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Internal Stability of Disturbance Feedback Control loops

Jan Bendtsen,* member, IEEE, and Fukiko Kawai†

Abstract

This paper deals with the so-called Disturbance Feedback Control concept, which is a technique to improve the disturbance rejection capabilities of existing control loops.

We perform an internal stability analysis of a generic DFC setup and identifies several stability conditions that the DFC law must satisfy in order to guarantee stable closed-loop operation, assuming all subsystem blocks are linear and time invariant. The validity of the stability conditions are illustrated on two examples, a simple scalar example that serves to illustrate instability in case of an open-loop unstable plant model, and a more involved example concerning a DFC design for a gantry crane considered in earlier publications.

1. INTRODUCTION

In many industrial applications, control systems are designed for a nominal plant model in a conservative manner and then subjected to extensive prototype testing. As products mature, the plants will eventually have to be commissioned by field engineers, who are expected to set up and start up the plants without fail under whatever exterior conditions their systems are subjected to [1, 2, 3]. Here, robustness is absolutely essential, whereas performance plays second fiddle. In the long run, however, it is often in the plant owner's interest to tune the control settings to obtain better performance once the baseline operation has been established. Also, hardware may degrade over time, and new software options that exploit new features of the system become available. Thus, it gradually becomes increasingly relevant to re-tune existing, stable control systems as more and more process data becomes available [4]. However, since the continued up-time of the plant is paramount, switching to new controller settings does represent a gamble to the plant owner; switching to new control parameters may yield better performance, but might also incur the risk of an unforeseen shut-down, resulting in significant losses. Hence, being able to quickly revert to the old, perhaps poorer but at least reliable, controller settings naturally provides a sense of safety in actual operation and consequently a greater willingness to attempt updating the controller(s) [5].

Disturbance Feedback Control (DFC) is a relatively little-known technique for improving the disturbance rejection performance of existing control loops pre-stabilized by simple and/or conservative controllers. The basic idea can be summarized as introducing an extra feedback, designed independently of the existing control loop specifically to attenuate external disturbances. It was introduced (and patented) in a basically ad-hoc fashion in the 1980's by the Japanese company Fuji Electric [6] and has been applied with success several times since then. Of particular interest to the present paper, Fuji Electric has developed various anti-sway control schemes for crane systems, including an observer based control scheme. Subsequently, this anti-sway control scheme was further improved to combine feedforward and state feedback control, and has been applied in commercial gantry cranes like the one shown in Fig. 1 [7]. More recently, DFC was added to the anti-sway control loop and tested successfully on a laboratory setup, as documented in [8]. Noticeable disturbance rejection performance improvements were observed, with little to no impact on the reference following capabilities of the existing controller.

However, although the DFC documented in [8] was designed using effective numerical techniques known from robust control, the impact of introducing the extra feedback on the overall stability of the closed-loop system has not been thoroughly analyzed as of yet.

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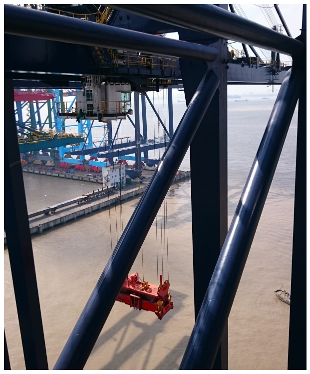

Figure 1. A gantry crane system [9].

The main contribution of the present paper is thus to identify any additional conditions that the DFC law must satisfy in order to guarantee stable closed-loop operation, assuming all subsystem blocks are linear and time invariant. In particular, it is found that the nominal model of the system must be open-loop stable for the scheme to have any chance of guaranteeing internal stability. The validity of the stability conditions are illustrated on two simulation examples, including the anti-sway control system previously discussed in [8].

The outline of the rest of the paper is as follows. Section 2 first recalls the notion of stability that will be employed in the subsequent analysis. Section 3 then describes the DFC design problem, whereupon Section 4 provides the main result of the paper, namely the specification of a set of operators involving the existing and new controllers that must be stable in order to guarantee stability for a given LTI plant. Section 5 then illustrates how the result can be applied to a very simple, 'toy' example and then to the aforementioned overhead crane system; and finally Section 6 contains some concluding remarks.

2. PRELIMINARIES

This section briefly recalls some important definitions and results on internal stability of closed-loop systems, which will be used in the sequel.

Firstly, for some function $f:\ \mathbb{R}_+ \cup \{0\} \to \mathbb{R}^m$, define the extended L_2^m norm as

$$\|f(t)\|_{2T} = \left(\int_0^T |f(t)|^\top |f(t)| dt\right)^{\frac{1}{2}}$$

where $|\cdot|$ denotes element-wise absolute value and $(\cdot)^\top$ denotes transpose. The space L_{2e}^m then denotes the extension of the Lebesgue space of square integrable functions L_2^m consisting of functions that satisfy $\|f(t)\|_{2T} < \infty$ for some fixed T. Needless to say, L_2^m is recovered by letting $T \to \infty$.

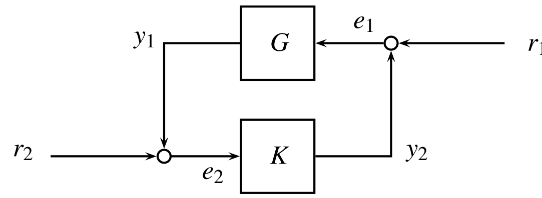

Figure 2. Basic interconnection of dynamic systems.

Next, let the operator $G : L_2^m \to L_2^p$ be a representation of a linear time invariant (LTI) system of the form

$$G: \quad \dot{x}(t) = Ax(t) + Bu(t) \qquad (1)$$
$$y(t) = Cx(t) + Du(t) \qquad (2)$$

where, for any given value of time $t \geq 0$, $x \in \mathbb{R}^n$ is the state of the system at time t, $u \in \mathbb{R}^m$ is the input to the system, $y \in \mathbb{R}^p$ is the measured output, and A, B, C, D are constant real matrices of appropriate dimensions. For convenience, all initial conditions are assumed to be zero unless otherwise stated.

The operator G is said to Bounded Input-Bounded Output stable if, for any input signal u with $\|u\|_\infty = \sup_{t\in\mathbb{R}_+} |u(t)| < \infty$, $y = Gu$ is similarly bounded. For stabilizable and detectable LTI systems, this is equivalent to the existence of a pair of symmetric and positive definite matrices $P = P^\top > 0, Q = Q^\top > 0$ satisfying the Lyapunov equation

$$PA + A^\top P = -Q \qquad (3)$$

which is in turn equivalent to all eigenvalues of A belonging to the open left half of the complex plane.

Finally, let $K : L_2^p \to L_2^m$ be another operator representing a LTI system and consider the interconnection shown in Figure 2.

From this block diagram it is readily seen that

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} (I-KG)^{-1} & (I-KG)^{-1}K \\ (I-GK)^{-1}G & (I-GK)^{-1} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}. \qquad (4)$$

For any $r_1 \in L_2^m, r_2 \in L_2^p$ the interconnection is stable if and only if all four entries in the closed-loop system matrix are stable. In this case, the closed-loop system is said to be internally stable.

3. DISTURBANCE FEEDBACK CONTROL

We now turn to the main subject of the paper.

In keeping with the setting described in the Introduction, we shall first consider systems of the form depicted in the top block diagram in Figure 3, where $K: L_2^p \to L_2^m$ is an existing controller, $G: L_2^m \to L_2^p$ is the controlled plant, and $r \in L_2^p$, $u \in L_2^m$ represent reference and control signals, respectively; $y \in L_2^p$ is the output to be controlled.

It is assumed that the existing controller K has been designed to stabilize G according to some appropriate criteria. However, exogenous disturbances may affect the existing control loop in an adverse manner, causing the intended performance to be deteriorated. Thus, an additional control loop is added to the existing closed-loop system as shown in the bottom block diagram.

Specifically, $\hat{G}: L_2^m \to L_2^p$ is a nominal model of G and $F: L_2^p \to L_2^m$ is a DFC law to be designed. The input signals d and v represent disturbance signals that should be attenuated—they are assumed to be bounded, but not necessarily in L_2—while $\tilde{u} \in L_2^m$ is the command signal from the existing controller. The DFC feeds back an additional control signal generated in response to the difference between the nominal model output, which in principle is unaffected by the disturbance, and the measured output, which is affected by the disturbance.

Note that DFC is different from Internal Model Control; where Internal Model Control has a similar model $\hat{G}$ that predicts the output, the prediction error is used as the only feedback to the controller (which is implemented as a so-called Youla-Kucera parameter, see e.g. [10]). This is clearly not the case in DFC, which maintains the ordinary output feedback, but then augments the input with an additive control signal.

Assume first $v \equiv 0, d \equiv 0$. From the block diagram, it is observed that

$$\begin{aligned} u &= F(y-\hat{y}) + K(r-y) \\ &= (F-K)y - F\hat{G}K(r-y) + Kr \\ &= (F-K+F\hat{G}K)y + (K-F\hat{G}K)r \end{aligned} \quad (5)$$

and

$$\begin{aligned} y &= Gu \\ &= G(F-K+F\hat{G}K)y + G(K+F\hat{G}K)r \\ &= \left(I - G(F-K+F\hat{G}K)\right)^{-1}(G(K-F\hat{G}K)r) \end{aligned} \quad (6)$$

which is similar to, but clearly more complicated than, the top row of (4). This indicates two things:

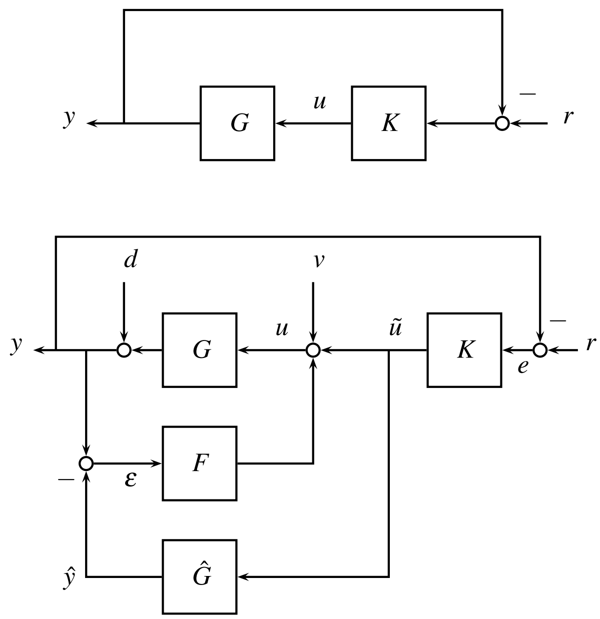

Figure 3. System under consideration; top: existing control system; bottom: with disturbance feedback.

firstly, the standard internal stability criterion (4) cannot be applied directly, and secondly, F cannot be chosen freely.

4. STABILITY ANALYSIS

In this section, we shall derive some necessary conditions on the design of F in order to ensure internal stability of the overall closed-loop system.

For ease of notation, we start by defining the 'auxiliary controller' $\mathscr{K}$ as follows:

$$\mathscr{K} = (I - F\hat{G})K. \quad (7)$$

The requirements for internal stability can now be formulated as follows.

Theorem 1. Consider the controlled system depicted in Figure 3 with bounded reference signal $r \in L_2^p$ and disturbances $v \in L_2^m$, $d \in L_2^p$. The system is internally stable if and only if the following operators are all stable:

$$e = (I - G(F-\mathscr{K}))^{-1}((I-GF)r - Gv - d) \quad (8)$$

$$\varepsilon = (I+\hat{G}K)(I - G(F-\mathscr{K}))^{-1}(G\mathscr{K}r + Gv + d) - \hat{G}Kr \quad (9)$$

$$u = (I - (F-\mathscr{K})G)^{-1}(\mathscr{K}r + v + (F-\mathscr{K})d). \quad (10)$$

Proof. With the notation introduced in (7) and for non-zero v, one has

$$u = (F-\mathscr{K})y + \mathscr{K}r + v$$

which with $y = Gu + d$ yields

$$u = (I - (F - \mathscr{K})G)^{-1} ((F - \mathscr{K})Gd + \mathscr{K}r + v)$$

i.e., (10).

Next, the control error is computed as

$$\begin{aligned} e &= r - y \\ &= r - GFy - G\mathscr{K}e - Gv - d \\ &= r - GF(r - e) - G\mathscr{K}e - Gv - d \\ &= (I - GF)r + G(F - \mathscr{K})e - Gv - d \\ &= (I - G(F - \mathscr{K}))^{-1} ((I - GF)r - Gv - d) \end{aligned}$$

which is (8); and finally, (9) is found as

$$\begin{aligned} \varepsilon &= y - \hat{y} \\ &= (I + \hat{G}K)y - \hat{G}Kr \\ &= (I + \hat{G}K) \left(I - G(F - K + F\hat{G}K)\right)^{-1} \times \\ &\quad (G\mathscr{K}r + Gv + d) - \hat{G}Kr \end{aligned}$$

using (6). ■

The requirements (8)–(10) are rather more complicated than one might initially expect, and there seems to be little hope of finding significant simplifications. However, the presence of the term $-\hat{G}Kr$ in (9), which is not multiplied by any inverse factors, is worthy of note. This immediately shows that $\hat{G}$ must be open-loop stable in order for any DFC scheme to remain stable, as it is not stabilized by K (unlike G itself).

5. NUMERICAL EXAMPLES

In this section we investigate the implications of Theorem 1 on two simple examples.

5.1. First-order system

To begin with, consider the following simple, scalar, first order system

$$\begin{aligned} G: \quad & \dot{x}(t) = -0.7x(t) + 0.1u(t), \quad x(0) = 0.2 \\ & y(t) = x(t) + d(t) \end{aligned}$$

where $d(t)$ is a disturbance signal. This system is stabilized by the static feedback

$$K: \quad \tilde{u}(t) = 10(r(t) - y(t)).$$

The output is predicted by the nominal model, which is essentially a copy of the actual system, but with an initial error in the state estimate:

$$\begin{aligned} \hat{G}: \quad & \dot{\hat{x}}(t) = -0.7\hat{x}(t) + 0.1\tilde{u}(t), \quad \hat{x}(0) = 0 \\ & \hat{y}(t) = \hat{x}(t) \end{aligned}$$

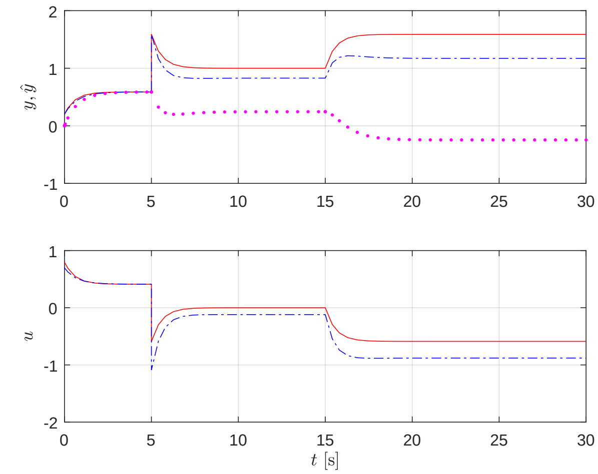

Figure 4. Simulation of the simple scalar example, stable case. Top plot: Red full line: system output y without DFC; blue dash-dotted line: system output y with DFC; magenta dotted line: model output $\hat{y}$ with DFC. Bottom plot: control signals

Finally, the DFC law is chosen as a simple negative feedback gain:

$$F: \quad u_F(t) = -5(y(t) - \hat{y}(t)).$$

Evaluating the operators specified in Theorem 1 yields double poles in $s = -1.7$ and $s = -1.2$, indicating that the system with DFC is indeed internally stable. This conclusion is confirmed by the simulation shown in Figure 4, where r, d and v are chosen as unit steps occurring at $t = 0$, $t = 5$ and $t = 15$, respectively. Notice how the model output of $\hat{G}$ converges to the output of G and the DFC contribution converges to 0 in the beginning. However, as the disturbances occur, the estimated and measured outputs deviate from each other, causing the DFC to correct the input signal and provide improved disturbance rejection.

If, on the other hand, the system is open-loop unstable, the DFC law in general cannot achieve internal stability. Take, for example, the test system

$$\begin{aligned} G: \quad & \dot{x}(t) = 0.7x(t) + 0.1u(t), \quad x(0) = 0.2 \\ & y(t) = x(t) + d(t) \end{aligned}$$

which can still be stabilized by the ordinary controller K above. In this case, however, evaluating the operators specified in Theorem 1 yields poles in $s = -0.3, s = 0.2$ and $s = 0.7$, which clearly implies instability—as also seen in Figure 5.

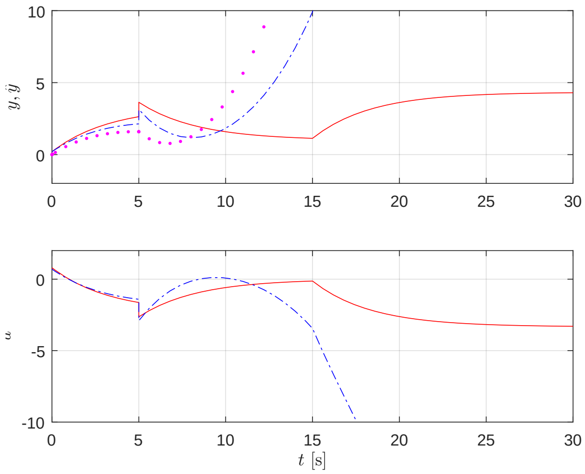

Figure 5. Simulation of the simple scalar example, unstable case. Top plot: Red full line: system output y without DFC; blue dash-dotted line: system output y with DFC; magenta dotted line: model output $\hat{y}$ with DFC. Bottom plot: control signals

5.2. Gantry crane

The crane system model considered in [8] was originally taken from [11]. It is described by the nonlinear coupled differential equations

$$(m_T + m_L)\ddot{\xi} + m_L l \ddot{\theta}\cos\theta - m_L l \dot{\theta}^2 \sin\theta = \gamma(-\dot{\xi} + u)$$
$$\ddot{\xi}\cos\theta + l\ddot{\theta} + g\sin\theta = 0$$

where m_T and m_L are masses of trolley and load, ξ is the trolley position along the supporting rail, θ is the angle of the load from vertical, l is the length of the suspension rope, g is the gravitational acceleration and γ is a constant gain translating command input u into force applied to the trolley via the drive train; see also Fig. 6.

Small-angle approximations yield the LTI description

$$\begin{aligned}\dot{\chi} &= A\chi + Bu \\ &= \begin{bmatrix} \frac{-\gamma}{m_T} & 0 & 0 & \frac{g}{l}\left(\frac{m_L}{m_T}\right) \\ \frac{\gamma}{m_T} & 0 & 0 & \frac{-g}{l}\left(\frac{m_L}{m_T}+1\right) \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\chi + \begin{bmatrix} \frac{\gamma}{m_T} \\ \frac{-\gamma}{m_T} \\ 0 \\ 0 \end{bmatrix} u \\ &= \begin{bmatrix} -6.25 & 0 & 0 & 1.960 \\ 6.25 & 0 & 0 & -2.352 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\chi + \begin{bmatrix} 6.25 \\ -6.25 \\ 0 \\ 0 \end{bmatrix} u\end{aligned}$$

where the state vector is defined as $\chi = [\dot{\xi},\, \dot{\zeta},\, \xi,\, \zeta]^\top$.

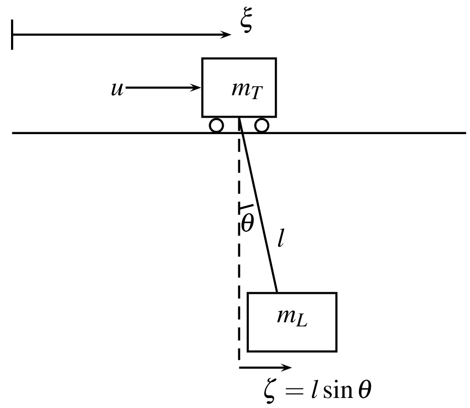

Figure 6. Free body diagram of gantry crane model.

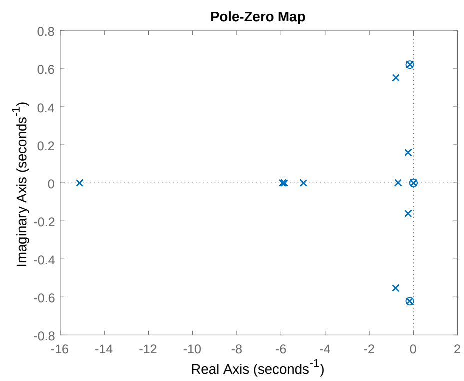

Figure 7. Poles ($\times$) and zeros ($\circ$) of the operators specified in Theorem 1.

The state feedback controller K was designed by standard pole placement:

$$K = \begin{bmatrix} 4.1603 & 3.8579 & 1.5625 & -0.5320 \end{bmatrix}$$

while the robust DFC gain was obtained as

$$F = \begin{bmatrix} 14.2388 & 11.9464 & 2.5123 & -10.7141 \end{bmatrix}$$

using Linear Matrix Inequality-based methods as described in [8].

The operators specified in (8)–(10) were evaluated using Matlab, and their poles and zeros are shown in Figure 7. As can be seen, all poles are in the open left half of the complex plane (barely), indicating that the system is internally stable. This fortunately matches with our previous results; examples of disturbance rejection with three different rope lengths are shown in Figure 8.

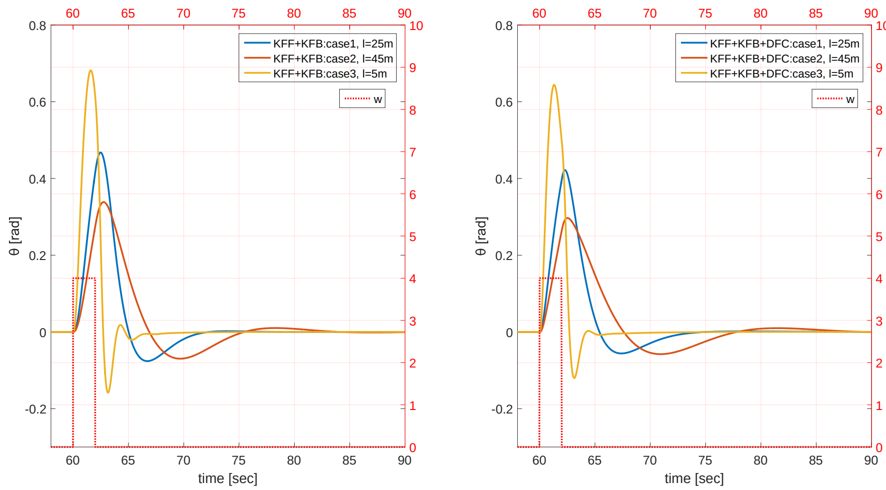

Figure 8. Response to disturbance without (left) and with (right) DFC for different rope lengths [8].

6. CONCLUSION

This paper performed an internal stability analysis for the so-called Disturbance Feedback Control concept and identified several stability conditions that the DFC law must satisfy in order to guarantee stable closed-loop operation, assuming all subsystem blocks are linear and time invariant. In particular, it was found that the nominal model of the system must be open-loop stable for the scheme to have any chance of guaranteeing internal stability in the face of exogenous disturbances. The validity of the stability conditions was illustrated on two examples, a simple scalar example that served to illustrate instability in case of an open-loop unstable plant model, and a more involved example concerning a DFC for a gantry crane.

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