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Multi-scenario Model Predictive Control of Combined Sewer Overflows in Urban Drainage Networks

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***Abstract*— Urban drainage networks (UDN) are among the most vital infrastructures within the natural water cycle. The most widely applied Real Time Control (RTC) on these systems is Model Predictive Control (MPC), which typically incorporates transport time delays and the effect of disturbances explicitly in the objectives and constraints. One of the greatest challenges in the control of UDNs is to formulate multiple control criteria regarding operational requirements of the network. Furthermore, MPC faces the challenge of handling uncertainty caused by disturbances, e.g. weather predictions.**

One way to incorporate the uncertainty in the decision making is to consider multiple scenarios, i.e. to generate different ensembles based on rain forecasts. To this end, we propose a Multi-scenario MPC (MS-MPC) approach, that deals with uncertainty in the expected inflow. First, a generic multi-objective MPC is established which deals with the time delays explicitly in the optimization. Then, this framework is extended to our formulation of the multiple scenario problem. The algorithm is verified through a case study by interfacing a high-fidelity simulator model of a sewer network as virtual reality.

I. INTRODUCTION

Combined sewers carry domestic wastewater and rain runoff towards treatment plants, where the sewage is treated before it is discharged to the environment [1]. Real-Time Control (RTC) of these networks is a challenging task since the system is characterized by large-scale dimensions, non-linear dynamics, and significant time-delays. Besides, UDNs are increasingly being pushed to their capacity limits due to changing weather conditions, resulting in increased amounts and more frequent Combined Sewer Overflows (CSO).

Constrained optimal control has been done in several works, mainly considering MPC. Due to the complexity and the large-scale nature of drainage networks, typically conceptualized control models are used, considering the available network volumes [2]. In [3] and [1], the volumetric storage of pipes, manholes, and retention tanks have been collectively modeled, while in [4] and [5], simplified hydraulic models were proposed, considering gravity-driven sewage pipes as simple delay elements without storage. In [6] and [7], the overflows have been conceptualized by introducing an artificial variable, indicating the average overflow over a specific horizon. Extending these previous frameworks, [8] and [9] used an indicator variable which was forecasting overflow only in case of an actual tank overflow. In [10], this previously-established, fast-solvable optimisation model has been successfully utilized in a simulation study, representing a real large-scale drainage network in Denmark.

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The control problem in UDNs consists of multiple criteria. For instance, [7] investigated various ways of weighting control objectives with regard to different rain conditions. The study in [11] proposed a systematic control design and focused on the multi-objective control performance regarding the choice of optimisation variables and the formulation of the objective function.

Nonetheless, the majority of the research reporting on MPC of UDNs investigates the performance by considering historical disturbances, i.e. historical rain data. Works on model-based optimization, taking into account the uncertainties, are relatively few. Runoff forecast uncertainties in risk-based optimization have been considered by using stochastic grey-box models in [12] and in [13]. In [14], an optimization framework has been introduced, which considered the estimated uncertainty of rain runoff forecasts, thereby estimating the risk of overflows based on the stored volumes in the system. This framework has used an optimization strategy with a simplified model, while the transport times have not been considered between pumping stations.

Another way to consider stochastic hydrological processes in optimization is to assume possible scenarios, estimate their likelihood and test the optimization under these assumptions. A flow control problem has been studied in [15], where a Multi-Scenario (MS-MPC) approach has been implemented on a simulation model of a dutch canal system [16]. In [17], a chance-constrained, tree-based and multi-scenario stochastic MPC approaches have been compared and applied to drinking water networks.

In the present paper, an MS-MPC approach is applied to a high-fidelity simulator model of a UDN, considering a simplified representation of the network. In contrast to [13], we implement a fast-solvable MPC strategy that considers the network delays in terms of the transport flows between pumping stations. Furthermore, we extend the work in [3] by evaluating the performance of MS-MPC, considering multiple operational and control objectives. We combine the results of [11], where the operational objectives and the tuning of the optimization parameters have been analyzed.

The remainder of the paper is structured as follows: In Section II, the preliminary introduction of UDNs and the simulation network are presented. Section III. reviews the simplified network models, whereupon Section IV introduces the generic MPC and the proposed MS-MPC control approaches. In Section V. numerical results and the applied scenarios are presented. Finally, Section VI. provides conclusions and sums up the contributions of the work.

a) *Nomenclature:* Throughout the paper, all quantities

mentioned are real. Boldface letters are used for sets, such as $\mathbf{s} = \{s_1, ...s_n\}$ as well as for vectors $\mathbf{x} = [x_1, ...x_n]^T \in \mathbb{R}^n$. Time dependent variables are denoted by $x(t)$ or $x(t_k)$, where $t \in \mathbb{R}^+$ and $t_k \in \mathbb{Z}^+$ are the continuous and discrete time variables, respectively.

II. Drainage Systems

Urban drainage systems typically consist of storage elements such as gravity pipes, retention tanks, catchment areas and one or several outlet points leading to the treatment plants. The most common actuators in these networks are pumps and gates. In the present work, networks with multiple retention tanks are considered, where the stored sewage volumes are controlled by pumps. Hence, the regulated variable is flow, provided by local, variable-speed pumps.

In order to make closed-loop control, a high-fidelity model is used in the MIKE URBAN[1] (MU) simulation environment. The network model is shown in Fig. 1.

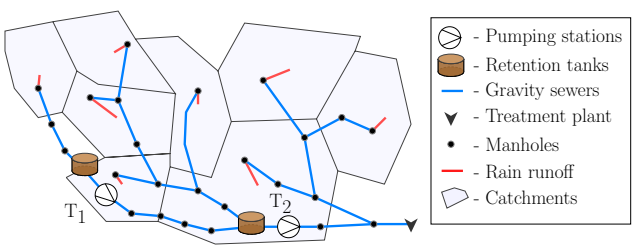

Fig. 1. Schematics of the high-fidelity simulator in MIKE URBAN.

The network consists of two pumping stations, equipped with retention tanks with a total storage capacity of about $30[m^3]$. The pumps are operated by local PID controllers. There is one outlet point representing the treatment plant and several catchment areas, where rainfall runs off and enters the system through manholes. The disturbances considered here are domestic sewage and rain infiltration. In the network, rainfall run-off flow enters the network through eight inlet points, distributed over the entire network.

III. Network Model

A. Gravity sewers

Gravity-driven flow in sewage pipes can be computed accurately by the well-known Saint-Venant partial differential equations [1]. Due to their computation burden and complexity, these equations are not well-suited for large-scale RTC applications. Instead, similarly to [4] and [6], the pipes are modelled as pure delay elements.

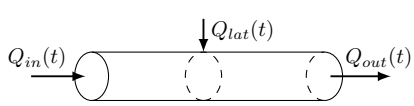

Fig. 2. Delay translation model.

Hence, outflows from a gravity pipe section are the delayed sums of controlled pump flows and uncontrolled lateral inflows, as shown in Fig. 2. (Lateral inflows are additional flows that enter the pipelines along the length of the channel.) The mass balance relation at time t is formulated as follows:

$$Q_{out}(t) = Q_{in}(t-\tau) + Q_{lat}(t-\tau_{lat}) \tag{1}$$

where $\tau \in \mathbb{R}_+$ and $\tau_{lat} \in \mathbb{R}_+$ are time lags measured from the upstream and from the point where lateral flows enter the pipeline, respectively. After discretization, delays are defined in δt sampling steps, hence the delayed flow is modeled with an augmented state vector consisting of the previous flows. The state equation, assuming $Q_{lat} = 0$ (to ease the notation), is given by:

$$\begin{bmatrix} Q_{out}(t+\delta t) \\ Q_{in}(t-\tau+2\delta t) \\ \vdots \\ \vdots \\ Q_{in}(t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} Q_{out}(t) \\ Q_{in}(t-\tau+\delta t) \\ Q_{in}(t-\tau+2\delta t) \\ \vdots \\ Q_{in}(t-\delta t) \end{bmatrix} + \mathbf{B}_u Q_{in}(t) \tag{2}$$

where Q_{in} inlet flow is subject to control, Q_{out} discharged flow is the output and the system matrices $\mathbf{A}$ and $\mathbf{B}_u$ are given by:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & \dots & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & & \ddots & 1 \\ 0 & 0 & \dots & \dots & 0 \end{bmatrix} \in \mathbb{R}^{\tau \times \tau}, \quad \mathbf{B}_u = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 1 \end{bmatrix} \in \mathbb{R}^{\tau}. \tag{3}$$

Note, that in case there are Q_{lat} inflows, the augmented state vectors are stacked together. This simple delay translation model is considered computationally beneficial and realistic enough for system-wide optimization, even though the physical phenomena such as flow attenuation and backwater effect are not incorporated in this formulation.

B. Retention tanks

Storage within the network is modeled by conceptual tanks that can account for overflows, as shown in Fig. 3.

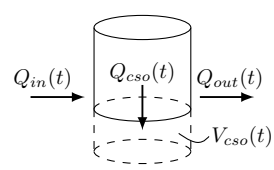

Fig. 3. Linear retention tank with V_{CSO} virtual overflow volume.

Flows to retention tanks (Q_{in}) are considered as (i) forecasted disturbances and (ii) controlled flows, coming from an upstream pumping station. The manipulated flow variables are denoted with Q_{out}, furthermore V represents the stored volume in the tank. The mass balance for each tank is:

$$\frac{dV(t)}{dt} = \sum_{n=1}^{N} Q_{in,n}(t) - \sum_{m=1}^{M} Q_{out,m}(t), \tag{4}$$

where N and M are the number of inlet points and number of pumps, respectively. The translation between volume and level is done by using the constant cross section area A.

In order to model overflows, the formulation in (4) is extended with a virtual volume, similarly as done in [8] and

[1]MIKE URBAN is a standard hydraulic simulation and modeling tool, used by operators at many water utilities. The MU simulation environment solves the full dynamic Saint-Venant equations for open-channel flow [18].

[9]. Hence, as depicted in Fig. 3, the storage model considers two joint volume elements:

$$V_t(t) \triangleq V(t) + V_{cso}(t), \tag{5}$$

where V is the physical volume of fluid and V_{cso} is the virtual volume accounting for overflows. To keep track of the physical volumes and to trigger an overflow in the tanks at the time when the physical limits are exceeded, the following restrictions apply to the storage model:

$$A\underline{h}+V_{cso}(t) \leq V_t(t) \leq A\overline{h}+V_{cso}(t), \quad \forall t_k = 1, ..., T, \tag{6}$$

where $\underline{h}$ and $\overline{h}$ are the physical lower and upper level bounds respectively and $V = Ah$. In case of an overflow event, V_{cso} increases both the lower and upper bounds, thereby keeping track of the physical storage and moving the overflow volume into the virtual storage at the bottom of the tank. Furthermore, the excess water leaves the system immediately. We assure this by letting $V_{cso} \geq 0$, meaning that the spilled sewage Q_{cso} spills to the environment, thus never flowing back to the retention tanks.

IV. Predictive Control

The terminology used in MPC of UDNs often differs in the literature coming from different backgrounds. For clarity, all the considered variables of UDNs are assigned to control-oriented variables, summarized in Table I.

Type of variable	Related symbols
System states (x)	V or equivalently h
Virtual states (z)	V_{cso} or equivalently h_{cso}
Control input (u)	Q_{out}
Disturbance (d)	Q_{in}
Output (y)	Q_W

TABLE I

Note, that the term disturbance represents the rain-runoff and domestic wastewater entering the network.

A. Multi-criteria MPC

In the optimization problem, $Q_{out,i}$ is considered as the decision variable, denoting the pumped flows of the ith pumping station. The physical system states are sewage levels h_j corresponding to the jth retention tank. In this study, similarly to [3], the objective function is formulated as a linearly weighted sum. The optimization is given by:

$$\min_{Q_{out}(0),...,Q_{out}(H_p)} \mathcal{L}(h, Q_{out}, Q_{in}, t_k) \triangleq \sum_{t_k=0}^{H_p-1} \sum_{j=1}^{\Gamma} \lambda_j \mathcal{F}_j(t_k), \tag{7}$$

where λ_j denotes the scaling weights, H_p is the prediction horizon, t_k is the discrete time index, and Γ is the number of control objectives. Note, that we write h tank levels as the system states, for the reason that levels are directly measurable in real life.

The first two terms $\mathcal{F}_1$ and $\mathcal{F}_2$ stand for overflow avoidance and tank emptying, respectively:

$$\mathcal{F}_1(t_k) \triangleq \sum_{i=1}^{P} V_{cso,i}(t_k)^2 \quad \text{and} \quad \mathcal{F}_2(t_k) \triangleq \sum_{i=1}^{P} V(t_k)^2, \tag{8}$$

where P is the number of overflow elements, i.e. retention tanks. Recall, that the overflow indicator $V_{cso} \geq 0$ is used to keep track of the water running out of the storage volume, as described in (6). Due to the fact that these physical level boundaries never decrease, V_{cso} has to be reset each time when the problem is resolved over H_p. The weights corresponding to overflows λ_1 are chosen to be significantly higher than the cost of other terms, making the usage of overflows undesirable if possible. Furthermore, we introduce $\mathcal{F}_2$ objective, as emptying the tanks is necessary to avoid odor problems occurring due to long retention times. Moreover, the weights on $\mathcal{F}_2$ allow to include the filling sensitivity of retention tanks, meaning that sensitive tanks are filled slower and emptied faster than less sensitive tanks.

The third objective $\mathcal{F}_3$ stands for minimizing the flow variation of the sewage leading to the treatment plant:

$$\mathcal{F}_3(t_k) \triangleq \Big(Q_W(t_k) - \frac{1}{H_p} \sum_{j=0}^{H_p-1} Q_W(j)\Big)^2, \tag{9}$$

where Q_W is the sum of controlled and disturbance flows leading to the treatment plant. Furthermore, the second term in (9) is considered as a reference flow, determined by the mean of the H_p-step ahead outlet flows towards the treatment plant. This formulation is inspired by [19], where the inlet flow variations to the treatment plant has been minimized over a daily horizon, assuming dry-weather conditions.

The fourth sub-goal $\mathcal{F}_4$ relates to the operation of actuators, where the control action is minimized. Hence, $\mathcal{F}_4$ is:

$$\mathcal{F}_4(t_k) \triangleq \sum_{l=1}^{L} Q_{out,l}(t_k)^2, \tag{10}$$

where L is the number of pumping stations and Q_{out} is the accumulated outflow, provided by pumps at the lth station.

The optimization problem in Equation (7) is formulated as a linear program, subject to the flow delays in Equation (1) and to the discretized tank dynamics in Equation (4). Furthermore, the equality constraint introduced in (5) and inequality constraint in (6) apply to the tank model, where V_{cso} is used as a virtual state in the optimization problem. Additionally, the control problem is subject to operational and physical constraints in the form:

$$\underline{Q}_{out} \leq Q_{out}(t_k) \leq \overline{Q}_{out}, \quad \forall t_k = 1, ..., T_k, \tag{11}$$

where $\underline{Q}_{out}$ and $\overline{Q}_{out}$ are the physical lower and upper bounds of the accumulated pump flows, respectively. Moreover, the rate of change of the control variables Q_{out} are constrained, in order to avoid deterioration of the pumps and pressure shocks in the following pressurized rising mains:

$$|Q_{out}(t_{k+1}) - Q_{out}(t_k)| \leq \overline{\Delta Q}_{out}, \quad \forall t_k = 1, ..., T_k, \tag{12}$$

where $\overline{\Delta Q}_{out}$ is the maximum allowed control input change, defined respectively for each pump. The operational constraint regarding the maximum inflow capacity of the treatment plant reads as follows:

$$Q_W(t_k) \leq \overline{Q}_W, \quad \forall t_k = 1, ..., T_k, \tag{13}$$

where $\overline{Q}_W$ is the maximum flow to the treatment plant.

B. Multi-Scenario MPC

Disturbances within an urban drainage framework include rainfall precipitation, groundwater infiltration, and domestic household sewage, among which rainfall is a stochastic hydrological process. The usage of various forecasting methods for rainfall infiltration, e.g. numerical weather predictions or radar rainfall estimates [14], implies that uncertainty is implicitly involved in the control of UDNs. For that reason, we extend the generic Multi-Criteria (MC-MPC) formulation and approximate the solution of the stochastic optimization problem with a Multi-Scenario (MS-MPC) approach. The control is then obtained by taking into account several forecasts, thereby making the decision making more robust towards weather prediction inaccuracies. To translate rainfall intensities to runoff flows, the catchment dynamics in the MIKE URBAN runoff environment are utilized. This engine makes several realizations of disturbance inflows based on forecasted rainfall intensities. The hierarchical structure of such control scheme is shown in Fig. 4.

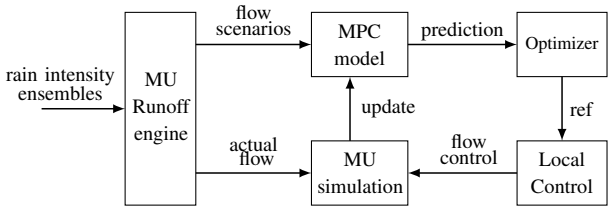

Fig. 4. Structure of the implemented MS-MPC approach.

The MU runoff engine incorporates the dynamics of catchments and produces a surface runoff hydrograph in response to a rain event, similarly as shown in Fig. 5. below.

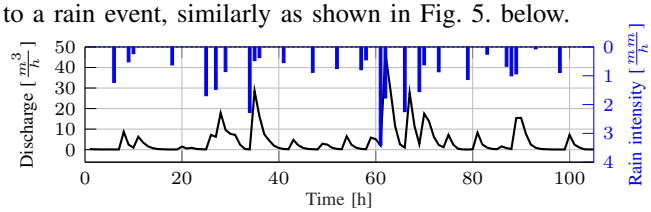

Fig. 5. Rain run-off computed by the MU engine. The reversed y-axis on the right denotes the intensity of rain, sampled at an hourly rate.

The optimization is reformulated such that the objectives of all scenarios are summed and weighted by the likelihood of occurrences. Hence, the MS-MPC can be recast as:

$$\min_{Q_{out}(0),\dots,Q_{out}(H_p)} \sum_{j=1}^{N_s} p_j \mathcal{L}_j(h_j, Q_{out}, Q_{in,j}, t_k), \quad (14)$$

where the subscript j represents the jth scenario, whereas p_j is the likelihood of occurrence. Moreover, $N_s \in \mathbb{Z}_+$ represents the number of scenarios used in the optimization. Note, that the cost functions differ in each scenario, as the different meteorological disturbances create different h_j future trajectories. Thus, there are dynamic and inequality constraints devoted to each scenario. To solve the MS-MPC problem, a common control Q_{out} is computed, which attempts to find the best decision for the most likely future states and prepare the system for possible worst-case events. For solving the problem, CVX is used [20] with the $SeDuMi$ solver [21].

An issue with the above formulation occurs in the case when a single ensemble does not predict rain at a certain time t_k, while the rest of the forecasts imply that there is a future storm event for which the system has to be prepared. In this case, the optimizer should act based on the likelihood of the events. However, the hard inequality constraint formulated in Equation (6), devoted to the no-rain scenario, does not allow to increase the control action, i.e. $Q_{out}(0), \dots, Q_{out}(H_p)$. This is also the case if one scenario has significantly smaller likelihood than any of the others, but requires to decrease the lower storage bounds $\underline{V}$ in order to increase the common control actions. We solve this problem by inserting a slack variable into the hard inequality constraint such that:

$$A\underline{h} + V_{cso}(t_k) - \epsilon(t_k) \leq Ah_t(t_k) \leq A\overline{h} + V_{cso}(t_k), \quad (15)$$

$\forall t_k = 1, \dots, T_k$, where $\epsilon \geq 0$. The slack variable is penalized and it is activated only when the likelihood of a no-rain event weights significantly less than ensembles predicting overflow. Hence, we avoid that an unlikely no-rain scenario restricts the usage of control actions when likely scenarios require to empty retention tanks due to heavy loads on the system.

V. Numerical results

The performance of the MS-MPC algorithm is assessed based on the high-fidelity model shown in Fig. 1. The control algorithm is tested against $N_s = 4$ different weather scenarios, covering a six days long wet-weather period. The test scenarios have been created based on rainfall intensities corresponding to realistic design storm events.[2]

It should be noted, that we do not aim to show how precisely the future is forecasted. Instead, the goal is to present how plausible future forecasts are embedded in a standard MPC problem. The combined run-off and domestic wastewater replicates are depicted in Fig. 6.

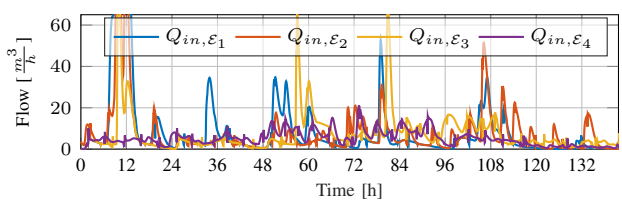

Fig. 6. Inflow scenarios computed by MU, using ensemble rain forecasts.

The signals $Q_{in,\mathcal{E}_1}, \dots, Q_{in,\mathcal{E}_4}$ represent four scenarios. The length of the simulation is $T = 6$ days. The longest travel time within the network is related to the connection between T_1 and T_2 stations and is approximately 90 minutes. An $H_p = 2$ [h] prediction horizon is used, with a sampling time and control step $\delta t = 5$ [min]. Moreover, the likelihoods $p_j = 0.25, \forall j = 1, \dots, 4$ are set equal. The weight parameters λ_1 corresponding to CSO prevention are equal for both T_1 and T_2 retention tanks, hence we do not prioritize overflows of one tank over another. (The optimization parameters are listed in the appendix.) The test results are shown in Fig. 7.

[2]The data is from https://www.silkeborg-vejret.dk/statistik.php. We deliberately chose rainy periods between 21-27 April, 2018, from Silkeborg, Denmark. The domestic wastewater inflow is artificially created and scaled.

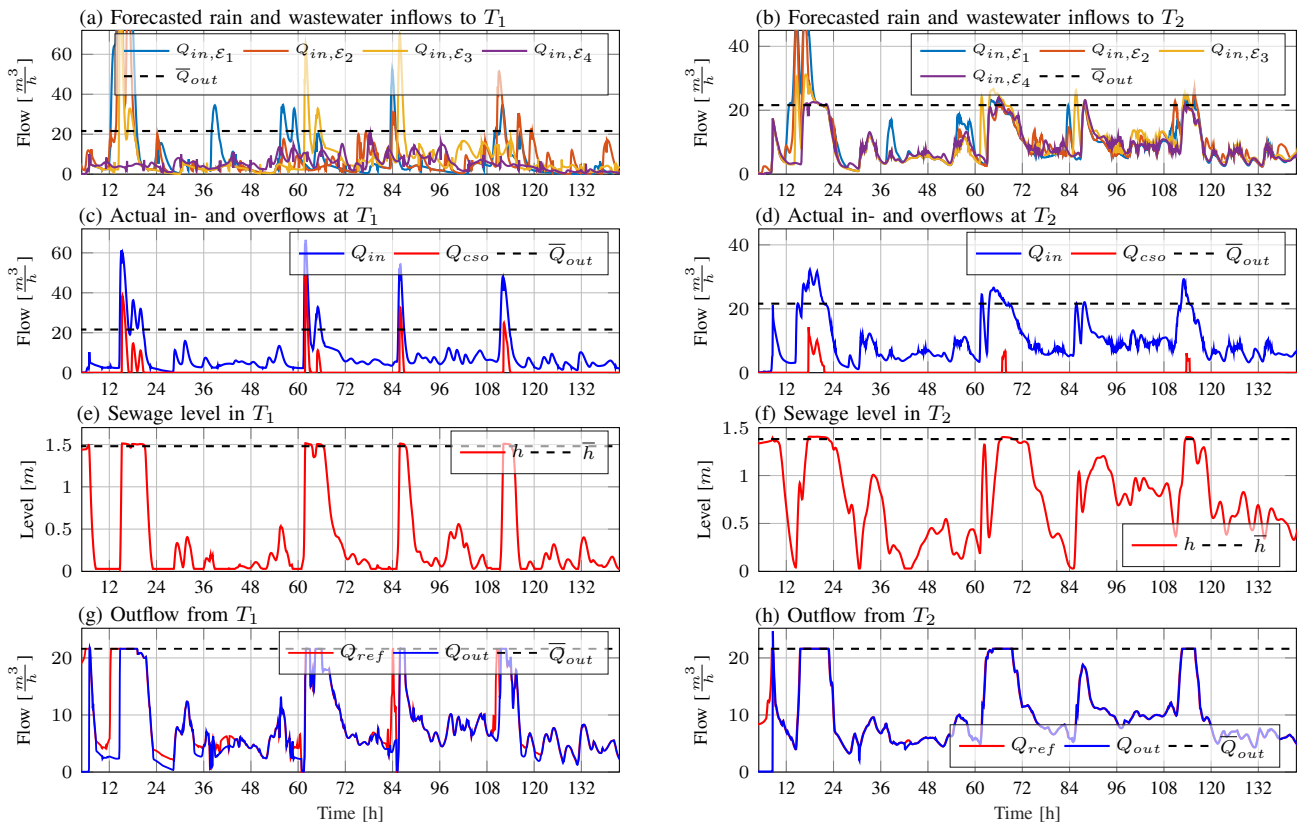

Fig. 7. Performance of the MS-MPC with equal likelihoods of four rain scenarios over a six days simulation period with $H_p = 2$ [h] and $\delta t = 5$ [min]. Results are shown in two columns, where signals corresponding to the upstream (T_1) and to the downstream (T_2) pumping stations are shown in the first and second columns, respectively. The plots (a) and (b) show possible inflow sequences, (c) and (d) the actual flows occurring in the system, (e) and (f) the current level in the retention tanks and finally, (g) and (h) the optimal flow set-points Q_{ref} and the actual control flows Q_{out} at the pumping stations.

The MS-MPC acts as an upper-level controller that computes optimal set-points (Q_{ref}) to local PID controllers. The signals in Fig. 7(c) and 7(d) show the actual accumulated inflows and the amount of overflows at T_1 and at T_2 stations. The control can account for the uncertainties in case the actual disturbances are close or within the range of the possible scenarios. For instance, as seen between 108 and 120 [h], T_2 overflows since the inflow exceeds the pumping capacities more than it was forecasted. Besides, (c) and (d) show overflows which could not be prevented due to the insufficient storage and pumping capacity of the network. Between 60 and 72 [h], the pumps at T_1 decrease the flow instead of further emptying the tank. This is for the reason that the delayed pumped sewage would arrive at T_2 in a high-inflow period, causing heavier overflows at the downstream.

Moreover, with the proposed method, overflows can be prioritized using λ_1 corresponding to Equation (8). The CSO reductions in two extreme cases are shown in Fig. 8.

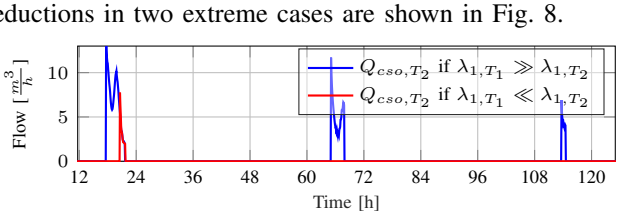

Fig. 8. CSO reduction at T_2 in case overflows are prioritized to T_1.

To protect T_2, the pumps at T_1 can hold the sewage back in heavy-load periods. Nevertheless, in the proposed framework, it should not be expected to obtain a universal solution that is optimal for all ensemble weather predictions, especially if there are conflicting objectives. Around 36 [h], for instance, the first ensemble predicts a potential overflow shown in (c), for which the pumps try to react by keeping the retention level at a minimum. Even though there is a nearly rain-free period during this time, the flow references to the pumps (red curves in (g) and (h)) indicate that the MS-MPC attempts to prepare the system for potential overflows. The same situation is observed at 84 [h], where a coupled rain event is expected earlier than it happens.

In addition to CSO prevention, the smoothing of the inflow to the treatment plant has been considered. This has been done by compensating the variances on the disturbance flows leading to the treatment plant over the prediction horizon. The performance is shown in Fig. 9.

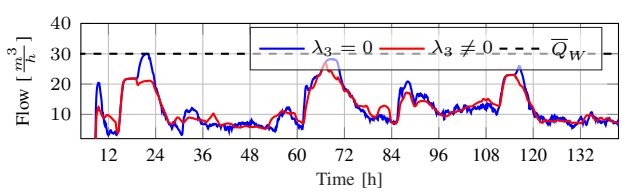

Fig. 9. Inlet flow (Q_W) to the treatment plant.

where $\overline{Q}_W$ is the maximum possible inlet flow capacity of the treatment plant and λ_3 is the weighting coefficient from

Equation (9). As seen, in contrast to maximizing the inlet flow towards the treatment plant, $\mathcal{F}_3$ attempts to even out the variations. The pumps at T_2 station need to consider the disturbances entering directly to the treatment plant and attempt to compensate for them. The peak flows in Fig. 9. correspond to periods when the overflow risk is high, therefore the pumps at T_2 move as much sewage as possible to avoid CSOs. This is for the reason that $\mathcal{F}_1$ and $\mathcal{F}_3$ are conflicting objectives and $\mathcal{F}_1$ is prioritized over the flow smoothing to the treatment plant. Hence, the highest potential for improving the quality of the treatment process is in dry-weather periods when the risk of overflooding is low.

VI. CONCLUSION

The presented paper studied how the hydrological uncertainties can be tackled in an RTC problem regarding the control of urban drainage networks. To this end, we proposed a Multi-Scenario approach as an extension of a standard, fast-solvable MPC framework. The method has been tested on a high-fidelity model of a test network and the implementation showed that both the simplified delay and retention tank models are feasible for on-line storage capacity optimization in UDNs. The MS-MPC has been tested on four different scenarios and the results showed that MS-MPC has a significant advantage over standard MPC methods that neglect uncertain weather forecasts. Although some scenarios can have a low probability of occurrence, the damage may be very high. Moreover, the results showed that the transport delays affect the MPC performance significantly, especially when prioritizing overflows and protecting sensitive waters.

In our future work, we focus on developing a systematic way of tuning the MS-MPC parameters, including the analysis of H_p size and the penalty gains on the objectives. Note, that the weights λ have been chosen based on pre-defined performance goals in a heuristic fashion. Furthermore, a natural extension of the treatment plant flow variation objective $\mathcal{F}_3$, formulated in (9), is to extend the prediction horizon to a daily scale, where varying sampling times are used. This allows for better optimization in dry-weather periods, where only domestic wastewater is considered with an inherent periodicity of one day.

APPENDIX

The optimization parameters are shown in Table II.

$\lambda_{1,T_1} = \lambda_{1,T_2} = 10^5[-]$	$\overline{h}_{T_1} = 1.5[m]$
$\lambda_{2,T_1} = \lambda_{2,T_2} = 10^3[-]$	$\overline{h}_{T_2} = 1.4[m]$
$\lambda_{4,T_1} = \lambda_{4,T_2} = 1[-]$	$\lambda_3 = 10^2[m]$
$\overline{Q}_{out,T_1} = \overline{Q}_{out,T_2} = 21.6[\frac{m^3}{h}]$	$N_s = 4[-]$
$\overline{\Delta Q}_{out,T_1} = \overline{\Delta Q}_{out,T_2} = 10[\frac{m^3}{h}]$	$\overline{Q}_W = 30[\frac{m^3}{h}]$

TABLE II

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