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Nasir, Mashood; zia, fahad

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Optimization Methods for Constrained Stochastic Wind Power Economic Dispatch

Fahad Zia[1]
[1]fahad.zia@umt.edu.pk,

Mashood Nasir[2]
[2]mashood.nasir@umt.edu.pk

Dr. Abdul Aziz Bhatti[3]
[3]drabhatti@umt.edu.pk,

University of Management and Technology
Lahore – Pakistan

***Abstract*— An economic load dispatch model developed for both wind turbines and thermal generators is presented in this paper. In the model, the wind power is considered as a constraint due to random nature of wind speed. This optimization problem is solved to obtain optimal power outputs of thermal generators, while taking into account of given constraints. All these optimal outputs and available wind power must satisfy the load generation balance of the power system. The closed form solution of this model is not possible due to the inequality constraints and this problem is solved by introducing a penalty function or barrier function in the model to take account of inequality constraints. The feasible ranges of optimum solutions are obtained by using the developed models based on penalty function and interior point methods.**

***Index Terms*— Economic Load Dispatch, interior point method, penalty function method, weibull distribution, wind power**

I. Introduction

Modern society highly relies on its energy supply, in particular the supply of electricity. Electricity consumption is strongly correlated with economic growth: economic growth increases the use of electric utilities which become the cause of increase in electricity demand. In the past three decades, economic growth has been an important factor in tripling the electricity consumption worldwide.

Demand for the utilization of renewable energy technologies such as biomass, geothermal, wind power, solar photovoltaics, tidal and wave power are increasing day by day. In the past decade, attention towards utilization of wind energy has increased more than any other renewable resources [1]. The main challenge in wind energy is efficient integration of its generated electricity into grids. Since the wind speed has stochastic nature, the output of wind turbines cannot be controlled as conventional generation technologies can be. Currently, conventional generation plays a vital role in maintaining the power balance between generation and demand. Wind power challenges power system balancing in two ways. On the one hand, wind power introduces additional variations and uncertainty. On the other hand, provided the wind is available for longer periods of time, the presence of wind power reduces the amount of conventional generation capacity scheduled and available for balancing purposes.

Due to randomness of wind power, the economic load dispatch model can be solved by the approches like stocahstic programming [2-3]. The present work focuses on solving economic load disptach model with random wind power constraint using penalty function and interior porint methods.

II. An ELD Model with Wind Power

The economic load dispatch problem is basically a mathematical optimization problem in electrical power systems. This optimization problem is solved to obtain optimal power outputs of thermal generators, while taking into account of given constraints. All these optimal outputs and available wind power must satisfy the load generation balance of the power system. The model can also include certain other constraints. These constraints are minimum and maximum limits of generator power outputs. In this paper, the quadratic cost function is selected which can be obtained by applying least square estimation method on measured data of thermal generators [4]. The ELD model is represented as:

$$\text{minimize} \qquad y = \sum_{i=1}^{n} a_{i0} + a_{i1}x_i + a_{i2}x_i^2$$

subject to

$$W + \sum_{i=1}^{n} x_i = P_d + P_{loss}$$

$$x_{min,i} \le x_i \le x_{max,i} \qquad i = (1,2,\dots,n).$$

where

a_{i0} , a_{i1}, a_{i2}	Cost coefficients for the ith thermal generator.
W	Wind power of a wind farm.
$x_{min,i}$, $x_{max,i}$	Minimum and maximum limit on ith thermal generator power output.
P_d	Load demand.
P_{loss}	System power losses.

III. Wind Speed Characterization

Wind speed is one of the most critical characteristics in wind power generation. In fact, wind speed varies in both time and space, determined by many factors such as geographic and weather conditions. Because wind speed is a random parameter, the instantaneous wind speed V changes with time, due to which it is averaged over a short time interval. Statisitcal methods are applied on these average figures for analysis of wind speed.

The wind speed, V, variations can be best described by Weibull distribution during a short period of time [5]. The probability density function (pdf) of wind speed random variable V is

$$f_V(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} exp\left[-\left(\frac{v}{c}\right)^{k}\right], \qquad v \geq 0 \tag{1}$$

where c is the scale factor used as a measure for wind speed v and k is the shape factor which describes the shape of distribution. These two parameters can be estimated by using statistical methods on available wind speed data.

The cumulative distribution function for a Weibull random variable, V, to find probability of wind speed is expressed as

$$F_V(v) = 1 - exp\left[-\left(\frac{v}{c}\right)^{k}\right]. \tag{2}$$

The Weibull distribution can degenerate into two special distributions, namely exponential distribution for which shape factor, k is 1 and the Rayleigh distribution for which shape factor, k is 2. Since observed wind data exhibits frequency distributions which are often well described by a Rayleigh distribution, this single parameter distribution is sometimes used by wind turbine manufacturers for calculation of normal operation factors for their machines.

On a global scale, the k factor varies significantly depending upon local weather conditions and the landscape. A low k factor, less than 1.8, is typical for wind climates with a high content of thermal winds. A high k factor, greater than 2.5, is representative for very constant wind climates, for example trade winds. Both Weibull c and k parameters are dependent on the height and are increasing up to 100 m above ground. Above 100 m the k parameter decreases.

Weibull distribution shape and scale factors can be estimated for the available wind speed data by using the methods given in [5]. The range of shape factor k is 1 to 3 while range of scale factor c is 5 to 20. Fig. 1 shows the wind speed probability density function with shape factor of 2 and scale factor of 5, 8 and 10.

The average and variance of wind speed are

$$E(V) = c\Gamma\left(1 + \frac{1}{k}\right) \tag{3}$$

$$\mathrm{var}(V) = c^2\Gamma\left(1 + \frac{2}{k}\right) - c^2\left[\Gamma\left(1 + \frac{1}{k}\right)\right]^2. \tag{4}$$

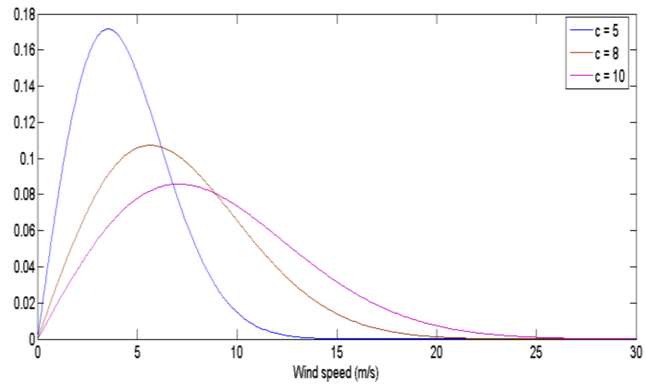

Fig. 1. Wind speed pdf for k =2

IV. Wind Power Probability Distribution

A general model for a wind turbine has been proposed in [6] to show the relation between wind power and wind speed.

$$W = \begin{cases} 0 & (V < v_i \; or \; V \geq v_o) \\ \omega_r & (v_r \leq V < v_o) \\ \dfrac{(V - v_i)\,\omega_r}{v_r - v_i} & (v_i \leq V < v_r) \end{cases} \tag{5}$$

Equation (5) shows that the wind power probability is mixed probability function due to the presence of both discrete and continuous events. The probability of event W = 0 is

$$\begin{aligned} \Pr(W = 0) &= \Pr(V < v_i) + \Pr(V \geq v_o) \\ &= 1 - exp\left[-\left(\frac{v_i}{c}\right)^{k}\right] \\ &\quad + exp\left[-\left(\frac{v_o}{c}\right)^{k}\right]. \end{aligned} \tag{6}$$

The wind power probability for $W = \omega_r$ is

$$\begin{aligned} \Pr(W = \omega_r) &= \Pr(v_r \leq V < v_o) \\ &= exp\left[-\left(\frac{v_r}{c}\right)^{k}\right] \\ &\quad - exp\left[-\left(\frac{v_o}{c}\right)^{k}\right]. \end{aligned} \tag{7}$$

For continuous part, the wind power Weibull probability density function is

$$\begin{aligned} f_W(\omega) = \frac{k\gamma v_i}{\omega_r\, c}\left(\frac{\left(1 + \frac{\gamma\omega}{\omega_r}\right)v_i}{c}\right)^{k-1} \\ \times \exp\left[-\left(\frac{\left(1 + \frac{\gamma\omega}{\omega_r}\right)v_i}{c}\right)^{k}\right] \end{aligned} \tag{8}$$

where $\gamma = (v_r/v_i) - 1$.

The average of wind power is

$$E(W) = \int_{v_i}^{v_r} \frac{(v - v_i)\,\omega_r}{v_r - v_i}\, f_V(v)dv + \int_{v_r}^{v_o} \omega_r\, f_V(v)dv$$

$$= \frac{\omega_r c}{(v_r - v_i)k}\left[\Gamma\left(\frac{1}{k}, \left(\frac{v_i}{c}\right)^k\right) - \Gamma\left(\frac{1}{k}, \left(\frac{v_r}{c}\right)^k\right)\right] - \omega_r\, exp\left[-\left(\frac{v_o}{c}\right)^k\right] \quad (9)$$

TABLE I. WIND PARAMTERS

v_i (m/s)	v_r (m/s)	v_o (m/s)
5	17	25

The cumulative wind power distribution function is represented by (10). Fig 2. Shows the cdf of wind power.

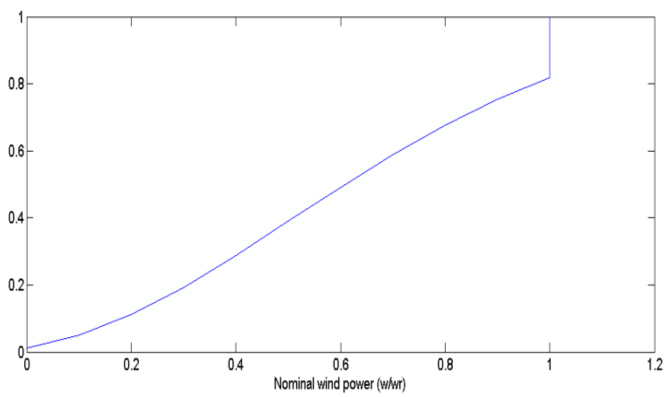

Fig. 2. Cumulative distribution of wind power

V. ELD MODEL WITH PENALTY AND BARRIER FUNCTIONS

Penalty functions have been a part of the constrained optimization for decades. There are two main types of penalty functions. They are exterior penalty functions and barrier functions.

The penalty function formulation introduces an increase in objective function value for infeasible points, the "penalty" added to the minimization problem. So, given a particular solution point, if an inequality constraint is satisfied, the resulting penalty to the objective value is zero. On the other hand, if the inequality constraint is violated, then a positive component multiplied by penalty parameter is added to the objective function [7]. The constrained problem has now been reduced to an unconstrained objective function.

A. *Solution of ELD model with Penalty Function Method*

In ELD Model, the random WP is used as a constraint instead of average WP due to its probabilistic infeasibility. Therefore, the optimal power output of thermal generators depends on random WP.

The ELD_EPF model with wind power as stochastic constraint is

$$\text{minimize} \qquad y = \sum_{i=1}^{n} a_{i0} + a_{i1}x_i + a_{i2}x_i^2 \qquad (11)$$

$$\text{subject to} \qquad W + \sum_{i=1}^{n} x_i = P_d \qquad (12)$$

$$x_{min,i} \le x_i \le x_{max,i} \qquad i = (1,2,\ldots,n). \qquad (13)$$

where W, random variable, represents the total wind power.
The system Power losses, P_{loss}, are neglected in ELD model.

The penalty functions developed to handle inequality constraints are

$$l_{1i}(x) = g_{1i}^2(x)\, h(g_{1i}) \qquad (14)$$

$$l_{2i}(x) = g_{2i}^2(x)\, h(g_{2i}) \qquad (15)$$

where

$$g_{1i}(x) = x_{min,i} - x_i \le 0 \qquad (16)$$

$$g_{2i}(x) = x_i - x_{max,i} \le 0 \qquad (17)$$

and $h(g_i)$ is a Heaviside function defined as

$$h(g_i) = \begin{cases} 0 & if\ g_i(x) \le 0 \\ 1 & if\ g_i(x) > 0 \end{cases}$$

Now, the objective function can be augmented as

$$F_w(\omega) = Pr(W \le \omega) = \begin{cases} 0 & (\omega < 0) \\ 1 - exp\left[-\left(\frac{\left(1+\frac{\gamma\omega}{\omega_r}\right)v_{w,in}}{c}\right)^k\right] + exp\left[-\left(\frac{v_{w,out}}{c}\right)^k\right] & (0 \le \omega \le \omega_r) \\ 1 & (\omega \ge \omega_r) \end{cases} \qquad (10)$$

$$\text{minimize} \quad y_g = \sum_{i=1}^{n}(a_{i0} + a_{i1}x_i + a_{i2}x_i^2) + \sum_{i=1}^{n}[\alpha_{1i}l_{1i}(x) + \alpha_{2i}l_{2i}(x)] \tag{19}$$

where α_{1i} and α_{2i} are penalty parameters. These parameters can be adjusted for solving ELD model using following classic penalty function optimization algorithm [7].

i) Choose a fixed sequence of α_{1i} and α_{2i}.
ii) For each α_{1i} and α_{2i}, find optimum solution of x_i.
iii) Terminate when $l_{1i}(x)$ and $l_{2i}(x)$ are sufficiently small.

TABLE II. THERMAL GENERATORS PARAMTERS

Generator index i	a_{i0}	a_{i1}	a_{i2}	$x_{min,i}$	$x_{max,i}$
1	10	200	100	0.02	0.5
2	10	150	120	0.03	0.7
3	20	180	40	0.05	1.2
4	10	100	60	0.06	1.5
5	20	180	40	0.05	1.2
6	10	150	100	0.03	0.7

TABLE III. OPTIMAL SOLUTIONS OF ELD_EPF MODEL ($\omega_r = 0.05$)

	Minimum (pu)	Average (pu)	Maximum (pu)
$x_{optm,1}$	0.3908	0.3994	0.4030
$x_{optm,2}$	0.5340	0.5412	0.5442
$x_{optm,3}$	1.2	1.2	1.2
$x_{optm,4}$	1.4846	1.4990	1.5
$x_{optm,5}$	1.2	1.2	1.2
$x_{optm,6}$	0.6408	0.6494	0.6530

TABLE IV. OPTIMAL SOLUTIONS OF ELD_EPF MODEL ($\omega_r = 0.2$)

	Minimum (pu)	Average (pu)	Maximum (pu)
$x_{optm,1}$	0.3693	0.3921	0.4030
$x_{optm,2}$	0.5161	0.5351	0.5442
$x_{optm,3}$	1.1732	1.2	1.2
$x_{optm,4}$	1.4488	1.4868	1.5
$x_{optm,5}$	1.1732	1.2	1.2
$x_{optm,6}$	0.6193	0.6420	0.6530

TABLE V. OPTIMAL SOLUTIONS OF ELD_EPF MODEL ($\omega_r = 0.4$)

	Minimum (pu)	Average (pu)	Maximum (pu)
$x_{optm,1}$	0.3482	0.3823	0.4030
$x_{optm,2}$	0.4985	0.5269	0.5442
$x_{optm,3}$	1.1206	1.2	1.2
$x_{optm,4}$	1.4137	1.4704	1.5
$x_{optm,5}$	1.1206	1.2	1.2
$x_{optm,6}$	0.5982	0.6322	0.6530

B. *Solution of ELD model with Interior Point Method*

The barrier functions are used to ensure that a feasible solution never becomes infeasible. In interior point method (barrier function method), a logarithmic function of slack variables multiplied by barrier paramter is augmented into the objective function and these slack variables are introduced into inequality constraints to convert them into equality constaints. Using this approach, the constrained ELD model with inequality constaints can be converted into a model with eqality constraints. The augmented ELD model (ELD_IPM) can be written as

$$\text{minimize} \quad y_g = \sum_{i=1}^{n}(a_{i0} + a_{i1}x_i + a_{i2}x_i^2) - \sum_{i=1}^{n}[\beta_{1i}\ln s_{1i} + \beta_{2i}\ln s_{2i}] \tag{20}$$

$$\text{subject to} \quad W + \sum_{i=1}^{n} x_i = P_d \tag{21}$$

$$x_{min,i} - x_i + s_{1i} = 0 \tag{22}$$

$$x_i - x_{max,i} + s_{2i} = 0 \tag{23}$$

where s_{1i} and s_{2i} are slack variables and they have positive vaules while β_{1i} and β_{2i} are barrier paramters.

The ELD_IPM model can be solved numerically in matlab using interior point algorithm for nonlinear optimization, developed in [8]. This algorithm is implemented on Matlab to solve ELD_IPM model.

TABLE VI. OPTIMAL SOLUTIONS OF ELD_IPM MODEL ($\omega_r = 0.05$)

	Minimum (pu)	Average (pu)	Maximum (pu)
$x_{optm,1}$	0.3907	0.3994	0.4030
$x_{optm,2}$	0.5340	0.5412	0.5441
$x_{optm,3}$	1.2	1.2	1.2
$x_{optm,4}$	1.4846	1.4990	1.5
$x_{optm,5}$	1.2	1.2	1.2
$x_{optm,6}$	0.6408	0.6494	0.6529

TABLE VII. OPTIMAL SOLUTIONS OF ELD_IPM MODEL ($\omega_r = 0.2$)

	Minimum (pu)	Average (pu)	Maximum (pu)
$x_{optm,1}$	0.3693	0.3920	0.4030
$x_{optm,2}$	0.5161	0.5350	0.5441
$x_{optm,3}$	1.1732	1.2	1.2

$x_{optm,4}$	1.4488	1.4867	1.5
$x_{optm,5}$	1.1732	1.2	1.2
$x_{optm,6}$	0.6193	0.6420	0.6529

TABLE VIII. OPTIMAL SOLUTIONS OF ELD_IPM MODEL ($\omega_r = 0.4$)

	Minimum (pu)	Average (pu)	Maximum (pu)
$x_{optm,1}$	0.3482	0.3822	0.4030
$x_{optm,2}$	0.4985	0.5268	0.5441
$x_{optm,3}$	1.1206	1.2	1.2
$x_{optm,4}$	1.4137	1.4704	1.5
$x_{optm,5}$	1.1206	1.2	1.2
$x_{optm,6}$	0.5982	0.6322	0.6529

VI. CASE STUDY

In this paper, both ELD models (ELD_EPF and ELD_IPM models) are applied on a power system consisiitng of six thermal generators and one wind farm. The wind parameters are listed in Table I while thermal parameters of the system are shown in Table II and these parameters are taken from [3]. The wielbull parameters to estimate wind speed are c = 2 and k = 8. The load demand is 5.5 MW. The base MVA of system is taken to be as 100 MVA. All power variables units are represented by p.u. system. The results of ELD_EPF and ELD_IPM models are listed in Tables III to VIII for different rated wind power $\omega_r = 0.05$, 0.2, 0.4 pu. By comparing Tables III to V of ELD_EPF model Tables VI to VIII of ELD_IPM model, both of these models have similar results. Fig. 3, 4 and 5 show the relation of maximum, minimum and average of optimal solution with wind power. The maximum optimal power outputs of thermal genrators represent the case when there is no wind power availability.. The results show that the maximum optimal solution does not depend on ω_r while minimum and average values of optimal soultion decrease with increase in ω_r .

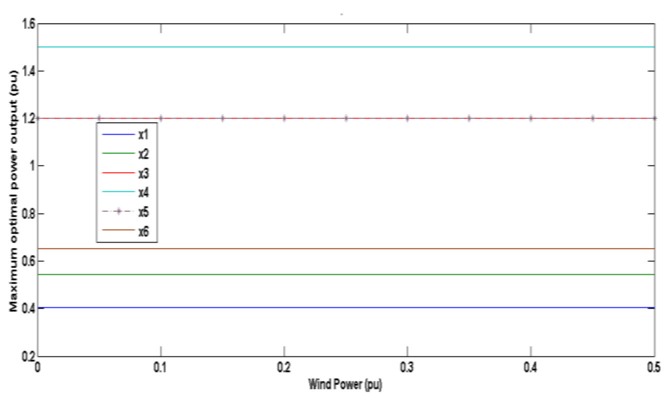

Fig. 3. Variation in maximum optimal solution with wind power

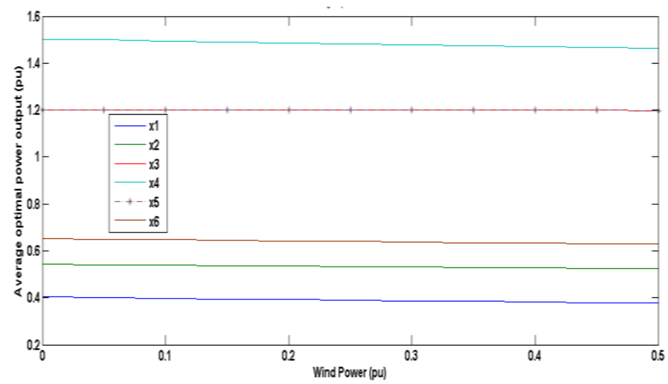

Fig. 4. Variation in average optimal solution with wind power

VII. CONCLUSION

In this paper, The random wind power is considered as a constraint in developing an economic load dispatch model for both wind turbines and thermal generators. A penalty function or barrier function is introduced in the model to take account of inequality constraints and to develop unconstrained problem by augmenting the objective function. Inequality constraint is defined as the restriction on the output power of thermal generators. The feasible ranges of optimum solutions are derived by penalty function and interior point methods.

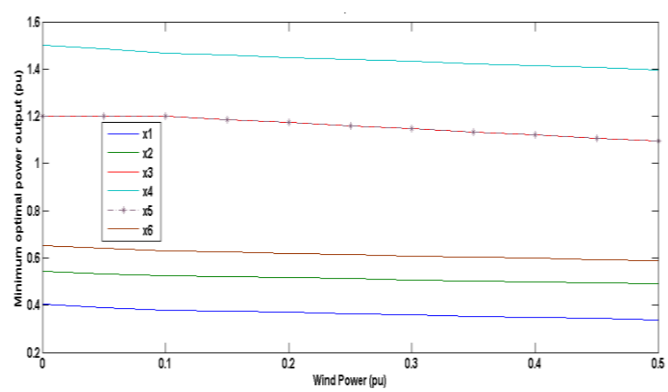

Fig. 5. Variation in minimum optimal solution with wind power

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