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# An Analytical AC Resistance Calculation Method for Multiple-Conductor Feeder Cables in Aircraft Electric Power Systems

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**Abstract**—In multi-phase AC power systems, e.g. aircraft electric power systems (EPS), the proximity effect between the feeder cables can be severe and complex due to the tightly wired cable bundles. Simulation software is widely used to analyze such effects in order to guarantee proper system operation, but it is highly time consuming and requires extensive computational resources. Accordingly, this paper proposes an analytical calculation method for estimating the ac resistance of closely bundled power cables, namely multi-conductor cables. Simulation and experimental tests are performed to validate the effectiveness and accuracy of the method, showing that the algorithm developed in this paper is easy to implement with a minimum accuracy of 92% in all the tests. It allows online estimation of cable resistances, which can be beneficial for advanced system monitoring and management to enhance safety and reduce losses. Aircraft EPS and cables are used as the example in this paper, while the proposed method is also applicable for other applications, such as shipboard EPS, renewable energy parks, or microgrids where multi-conductor cables are used.

**Index Terms**—ac resistance; feeder cables; equivalent charge point; analytical calculation method

## I. INTRODUCTION

THE electrification of all the energy sectors (such as transportation and heating systems) has led to the increasing of system power levels. Taking aircrafts as examples, the Boeing 787 has a total power capacity of 1000kVA, and A380 electric power system (EPS) is at the power capacity of 600kVA. As a consequence, the length and capacity of the power cables in such systems are also increased considerably. The characteristics of the cables certainly have significant influence on system performance, safety and power losses [1]. In multi-phase EPSes, the skin effect and the proximity effect are the major concerns due to the feature that the cables are usually tightly bundled for space saving purposes. The consequences of the two effects mentioned above include

potential higher power losses, uneven distribution of current/power in paralleled branches, as well as voltage unbalances between phases. These effects are more severe in variable frequency AC (VFAC) EPSes, such as aircraft EPS, the frequency of which is 360~800 Hz. Thus, it is undoubtedly important to take cable characteristics into consideration in system design, control and supervision. In addition, the skin effect and proximity effect are also noticeable in transient processes such as startup or switching of large capacity loads, faults in generators or distribution lines, etc. Severe faults can easily cause the voltage transients to go beyond the limits of the safety range specified in GJB-181A [2] and MIL-STD-704F [3], thereby, destroying the sensitive equipment. Furthermore, in variable frequency systems, the increase of the feeder cable reactance at higher frequencies (mainly caused by the skin effect and the proximity effect) increases the voltage imbalances [4][5]. Thus, in order to improve the power quality, safety and efficiency of such EPSes, it is essential to study the AC impedance of feeder cables, especially taking into account the resistive nature of feeder lines in small to medium scale EPS.

Vehicle EPSes are commonly small to medium scale, and the impedance of feeder cables is mainly resistive. In aircraft VFAC EPS with a frequency range of 360~800Hz, considering the influence of odd harmonics current up to 25th harmonic, a frequency range of 0~20 kHz needs to be fully covered for evaluating the cable ac resistance. Although the eddy current field module in ANSOFT can be used to estimate the cable resistance with full consideration of skin and proximity effects, it is highly time consuming requiring huge amount of computational resources. Hence, it is of great significance to develop an analytical calculation method for the ac resistance estimation in wide frequency ranges.

The proposed method is generally applicable to all types of multi-phase AC systems. This paper emphasizes the application in aircraft systems due to the considerations that the cables in aircraft EPS are tightly bundled due to limited space and mainly resistive due to small system size, the frequency in aircraft EPS (360~800Hz) is much higher than utility power grid (50/60Hz). Thus, the skin effect and proximity effect in aircraft system are more severe than in other systems. Moreover, the method is also useful for other occasions, such as the shipboard EPS, 60Hz or 400Hz, 440V or 115V, and microgrids, 50/60Hz, small scale and resistive network. In

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summary, the method is widely applicable to multi-phase power systems with resistive and closely bundled cables. It is especially useful in aircraft EPS due to the variable frequency range, resistive feature and tightly bundled cables.

A large number of research works have been carried out for this purpose. The influence of skin effect and proximity effect on the cable resistance was firstly considered in 1873 by Clerk Maxwell who calculated the ac resistance of a single cable. In 1886, a complete calculation method has been presented by Lord Rayleigh for the skin effect of infinite width strip conductors. In 1889, the skin factor formula based on Bessel function was proposed by Lord Kelvin. In 1915, A.E.Kennelly analyzed the influence of skin effect and proximity effect on conductors with different shapes through experimental tests, and verified the skin factor formula based on Bessel function [6]. In addition, some scholars have studied the influence of skin effect and proximity effect on the ac resistance of different cables. H. B. Dwight from Westinghouse carried out theoretical calculation of the impedance for two closely bundled cables [7][8]. In [9], the relationship between conductor diameter and proximity effect was analyzed, and it was concluded that the loss caused by skin effect and proximity effect cannot be ignored. In 1948, Samuel d. Summers from United States Naval Research Laboratory analyzed the variation of proximity effect against the cable spacing and current frequency [10]. The skin effect and the proximity effect of two cables under high frequency power transmission were analyzed in [11]. Reference [12] and [13] give the calculation formula of bundled cables considering proximity effect. This method is based on the electromagnetic field theory and Bessel function. However, the calculation formula was highly complex which makes it hard to be applied in real time applications. The skin factor of hollow circular conductor was obtained by means of layered numerical calculation in [14], but the proximity factor was not involved in this method. Reference [15] calculated ac impedance of solid and tubular cylindrical conductors based on the Bessel function, but the formula was also complex and cannot be used for the calculation of proximity effect between two conductors or multi-conductors. In 2014, Alan Payne obtained the ac resistance of a single conductor based on the equivalent conductive area theory, and the proximity factors between two conductors were also estimated and verified. The new formula was simpler and easier than the formula based on Bessel function [16].

In summary, the skin effect and factor estimation technic is quite mature, especially the one for the solid cylindrical conductor that can be commonly found in fundamental text books [17][18]. However, the proximity factor estimation is still an open challenge due to the complexity it brings to the calculation. The key parameters that need to be considered include the current frequency, the conductor size, and the conductor material, as well as the distance between conductors, the relative positions of multi-conductors, and the current-phase. Thus, in most existing applications, the ac resistance of feeder cables is obtained majorly by finite element simulation software that is extensively time taking when the analyzed frequency range is widened.

In order to achieve efficient and accurate estimation of the feeder resistance under complicated wiring conditions, this paper develops an analytical calculation algorithm based on the fundamentals of skin effect and proximity effect. Three-phase VFAC (360~800Hz) power system, which is widely applied in aircraft electric power systems (EPS), is taken as the application background. Triangle wiring of the feeder cables is commonly used in these systems, and thus is used as the test case to verify the accuracy of the proposed approach. The paper is organized as follows. Section II presents the principle of skin effect and proximity effect. The ac resistance calculation method of multiple conductors is proposed and discussed in Section III. In Section IV, the experiment results are given and analyzed. Conclusion is drawn in Section V.

## II. THE PRINCIPLE OF SKIN EFFECT AND PROXIMITY EFFECT

### A. The skin factor for a single conductor

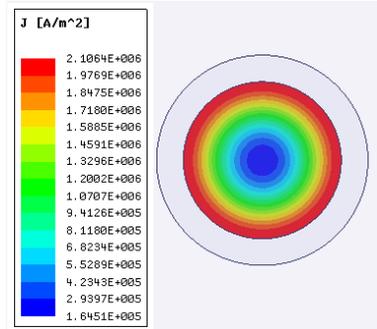


Fig. 1. Current density distribution of single conductor

According to the definition of skin effect, as the current frequency increases, the current is majorly concentrated near the surface of the conductor, and the effective conducting area is thus reduced. Fig. 1 shows an example case obtained in ANSOFT. The simulated cable is the single AWG#4/0 cable. The resistance of conductor is increased due to the reduction of its cross-sectional area.

A skin factor is defined as follows [19]:

$$\alpha_R = \frac{R_0}{R_{DC}} \quad (1)$$

where,  $R_0$  is the ac resistance of the conductor considering skin effect, and  $R_{DC}$  is the DC resistance.

The skin factor of a conductor depends on its material and physical features, and can be obtained by [16]:

$$\alpha_R = \begin{cases} 1 & f < 1/\pi\mu\sigma a_w^2 \\ 1 + [d_w/(4\delta)]^4/3 & 1/\pi\mu\sigma a_w^2 \leq f \leq 16/d_w^2\pi\mu\sigma \\ 0.25(d_w/\delta)^2/(d_w/\delta - 1) & f > 16/d_w^2\pi\mu\sigma \end{cases} \quad (2)$$

where,  $a_w$  is the conductor radius,  $d_w$  is the conductor diameter, the  $\mu$  is permeability, and  $\sigma$  is conductivity, the  $\delta$  is the skin depth, defined as:

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \quad (3)$$

### B. Calculation method of proximity factor for two conductors

When the alternating current flows through adjacent conductors, the phenomenon that the current tends to shift towards one side of the conductor due to electromagnetic effect is called proximity effect, as shown in Fig.2.

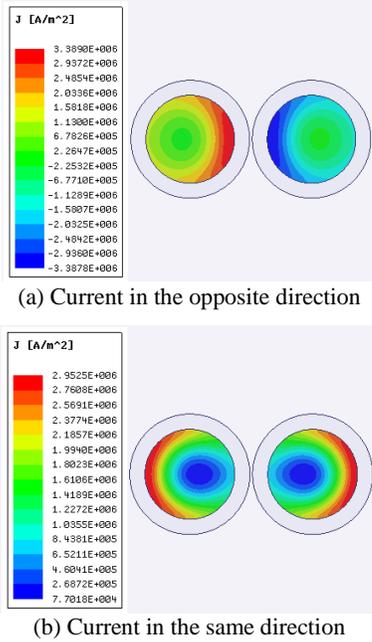


Fig.2. Current density distribution of two adjacent conductors

The principle of proximity effect is similar to that of skin effect. The eddy currents generated by the alternating magnetic forces between adjacent conductors are superimposed and offset with their own current, resulting in uneven distribution of current. The current distribution caused by proximity effect is also related to the direction of the current in conductors. As shown in Fig. 2 (simulation software ANSOFT, simulated cable AWG#4/0), when the direction of the current in the two adjacent conductors is opposite, the current flow is concentrated in the inner side of the two conductors, and when the direction of the current is same, the current flow is concentrated in the outer side of the two conductors.

The proximity factor is defined as:

$$\beta_R = \frac{R}{R_0} \quad (4)$$

where, R is the ac resistance between two conductors.

The proximity factor of conductor is affected by many factors, thus, most of the calculation methods for proximity factor are complicated. The PAYNE, British Electrical Engineer, proposed a simple proximity factor calculation method for two conductors [16]:

$$\beta_R = \left( 1 + \left( \frac{a'_w}{r'_1} - \frac{a'_w}{r'_2} \right) \right) \quad (5)$$

where,  $a'_w$  is the conductor equivalent radius,  $a'_w = a_w - \delta/2$ ,  $a_w$  is the conductor's radius,  $r'_1$  and  $r'_2$  are equivalent distance.

This paper takes (4) and (5) as the basis, and develops an analytical resistance calculation method for multiple conductors.

### III. THE PROPOSED ANALYTICAL CALCULATION METHOD FOR MULTI-CONDUCTORS AC RESISTANCE

In 230V AC, 360Hz~800Hz, 250kVA aircraft EPS, the power of the main generator is 250kVA, the phase current is 363A, and the maximum harmonic frequency of generator is 20 kHz. The influence of skin effect and proximity effect on the feeder cables is critical. With the development of large aircraft, carbon fiber composite materials are widely used for the aircraft shell. As a consequence, an independent neutral wire also exists along with the three-phase wires. Therefore, the feeder cable is composed of four conductors which are laid typically with triangle wiring strategy as shown in Fig. 3. This paper takes AWG#4/0 wire as an example, which is commonly used three-phase four-wire aircraft EPS. Its conductive radius is 5.842mm. The specific process of the analytical calculation method for the ac resistance of multi-conductors is given.

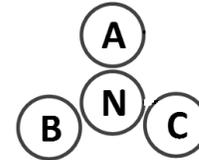


Fig.3. The triangle wiring strategy of main feeder

#### A. Phase division according to direction of current

In this paper, the balance three-phase current is considered. The three phase current in balanced condition (no zero-sequence current flow in neutral) is shown in Fig. 4.

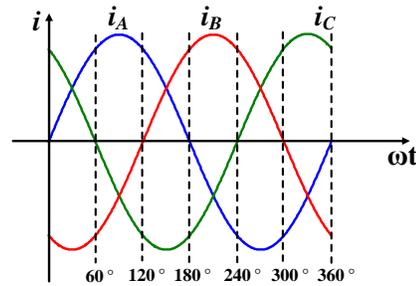


Fig.4. The three-phase current waveform

The skin effect and proximity effect are closely related to the direction, phase and frequency of the current, while the amplitude does not have significant influence. Thus, the three-phase currents are first segmented according to the direction of the current. Taking the zero crossing point of current as the dividing point, the 0~360 degree is divided into 6 segments, as shown in Table I.

TABLE I  
THE SEGMENT OF THREE-PHASE CURRENT

	0~60°	60~120°	120~180°
Positive	A、C	A	A、B
Negative	B	B、C	C
	180~240°	240~300°	300~360°
Positive	B	B、C	C
Negative	A、C	A	A、B

It can be seen from Table I that the relative direction of the current between conductors in the range of 180~360 degrees is the same as that in the range of 0~180 degree. Accordingly, the following analysis is conducted for 0~180 degree, which can be similarly extended for 180~360 degree range.

### B. The position of the equivalent charged point

Taking the two conductors with the reverse current as an example, the distribution of the current density is shown in Fig. 2(a). According to the “electric axis method”, the charge effect on the surface of two cylindrical conductors can be considered equivalent to two charged thin lines [19]. Due to the influence of proximity effect, the position of the equivalent charged thin lines will deviate from the center of the conductor. As shown in Fig. 5, “A1” and “B1” are the actual positions of the equivalent charged thin lines that have been represented by equivalent charged points on the cross-section of the conductor. In this figure,  $r'_1 = r_1 - d$ ,  $r'_2 = r_2 - d$ , “d” is the offset calculated by the electric axis method.

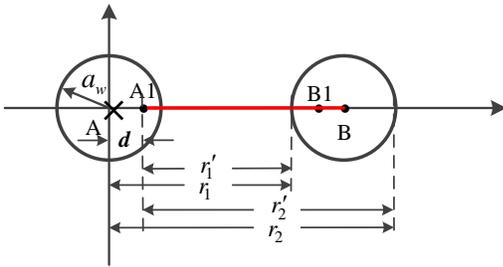


Fig.5. Equivalent spacing between two conductors (reverse current)

The key for ac resistance calculation of multiple conductors is to determine the position of the equivalent charged thin lines according to current density distribution. Based on that, the equivalent center distance between the conductors can be obtained. Afterwards, the values of the ac resistance between A & B conductors, B & C conductors, and A & C conductors are calculated respectively. Finally, by considering the mutual influence between multiple conductors, the total resistance of the multiple conductors is obtained.

The calculation formula is related to the direction of the current in two conductors. Taking the 0~60 degree range as an example, the direction of Phase-A current and Phase-C current are the same. The direction of Phase-A current and Phase-B current are opposite, as well as Phase-B current and Phase-C current. The interaction of Phase-A, Phase-B and Phase-C will lead to uneven distribution of current density. In the electromagnetic field, the direction of current determines the direction of magnetic field lines, and consequently the direction of induced current in the conductor. The ultimate impact is the uneven current density distribution. Vector superposition method is used in this paper to analyze and clarify the interaction between conductors.

In the segment range 0-60 degree (as shown in Table I), the direction of Phase-A current and Phase-B current are opposite. According to the uniqueness theorem [19], the proximity effect from Phase-B tends to shift the charged thin line of Phase-A to the new point  $A_{B1}$ , and the proximity effect from Phase-C tends to shift the charged thin line of Phase-A to the new point  $A_{C1}$ .

The superposition of the proximity effect from Phase-B and Phase-C results in the estimated final position  $A_1$ , as shown in Fig. 6. The green thin line represents the equivalent center distance associated with  $A_1$ .

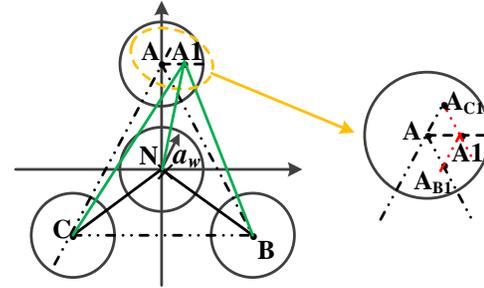


Fig.6. The position for equivalent charged thin line of Phase-A

In the same way, the positions of the equivalent charged thin line for Phase-B and Phase-C are respectively at points  $B_1$  and  $C_1$ . The details are shown in the Fig. 7. The blue thin line represents the equivalent center distance associated with  $B_1$ . The red thin line represents the equivalent center distance associated with  $C_1$ . In this way, the influence of proximity effect is considered.

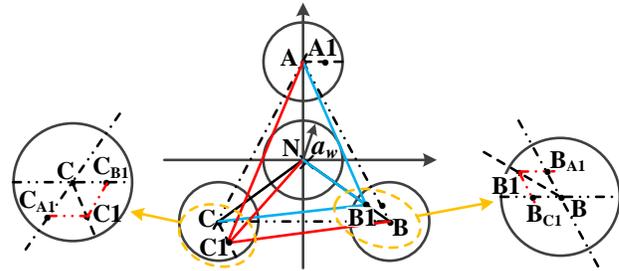


Fig.7. The position for equivalent charged thin line of Phase-B and Phase-C

After determining the position of the equivalent charged thin line, the equivalent center distance between can be calculated.

### C. The calculation of equivalent center distance

The equivalent center distance is the distance between equivalent charged point of conductor and center point of the other conductor. Taking the Phase-A as an example,  $p_{AIB}$  is the equivalent center distance between Phase-A & Phase-B,  $p_{AIC}$  is the equivalent center distance between Phase-A & Phase-C, and  $p_{AIN}$  is the equivalent center distance between Phase-A & Phase-N.

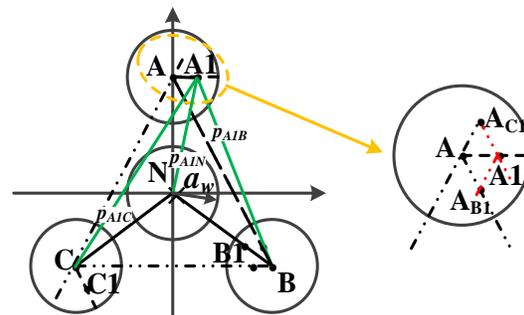


Fig.8. The equivalent center distance for Phase-A

According to the geometrical fundamentals, the equivalent center distance  $p_{A1B}$  is:

$$p_{A1B} = \sqrt{p_{AB}^2 + p_{AA1}^2 - 2p_{AB}p_{AA1} \cos(\angle A1AB)}. \quad (6)$$

According to the symmetry of the triangular structure, the angle relationship is:  $\angle A_{C1}AA1 = \angle A_{B1}AA1 = \angle BAA1 = 60^\circ$ , and the side length relationship as:  $p_{AB} = \sqrt{3}p_{AN}$ . According to the property of parallelogram  $AA_{C1}A1A_{B1}$ , the side length relationship:  $p_{AA1} = p_{A_{B1}A} = p_{A_{C1}A} = d$ ,  $d$  can be obtained from the uniqueness theorem as follows:

$$d = p_{AB}/2 - \sqrt{(p_{AB}/2)^2 - a_w^2}/2}. \quad (7)$$

Then,

$$p_{A1B} = \sqrt{p_{AB}^2 + d^2 - 2p_{AB}d \cos(60^\circ)}. \quad (8)$$

In the same way:

$$p_{A1C} = \sqrt{p_{AC}^2 + d^2 - 2p_{AC}d \cos(120^\circ)}. \quad (9)$$

$$p_{A1N} = \sqrt{p_{AN}^2 + d^2}. \quad (10)$$

The equivalent center distance solving process of Phase-B and Phase-C is similar with that of Phase-A.

#### D. Study on the calculation method of the proximity factor between multi-conductors

The typical triangle wiring of feeder cables is composed of four conductors. When symmetric current is flowing through the three-phase conductors, no current should flow through the neutral wire. However, an induced current still flows through the neutral wire. This induced current will also affect the current density distribution in Phase-A, Phase-B and Phase-C.

In order to calculate the ac resistance of Phase-A, it is necessary to include the influence of the proximity effect from other phase conductors i.e., Phase-B, Phase-C and neutral wire, while performing the calculations, as shown in (11).

$$R_A/R_0 = 1 + \beta_{A1B} + \beta_{A1C} + \beta_{A1N}. \quad (11)$$

where,  $\beta_{A1B}$ ,  $\beta_{A1C}$ ,  $\beta_{A1N}$  are the proximity factors of Phase-A & Phase-B, Phase-A & Phase-C, and Phase-A & Phase-N, respectively. The formulas are as follows:

$$\beta_{A1B} = \frac{1}{p_{A1B}/a'_w - 1} - \frac{1}{p_{A1B}/a'_w + 1}. \quad (12)$$

$$\beta_{A1C} = \frac{1}{p_{A1C}/a'_w - 1} - \frac{1}{p_{A1C}/a'_w + 1}. \quad (13)$$

$$\beta_{A1N} = \frac{1}{p_{A1N}/a'_w - 1} - \frac{1}{p_{A1N}/a'_w + 1}. \quad (14)$$

The ac resistance of Phase-A can be obtained. And the ac resistance of Phase-B and Phase-C can be obtained by the same method.

$$R_B/R_0 = 1 + \beta_{AB1} + \beta_{B1C} + \beta_{B1N}. \quad (15)$$

$$R_C/R_0 = 1 + \beta_{AC1} + \beta_{BC1} + \beta_{C1N}. \quad (16)$$

The influence of proximity effect on neutral wire also needs to be taken into account. Since there is no current flowing through the neutral wire under balanced conditions, the DC

resistance is not considered. The influence of proximity effect from Phase-A, Phase-B and Phase-C is weak, the formula is as follows.

$$R_N/R_0 = (\beta_{AN} + \beta_{BN} + \beta_{CN})/6. \quad (17)$$

where:

$$\beta_{AN} = \frac{1}{p_{AN}/a'_w - 1} - \frac{1}{p_{AN}/a'_w + 1}. \quad (18)$$

$$\beta_{BN} = \frac{1}{p_{BN}/a'_w - 1} - \frac{1}{p_{BN}/a'_w + 1}. \quad (19)$$

$$\beta_{CN} = \frac{1}{p_{CN}/a'_w - 1} - \frac{1}{p_{CN}/a'_w + 1}. \quad (20)$$

The average resistance  $R_1$  in the range of 0~60 degree is:

$$R_1 = (R_A + R_B + R_C)/3 + R_N. \quad (21)$$

In the range of 60~120 degree, the current of Phase-B and Phase-C are in the same direction, the current of Phase-A and Phase-B are in the opposite direction as well as that of Phase-A and Phase-C. The position of equivalent charged thin line and equivalent center distance are shown in Fig. 9. Similarly, the green, blue and red thin line represents respectively the equivalent center distance associated with A2, B2 and C2.

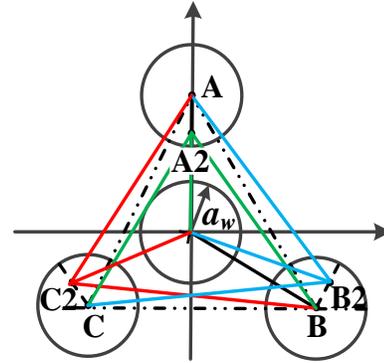


Fig.9. The equivalent center distance in the range of 60~120 degree

In the range of 120~180 degree, the current of Phase-A and Phase-B are in the same direction. The current of Phase-A and Phase-C are in the opposite direction as well as that of Phase-B and Phase-C. The position of equivalent charged thin line and equivalent center distance are shown in the Fig. 10. Similarly, the green, blue and red thin line represents respectively the equivalent center distance associated with A3, B3 and C3.

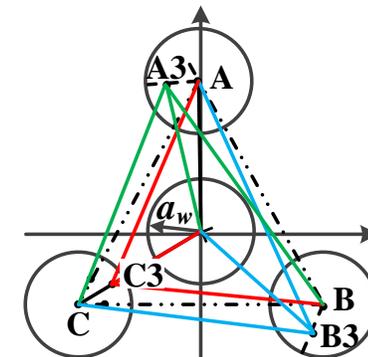


Fig.10. The equivalent center distance in the range of 120~180 degree

The average resistance in the range of 60~120 degree is represented by  $R_2$ , and that in the range of 120~180 degree is represented by  $R_3$ . The calculation method for  $R_2$  and  $R_3$  is the same as that for  $R_1$ , shown in (21).

Due to the symmetrical structure of the triangle wiring strategy and the symmetry characteristic of current, the resistance of triangle wiring strategy can be obtained by taking the average of  $R_1$ ,  $R_2$ , and  $R_3$ , as:

$$R = (R_1 + R_2 + R_3) / 3. \quad (22)$$

The ac resistance values calculated with formula (22) are compared with the simulation values obtained using ANSOFT software, and the error curve is shown in Fig. 11. AWG#4/0 cable with conductive radius of 5.842mm is used for comparison. The center distance of conductors is set 15.82mm, 20mm, 30mm, 70mm and 100mm, respectively. As can be seen in Fig. 11 that, most of the error values are within 2% for frequency range above 1800 Hz. However, the error is larger in the lower frequency range with maximum error of about 7% at 600 Hz.

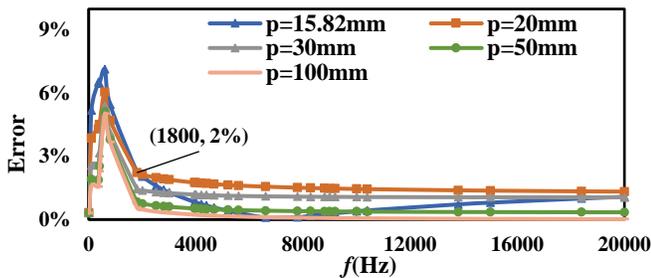


Fig.11. The ac resistance error curve of triangle wiring strategy

### E. AC resistance calculation method for unsymmetrical systems

For unsymmetrical systems, the three phase quantities can be decomposed to three balanced systems, namely positive, negative and zero sequence systems, and analyzed respectively. The vector diagram of an example unbalanced system is given in Fig. 12. The unsymmetrical current in the feeder cables is decomposed into three sets of symmetric three-phase current: the positive-sequence current ( $i_A^+$ ,  $i_B^+$ ,  $i_C^+$ ), negative-sequence current ( $i_A^-$ ,  $i_B^-$ ,  $i_C^-$ ) and zero-sequence current ( $i_A^0$ ,  $i_B^0$ ,  $i_C^0$ ).

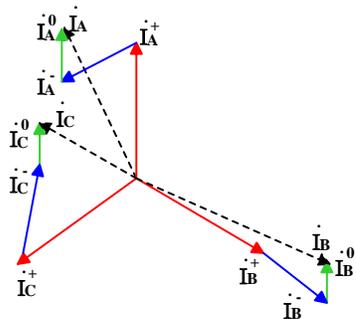


Fig.12. The vector diagram of three-phase current

The skin effect and proximity effect are closely related to the direction, phase and frequency of the current, while the amplitude does not have significant influence. Thus, the calculation process of negative-sequence resistance is the same

as that of positive-sequence resistance. The zero-sequence current has the same amplitude and same phase, but the neutral conductor carries three times reverse current. The calculation process of zero-sequence resistance is different. However, the proposed approach in this paper is also applicable. The zero-sequence resistance calculation process of the triangle wiring strategy is given below.

In the case of zero-sequence current, the direction of Phase-A current, Phase-B and Phase-C current are the same. The direction of Phase-A current and Phase-N current are opposite, as well as Phase-B current and Phase-N current, Phase-C current and Phase-N current. Considering the symmetry, the positions of equivalent charged thin line overlap with the original center point, as shown in Fig. 13.

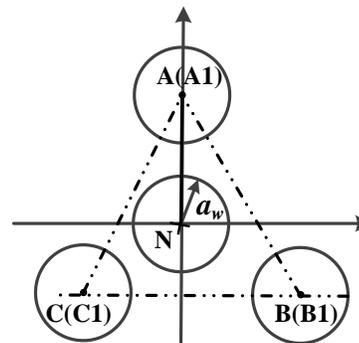


Fig.13. The position for equivalent charged thin line

The ac resistance values calculated based on the proposed approach are compared with the simulation values from ANSOFT software, and the error curve is shown in Fig. 14. AWG#4/0 cable with conductive radius of 5.842mm is used for comparison. The center distance of conductors is set to 15.82mm, 20mm, 30mm, 50mm and 100mm, respectively. As can be seen in Fig. 14 that, the error values are within 8%.

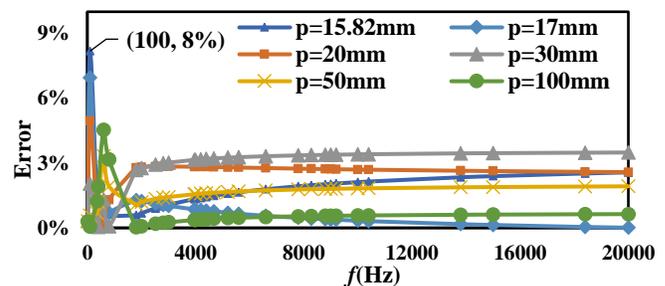


Fig.14. The ac resistance error curve of triangle wiring strategy

In the unsymmetrical three-phase system, the influence of negative-sequence current and zero-sequence current is considered. The calculation of negative-sequence resistance and zero-sequence resistance can also be realized based on the proposed approach in this paper. And the error values are within 8%.

### F. Verification for ac resistance analytical calculation method of multi-conductors

#### 1) Different wiring strategies

The validness of the proposed method is tested under different wiring strategies, such as paralleled wiring. The paralleled wiring strategy is composed of four conductors. The

Phase-A, Phase-B, Phase-C and Phase-Neutral are arranged in parallel as shown in Fig.15 . By applying the method described in the Section III of the paper, the positions of equivalent charged thin line (A1,B1 and C1) are given as an example in Fig.15.

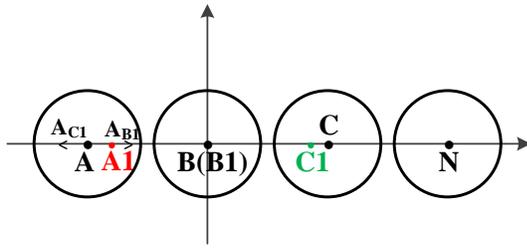


Fig.15. The position for equivalent charged thin line

The ac resistance values calculated based on the proposed approach are compared with the simulation values from ANSOFT software, and the error curve is shown in Fig. 16. AWG#4/0 cable with conductive radius of 5.842mm is used as the test objective. The center distance of conductors is set to 16mm, 20mm, 30mm, 60mm and 100mm, respectively. As can be seen in Fig.16 that, most of the error values are within 2% for frequency range above 1800 Hz. However, the error is larger in the low frequency range with maximum error around 6% at 600 Hz.

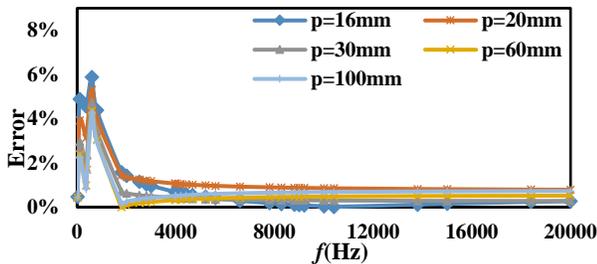


Fig.16. The ac resistance error curve of paralleled wiring strategy

## 2) Different conductor radiuses and distances

The validness of the proposed method is also tested under different conductor radiuses and distances. The ac resistance calculation method shown in (22) with different center distance has been verified (cable AWG#4/0 is used as the study case).

TABLE II  
PARAMETERS OF THE CABLES

	Conductive Radius(mm)	Outer Radius(mm)	Center Distance(mm)
AWG 0#	4.126	5.614	12, 15, 20, 50, 100
AWG 6#	2.0575	2.94	6, 10, 15, 50 100

Next, the cables AWG 0# and AWG 6# are analyzed respectively. Based on the parameters of the cables given in Table II, the ac resistances of triangle wiring strategy can be calculated by (22). The calculation results are compared with ANSOFT simulation results and shown in Table III-IV. The 12mm and 20mm represent the center distances of cables AWG 0#, the 6mm and 15mm represent the center distances of cables AWG 6#.

TABLE III  
AC RESISTANCE COMPARISON OF AWG 0#

Frequency/Hz	Resistance Comparison (mΩ /m)			
	12mm		20mm	
	Simulation	Calculation	Simulation	Calculation
10	0.323	0.322	0.323	0.322
100	0.326	0.322	0.325	0.322
400	0.375	0.383	0.351	0.354
800	0.473	0.463	0.412	0.413
2000	0.733	0.761	0.600	0.618
5600	1.223	1.240	0.959	0.972
10000	1.634	1.645	1.257	1.272
20000	2.314	2.316	1.748	1.768

Frequency/Hz	Resistance Comparison (mΩ /m)			
	6mm		15mm	
	Simulation Results	Calculation Results	Simulation Results	Calculation Results
10	1.297	1.293	1.297	1.293
100	1.298	1.293	1.297	1.293
400	1.313	1.315	1.303	1.300
800	1.357	1.399	1.320	1.324
2000	1.599	1.610	1.425	1.428
5600	2.454	2.379	1.930	1.988
10000	3.283	3.378	2.498	2.541
20000	4.628	4.698	3.399	3.426

TABLE IV  
AC RESISTANCE COMPARISON OF AWG 6#

The ac resistance calculation error of the cables AWG 0# and AWG 6# are shown in Fig.17 and Fig.18.

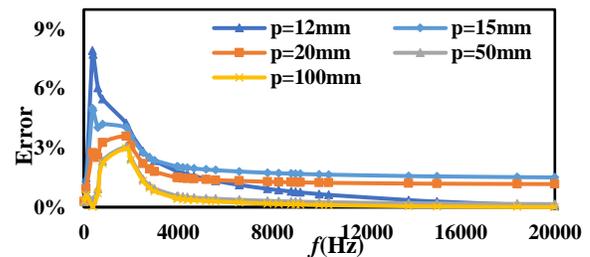


Fig.17. The ac resistance error curve of triangle wiring strategy (AWG 0#)

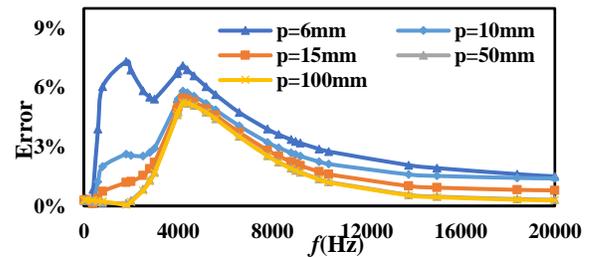


Fig.18. The ac resistance error curve of triangle wiring strategy (AWG 6#)

As can be seen from Fig. 17 and Fig. 18 that, the error of ac resistance calculation is no bigger than 8% in the frequency range 0~20 kHz.

## G. Discussion and analysis on the calculation error

Based on the above analysis, the maximum error of the proposed method is 8%. In order to make the proposed method more applicable, the error of the method under different frequency, cable type and wiring strategy shall be documented and provided as a manual so that certain errors can be taken into consideration in real world applications. In addition, the errors are also analyzed.

As indicated in this paper, the errors are bigger under a certain frequency range, and the maximum error appears at different frequency under different conductor radius. This

phenomenon is caused by the calculation error of single conductor ac resistance  $R_0$ . As described in the paper, the calculation formula of single conductor ac resistance is a piecewise function which is related to the frequency:

$$R_0 = \begin{cases} R_{DC} & f < f_1 \quad (F.1) \\ \left(1 + [d_w / (4\delta)]^4 / 3\right) R_{DC} & f_1 \leq f \leq f_2 \quad (F.2) \\ \left(0.25(d_w / \delta)^2 / (d_w / \delta - 1)\right) R_{DC} & f > f_2 \quad (F.3) \end{cases} \quad (23)$$

where:

$$f_1 = 1 / a_w^2 \pi \mu \sigma \quad (24)$$

$$f_2 = 16 / a_w^2 \pi \mu \sigma \quad (25)$$

The calculation errors of the formula (23) are large at the subsection frequency of Eq. (F.2) and Eq. (F.3), and the subsection frequency  $f_2$  is a radius-related parameter. The subsection frequencies vary with the radiuses, as shown in Table V.

TABLE V  
SUBSECTION FREQUENCY OF DIFFERENT RADIUSSES

	AWG 4/0#	AWG 0#	AWG 2#
	5.842mm	4.126mm	3.272mm
$f_2$	510Hz	1024Hz	1628Hz
	AWG 4#	AWG 6#	AWG 8#
	2.595mm	2.058mm	1.632mm
$f_2$	2589Hz	4116Hz	6542Hz

In order to verify the relationship between conductor radius and the frequency where the maximum error appears, the error curve of single conductor ac resistance is given in Fig. 19.

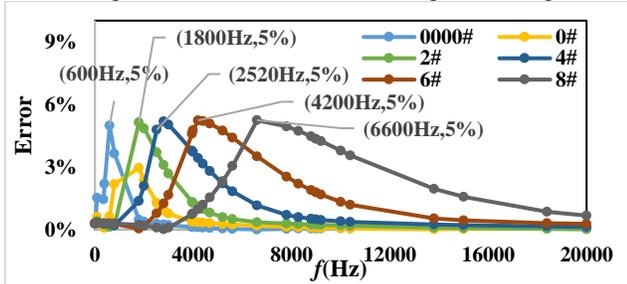


Fig.19. The ac resistance error curve of single conductor

As can be seen from Table V and Fig. 19 that, the frequency where the maximum error occurs is similar to the subsection frequency  $f_2$  of the piecewise function at each radius. That is, the errors of single conductor ac resistance result in the different frequencies where the maximum error occurs, and the maximum error is 5%. Combined with the proposed method in this paper, the maximum error becomes 8%.

#### IV. THE EXPERIMENTAL VERIFICATION

Experiments are also conducted to validate the effectiveness and accuracy of the proposed method. The electrical connection diagram of the impedance measurement platform is shown in Fig. 20, which is mainly composed of four parts: source, loads, test cables and measurement circuit. The source is variable frequency AC power supply, the voltage is 270V, and the power supply structure is three-phase four-wire system. By changing the load value, the current in the circuit can be adjusted. The measuring circuit collects the voltage and current

of test cables, the ac resistance value of teste cable can be obtained by calculation.

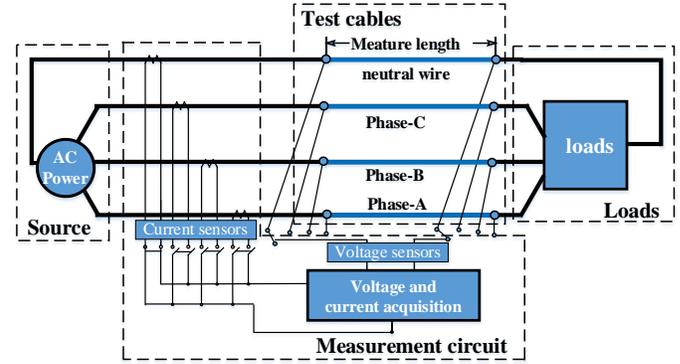


Fig.20. The electrical connection of impedance measurement platform

The length of test cables in the experiment are 9.2m, with conductive area of 50mm<sup>2</sup> and center distance of 15mm between two cables. The measured current frequency range is 360~800 Hz, which is the main frequency range of main feeder current. The experimental platform and partial cable laying diagram are shown in Fig. 21.

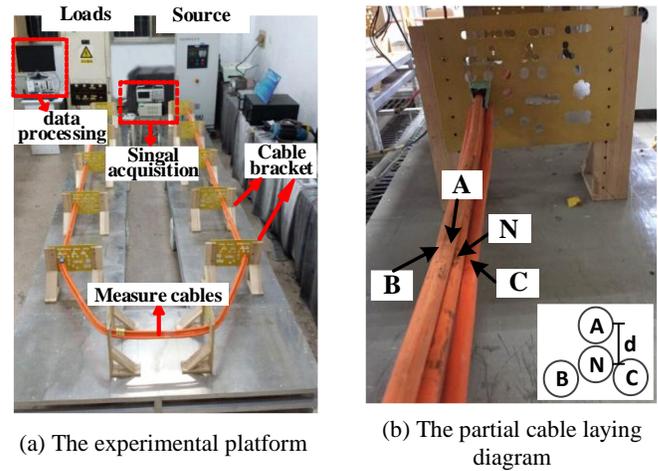


Fig.21. The experimental measurement platform of ac resistance

The experimental, simulation and formula values of ac resistance are shown in Table VI.

TABLE VI  
THE DATA UNDER DIFFERENT CONDITIONS

Frequency/Hz	Experiment Value (mΩ)	Simulation Value (mΩ)	Formula Value (mΩ)
360	3.425	3.472	3.609
400	3.459	3.533	3.666
600	3.774	3.875	4.035
800	4.267	4.246	4.441

TABLE VII  
THE ERROR UNDER DIFFERENT CONDITIONS

Frequency/Hz	E1	E2
360	1%	5%
400	2%	6%
600	3%	7%
800	1%	4%

In Table VII, the “E1” represents the error between the experimental value and the simulation value, and the “E2” represents the error between the experimental value and the formula value.

Then we draw the data shown in Table VI into the form of chart, shown as follows.

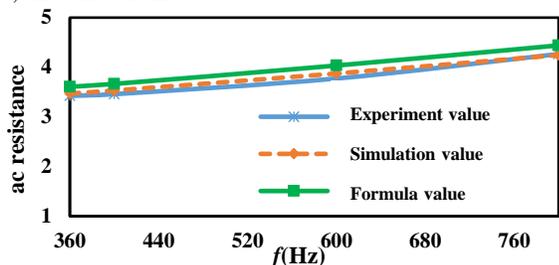


Fig.22. The data comparison under different conditions

It can be seen that the experimental results verify the correctness of the simulation value and the analytical calculation method. The error of the simulation value and the experimental value is kept within 3%, which proves the credibility and authenticity of the simulation calculation results based on the ANSOFT software. And the error of analytical method and experimental result is within 7%, which indicates the ac resistance analysis method given in this paper can effectively estimate the ac resistance of multi-conductors.

## V. CONCLUSION

This paper mainly derives an analytical ac resistance calculation method for multiple-conductor feeder cables. It can calculate the ac resistance directly under specified frequency, conductor radius and conductor center distance. The error is less than 8%, which can replace the tedious finite element simulation process, and simplify the calculation process of the ac resistance. The result has important engineering application value.

In future work, the authors will also try to improve the accuracy by optimizing the calculation formula of single ac resistance, considering the iterative influence of equivalent charging point, etc. The method will be further analyzed and verified.

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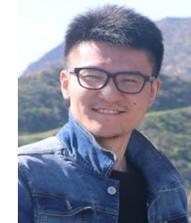
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