-3} D i_{corr}(t) \tag{8}$$

By integrating (7)

$$W_{rust}^2(t) = 2\int_0^t k_{rust}(t)dt \tag{9}$$

5. ESTIMATION OF TIME TO CRACKING

Let $i_{corr}(t)$ be modelled by a time-independent normally distributed stochastic variable N(3 , 0.3) (μ A/cm^2) then the time to cracking T_{crack} can be estimated by (9) by setting $W_{rust}^2(T_{crack}) = W_{crit}^2$.

$$T_{crack} = \frac{W_{crit}^2}{2k_{rust}} = \frac{W_{crit}^2}{2 \times 0.383 \times 10^{-3} D i_{corr}} \tag{10}$$

Then by Monte Carlo simulation it can be shown that T_{crack} with a good approximation can be modelled by a Weibull distribution W(3.350, 1.944, 0) years, see figure 6. The mean is 2.95 years and the standard deviation 1.58 years.

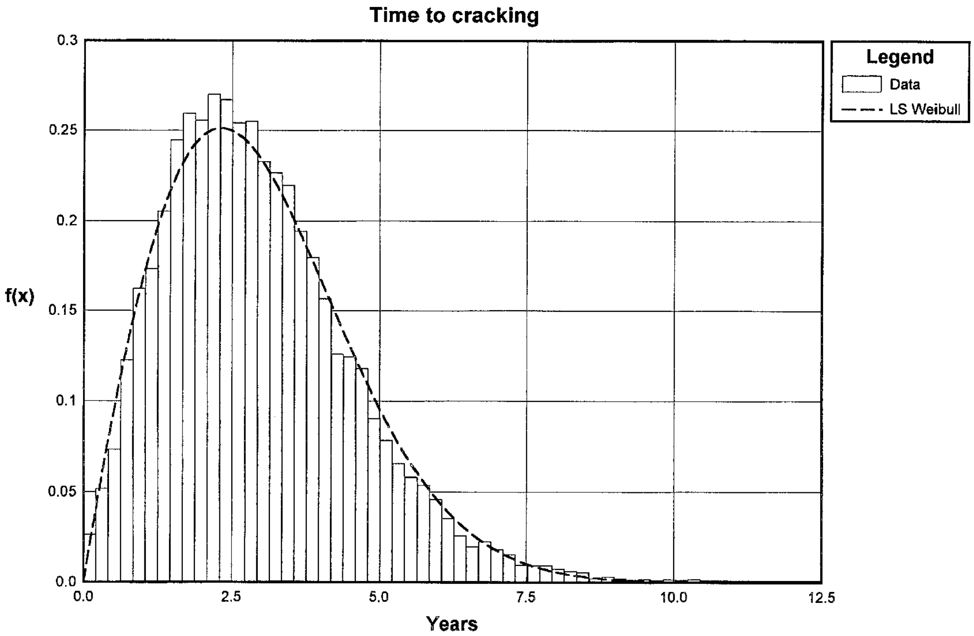

Figure 6. Stochastic modelling of T_{crack}.

The mean value of T_{crack} is of the same order as the experimental values and the deterministic values obtained by see Y. Liu & R.E. Weyers [2]. This is not surprising since almost the same data are used.

6. CONCLUSIONS

A procedure by which the time to corrosion cracking can be estimated is presented. By an example it is shown that the time to corrosion cracking can be modelled by a Weibull distribution.

It is also shown that the time to corrosion cracking is small compared with the time to corrosion initiation. In the example the mean value of the time to corrosion cracking is 2.95 years and the standard deviation 1.58 years. This conclusion is of course limited to the example where a number of relatively crude assumptions are made. When more reliable data are available a better estimation can be made.

REFERENCES

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[2] Liu, Y. & R.E. Weyers. Modeling of the Time-to Corrosion Cracking in Chloride Contaminated Reinforced Concrete Structures. ACI Materials Journal, Vol. 95, 1998, pp. 675- 681.