Jens Mammen, January 2016.

Proof of independence of axioms for sense and choice categories in Mammen (2016)

This proof of independence of axioms in Mammen (2016) is a free translation from Mammen (1996, pp. i-xiii), with a few omissions and some changes in terminology and of the naming of the axioms. The correspondence between the naming of the axioms in Mammen (1996) and in Mammen (2016) is listed in Mammen, Engelsted *et al.* (2000, p. 259-261). There are 11 axioms:

Ax. 1: There is more than one object in Ù

Ax. 2: The intersection of two sense categories is a sense category

Ax. 3: The union of any set of sense categories is a sense category

Ax. 4 (Hausdorff): For any two objects in Ù there are two disjunct sense categories so that one object is in the one and the other object in the other one

Ax. 5 (perfectness): No sense category contains just one object

Ax. 6: No non-empty choice category is a sense category

Ax. 7: There exists a non-empty choice category

Ax. 8: Any non-empty choice category contains a choice category containing only one object

Ax. 9: The intersection of two choice categories is a choice category

Ax. 10: The union of two choice categories is a choice category

Ax. 11: The intersection of a choice category and a sense category is a choice category

The proof uses the method of "models", i.e. referring to examples of mathematical "spaces" on a point-set or "universe" Ù of points or objects where some of Ù's subsets are appointed sense categories and other subsets are appointed choice categories. In each of the examples S denotes the set of all sense categories and C the set of all choice categories. S is not necessarily defining a topology in Ù.

The logic of the proof is that consistency of a set of axioms is proven if there exists an example of a space where they are all valid. Given this proof it is further proven that an axiom is independent of the other axioms if there exists an example of a space where all axioms are valid except the one in question. So we need 12 examples of spaces for the proof, one for the entire set, and one for each of the axioms.

The examples are all referring to subsets of the real axis, here denoted R. The subset of all rational numbers in R is denoted Q. The empty set is denoted Ø. The subset of all unions of open intervals in R including Ø is denoted O. The examples have no interpretative relation to the set of objects treated in Mammen (2016) which are neither points in nor parts of R. The examples are chosen only for the purpose of technical proof, and an infinity of other examples could have served the same purpose.

Here are the 12 examples:

All axioms valid: Ù = R; S = O; C = all finite subsets in R.

All except Ax. 1: Ù = {1}; S = Ø; C = Ø and {1}.

All except Ax. 2: Ù = R; S = O except Ø; C = Ø and {1}.[1]

All except Ax. 3: Ù = R; S = all open intervals in R and Ø; C = Ø and {1}.[2]

All except Ax. 4: Ù = R; S = all intersections of Q and O; C = Ø and {1}.[3]

All except Ax. 5: Ù = R; S = O and all unions of O and {2}; C = Ø and {1}.[4]

All except Ax. 6: Ù = R; S = O; C = all subsets in R.

All except Ax. 7: Ù = R; S = O; C = Ø.

All except Ax. 8: Ù = R; S = O; C = all intersections of O and Q.[5]

All except Ax. 9: Ù = R; S = O; C = Ø and all unions of a finite non-empty subset in R not including {0} and an intersection of O and subsets of $q \in$ Q defined as $-\infty < q \leq 0$ and/or $0 \leq q < \infty$.[6]

All except Ax. 10: Ù = R; S = O; C = Ø, {1} and {2}.[7]

All except Ax. 11: Ù = R; S = O; C = {1}.[8]

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[1] The intersection of two disjunct open intervals is Ø which is not in S.
[2] The union of two disjunct open intervals is not an open interval and hence not in S.
[3] Irrational numbers are not member of a subset in S.
[4] {2} is the union of Ø in S and {2} and hence itself in S.
[5] No subset in C is finite.
[6] The intersection of a subset in C defined by $-\infty < q \leq 0$ and a subset defined by $0 \leq q < \infty$ is {0} which is not in C. On the other hand no intersection with a subset in S can “isolate” {0} as member of C using Ax. 11.
[7] {1,2}, the union of {1} and {2}, is not a choice category.
[8] Ø is in S and its intersection with {1} is Ø which is not in C. This example is a correction to the example in Mammen (1996) due to an adjustment of Ax. 8 (see Mammen, Engelsted *et al.*, 2000, pp. 260-261).

References

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