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CHAPTER 107

MODELLING OF THE DETERIORATION OF REINFORCED CONCRETE STRUCTURES¹

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ABSTRACT

Stochastic modelling of the deterioration of reinforced concrete structures is addressed in this paper on the basis of a detailed modelling of corrosion initiation and corrosion cracking. It is proposed that modelling of the deterioration of concrete should be based on a sound understanding of the physical and chemical properties of the concrete. The relationship between rebar corrosion and crack width is investigated. A new service life definition based on the evolution of the corrosion crack width is proposed.

1. INTRODUCTION

In an earlier paper by Thoft-Christensen [1], the service life $T_{service}$ for a reinforced concrete structure is defined as the initiation time for corrosion T_{corr} of the reinforcement.

$$T_{service} = T_{corr} \quad (1)$$

Using diffusion modelling of the chloride penetration of the concrete it is shown based on simulation data that the corrosion initiation time for the considered example can be modelled by a Weibull distribution. This approach based on diffusion theory seems to have reached general acceptance among researchers in this field.

In a later paper by Thoft-Christensen [2], the service life has been modified so that the time Δt_{crack} from corrosion initiation to corrosion crack initiation in the concrete is included. The service life is then defined as

$$T_{service} = T_{crack} = T_{corr} + \Delta t_{crack} \quad (2)$$

¹ Proceedings IFIP WG 7.5 Conference on "Reliability and Optimization of Structural Systems", Ann Arbor, USA, September 25-27, 2000, pp. 15-26.

In the paper a stochastic model for Δt_{crack} is developed on the basis of existing deterministic theories for crack initiation, see Liu & Weyers [3]. The corrosion-cracking model is restricted to the stresses resulting from the expansion of the corrosion products. Three stages are considered in the model:

- Free expansion
- Stress initiation
- Crack initiation

In the present paper the model is extended to include the situation after initiation of corrosion cracking. The reliability profile (reliability as a function of time) is estimated by first relating the amount of corrosion products to time and then the reliability and the crack width to the amount of corrosion products. The final goal is to be able to estimate the reliability of a given structure on the basis of measurements of the crack widths. It is believed that such a methodology may be useful in rating corroded concrete bridges and therefore become an important tool in optimal repair strategies for reinforced concrete bridges.

A new service life is defined in this paper. The definition is

$$T_{service} = T_{crack} + \Delta T_{cr} = T_{corr} + \Delta t_{crack} + \Delta T_{cr} \quad (3)$$

where ΔT_{cr} is the time from initial cracking to a critical crack is developed.

2. RELIABILITY PROFILES

Reliability profiles can be based directly on observations if such data are available using curve fitting and a given functional form with a number of parameters. If the functional form is chosen without any physical arguments then the parameters will also be without a physical meaning. This is a clear disadvantage since a direct estimation of the parameters is not possible in general.

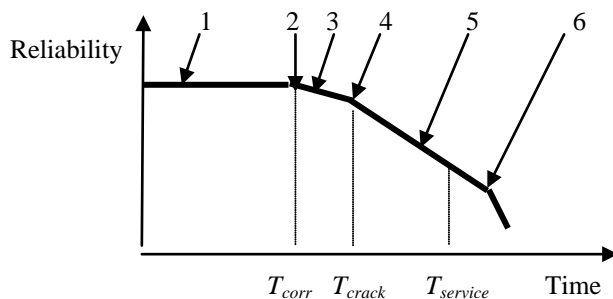


Figure 1. Deterioration steps.

In this paper it is proposed to use simple physical modelling as a basis for estimation of the reliability profiles. Only corrosion of the reinforcement due to chloride penetration of the concrete and cracking of the concrete due to corrosion of the reinforcement is considered in this paper. The following deterioration steps are considered, see figure 1:

1. Chloride penetration of the concrete
2. Initiation of the corrosion of the reinforcement
3. Evolution of corrosion of the reinforcement
4. Initial cracking of the concrete
5. Evolution of cracks in the concrete.
6. Spalling

Deterioration steps 1-3 are well understood, and are presented in numerous papers, e.g. Thoft-Christensen [1]. Step 4 has been treated in Thoft-Christensen, 2000, but steps 5 and 6 do not seem to have been investigated before in this connection.

3. STEP 1. CHLORIDE PENETRATION OF THE CONCRETE

Corrosion initiation period refers to the time during which the passivation of steel is destroyed and the reinforcement starts corroding actively. Fick's law of diffusion can represent the rate of chloride penetration into concrete, as a function of depth from the concrete surface and as a function of time

$$\frac{dC(x,t)}{dt} = D_c \frac{d^2C(x,t)}{dx^2} \quad (4)$$

where $C(x,t)$ is the chloride ion concentration, as % by weight of cement, at a distance of x cm from the concrete surface after t seconds of exposure to the chloride source. D_c is the chloride diffusion coefficient expressed in cm^2/sec . The solution of the differential equation (1) is

$$C(x,t) = C_0 \left\{ 1 - \operatorname{erf} \left(\frac{x}{2\sqrt{D_c \cdot t}} \right) \right\} \quad (5)$$

where C_0 is the equilibrium chloride concentration on the concrete surface, as % by weight of cement, erf is the error function.

4. STEP 2. INITIATION OF CORROSION

In a real structure, if C_{cr} is assumed to be the chloride corrosion threshold and d is the thickness of concrete cover, then the corrosion initiation period T_{corr} can be calculated. The time T_{corr} to initiation of reinforcement corrosion is

$$T_{corr} = \frac{d^2}{4D_c} \left(\operatorname{erf}^{-1} \left(\frac{C_{cr} - C_0}{C_i - C_0} \right) \right)^2 \quad (6)$$

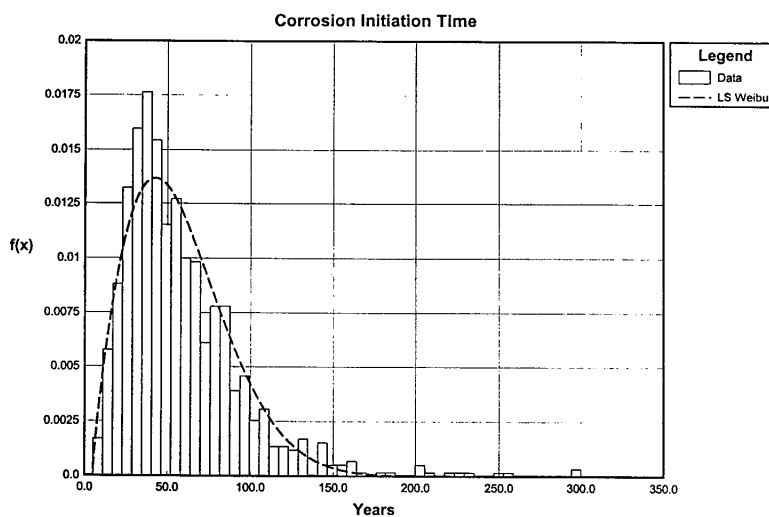


Figure 2. Density function of the corrosion initiation time T_{corr} .

On the basis of equation (3) outcomes of the corrosion initiation time T_{corr} have

been performed on the basis of the following data by simple Monte Carlo simulation:

Initial chloride concentration:	0%
Surface chloride concentration:	Normal (0.650 ; 0.038)
Diffusion coefficient:	Normal (30 ; 5)
Critical concentration:	Normal (0.3 ; 0.05)
Cover:	Normal (40 ; 8)
Number of samples:	1000

A Weibull distribution can be used to approximate the distribution of the simulated data. The Weibull distribution is $W(x; \mu, k, \varepsilon)$, where $\mu = 63.67$, $k = 1.81$ and $\varepsilon = 4.79$. The corresponding histogram and the density function are shown in figure 2.

5. STEP 3. EVOLUTION OF CORROSION OF THE REINFORCEMENT

When corrosion is initiated the diameter $D(t)$ of the reinforcement bars at the time t is modelled by

$$D(t) = D_0 - c_{corr} i_{corr} t \quad (7)$$

where D_0 is the initial diameter, c_{corr} is a corrosion coefficient, and i_{corr} is the rate of corrosion. Based on a survey, three models for chloride penetration have been proposed (the initial chloride is assumed to be zero): low, and high deterioration; Thoft-Christensen & Jensen [4].

It is assumed that there is a porous zone around the steel/concrete surface caused by the transition from paste to steel, entrapped/entrained air voids, and corrosion products diffusing into the capillary voids in the cement paste. When the total amount of corrosion products exceeds the amount of corrosion products needed to fill the porous zone around the steel, the corrosion products create expansive pressure on the surrounding concrete.

Close to reinforcement bars the concrete has some porosity. Very close to the bars the porosity is close to one, but the porosity decreases with the distances from the bars. The porosity is typically of the order of 0.5 about 10-20 μ m from the bars so the porous zone is very narrow. Let t_{por} be the thickness of an equivalent zone with porosity one around a steel bar. Then the amount of corrosion products necessary to fill the porous zone can be written

$$W_{porous} = \pi \rho_{rust} t_{por} D \quad (8)$$

where D is the diameter of the reinforcement bar and ρ_{rust} the density of the corrosion products.

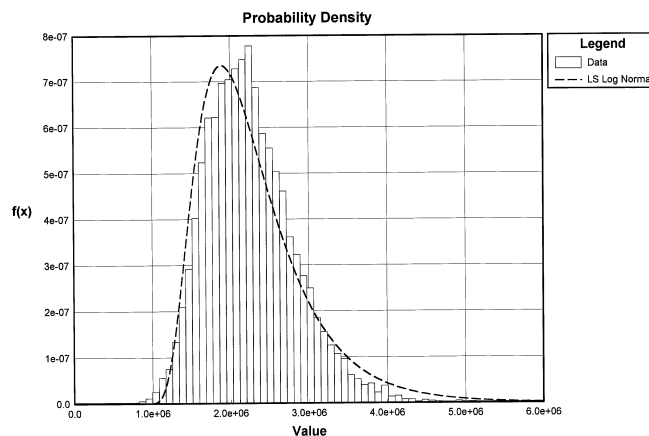


Figure 3. Stochastic modelling of W_{porous} .

For illustration, let t_{por} be modelled by a lognormal distribution with the mean $12.5 \mu\text{m}$ and a standard deviation of $2.54 \mu\text{m}$. Further, let ρ_{rust} and D be modelled by normal distributions $N(3600,360) \text{ kg/m}^3$ and $N(16,1.6) \text{ mm}$, respectively.

Then by Monte Carlo simulation it can be shown that W_{porous} with a good approximation can be modelled by a shifted lognormal distribution with a mean $2.14e-03 \text{ kg/m}$, a standard deviation $0.60e-03 \text{ kg/m}$ and a shift of $0.82e-03 \text{ kg/m}$, see figure 3.

6. STEP 4. INITIAL CRACKING OF THE CONCRETE

After corrosion initiation the rust products will initially fill the porous zone and then result in an expansion of the concrete near the reinforcement. As a result of this, tensile stresses are initiated in the concrete. With increasing corrosion the tensile stresses will reach a critical value and cracks will be developed.

During this process the corrosion products at initial cracking of the concrete will occupy three volumes, namely the porous zone, the expansion of the concrete due to rust pressure, and the space of the corroded steel. The corresponding total amount of critical rust products W_{crit} to fill these volumes is

$$W_{crit} = W_{porous} + W_{expan} + W_{steel} \quad (9)$$

where W_{expan} is the amount of corrosion products needed to fill in the space due to the expansion of the concrete around the reinforcement, and W_{steel} is the amount of corrosion products that generate the cracking.

Let the expansion of the concrete around the reinforcement have the thickness t_{expan} , then W_{expan} can be written

$$W_{expan} = \rho_{rust} \pi (D + 2t_{por}) t_{crit} \quad (10)$$

where t_{crit} is the thickness of the expansion at crack initiation.

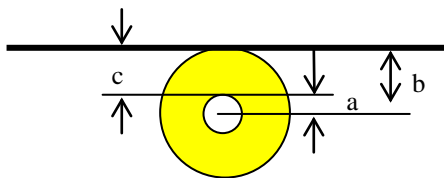


Figure 4. Idealization of the concrete around the reinforcement by a thick-walled cylinder.

Liu & Weyers [3] have estimated t_{crit} by assuming that the concrete is a homogeneous elastic material and can be approximated by a thick-walled concrete cylinder with inner radius $a = (D + 2t_{por})/2$ and outer radius $b = c + (D + 2t_{por})/2$ where c is the cover depth, see figure 4. Then the approximate value of the critical expansion t_{cr} is

$$t_{crit} = \frac{cf_t'}{E_{ef}} \left(\frac{a^2 + b^2}{b^2 - a^2} + \nu_c \right) \quad (11)$$

where E_{ef} is the effective elastic modulus of the concrete and f_t' is the tensile strength of the concrete. ν_c is Poisson's ratio of the concrete. In this paper E_{ef} , ν_c , and c are considered deterministic with values 10 GPa , 0.25 , and 60 mm , respectively. The tensile strength f_t' is modelled as a normally distributed variable with the mean value 4 MPa and the standard deviation 0.6 MPa .

Then by Monte Carlo simulation it can be shown that W_{expan} with a good approximation can be modelled by a normal distribution $N(0.0047, 0.0011)$ kg/m, see figure 5.

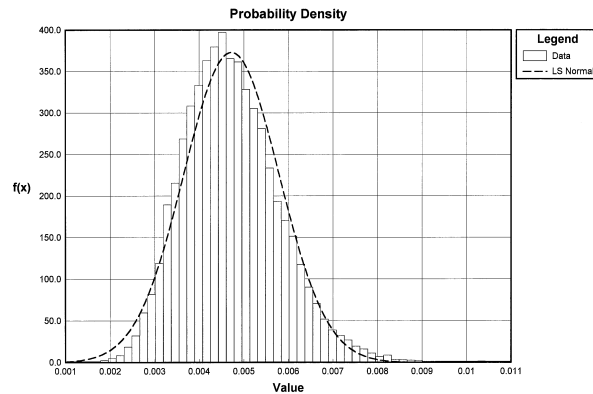


Figure 5. Stochastic modelling of W_{expan} .

Finally, W_{steel} can be written

$$W_{steel} = \frac{r_{rust}}{r_{steel}} M_{steel} \quad (12)$$

where ρ_{steel} is the density of steel and M_{steel} is the mass of the corroded steel. Clearly, M_{steel} is proportional to W_{crit} . Liu & Weyers [3] have calculated the factor of proportionality for two kinds of corrosion products as 0.523 and 0.622. For simplicity, it will be assumed that $M_{steel} = 0.57W_{crit}$. Therefore, equation (9) can be rewritten

$$W_{crit} = \frac{\rho_{steel}}{\rho_{steel} - 0.57\rho_{rust}} (W_{porous} + W_{expan}) \quad (13)$$

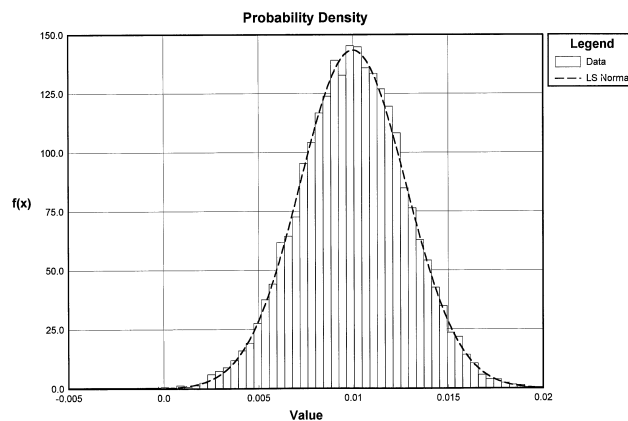


Figure 6. Stochastic modelling of W_{crit} .

Let ρ_{steel} be modelled by a normal distribution $N(8000; 800)$ kg/m³. Then by Monte Carlo simulation it can be shown that W_{crit} with a good approximation can be modelled by a normal distribution $N(0.010; 0.0027)$ kg/m, see figure 6.

The rate of rust production as a function of time t (years) from corrosion initiation can (Liu & Weyers [3]) be written

$$\frac{dW_{rust}(t)}{dt} = k_{rust}(t) \frac{I}{W_{rust}(t)} \quad (14)$$

i.e. the rate of corrosion is inversely proportional to the amount of rust products W_{rust} (kg/m). The factor $k_{rust}(t)$ ($\text{kg}^2/\text{m}^2\text{year}$) is assumed to be proportional to the annual mean corrosion rate $i_{cor}(t)$ ($\mu\text{A}/\text{cm}^2$) and the diameter D (m) of the reinforcement. The proportionality factor depends on the types of rust products, but is here taken as $0.383e-3$.

$$k_{rust}(t) = 0,383 \times 10^{-3} Di_{corr}(t) \quad (15)$$

By integration

$$W_{rust}^2(t) = 2 \int_0^t k_{rust}(t) dt \quad (16)$$

Let $i_{cor}(t)$ be modelled by a time-independent normally distributed stochastic variable $N(3 ; 0.3)$ ($\mu\text{A}/\text{cm}^2$) then the time from corrosion initiation to cracking Δt_{crack} can be estimated by (16) by setting $W_{rust}(\Delta t_{crack}) = W_{crit}$.

$$\Delta t_{crack} = \frac{W_{crit}^2}{2k_{rust}} = \frac{W_{crit}^2}{2 \times 0.383 \times 10^{-3} Di_{corr}} \quad (17)$$

Then by Monte Carlo simulation it can be shown that Δt_{crack} with a good approximation can be modelled by a Weibull distribution $W(3.350 ; 1.944 ; 0)$ years, see figure 7. The mean is 2.95 years and the standard deviation 1.58 years.

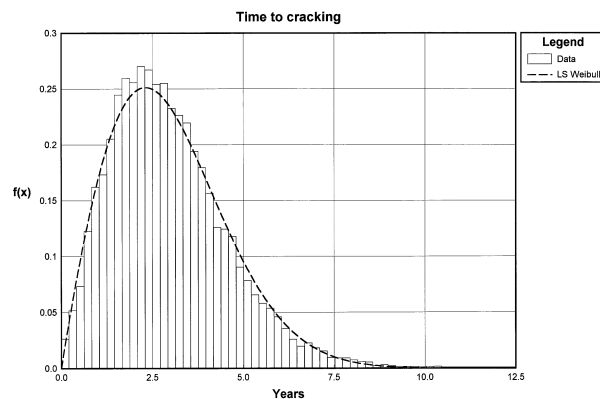


Figure 7. Stochastic modelling of Δt_{crack} .

The mean value of T_{crack} is of the same order as the experimental values (and the deterministic values) obtained by Liu & Weyers [3].

7. STEP 5. EVOLUTION OF CRACKS IN THE CONCRETE

Andrade, Alonso & Molina [5] have investigated experimentally the evolution of corrosion cracks in reinforced concrete beams. After formation of the initial crack the rebar cross-section is further reduced due to the continued corrosion, and the width of the crack is increased. In the paper four simple test specimens have been investigated.

The specimens are simplified small reinforced concrete beams with only a single rebar and 2 or 3 cm of cover. An impressed current artificially corrodes the beams. The loss of bar sections is monitored and the corresponding crack evolution is measured by the use of strain gauges attached to the surface of the beams. In all four experiments the function between the reduction of the rebar diameter and the maximum crack width

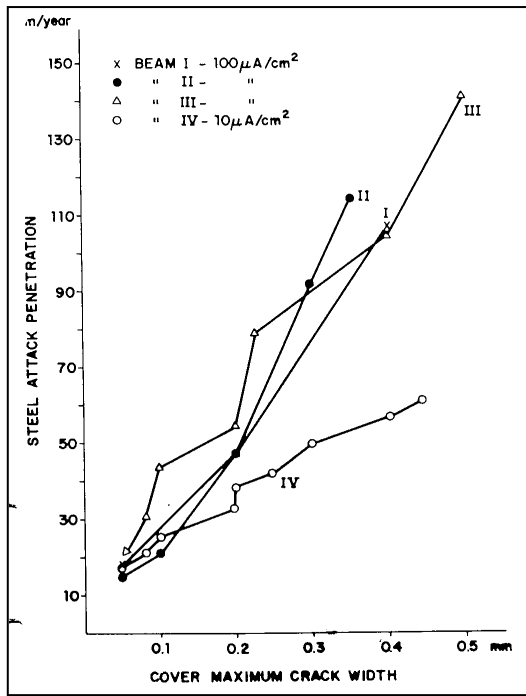


Figure 8. Loss in rebar diameter versus the crack width. Andrade et al. [5].

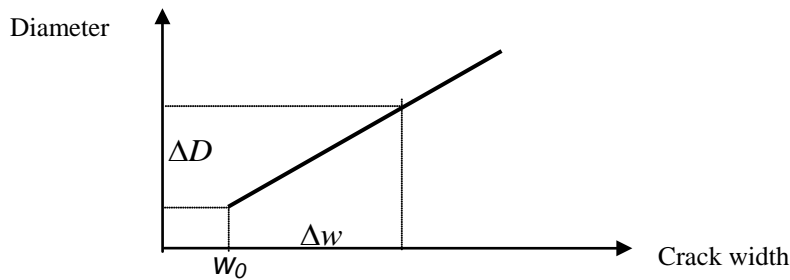


Figure 9. Relation between reduction in diameter and the crack width.

A simple approximate estimation of γ for a given beam cross-section may be performed as shown below. For illustration, assume that the diameter of the rebar at the time of crack initiation is 16 mm and that the cover is 30 mm, see figure 10.

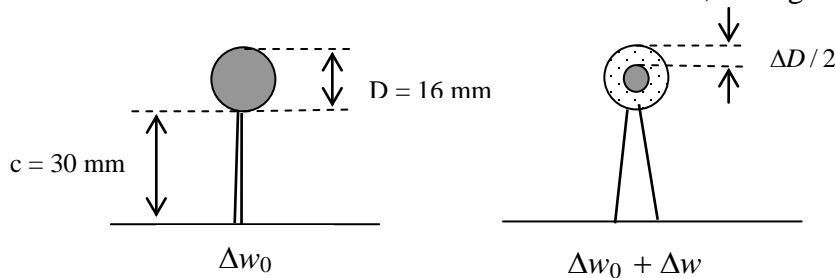


Figure 10. Evolution of cracks.

measured in the surface of the concrete specimen can be approximated by a linear function, see figure 8.

Let Δw be the increase in crack width in the time interval Δt and let the corresponding loss of rebar diameter be ΔD . Then, see figure 9

$$\Delta w = \gamma \Delta D \tag{18}$$

where γ is of the order 1.5 to 5 in the experiments reported by Andrade et al. [6]. The factor γ depends on the applied current and on the cross-sectional data.

To the left in figure 10 the crack at the time of the initial crack forming is shown. The initial crack width is Δw_0 . To the right the assumed crack configuration at the time when the diameter loss is ΔD is shown. The crack width is now $\Delta w_0 + \Delta w$. Assuming that the increase in the volume of the crack is equal to the volume of the corrosion products produced when the diameter is reduced to $D - \Delta D$, the relationship between Δw and D can be obtained approximately by

$$\frac{1}{2} \left(\frac{D/2}{D/2+c} + 1 \right) c \Delta w = (\alpha - 1) \pi D \frac{\Delta D}{2} \quad (19)$$

where $\alpha = \rho_{rust} / \rho_{steel}$ (the relation between the densities of the rust product and the steel) depends on the type of corrosion products. Typical values are 2 - 4. By inserting the above-mentioned data one obtains for this case $\gamma = 1.4 - 4.2$ in good agreement with the experimental results described by Andrade et al. [5].

8. STEP 6. SPALLING

Using FEM the procedure presented above can be extended to estimating the time for corrosion based spalling of concrete for e.g. slabs and beams, see figure 11.

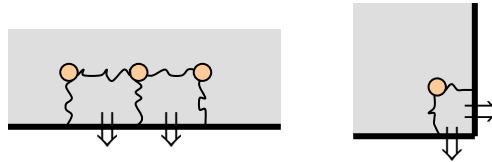


Figure 11. Typical examples of concrete spalling.

9. SERVICEABILITY LIMIT STATES (SLS)

Limit states are, according to EUROCODE 2, states beyond which the structure no longer satisfies the design performance requirements. *Ultimate limit states* (ULS) are those associated with collapse or with other forms of structural failure, which may endanger the safety of people. Examples are loss of equilibrium of the structure or failure due to excessive deformation, rupture or loss of stability. *Serviceability limit states* (SLS) correspond to states beyond which specified service requirements are no longer met. Examples are deformations or deflections, vibrations, cracking of concrete, or damaging of concrete.

A corrosion based crack based serviceability limit state can be formulated as

$$M(t) = w_{service} - w(t) \quad (20)$$

where the critical serviceability corrosion crack width $w_{service}$ is e.g. 0.3 mm and

$$w(t) = w_0 + \gamma(D(T_{crack} - D(t))) = w_0 + \gamma c_{corr} i_{corr} (t - T_{crack}) \quad (21)$$

where T_{crack} is the time of initial cracking. The service life $T_{service}$ may then be calculated by

$$w(T_{service}) = w_{service} \Rightarrow T_{service} = \frac{w_{service} - w_0}{\gamma c_{corr} i_{corr}} + T_{crack} \quad (22)$$

Using Monte Carlo simulation the distribution function of $T_{service}$ can then for a

given structure be estimated for any value of the serviceability crack width.

10. RELIABILITY CONTRA CORROSION CRACK WIDTH

A major problem in connection with reliability-based maintenance is the need during the inspection to estimate the reliability of a structure (structural element). A direct estimation will require information on the degree of corrosion in the reinforcement, but this information is difficult or at least in most cases expensive to obtain. By relating the reliability to the corrosion crack width it would be easier simply by measurements of the crack width to get an estimate of the reliability.

For a reinforced bridge slab the bending strength is proportional to the area of the reinforcement that is proportional to the rebar diameter D . After the initial cracking of the concrete the reduction of the diameter D is proportional to the corrosion crack width w . Therefore, the bending strength is reduced proportionally to w . As a consequence of this, the reliability with regard to the considered failure mode can be directly related to the crack width. In principle, the reliability can be estimated simply by measuring of the crack width.

11. CONCLUSIONS

Estimation of the reliability profile for a reinforced concrete structure is divided into six steps: Chloride penetration, Corrosion initiation, Corrosion evolution, Initial cracking, Crack evolution, Spalling. The first three steps are based on chloride penetration into the concrete using diffusion modelling. The remaining three steps are analysed by estimating the evolution of corrosion products and the effect of corrosion products on the concrete. A new model for the last three steps is presented. A new service life related to a serviceability limit state is proposed.

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