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# Vector Fitting-Based Reduced Order Modelling Method for Power Cables

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**Keywords:** Balanced truncation, power cable, Prony analysis, reduced order model, vector fitting.

## Abstract

To study the frequency and damping characteristics of power cable, high-order RLC circuit models are commonly required, which involves high computational burdens for modelling and analysis. In this paper, a reduced-order modelling method for power cable is proposed on the basis of Prony analysis (PA), vector fitting (VF) algorithm and balanced truncation (BT) algorithm. The state space model of power cable is first obtained by fitting terminal frequency responses using PA and VF instead of directly building mathematical model; Then the low-frequency characteristics is maintained using BT algorithm. The fitting accuracy can be improved by increasing the order. The proposed method is able to reduce computational burdens for power cable modelling, which thus improves modelling and analysis efficiency. Simulation results are given to validate the effectiveness of the proposed modelling method.

## 1 Introduction

Modelling of power cable is one of important aspects for electromagnetic transient analysis in power systems. To study the frequency and damping characteristics of power cable, high-order RLC circuit models are commonly required, which involves high computational burdens for modelling and analysis [2]. Therefore, its important to establish a computationally-efficient modelling method for efficiently investigating the frequency and damping characteristics of power cable.

Vector fitting (VF) algorithm was first proposed in [1], and it has been widely used to fit state space models of frequency dependent network equivalents (FDNEs) used in power system electro-magnetic transient (EMT) simulations [2–5]. In principal, VF is a black-box algorithm, and it can generate a state space model/ transfer function based on the terminal frequency responses and the desired order. Therefore, conventional VF has to be implemented multiple times until the optimal order is found [1, 4]. It's obvious that the optimal order determination process is time-consuming, especially if the FDNE is complex, and the system order is high. A Prony analysis (PA) based fast order identification method is

proposed in [6], where the number of resonance peaks provides initial assumption about the minimum order. Compared with the trial-and-error method, the improved method only applies VF twice. However, this method relies on the resonance peaks number, and no further order reduction is performed. This paper proposes a further convenient and efficient method. Prony analysis (PA) is first used to determine the minimum system order based on the terminal frequency responses, and VF is used to generate a state space model/transfer function, where no resonance peaks information is needed, and VF is only used once.

In addition, the order of the fitted state space model/ transfer function of power cable can be very high, since the parasitic capacitance exists along the power cable [7, 8, 12]. There are numerous classical model order reduction (MOR) methods for linear time invariant (LTI) systems [6, 7, 10, 11, 13–17], among which the singular value decomposition (SVD)-based approximation method has been gained much population [11, 14]. SVD-based MOR methods mainly consist of proper orthogonal decomposition (POD) technique, balanced truncation (BT) approach. POD technique can obtain a basis from a series of responses, if the inputs and the initial conditions of the system are determined. However, the obtained reduced order model is not the same once the inputs and initial conditions change. BT approach first uses a transformation matrix to transform the original state space model to an internally balanced system, which means the controllability Gramian and the observability Gramian are the same and are diagonal [17]. The elements of the balanced diagonal matrix are the Hankel singular values (HSVs) representing the importance of the state variables on the input-output behavior [15]. The significantly smaller states can be directly eliminated, and the dynamic performance of the reduced order model is almost the same as the original system. In this paper, BT is used to reduce the order of the fitted state space model/transfer function.

This paper proposes a reduced-order modelling method for power cable, where PA, VF and BT are used sequentially. PA first determines the minimum order directly based on the terminal frequency responses. Then, VF fits a state space model for the terminal frequency responses using the minimum order. BT is lastly used to reduce the model of the fitted state space model by maintaining the low-frequency

range characteristics.

## 2 Terminal impedance characteristics of power cable

The cross section of the power cable containing an inner copper conductor with an outer screen conductor, using cross-linked polyethylene (XLPE) insulator material is shown in Fig. 1. In practical, the screen conductor layer of the power cable is often grounded at the sending end and the receiving end, and additional points along the power cable make the screen conductor layers at ground potential. Therefore, the screen conductor voltage is much smaller than phase conductor voltage. The power cable can be modelled as the single  $\Pi$  circuit shown as in Fig. 2. The series impedance  $Z_s$  and parallel conductance  $Y_p$  can be expressed as,

$$Z_s = z(\omega)L \frac{\sinh(\gamma(\omega)L)}{\gamma(\omega)L} \quad (1)$$

$$Y_p = y(\omega)L \frac{\tanh(\gamma(\omega)L/2)}{\gamma(\omega)L/2} \quad (2)$$

where  $z(\omega)$  and  $y(\omega)$  are the per unit length (p.u.l.) power cable impedance and conductance, respectively, and  $L$  is the cable length. Further,

$$z(\omega) = r(\omega) + j\omega l(\omega) \quad (3)$$

$$y(\omega) = g(\omega) + j\omega c(\omega) \quad (4)$$

$$\gamma(\omega) = \sqrt{z(\omega)y(\omega)} \quad (5)$$

where  $r(\omega)$ ,  $l(\omega)$ ,  $g(\omega)$ ,  $c(\omega)$  are the p.u.l. power cable resistance, inductance, conductance and capacitance, respectively, and  $\gamma(\omega)$  is the propagation constant.

The parameters of the power cable are listed in Table 1. The matlab toolbox *power\_cableparam* is used to calculate the frequency dependent p.u.l. parameters [18]. The calculated p.u.l. resistance and inductance are shown in Fig. 3, and the calculated p.u.l. capacitance is  $0.27331 \mu F/km$ . These p.u.l. parameters are then used to calculate the power cable series impedance  $Z_s$  using (1), where cable length  $100km$  is used. The Bode diagram of the derived cable impedance with one terminal short-circuit is shown as the blue line in Fig. 6. It can be seen from Fig. 6 that the terminal impedance of the power cable has multiple resonance peaks, among which the peak magnitudes decrease when frequency increases due to the inherent damping characteristics. It should be noted that the terminal frequency response of the power cable is obtained directly using (1) and (2) instead of field measurement which may bring measurement noise, since this paper focuses on the proposed MOR algorithm. To show how well the model order reduction algorithm can identify and maintain the dominant resonance peaks, the nine resonance peaks are listed here: 100Hz, 309Hz, 523Hz, 739Hz, 957Hz, 1177Hz, 1398Hz, 1621Hz and 1844Hz.

Table 1: The parameters of the power cable.

Components	Parameters	Values
Phase conductor	Strands number	58
	Strand diameter	2.7 mm
	Resistivity	$1.78e-8 \Omega/m$
	Relative permeability	1
	External diameter	20.9 mm
Screen Conductor	Resistivity	$1.78e-8 \Omega/m$
	Total Section	$0.000169 m^2$
	Internal Diameter	65.8 mm
	External Diameter	69.8 mm
Phase-Screen Insulator	Relative Permeability	2.3
	Internal Diameter	23.3 mm
	External Diameter	60.6 mm
Outer Screen Insulator	Relative Permeability	2.25
	Internal Diameter	69.8 mm
	External Diameter	77.8 mm
	Ground Resistivity	$100 \Omega/m$

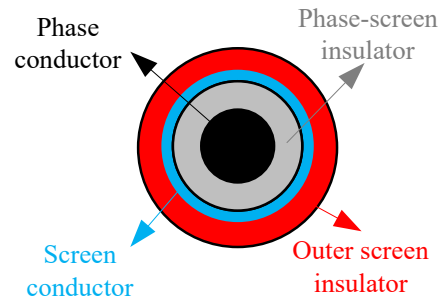


Figure 1: The cross section of the power cable with two conducting layers.

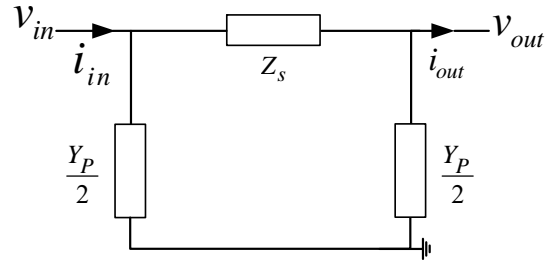


Figure 2: Single  $\Pi$  circuit model of the power cable.

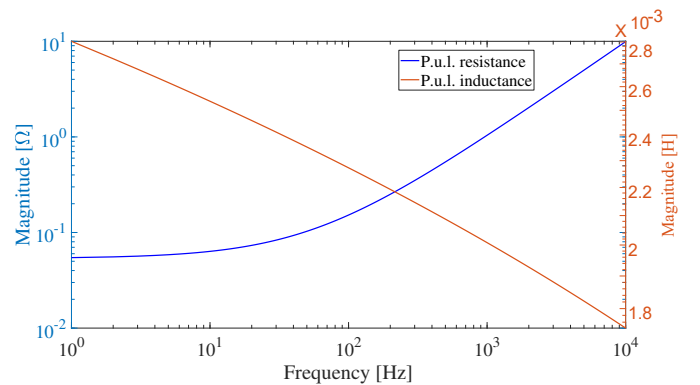


Figure 3: P.u.l. resistance and inductance of the power cable.

### 3 Implementation of the proposed MOR approach

2000 sample frequencies are evenly spaced in the frequency range 1-2000Hz and will be used in the following demonstration. To show the effectiveness of the proposed MOR algorithm, the simulation results using the conventional trial-and-error approach, the proposed MOR approach and the method proposed in [6] are presented in the rest of this section.

#### 3.1 High order determination stage

To show that the proposed high order determination approach can obtain the system order much faster without loss of accuracy, the conventional trial-and-error approach is first performed, followed by the proposed method.

##### 3.1.1 Conventional trial-and-error approach

The conventional trial-and-error approach works by increasing the order step by step until the error requirement is met. The flowchart of the approach is shown in Fig. 4, where the error between the fitted  $N$ -order state space model/transfer function and the original frequency response is defined in VF algorithm, shown as follows,

$$Err_N = \sqrt{\sum_{k=1}^P (fit_N(k) - f(k))^2 / P} \quad (6)$$

where  $P$  is the number of sample frequencies,  $fit_N(k)$  and  $f(k)$  are the  $k$ th fitted value in the  $N$ -order fitted model and the  $k$ th original sample frequency response, respectively.

The error curve for different-order fitted models using VF is shown in Fig. 5. It shows that the fitted error roughly decreases as the order increases, excluding some eccentric orders 52, 53, 56, 57, etc. In addition, the Bode diagrams of the frequency responses of the original model and the fitted 26-order model are shown in Fig. 6. It can be seen from both Fig. 5 and 6 that the error of the fitted 26-order model is small enough. However, the conventional trial-and-error method has to iterate 25 times until the error limitation is met.

##### 3.1.2 Proposed high order determination approach

Conjugate symmetry forms of the 2000 frequency responses are first added to the frequency series. Then, PA extracts out the error-bonded model order from the corresponding time-domain series which are derived by applying IFFT algorithm on the frequency series. It should be noted that PA is based on SVD, a threshold value  $\Delta$  near to 1 for truncating the less dominant state variables is needed, shown as follows,

$$(\sigma_1^2 + \sigma_2^2 + \dots + \sigma_k^2) / (\sigma_{k+1}^2 + \sigma_{k+2}^2 + \dots + \sigma_s^2) \gg \Delta^2 \quad (7)$$

where  $k$  is the determined order. Using (7),  $k$  is 20, 23, 26, 30, 32 when  $\Delta$  is 0.9999, 0.999999, 0.99999999, 0.9999999999, 0.999999999999, respectively. It means that the model order

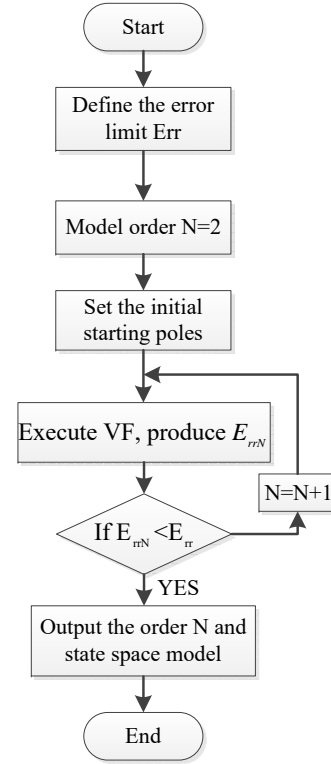


Figure 4: The flowchart of the conventional trial-and-error approach.

can be determined directly based on the given error limitation  $\Delta$ , and the fitted order increases if  $\Delta$  increases. The frequency response of the fitted 32-order model using VF is also shown in Fig. 6.

To quantitatively show that the proposed high order determination approach is faster than the conventional trial-and-error algorithm, the running time of the latter algorithm is shown in Fig. 7. It can be seen that the running time is proportional with the desired order. On the other hand, the running times of the proposed algorithm are only 8.08s, 8.40s, 8.52s, 7.91s, 8.22s for the above mentioned different  $\Delta$  values (Inter(R) Core(TM) i7-7600U processor, 8GB RAM and 64-bit operating system), which means that the running time of the proposed algorithm is almost the same regardless of the desired accuracy.

#### 3.2 MOR stage

The reduced-order model is obtained by applying the BT algorithm to the high-order model which is determined in the high order determination stage. The dominant state variables are preserved, and the less dominant state variables are eliminated.

BT algorithm consists of two main steps which are balance step and truncation step. The two steps can be implemented using *balreal* command and *modred* command in Matlab, respectively. The detailed implementation procedure of MOR stage is shown in Algorithm 1. The fitted 32-order model in

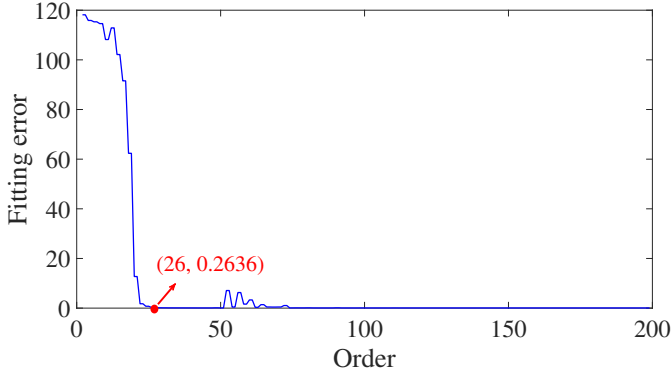


Figure 5: The error curve for different-order fitted models using VF algorithm.

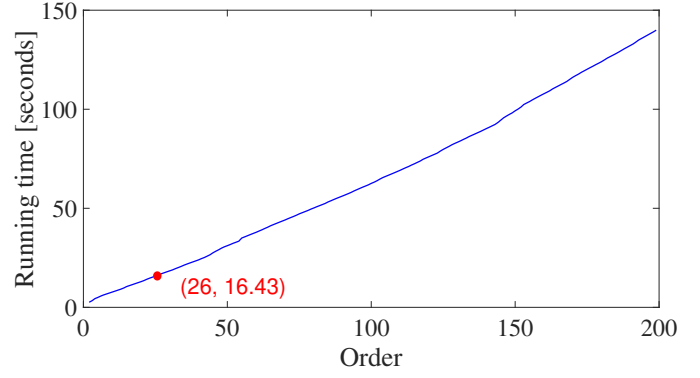


Figure 7: Running time of the conventional trial-and-error algorithm.

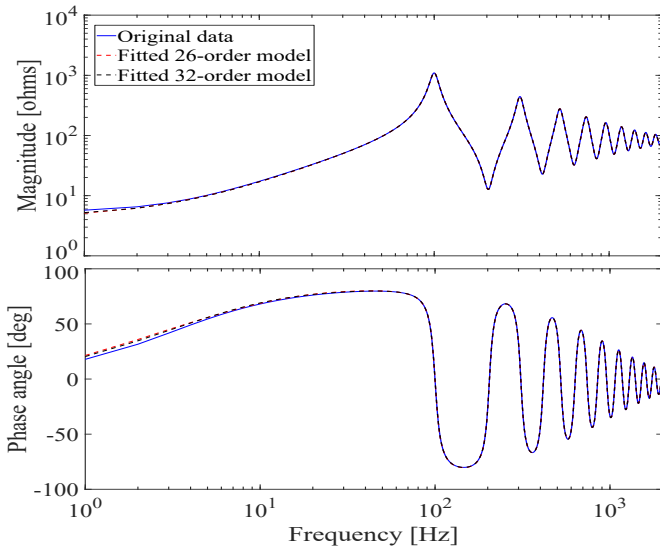


Figure 6: Bode diagrams of the terminal frequency responses of the original, fitted 26-order and fitted 32-order models.

the high order determination stage is reduced here using BT. Different reduced-order models can be obtained by setting different threshold value  $\Delta$ , shown as line 9 in Algorithm 1. The Bode diagrams of the original model, the reduced 6-order, 12-order and 18-order models are shown in Fig. 8. It can be seen that the reduced 6-order, 12-order and 18-order models can capture the first 2, 5, 7 resonance peaks, respectively. The fitting ability for the high-frequency range increases if the order of the reduced model increases.

For comparison, a 12-order model fitted directly by VF algorithm is shown in Fig. 9(a). It can be seen that the error between the Bode diagrams of the original frequency responses and the 12-order model is much larger than that in Fig. 8(b). In addition, since the lowest system order to recovering all of the resonance peaks is double of the number of resonance peaks, the state space model is first established using VF with order 18 (The number of the resonance peaks is 9). Then BT is used to reduce the order from 18 to 12. The Bode diagrams of the 18-order and the 12-order models are shown in Fig. 9(b) and

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**Algorithm 1:** Implementation procedure of BT

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**Result:** reduced-order state space model *sysr*

```

1 sys=ss(A, B, C, D);
2 [sysb, g]=balreal(sys);
3 set elim=[];
4 set  $S = S_a = S_b = i = j = 0$ ;
5 while  $i < size(g)$  do
6   |  $S = S + g_i^2$ ;
7   |  $i ++$ ;
8 end
9 while  $S_a / (S - S_a) < \Delta$  do
10  |  $S_a = S_a + g_j^2$ ;
11  |  $j ++$ ;
12 end
13 while  $j < order(A)$  do
14  | elim=[elim, j];
15  |  $j ++$ ;
16 end
17 sysr=modred(sysb, elim);

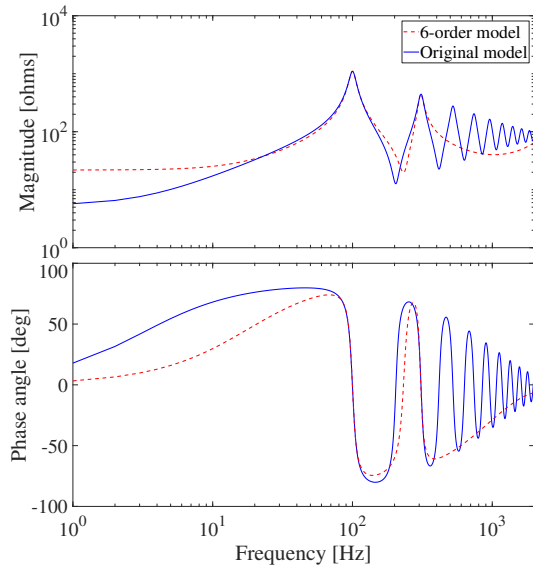
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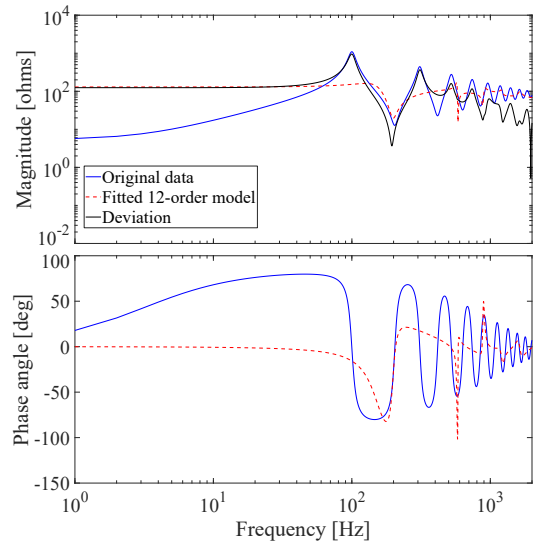
Fig. 9(c), respectively. Comparing Fig. 9(b) with Fig. 8(c), Fig. 9(c) with Fig. 8(b), it can be seen that, for the same order, the proposed algorithm which obtains a reduced-order model based on the high-order model can maintain the original frequency responses much better than the conventional algorithm which uses two times resonance peaks as the model order. It also shows that, if a low-order model is desired, a higher-order model is necessary to be established first to reproduce the low-frequency range characteristics.

### 3.3 Time-Domain Dynamic Performance Comparisons

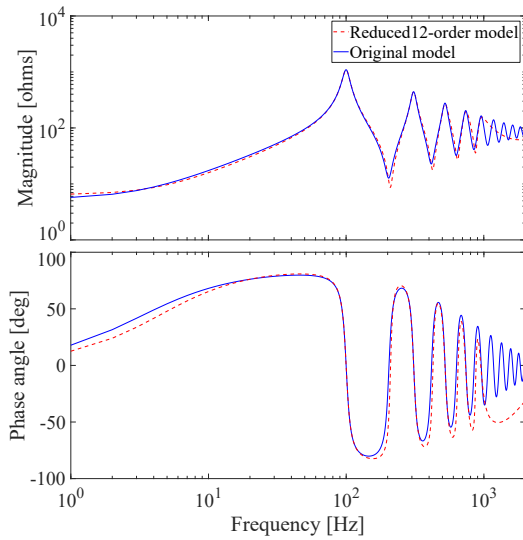
To further analyze and compare the approximation ability of different reduced-order models, the time-domain dynamic performances of the different reduced-order models are compared. It can be seen from Fig. 10(a) and (b) that the step responses of the 32-order transfer function approximates the actual situation well. In addition, the step responses of the 6 order and the 12 order transfer functions which are reduced from the 32



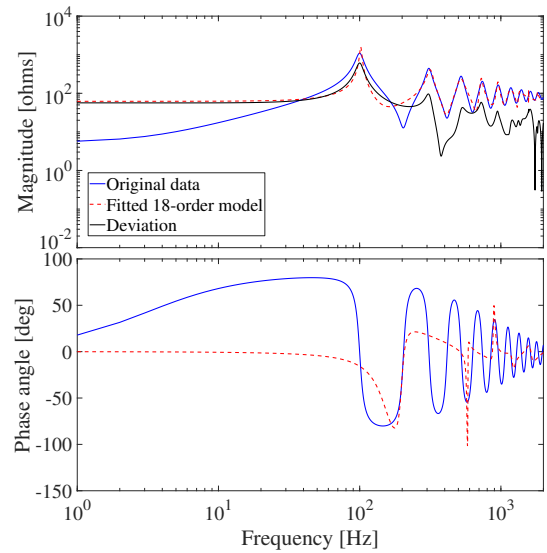
(a)



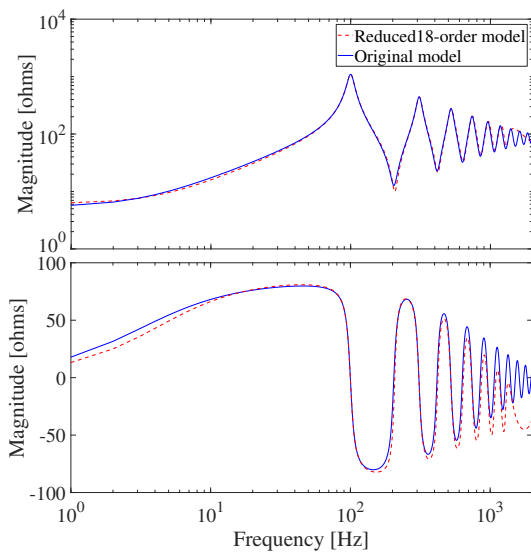
(a)



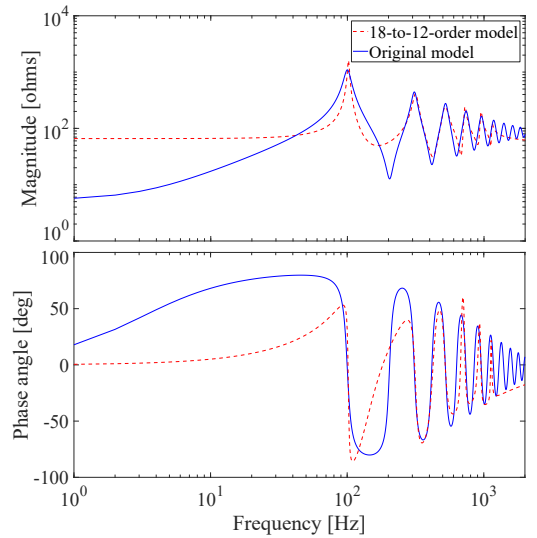
(b)



(b)



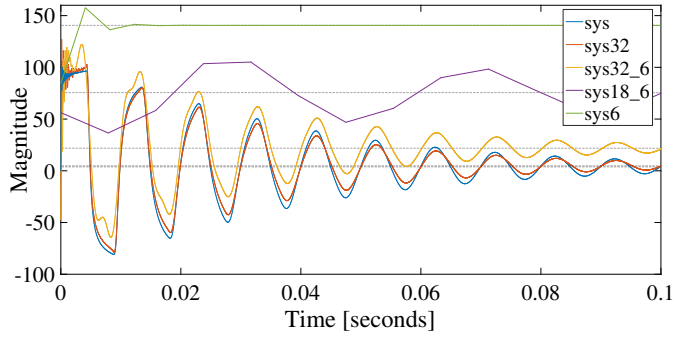
(c)



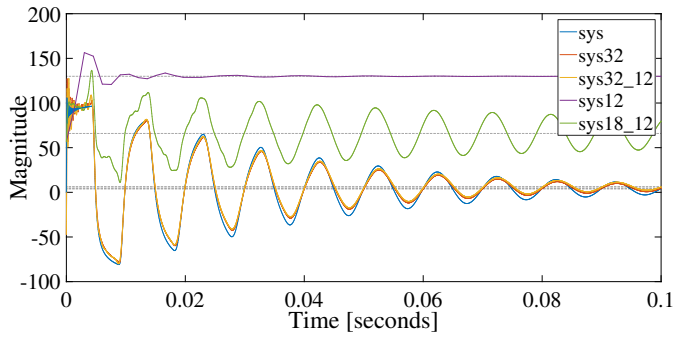
(c)

Figure 8. The Bode diagrams of different reduced-order models. (a) Reduced 6-order model; (b) Reduced 12-order model; (c) 5

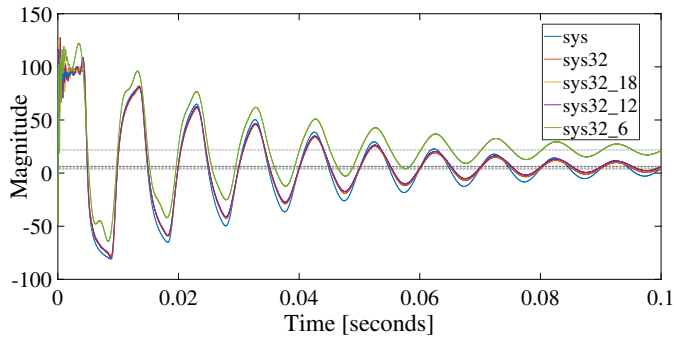
Figure 9. The Bode diagrams of different-order models. (a) 12-order model directly fitted by VF; (b) 18-order model directly fitted by VF; (c) 12-order model reduced from 18-order model.



(a)



(b)



(c)

Figure 10. Step responses of different reduced-order models. (a) 6-order models; (b) 12-order models; (c) 18-order models.

order transfer function approximate the actual situation better than the 6 order and 12 order transfer functions which are generated directly using VF. The simulation results agree with the frequency-domain simulation results. Fig. 10(c) shows that the time-domain approximation ability of the reduced-order model increases as the order increases. However, the model will become more complicated.

## 4 Conclusions

The proposed reduced order modelling method only uses the terminal frequency responses, and avoids deriving the frequency-dependent cable impedance from physical parameters, simplifying the modelling process significantly. Besides, the proposed algorithm can obtain the error-banded high order using PA instead of conventional trial-and-error method, which is much more time-saving. In addition, the reduced-

order model has almost the same dynamic performance as the original model, achieving the trade off between accuracy and simplification. Simulation results of power cable verify the effectiveness of the proposed algorithm.

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