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## Environmental vibration reduction utilizing resonant periodic-mass scatterers

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### Abstract

Ground vibration generated by rail and road traffic is a major source of environmental noise and vibration pollution in the low-frequency range. A promising and cost effective mitigation method can be the use of heavy masses placed as a periodic array on the ground surface near the road or track (*e.g.* concrete or stone blocks, specially designed brick walls, etc.). The natural frequencies of vibration for such blocks depend on the local ground stiffness and on the mass of the blocks which can be chosen to provide resonance at specified frequencies. This work concerns the effectiveness of such “blocking” resonating masses. A semi-analytical lumped-parameter method is utilized assuming that the blocks are point masses situated on an elastic half-space. The work is enhanced by examples highlighting advantages and disadvantages of single-mass scatterers and periodic scatterers.

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*Keywords:* Elastic half-space; blocking masses; mobility methods

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### 1. Introduction

One of the major sources of environmental noise and vibration pollution is human-induced ground vibration. With increasing densification of the cities, it becomes increasingly important to assess ground vibration resulting from construction work, heavy traffic and factory machinery. Most of the energy of such vibration is concentrated in the low frequency range, and the complex nature of wave propagation in the ground does not permit direct analogies to acoustic screening. Instead, numerous methods for analysis of dynamic soil–structure interaction (SSI) have been proposed, and various techniques have been suggested to mitigate ground vibration.

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Most such vibration mitigation techniques aim to protect sensitive buildings by creating a “shadow zone” within the propagation path beyond an “isolation” element: an isolating screen, a trench or wave barrier, or a wave-impeding block (WIB), see Ref. [1]. In this study, the emphasis is on WIBs and their location on the ground surface relative to a vibration source and a receiver. Due to its large mass, a WIB constructed as a heavy rigid block imposes a rigid-like boundary condition over the ground surface at its footprint. This has the effect of impeding the Rayleigh wave transmission but has, in itself, limited efficiency due to other ground waves that have the possibility to “diffract” around the rigid boundary. However, a second effect—introduced by the ground—is the resultant mass-stiffness system which has the ability to store and release energy similar to a single-degree-of-freedom (SDOF) system. Given certain site-specific characteristics, it could be possible to control the isolation efficiency.

Many researchers have considered the dynamic behaviour of the ground surface and its influence on soil coupling to structures. One of the earliest contributions was by Warburton *et al.* [2] who considered the interaction of two rigid circular foundations using a mixed integral equation approach. Also Krylov [3] studied the effect of blocking masses such as concrete masses placed on the ground. More recently Dijckmans *et al.* [4] studied an array of blocks placed alongside a railway track. The principle of this solution was to modify the wave propagation regime of the ground by introducing an inertial mass near the load.

In this paper, the transmission and reduction of vibration transmission to the far field of the surface of the ground is investigated using a simple model. Section 2 outlines the applied methodology, and Section 3 presents a study of the insertion loss provided by an array of blocks placed on the ground surface and an example of vibration efficiency of a wall compared to a rigid slab. Finally, a short summary and conclusions are given in Section 4.

## 2. Methodology

Analytical or semi-analytical approaches have been used extensively for analysis of dynamic SSI problems in the past. Depending on excitation frequency, a strip in two dimensions or a foundation in three dimensions serves as a resonant wave scatterer. When subjected to ground vibration, the mass will start vibrating, and over time the stresses at the interface towards the soil will reintroduce energy into the ground in the form of a wave field which propagates into the half-space. For each frequency, the induced vibration field on the surface of the half-space is determined by an inverse Fourier spatial transform as proposed by D.V. Jones [5]. The transform variables are originally formulated via a dynamic stiffness-matrix problem which yields simple expressions in the spatial transform domain. The mass is situated on the surface and is treated as a point mass, and the dynamic system consists of the load (L), mass (M) and receiver (R), see Fig. 1. For modelling and trend analyses, the effect of a point-mass model can be obtained easily from a simplified two-dimensional (2-D) plane-strain half-space model as outlined below.

Consider first a harmonic force acting upon a mass which rests at a point on the surface of the ground. The structure of the ground can be any model for a 2-D half-space, and it can easily be extended to a three-dimensional system. It is worthwhile to note here that the only underlying geometrical restriction is that the half-space is homogeneous in the longitudinal or vertical directions. Essentially this means that these methods support transversely isotropic material, layered half-spaces and bedrock structures.

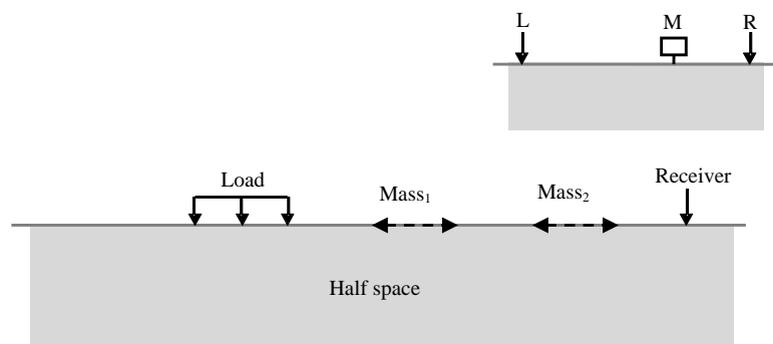


Fig. 1. Insert showing a lumped point-mass model (L-Load, M-Mass and R-Receiver) while the main figure illustrates a typical geometry utilized in a semi-analytical model. Masses are modelled as uniform mass-loads prescribed over a finite strip.

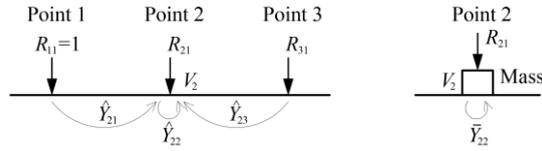


Fig. 2. Transfer mobilities used to find  $V_2$ .

An analytical or semi-analytical approach has the advantage that results may easily be determined for simple models of half-spaces, bedrock, layered ground or in some cases inhomogeneous soils. Fig. 1 shows the 2-D configuration for a mass sitting on a half-space. The inset model shows the simple analytical half-space model employed to derive frequency response functions for the ground surface at various distances from a vibrational force applied to the ground surface. The main figure illustrates the scheme used in the numerical implementation. Fig. 2 illustrates the situation where we have a load at Point 1 and masses at Points 2 and 3. We shall not include details regarding the lumped parameter modelling but only consider the situation for finding the transfer mobilities,  $V_2/F_1$  and  $V_3/F_1$ , defined by the velocities under the two masses. Utilizing the convention  $R_{ij} = F_i/F_j$  as the quotient of forces, and  $\hat{Y}_{ij}$  and  $\bar{Y}_{ij}$  as transfer mobilities between points  $i$  and  $j$  on the ground surface and mass, respectively, the following expression for point 2 can be obtained:

$$V_2/F_1 = \hat{Y}_{21} + \hat{Y}_{22}(R_{21}) + \hat{Y}_{23}(R_{31}), \quad V_2/F_1 = -\bar{Y}_{22}(R_{21}), \quad (1)$$

Transfer mobilities used are illustrated in Fig. 2. Further using a similar approach for point 3:

$$V_3/F_1 = \hat{Y}_{31} + \hat{Y}_{32}(R_{21}) + \hat{Y}_{33}(R_{31}), \quad V_3/F_1 = -\bar{Y}_{33}(R_{31}). \quad (2)$$

This yields a convenient system of 2x2 equations:

$$\begin{pmatrix} \bar{Y}_{22} + \hat{Y}_{22} & \hat{Y}_{23} \\ \hat{Y}_{32} & \bar{Y}_{33} + \hat{Y}_{33} \end{pmatrix} \begin{pmatrix} R_{21} \\ R_{31} \end{pmatrix} = \begin{pmatrix} -\hat{Y}_{21} \\ -\hat{Y}_{31} \end{pmatrix}. \quad (3)$$

The matrix characterises the relation between the two masses, located on the surface. In essence, this dependency relates the mass values, the frequency, the width of the strip and the distance between the two masses. The behaviour of the coupled masses can be characterised by the determinant which is given by:

$$\Delta = (\bar{Y}_{33} + \hat{Y}_{33})(\bar{Y}_{22} + \hat{Y}_{22}) - \hat{Y}_{32}\hat{Y}_{23}. \quad (4)$$

To obtain the vibration response at a receiver position, a discussion including examples is provided in Section 3. Once the values for the coupled terms have been determined at receiver point 4 with load applied at point 1, and with two intervening masses at points 2 and 3, the transfer mobility can be calculated as:

$$V_4/F_1 = \hat{Y}_{41} + \hat{Y}_{42}(R_{21}) + \hat{Y}_{43}(R_{31}). \quad (5)$$

For the case of many receiver positions, the mobility matrix in Eq. (3) needs only be calculated once for each frequency. Hence, this method proves to be a convenient and efficient approach for determining vibration velocities at many receivers beyond and nearby the loads and masses.

### 3. Results

The parameters of two sites are given in Tab. 1. For simplicity a loss factor which is constant with frequency has been used rather than the frequency proportional value that is shown to be more appropriate for accurate modelling of the actual site. This does not make a significant difference to the calculated insertion losses, and the primary objective is to compare the results obtained with different configurations of the masses and soil.

Tab. 1. Properties of the soil.

| Soil                      | Young's modulus (MPa) | Poisson's ratio | Density (kg/m <sup>3</sup> ) | Loss factor | Rayleigh wave speed (m/s) | Distance to Mass <sub>1</sub> from load, centre to centre (m) |
|---------------------------|-----------------------|-----------------|------------------------------|-------------|---------------------------|---|
| Homogeneous half-space, A | 269                   | 0.257           | 1550                         | 0.1         | 241                       | 3.0   |
| Homogeneous half-space, B | 213                   | 0.333           | 2000                         | 0.02        | 186                       | 4.0   |

### 3.1. Example of transfer mobilities insertion losses for single masses and arrays of wave impedance blocks

For Soil A, the distance between the blocks or WIBs, centre to centre, is 1.0 m, and for Soil B it is 4.0 m. The blocks are considered rigidly attached to the ground surface. For soil A, the blocks measure 0.1 m in the longitudinal direction (along the array) but they are 2.0 m long for soil B. Figs. 2 and 3 show transfer mobilities at two receiver locations with and without (reference) the closest block for both cases. For soil A, the presence of a WIB resonance is clear around 20 Hz. Beyond this, due to the WIB, the mobility reduces across the whole frequency range. Now, increasing the number of masses from one to three blocks in the array, this determines the insertion loss for the frequency range [0 Hz;100 Hz]. Subject to harmonic excitation, the resulting vertical insertion loss for one mass, two and three masses are shown for receiver positions beyond the periodic WIB arrays.

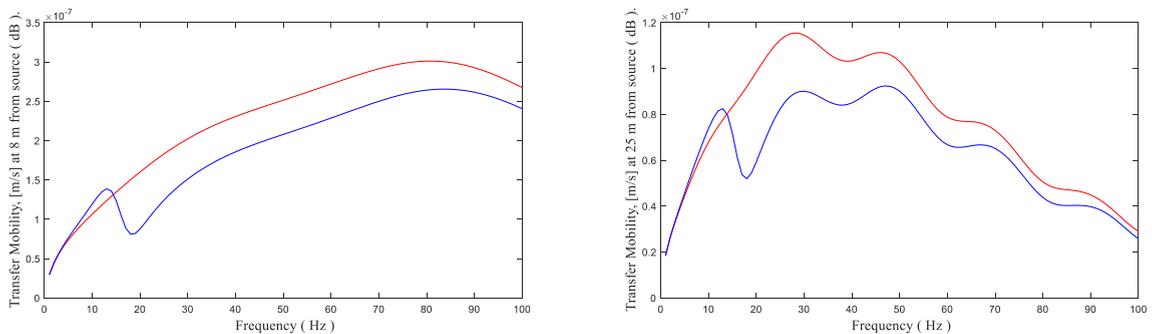


Fig. 2. Vertical transfer mobilities for soil A without mass-loading (red lines) and with mass-loading (blue lines). A single mass of 9600 kg is placed 3 m away from the centre of the load. Left: The receiver point is placed 8 m away from the centre of the load. Right: The receiver point is placed 25 m away from the centre of the load.

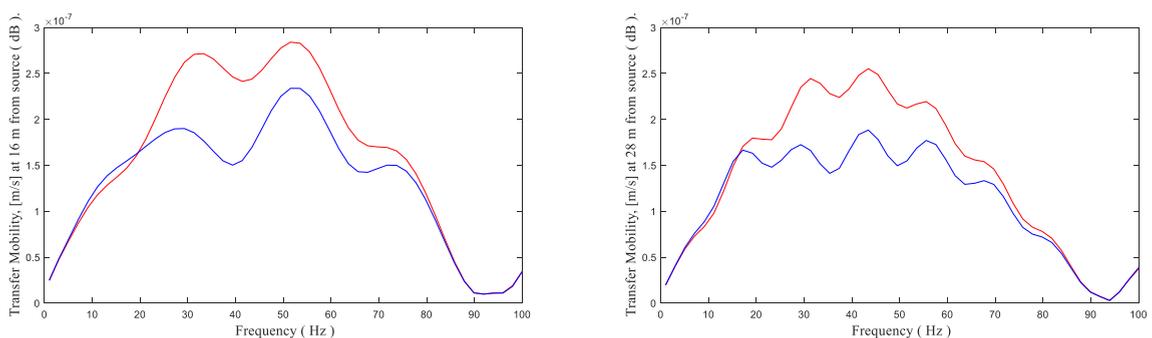


Fig. 3. Vertical transfer mobilities for soil B without mass-loading (red lines) and with mass-loading (blue lines). A single mass of 9600 kg is placed 4 m away from the centre of the load. Left: The receiver point is placed 16 m away from the centre of the load. Right: The receiver point is placed 28 m away from the centre of the load.

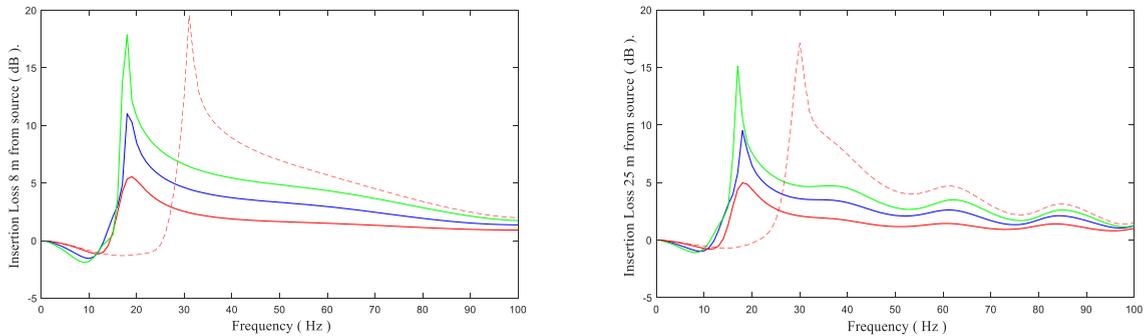


Fig. 4. Vertical insertion losses in dB for soil A at receiver positions beyond 9,600 kg regular mass placements: Full red line (1 mass); full blue line (2 masses); full green line (3 masses); and dashed red line (3 $\times$ 3,200 kg masses). The linear array starts 3 m away from the centre of the load. Left: Receiver position 8 m away from the centre of the load. Right: Receiver position 25 m away from the centre of the load.

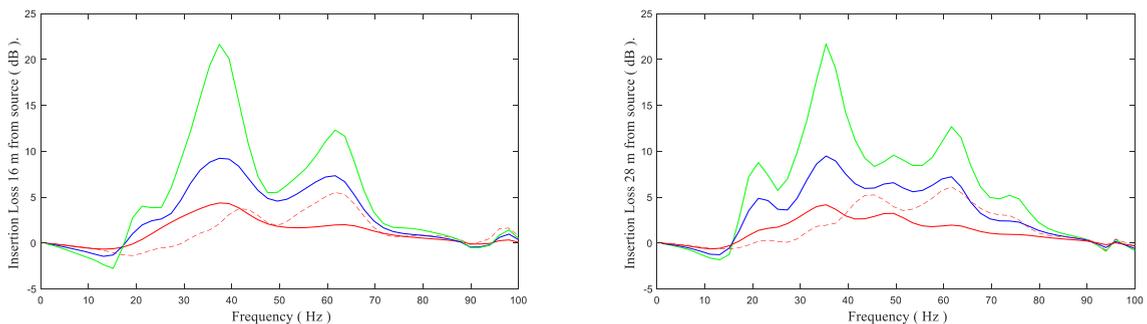


Fig. 5. Vertical insertion losses in dB for soil B at receiver positions beyond 9,600 kg regular mass placements: Full red line (1 mass); full blue line (2 masses); full green line (3 masses); and dashed red line (3 $\times$ 3,200 kg masses). The linear array starts 4 m away from the centre of the load. Left: Receiver position 16 m away from the centre of the load. Right: Receiver position 28 m away from the centre of the load.

In Figs. 4 and 5, blocks with mass 9,600 kg are placed at regular intervals with an extra curve shown for all three blocks having an equal 3,200 kg mass. Example results show that, in particular for soil A, the spectra of insertion loss for the three cases at the two receiver locations (see Fig. 4) are not significantly different. However, the insertion-loss peak is shifted to the right in the red dashed lines representing the three lower mass blocks, which is reasonable. Nevertheless, the maximum insertion loss for this case is comparable with the maximum insertion loss observed for three much heavier blocks. Since the study involves periodic arrays, some evidence of periodic behaviour is visible in Fig. 5, comparing a single mass to the cases in which extra masses are included in an array. Evidence for blocking vibration transmission at 60 Hz is clear in Fig. 5.

### 3.2. Example of the influence of a wall or a slab located on the surface

The given blocking-masses, with a certain footprint, can provide a positive benefit in terms of reducing vibration transmission across the ground surface. It seems reasonable to assume that a slab lying on the surface of the ground, with a relatively large footprint, provides a greater benefit in the low-frequency end of the spectrum compared to a wall of equal mass which has a much smaller footprint compared to the slab. For soil A, Fig. 6 indicates that the vibration reduction capability of a wall (thickness 0.5 m, mass 3,200 kg) reaches a possible 4 dB limit at around 30 Hz, and beyond this limit the insertion loss steadily decreases. However, a slab with larger footprint (1.6 m long) proves marginally less efficient at low frequencies; but there is evidence to suggest that its presence increases performance significantly towards 100 Hz, reaching a 10 dB vibration reduction at 100 Hz.

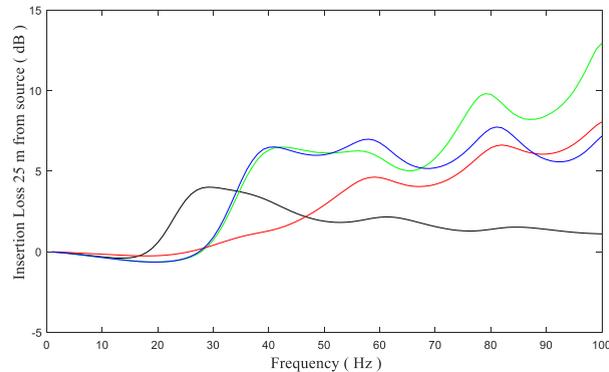


Fig. 6. Vertical insertion losses for soil A with various mass-load configurations observed at 25 m from the centre of the load, starting with single masses. Black line: 6,400kg wall at 3 m; Red line 6,400 kg slab at 3 m; Green line: 3,200 kg slab and wall at 3 m and 5 m, respectively; and Blue line: 3,200 kg wall and slab at 3 m and 5 m, respectively.

Next, consider the case where the wall and slab are placed adjacent, with a 2.0 m gap centre to centre. The difference in insertion loss is only marginal but both cases show a significant increase of insertion loss from 6 dB upwards. This may not be a practical solution to an engineering problem but it does show that coupling between masses can significantly improve the vibration field to suit the circumstances involved.

#### 4. Conclusions

In this paper, an analytical model that predicts ground vibration from rigid strip loads has been extended to estimate the impact of heavy point masses on the vibration transmission beyond the load. It was found that generally heavy masses can yield positive insertion losses for a broad frequency range and the interaction between periodically aligned masses can produce a beneficial effect.

Further research, comparing the results of the proposed simple analytical model with those from a model based on the finite-element method and/or the boundary-element method, would complement the results shown here. Also experiments should be conducted to validate the models for real-life scenarios where complex soil conditions, including stratification and inhomogeneities, are encountered.

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