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A Consensus-Based Algorithm for Power Sharing and Voltage Regulation in DC Microgrids

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Abstract—In DC microgrids, load power sharing and bus voltage regulation are two common control objectives. In this paper, a consensus-based algorithm is presented to achieve proportional power sharing and regulation of weighted geometric mean of bus voltages in DC microgrids with ZIP (constant impedance, constant current, and constant power) loads simultaneously. By using the virtue of the Laplacian matrices of undirected connected graphs, a lemma is derived to assist the stability analysis of the DC microgrids. Thus, a sufficient condition that stabilizes the system with ZIP loads is established. In addition, with the help of a distributed voltage regulation error estimator, the tuning of the bus voltages can be realized without the requirement on initial voltage conditions. Through Lyapunov synthesis, the large-signal stability of the closed-loop system is theoretically ensured for a wide range of load conditions. Finally, simulation studies are performed to validate the merits of the proposed consensus-based algorithm.

Index Terms—Power sharing, voltage regulation, DC microgrid, Laplacian matrix, consensus algorithm.

I. INTRODUCTION

A MICROGRID generally consists of distributed generations (DGs), energy storage systems, and a collection of loads, operating either in grid-connected mode or island one [1]–[3]. Compared with AC microgrids [4], [5], DC microgrids offer a more desirable choice to integrate DC energy generations and DC loads by avoiding additional AC/DC conversion stages [6]–[8]. Besides, DC microgrids do not need to deal with frequency regulation, synchronization, and reactive power flow issues existing in AC microgrids [9], [10].

In the operation of a DC microgrid, load sharing (current or power sharing) and bus voltage regulation are two common control objectives considered for DG units [11], [12]. Traditionally, these objectives can be realized by a decentralized control algorithm, i.e., droop control, owing to its reliability and expansibility [13]. However, an inherent problem still exists in traditional decentralized method [14], where the

steady-state voltage deviation comes into conflict with the load sharing accuracy [15].

In order to achieve accurate load current sharing and satisfactory bus voltage regulation performance, a centralized control scheme including multiple control levels that act at different timescales is proposed in [16]. However, the centralized control scheme lacks flexibility and is susceptible to a single point of failure [17]. Therefore, sparse communication-based distributed control algorithms are alternatively developed to improve the reliability of microgrids [18]–[20]. In [21], a distributed algorithm is developed for two-bus microgrids to improve control performance. Unfortunately, its extension to multiple-bus microgrids is not provided clearly. In [22], such control objectives are achieved by adding two correction terms to droop algorithms, which are generated by proportional integral (PI) laws. However, the large-signal stability of the distributed control algorithms [16], [18], [22] is not quantitatively analyzed.

To overcome the aforementioned theoretical problems, a consensus-based control algorithm with rigorous stability analysis is presented in [23]. Furthermore, an event-triggered control algorithm is proposed in [24] to reduce the communication burden [25], [26]. Thereafter, in [27], a periodic event-triggered algorithm is developed based on the discrete-time DC microgrid model to meet the inherent requirement of discrete-time implementation of control algorithms. However, the steady-state bus voltages are highly dependent on their initial conditions [27]. No active voltage regulation algorithm is presented in [23], [24]. Recently, distributed consensus algorithms are developed in [28], [29] to achieve proportional load current sharing and weighted average bus voltage regulation without the requirement on voltage initial values.

Nevertheless, relatively less research on distributed control algorithms with large-signal stability analysis has been reported for the load power sharing problem in DC microgrids, due to the fact that the power flow equations are nonlinear in nature, which makes the control problem much more complex [30]. To attain desired load power sharing performance, a nonlinear consensus algorithm is developed in [31] for DC microgrids with ZIP (constant impedance, constant current, and constant power) loads. However, the bus voltage regulation with a user-defined reference is not guaranteed [31]. Moreover, although the load current sharing and bus voltage regulation have been achieved by the aforementioned method [28], [29], how to simultaneously achieve load power sharing and bus voltage regulation in DC microgrids with rigorous stability

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analysis is still an open problem.

Therefore, in this paper, a consensus-based algorithm is developed and analyzed to attain the proportional power sharing and regulation of the weighted geometric mean of the bus voltages in DC microgrids. Firstly, a nonlinear dynamic model based on nonlinear power flow equations of DC microgrids is derived. Thereafter, a lemma based on the properties of the Laplacian matrices of undirected connected graphs is discovered as a guideline for designing controllers. Subsequently, a distributed algorithm is presented to achieve the aforementioned control objectives. The voltage regulation error is obtained in a distributed way for each DG's controller to assist the tuning of the bus voltages. Through the Lyapunov synthesis, the power sharing and voltage regulation errors are mathematically proved to converge to zero asymptotically. Finally, the merits of the proposed control algorithm is validated by simulation results.

The contributions of this paper can be summarized as

- (i) Proportional power sharing and regulation of the weighted geometric mean of the bus voltages are achieved for DC microgrids with ZIP loads, simultaneously. The requirement on initial bus voltage conditions is relaxed;
- (ii) A sufficient condition that stabilizes the system with ZIP loads is investigated. A lemma using the virtue of the Laplacian matrices of undirected connected graphs is studied to aid the stability analysis of the system. Hence, the impact of the non-symmetric and indefinite Jacobian matrix on the stability can be melted;
- (iii) The large-signal stability of the closed-loop system is theoretically ensured for a wide range of load conditions. Both the power sharing and voltage regulation errors are proved to converge to zero asymptotically.

The rest of this paper is organized as follows. In Section II, the problem formulation of the DC microgrid is given. Section III studies the properties of the Laplacian matrix, based on which a key lemma is established. In Section IV, a consensus-based algorithm along with the stability analysis for the DC microgrid with ZIP loads is proposed. In Section V, simulation results are provided to show the effectiveness of the proposed control algorithm, followed by some concluding remarks in Section VI.

Notation: Let $\mathbf{0}_n = [0, 0, \dots, 0]^T \in \mathbb{R}^n$, $\mathbf{1}_n = [1, 1, \dots, 1]^T \in \mathbb{R}^n$, and $\mathbf{I}_n \in \mathbb{R}^{n \times n}$ denote the identity matrix. Given a vector $z \in \mathbb{R}^n$, $\langle z \rangle$ denotes a diagonal matrix whose diagonal entries are the components of z , and $\ln z$ denotes the element-wise natural logarithm of z . For a real symmetric matrix A , denote the eigenvalues of A by $\lambda_i(A)$, $i = 1, 2, \dots, n$, satisfying $\lambda_1(A) \leq \lambda_2(A) \leq \dots \leq \lambda_n(A)$.

II. PROBLEM FORMULATION

In Fig. 1, an example of the considered DC microgrid comprising an electrical network and a communication network is presented. The electrical network is a physical grid for delivering electric power from DGs to loads, while the communication network is a sparse one for information sharing among DGs' controllers [32].

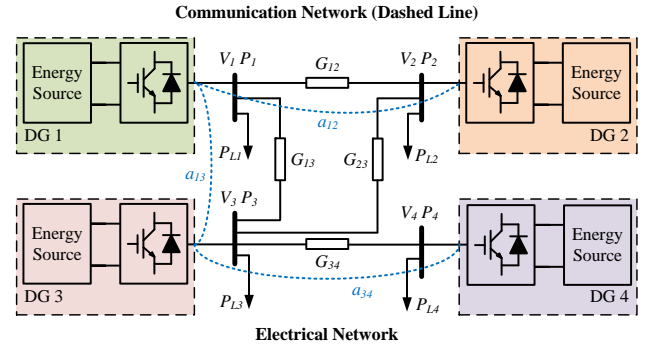


Fig. 1. An example of the considered DC microgrid.

A. Electrical Network Model

The electrical network can be modeled as an undirected connected graph $\mathcal{G}_E = (\mathcal{V}_E, \mathcal{E}_E)$, where $\mathcal{V}_E = \{1, 2, \dots, n\}$ represents the n buses with DGs and possible loads, and $\mathcal{E}_E \subseteq \mathcal{V}_E \times \mathcal{V}_E$ represents the distribution lines of the DC microgrid. Define $G_{ij} = G_{ji} \in \mathbb{R}^+$ as the conductance of the distribution line if $(i, j) \in \mathcal{E}_E$, and $G_{ij} = G_{ji} = 0$, otherwise. In addition, the Laplacian matrix of graph \mathcal{G}_E is defined as $\mathcal{L}_E = \{l_{Eij}\} \in \mathbb{R}^{n \times n}$ with $l_{Eii} = \sum_{j=1}^n G_{ij}$ and $l_{Eij} = -G_{ij}$ for $i \neq j$.

Let V_i , P_i , and P_{Li} denote the voltage, the DG's output power, and the load power of bus i , respectively. The power flow equations of the DC microgrid can be given as

$$P_i = V_i \sum_{j=1}^n G_{ij}(V_i - V_j) + P_{Li} = V_i \sum_{j=1}^n l_{Eij}V_j + P_{Li} \quad (1)$$

for $i \in \mathcal{V}_E$.

Depending on specific load models, the load power P_{Li} takes different forms. In DC microgrids, constant impedance (Z), constant current (I), and constant power (P) loads are usually of interest [23], [27]–[29], [31], whose models are given as

- (i) constant impedance loads: $P_{Li}^Z = G_i^* V_i^2$ with $G_i^* \in \mathbb{R}_{\geq 0}$ being the load conductance;
- (ii) constant current loads: $P_{Li}^I = I_i^* V_i$ with $I_i^* \in \mathbb{R}_{\geq 0}$ being the load current;
- (iii) constant power loads: $P_{Li}^P = P_i^*$ with $P_i^* \in \mathbb{R}_{\geq 0}$ being the load power.

Thereafter, the total load power can be expressed as

$$P_{Li} = P_{Li}^Z + P_{Li}^I + P_{Li}^P = G_i^* V_i^2 + I_i^* V_i + P_i^*. \quad (2)$$

Combining (1) and (2) yields

$$P_i = V_i \sum_{j=1}^n l_{Eij}V_j + G_i^* V_i^2 + I_i^* V_i + P_i^* \quad (3)$$

and in a compact form

$$P = \langle V \rangle \mathcal{L}_E V + G^* \langle V \rangle V + I^* V + P^* \quad (4)$$

where $P = [P_1, P_2, \dots, P_n]^T$ and $V = [V_1, V_2, \dots, V_n]^T$ are the vectors of the DGs' output powers and the bus voltages, respectively, $G^* = \text{diag}\{G_1^*, G_2^*, \dots, G_n^*\}$, $I^* = \text{diag}\{I_1^*, I_2^*, \dots, I_n^*\}$, and $P^* = [P_1^*, P_2^*, \dots, P_n^*]^T$ are load parameters.

By taking the time derivative of (4), the dynamic model of the DC microgrid can be derived as

$$\dot{P} = F(V)\dot{V} \quad (5)$$

where \dot{V} is the control input, and $F(V) = [\langle V \rangle \mathcal{L}_E + \langle \mathcal{L}_E V \rangle + 2G^* \langle V \rangle + I^*]$ is the equivalent Jacobian matrix of the DC microgrid, which is non-symmetric and indefinite.

In this study, the local loads are assumed to satisfy the following assumption.

Assumption 1: For V_i within normal operating range, there exists a constant $\epsilon \in \mathbb{R}^+$ such that

$$P_i + G_i^* V_i^2 - P_i^* \geq - \left(\frac{\lambda_2(\mathcal{L}_E)}{\max\{P_{Ci}/V_i\}} - \epsilon \right) P_{Ci} V_i \quad (6)$$

holds for all $i \in \mathcal{V}_E$, where $P_{Ci} \in \mathbb{R}^+$ is the constant power sharing ratio.

Remark 1: In this study, Assumption 1 is a sufficient condition that stabilizes the system as illustrated in Section IV-C, which can be verified locally if the algebraic connectivity of the electrical network, i.e., $\lambda_2(\mathcal{L}_E)$, is known. In practice, since $P_i \geq 0$ under normal operating conditions and $\lambda_2(\mathcal{L}_E) > 0$, a conservative version of (6) can be derived as $G_i^* - P_i^*/V_i^2 \geq 0$, i.e., the equivalent conductance of each local load is positive, which is the condition applied in [31].

Remark 2: The quasi-stationary line (QSL) approximations is applied in this study, which assumes that the power lines are mainly resistive. The approximated electrical network model can be justified regarding the singular perturbation theory [33], [34].

B. Communication Network Model

The communication network among the controllers is also considered as an undirected connected graph $\mathcal{G}_C = (\mathcal{V}_C, \mathcal{E}_C)$ with the set of nodes $\mathcal{V}_C = \{1, 2, \dots, n\}$ representing the n controllers, and $\mathcal{E}_C \subseteq \mathcal{V}_C \times \mathcal{V}_C$ representing the communications links among these controllers. The adjacency matrix of graph \mathcal{G}_C is defined as $\mathcal{A}_C = \{a_{ij}\} \in \mathbb{R}^{n \times n}$, where $a_{ij} \in \mathbb{R}^+$ is a constant if $(i, j) \in \mathcal{E}_C$, and $a_{ij} = 0$ otherwise. Assume that no self edge exists in graph \mathcal{G}_C , i.e., $a_{ii} = 0$. For an undirected graph, $a_{ij} = a_{ji}$. Besides, the Laplacian matrix of graph \mathcal{G}_C is defined as $\mathcal{L}_C = \{l_{Cij}\} \in \mathbb{R}^{n \times n}$, with $l_{Cii} = \sum_{j=1}^n a_{ij}$ and $l_{Cij} = -a_{ij}$ for $i \neq j$. The neighbor set of node i is defined as $\mathcal{N}_i = \{j \in \mathcal{V}_C | (i, j) \in \mathcal{E}_C\}$.

C. Control Objectives

Two control objectives are considered in this study. The first objective is load power sharing, which is crucial for maintaining the safety of the microgrid to prevent the possibility that local loads overload local DGs and eventually lead to failures [23].

Objective 1: For a DC microgrid, load power sharing is achieved if the overall load power is proportionally shared among multiple DGs at steady-state, i.e.

$$\frac{P_i(t)}{P_{Ci}} = \frac{P_j(t)}{P_{Cj}}, \quad \forall i, j \in \mathcal{V}_E \quad (7)$$

when $t \rightarrow \infty$.

Usually, P_{Ci} is set according to the rated power of local DGs, implying that the total load power is shared proportionally to the DGs capacities [23].

The second control objective is bus voltage regulation. Often the desired bus voltages are all set identically to the voltage level of the microgrid [16]. However, the power sharing objective usually does not permit such way of voltage regulation. Additionally, the DG with a relatively large generation capacity should have a relatively small voltage deviation to ensure the overall voltage performance, since it usually determines the grid voltage level [28]. Therefore, other alternatives are to keep the overall voltage profiles at the steady-state, such as the weighted average value [23], [28] or the weighted geometric mean of the bus voltages [31], identical to the desired reference. In this study, the regulation of the weighted geometric mean of the bus voltages is considered, which is defined as [31]

Objective 2: For a DC microgrid, bus voltage regulation is achieved if the bus voltages satisfy

$$\lim_{t \rightarrow \infty} v_m = v_m^* \quad (8)$$

where v_m is the weighted geometric mean of the bus voltages, expressed as

$$v_m = \sqrt[n]{\prod_{i=1}^n V_i^{P_{Ci}}} \quad (9)$$

and v_m^* is a user-defined reference for v_m .

Traditionally, v_m^* is generated by upper control levels and is available to all DGs [35].

III. PROPERTIES OF THE LAPLACIAN MATRIX

The Laplacian matrices of undirected connected graphs have some key properties summarized in the following proposition [36].

Proposition 1: The Laplacian matrix \mathcal{L} of an undirected connected graph \mathcal{G} has the following properties:

- (i) \mathcal{L} is a symmetric positive semi-definite matrix, i.e., $\mathcal{L} = \mathcal{L}^T \geq 0$;
- (ii) 0 is the simple eigenvalue of \mathcal{L} with $\text{span}\{\mathbf{1}_n\}$ being the null space of \mathcal{L} , i.e., $\mathcal{L}\mathbf{1}_n = \mathbf{0}_n$;
- (iii) $0 = \lambda_1(\mathcal{L}) < \lambda_2(\mathcal{L}) \leq \lambda_3(\mathcal{L}) \leq \dots \leq \lambda_n(\mathcal{L})$.

Based on Proposition 1, the following lemma is introduced.

Lemma 1: Given a symmetric matrix $N \in \mathbb{R}^{n \times n}$, and a positive definite diagonal matrix $D = \text{diag}\{d_1, d_2, \dots, d_n\} \in \mathbb{R}^{n \times n}$, if there exists a constant $\epsilon \in \mathbb{R}^+$ such that

$$N \geq - \left(\frac{\lambda_2(\mathcal{L}_E)}{\max\{d_i\}} - \epsilon \right) D, \quad (10)$$

then the following inequality

$$\mathcal{L}_C D^{-1} (\mathcal{L}_E + N) D^{-1} \mathcal{L}_C \geq \epsilon \mathcal{L}_C D^{-1} \mathcal{L}_C \quad (11)$$

holds.

Proof: Firstly, define $\bar{\mathcal{L}}_C = D^{-1/2} \mathcal{L}_C D^{-1/2}$, $\bar{\mathcal{L}}_E = D^{-1/2} \mathcal{L}_E D^{-1/2}$, and $\bar{N} = D^{-1/2} N D^{-1/2}$. By evoking (10), one has

$$\bar{N} \geq - \left(\frac{\lambda_2(\mathcal{L}_E)}{\max\{d_i\}} - \epsilon \right) \mathbf{I}_n. \quad (12)$$

$$\begin{aligned}
\bar{\mathcal{L}}_C(\bar{\mathcal{L}}_E + \bar{N})\bar{\mathcal{L}}_C &\geq U \begin{bmatrix} 0 & \mathbf{0}_{n-1}^T \\ \mathbf{0}_{n-1} & \bar{\mathcal{L}}_{Csub} \end{bmatrix} U^T \left\{ U \begin{bmatrix} 0 & \mathbf{0}_{n-1}^T \\ \mathbf{0}_{n-1} & \bar{\mathcal{L}}_{Esub} \end{bmatrix} U^T - (\lambda_2(\bar{\mathcal{L}}_E) - \epsilon)\mathbf{I}_n \right\} U \begin{bmatrix} 0 & \mathbf{0}_{n-1}^T \\ \mathbf{0}_{n-1} & \bar{\mathcal{L}}_{Csub} \end{bmatrix} U^T \\
&= U \begin{bmatrix} 0 & \mathbf{0}_{n-1}^T \\ \mathbf{0}_{n-1} & \bar{\mathcal{L}}_{Csub} \end{bmatrix} \begin{bmatrix} -(\lambda_2(\bar{\mathcal{L}}_E) - \epsilon) & \mathbf{0}_{n-1}^T \\ \mathbf{0}_{n-1} & \bar{\mathcal{L}}_{Esub} - (\lambda_2(\bar{\mathcal{L}}_E) - \epsilon)\mathbf{I}_{n-1} \end{bmatrix} \begin{bmatrix} 0 & \mathbf{0}_{n-1}^T \\ \mathbf{0}_{n-1} & \bar{\mathcal{L}}_{Csub} \end{bmatrix} U^T \\
&= U \begin{bmatrix} 0 & \mathbf{0}_{n-1}^T \\ \mathbf{0}_{n-1} & \bar{\mathcal{L}}_{Csub} \end{bmatrix} \begin{bmatrix} \epsilon & \mathbf{0}_{n-1}^T \\ \mathbf{0}_{n-1} & \bar{\mathcal{L}}_{Esub} - (\lambda_2(\bar{\mathcal{L}}_E) - \epsilon)\mathbf{I}_{n-1} \end{bmatrix} \begin{bmatrix} 0 & \mathbf{0}_{n-1}^T \\ \mathbf{0}_{n-1} & \bar{\mathcal{L}}_{Csub} \end{bmatrix} U^T. \quad (19)
\end{aligned}$$

According to Corollary 4.4.11 and Fact 8.18.17 in [37], the following inequality holds

$$\lambda_2(\bar{\mathcal{L}}_E) = \lambda_2(\bar{\mathcal{L}}_E D) \leq \max\{d_i\} \lambda_2(\bar{\mathcal{L}}_E). \quad (13)$$

Next, substituting (13) into (12) yields

$$\bar{N} \geq -(\lambda_2(\bar{\mathcal{L}}_E) - \epsilon)\mathbf{I}_n. \quad (14)$$

Then the following inequality can be derived

$$\bar{\mathcal{L}}_C(\bar{\mathcal{L}}_E + \bar{N})\bar{\mathcal{L}}_C \geq \bar{\mathcal{L}}_C [\bar{\mathcal{L}}_E - (\lambda_2(\bar{\mathcal{L}}_E) - \epsilon)\mathbf{I}_n] \bar{\mathcal{L}}_C. \quad (15)$$

According to Proposition 1, one has that $\bar{\mathcal{L}}_E$ is a positive semi-definite matrix, whose null space is $\text{span}\{D^{1/2}\mathbf{1}_n\}$. Hence, there exists a unitary matrix such that $\bar{\mathcal{L}}_E$ is diagonal, i.e.,

$$\bar{\mathcal{L}}_E = U \begin{bmatrix} 0 & \mathbf{0}_{n-1}^T \\ \mathbf{0}_{n-1} & \bar{\mathcal{L}}_{Esub} \end{bmatrix} U^T \quad (16)$$

where $\bar{\mathcal{L}}_{Esub} = \text{diag}\{\lambda_2(\bar{\mathcal{L}}_E), \lambda_3(\bar{\mathcal{L}}_E), \dots, \lambda_n(\bar{\mathcal{L}}_E)\}$ is a diagonal matrix, and the unitary matrix $U = [u_1, u_2, \dots, u_n]$, with $u_1 = D^{1/2}\mathbf{1}_n / \sqrt{\mathbf{1}_n^T D \mathbf{1}_n}$, and $u_i \in \mathbb{R}^n$, $i = 2, 3, \dots, n$ being the eigenvector of $\bar{\mathcal{L}}_E$ associated with the eigenvalue $\lambda_i(\bar{\mathcal{L}}_E)$.

Further, with the help of Proposition 1, the null space of $\bar{\mathcal{L}}_C$ is also $\text{span}\{D^{1/2}\mathbf{1}_n\}$. Therefore, $\bar{\mathcal{L}}_C$ can be expressed as

$$\bar{\mathcal{L}}_C = U \begin{bmatrix} 0 & \mathbf{0}_{n-1}^T \\ \mathbf{0}_{n-1} & \bar{\mathcal{L}}_{Csub} \end{bmatrix} U^T \quad (17)$$

with

$$\bar{\mathcal{L}}_{Csub} = \begin{bmatrix} u_2^T \bar{\mathcal{L}}_C u_2 & \cdots & u_2^T \bar{\mathcal{L}}_C u_n \\ \vdots & \ddots & \vdots \\ u_n^T \bar{\mathcal{L}}_C u_2 & \cdots & u_n^T \bar{\mathcal{L}}_C u_n \end{bmatrix} \quad (18)$$

being a positive definite matrix.

Substituting (16), (17) into (15) yields (19). Notice that

$$\begin{aligned}
&\bar{\mathcal{L}}_{Esub} - (\lambda_2(\bar{\mathcal{L}}_E) - \epsilon)\mathbf{I}_{n-1} \\
&\geq \lambda_2(\bar{\mathcal{L}}_E)\mathbf{I}_{n-1} - (\lambda_2(\bar{\mathcal{L}}_E) - \epsilon)\mathbf{I}_{n-1} = \epsilon\mathbf{I}_{n-1}. \quad (20)
\end{aligned}$$

Combining (19) and (20) yields

$$\begin{aligned}
&\bar{\mathcal{L}}_C(\bar{\mathcal{L}}_E + \bar{N})\bar{\mathcal{L}}_C \\
&\geq \epsilon U \begin{bmatrix} 0 & \mathbf{0}_{n-1}^T \\ \mathbf{0}_{n-1} & \bar{\mathcal{L}}_{Csub} \end{bmatrix} \begin{bmatrix} 0 & \mathbf{0}_{n-1}^T \\ \mathbf{0}_{n-1} & \bar{\mathcal{L}}_{Csub} \end{bmatrix} U^T \\
&= \epsilon \bar{\mathcal{L}}_C \bar{\mathcal{L}}_C. \quad (21)
\end{aligned}$$

Hence, according to the definition of $\bar{\mathcal{L}}_C$, $\bar{\mathcal{L}}_E$, and \bar{N} , (21) becomes

$$\bar{\mathcal{L}}_C D^{-1}(\bar{\mathcal{L}}_E + \bar{N})D^{-1}\bar{\mathcal{L}}_C \geq \epsilon \bar{\mathcal{L}}_C D^{-1}\bar{\mathcal{L}}_C. \quad (22)$$

Thus, (11) holds. ■

IV. A CONSENSUS-BASED ALGORITHM FOR DC MICROGRIDS

A. Error Dynamics

1) *Load power sharing error:* Based on the control objectives, define the power sharing error for DG i as

$$e_{Pi} = \sum_{j \in \mathcal{N}_i} a_{ij} \left(\frac{P_i}{P_{Ci}} - \frac{P_j}{P_{Cj}} \right), \quad \forall i \in \mathcal{V}_E \quad (23)$$

and in a compact form

$$e_P = \mathcal{L}_C P_C^{-1} P \quad (24)$$

with $P_C = \text{diag}\{P_{C1}, P_{C2}, \dots, P_{Cn}\}$, and $e_P = [e_{P1}, e_{P2}, \dots, e_{Pn}]^T$. Substituting (5) into the time derivative of e_P yields

$$\dot{e}_P = \mathcal{L}_C P_C^{-1} F(V) \dot{V}. \quad (25)$$

2) *Bus voltage regulation error:* Define the voltage regulation error as

$$e_V = \ln v_m - \ln v_m^*. \quad (26)$$

Substituting (9) into (26) yields

$$e_V = \frac{\mathbf{1}_n^T P_C \ln V}{\mathbf{1}_n^T P_C \mathbf{1}_n} - \ln v_m^* \quad (27)$$

whose time derivative is

$$\dot{e}_V = \frac{\mathbf{1}_n^T P_C \langle V \rangle^{-1} \dot{V}}{\mathbf{1}_n^T P_C \mathbf{1}_n}. \quad (28)$$

Thereafter, the power sharing and the voltage regulation in DC microgrids can be realized by maneuvering e_P and e_V to zero.

It is noticed that the voltage regulation error is not known to all DGs since the calculation of v_m requires all bus voltages. In Section IV-B, e_V is estimated in a distributed manner for each DG i , which allows the regulation of the weighted geometric mean of the bus voltages without centralized communication network.

B. Consensus-Based Algorithm Development

The distributed control algorithm of DG i is designed as

$$\dot{V}_i = -k_P V_i P_{Ci}^{-1} e_{Pi} - k_V V_i \hat{e}_{Vi} \quad (29)$$

where $k_P, k_V \in \mathbb{R}^+$. \hat{e}_{Vi} is the estimate of e_V of DG i , which is obtained from

$$\hat{e}_{Vi} = \ln V_i + k_P P_{Ci}^{-1} \Omega_i - \ln v_m^* \quad (30)$$

with Ω_i governed by

$$\dot{\Omega}_i = e_{Pi}, \quad \Omega_i(0) = 0. \quad (31)$$

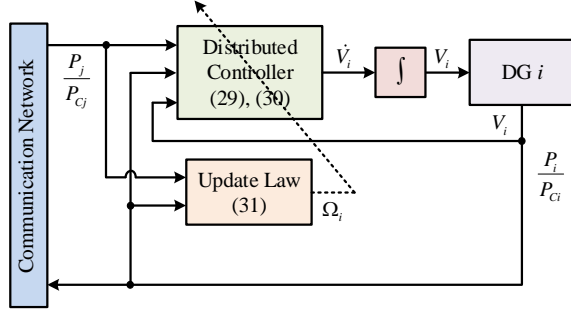


Fig. 2. Diagram of the distributed controller for DG i .

Rewriting (29) in a compact form gives

$$\dot{V} = -k_P \langle V \rangle P_C^{-1} e_P - k_V \langle V \rangle \hat{e}_V \quad (32)$$

where $\hat{e}_V = [\hat{e}_{V1}, \hat{e}_{V2}, \dots, \hat{e}_{Vn}]^T$ is expressed as

$$\hat{e}_V = \ln V + k_P P_C^{-1} \Omega - (\ln v_m^*) \mathbf{1}_n \quad (33)$$

with $\Omega = [\Omega_1, \Omega_2, \dots, \Omega_n]^T$ governed by

$$\dot{\Omega} = e_P, \quad \Omega(0) = \mathbf{0}_n. \quad (34)$$

Next, substituting (32) into (25) and (28) yields the closed-loop system dynamics

$$\dot{e}_P = -\mathcal{L}_C P_C^{-1} F(V) \langle V \rangle (k_P P_C^{-1} e_P + k_V \hat{e}_V) \quad (35)$$

and

$$\begin{aligned} \dot{e}_V &= -\frac{\mathbf{1}_n^T P_C \langle V \rangle^{-1}}{\mathbf{1}_n^T P_C \mathbf{1}_n} (k_P \langle V \rangle P_C^{-1} e_P + k_V \langle V \rangle \hat{e}_V) \\ &= -k_V \frac{\mathbf{1}_n^T P_C \hat{e}_V}{\mathbf{1}_n^T P_C \mathbf{1}_n}. \end{aligned} \quad (36)$$

The diagram of the proposed distributed controller for DG i is illustrated in Fig. 2. The local information of DG i and the information from its neighbors are used to govern Ω_i and to calculate the bus voltage reference via the distributed control algorithm (29) and (30). Eventually, the proposed algorithm is distributed.

Remark 3: Notice that the proposed algorithm is independent of the distribution line and ZIP load parameters. Hence, with the Kron reduction technique [38], [39], the proposed algorithm can be extended to a general DC microgrid with zero injection buses or simple load ones to achieve load power sharing and source bus voltage regulation.

C. Stability Analysis

In this section, the stability of the closed-loop systems (35) and (36) is analyzed. The following lemmas are introduced to aid the proof of the main theorem.

Lemma 2: Consider $\Omega(t)$, whose update law is designed in (34), then $\mathbf{1}_n^T \Omega(t) = 0, \forall t \in \mathbb{R}_{\geq 0}$ holds.

Proof: According to Proposition 1, left-multiplying both side of (34) yields

$$\begin{aligned} \mathbf{1}_n^T \dot{\Omega}(t) &= \mathbf{1}_n^T e_P(t) = \mathbf{1}_n^T \mathcal{L}_C P_C^{-1} P(t) \\ &= \mathbf{1}_n^T \mathcal{L}_C^T P_C^{-1} P(t) = \mathbf{0}_n^T P_C^{-1} P(t) = 0. \end{aligned} \quad (37)$$

Since the initial value of Ω equals to $\mathbf{0}_n$, integrating both side of (37) yields

$$\mathbf{1}_n^T \Omega(t) = 0 + \mathbf{1}_n^T \Omega(0) = 0. \quad (38)$$

Thus, Lemma 2 holds. ■

Lemma 3: Let Assumption 1 hold. Then

$$\mathcal{L}_C P_C^{-1} F(V) \langle V \rangle P_C^{-1} \mathcal{L}_C \geq \epsilon \mathcal{L}_C P_C^{-1} \langle V \rangle \mathcal{L}_C \quad (39)$$

holds.

Proof: Recalling the definition of $F(V)$ in (5) gives

$$\begin{aligned} F(V) \langle V \rangle &= [\langle V \rangle \mathcal{L}_E + \langle \mathcal{L}_E V \rangle + 2G^* \langle V \rangle + I^*] \langle V \rangle \\ &= \langle V \rangle \mathcal{L}_E \langle V \rangle + \langle V \rangle \langle \mathcal{L}_E V \rangle + 2G^* \langle V \rangle^2 + I^* \langle V \rangle \\ &\quad + \langle P \rangle - \langle P^* \rangle. \end{aligned} \quad (40)$$

Substituting (4) into (40) yields

$$\begin{aligned} F(V) \langle V \rangle &= \langle V \rangle \mathcal{L}_E \langle V \rangle + \langle P \rangle + \langle V \rangle \langle \mathcal{L}_E V \rangle + 2G^* \langle V \rangle^2 \\ &\quad + I^* \langle V \rangle - \langle V \rangle \langle \mathcal{L}_E V \rangle - G^* \langle V \rangle^2 - I^* \langle V \rangle - \langle P^* \rangle \\ &= \langle V \rangle \mathcal{L}_E \langle V \rangle + \langle P \rangle + G^* \langle V \rangle^2 - \langle P^* \rangle. \end{aligned} \quad (41)$$

According to (6), the following implication can be obtained

$$\begin{aligned} \langle P \rangle + G^* \langle V \rangle^2 - \langle P^* \rangle &\geq -\left(\frac{\lambda_2(\mathcal{L}_E)}{\max\{P_{Ci}/V_i\}} - \epsilon \right) P_C \langle V \rangle \\ \Rightarrow \langle V \rangle^{-1} [\langle P \rangle + G^* \langle V \rangle^2 - \langle P^* \rangle] \langle V \rangle^{-1} &\geq -\left(\frac{\lambda_2(\mathcal{L}_E)}{\max\{P_{Ci}/V_i\}} - \epsilon \right) P_C \langle V \rangle^{-1}. \end{aligned} \quad (42)$$

Thereafter, by letting $D = P_C \langle V \rangle^{-1}$ and $N = \langle V \rangle^{-1} [\langle P \rangle + G^* \langle V \rangle^2 - \langle P^* \rangle] \langle V \rangle^{-1}$, (41) and (42) become

$$F(V) \langle V \rangle = \langle V \rangle (\mathcal{L}_E + N) \langle V \rangle \quad (43)$$

and

$$N \geq -\left(\frac{\lambda_2(\mathcal{L}_E)}{\max\{d_i\}} - \epsilon \right) D. \quad (44)$$

Next, according to Lemma 1, the following inequality can be derived

$$\begin{aligned} \mathcal{L}_C P_C^{-1} F(V) \langle V \rangle P_C^{-1} \mathcal{L}_C &= \mathcal{L}_C D^{-1} (\mathcal{L}_E + N) D^{-1} \mathcal{L}_C \\ &\geq \epsilon \mathcal{L}_C D^{-1} \mathcal{L}_C = \epsilon \mathcal{L}_C P_C^{-1} \langle V \rangle \mathcal{L}_C. \end{aligned} \quad (45)$$

Thus, (39) holds. ■

Next, the main results for the DC microgrid with the designed distributed control algorithm (32) are stated in the following theorem.

Theorem 1: Consider a DC microgrid modeled by (4) satisfying Assumption 1. If the distributed control algorithm is designed as in (32) with the update laws (33) and (34), then the power sharing error e_P and the voltage regulation error e_V can converge to zero asymptotically. The control objectives defined in Section II-C can be achieved.

Proof: Firstly, define a Lyapunov function of e_P, e_V , and \hat{e}_V as

$$W = W_1 + W_2 + W_3 \quad (46)$$

where

$$W_1 = \frac{1}{2} e_P^T Q e_P \quad (47)$$

with Q is a positive definite matrix satisfying $\mathcal{L}_C Q \mathcal{L}_C = \mathcal{L}_C$ [40],

$$W_2 = \frac{1}{2} e_V^2 \quad (48)$$

and

$$W_3 = \frac{\rho}{2} \hat{e}_V^T \hat{e}_V. \quad (49)$$

With the help of (24), substituting (35) into the time derivative of W_1 yields

$$\begin{aligned} \dot{W}_1 &= -e_P^T Q \mathcal{L}_C P_C^{-1} F(V) \langle V \rangle (k_P P_C^{-1} e_P + k_V \hat{e}_V) \\ &= -P^T P_C^{-1} \mathcal{L}_C P_C^{-1} F(V) \langle V \rangle (k_P P_C^{-1} e_P + k_V \hat{e}_V) \\ &= -k_P e_P^T P_C^{-1} F(V) \langle V \rangle P_C^{-1} e_P \\ &\quad - k_V e_P^T P_C^{-1} F(V) \langle V \rangle \hat{e}_V. \end{aligned} \quad (50)$$

According to Lemma 3 and the definition of e_P in (24), the first term in (50) becomes

$$\begin{aligned} &e_P^T P_C^{-1} F(V) \langle V \rangle P_C^{-1} e_P \\ &= P^T P_C^{-1} \mathcal{L}_C P_C^{-1} F(V) \langle V \rangle P_C^{-1} \mathcal{L}_C P_C^{-1} P \\ &\geq \epsilon P^T P_C^{-1} \mathcal{L}_C P_C^{-1} \langle V \rangle \mathcal{L}_C P_C^{-1} P = \epsilon e_P^T P_C^{-1} \langle V \rangle e_P. \end{aligned} \quad (51)$$

Thereafter, substituting (51) into (50) yields

$$\begin{aligned} \dot{W}_1 &\leq -\epsilon k_P e_P^T P_C^{-1} \langle V \rangle e_P \\ &\quad - k_V e_P^T P_C^{-1} F(V) \langle V \rangle \hat{e}_V \\ &\leq -\epsilon k_P e_P^T P_C^{-1} \langle V \rangle e_P + \frac{\rho k_V}{2} \hat{e}_V^T \hat{e}_V \\ &\quad + \frac{k_V}{2\rho} e_P^T P_C^{-1} \langle V \rangle F^T(V) F(V) \langle V \rangle P_C^{-1} e_P \end{aligned} \quad (52)$$

where $\rho \in \mathbb{R}^+$ is a constant satisfying

$$k_V F^T(V) F(V) \leq \rho \epsilon k_P P_C \langle V \rangle^{-1}. \quad (53)$$

Hence, (52) becomes

$$\dot{W}_1 \leq -\frac{\epsilon k_P}{2} e_P^T P_C^{-1} \langle V \rangle e_P + \frac{\rho k_V}{2} \hat{e}_V^T \hat{e}_V. \quad (54)$$

Next, by recalling (36), the time derivative of W_2 can be expressed as

$$\dot{W}_2 = -k_V e_V \frac{\mathbf{1}_n^T P_C \hat{e}_V}{\mathbf{1}_n^T P_C \mathbf{1}_n}. \quad (55)$$

With the help of Lemma 2, substituting (33) into (55) yields

$$\begin{aligned} \dot{W}_2 &= -k_V e_V \left[\frac{\mathbf{1}_n^T P_C \ln V}{\mathbf{1}_n^T P_C \mathbf{1}_n} + k_P \frac{\mathbf{1}_n^T \Omega}{\mathbf{1}_n^T P_C \mathbf{1}_n} \right. \\ &\quad \left. - \frac{\mathbf{1}_n^T P_C \mathbf{1}_n}{\mathbf{1}_n^T P_C \mathbf{1}_n} \ln v_m^* \right] \\ &= -k_V e_V (\ln v_m - \ln v_m^*) = -k_V e_V^2. \end{aligned} \quad (56)$$

Thereafter, by evoking (33), the time derivative of (49) is

$$\dot{W}_3 = \rho \hat{e}_V^T [\langle V \rangle^{-1} \dot{V} + k_P P_C^{-1} \dot{\Omega}]. \quad (57)$$

Substituting (32) and (34) into (57) yields

$$\begin{aligned} \dot{W}_3 &= \rho \hat{e}_V^T (-k_P P_C^{-1} e_P - k_V \hat{e}_V + k_P P_C^{-1} e_P) \\ &= -\rho k_V \hat{e}_V^T \hat{e}_V. \end{aligned} \quad (58)$$

TABLE I
DC MICROGRID SYSTEM PARAMETERS

	Parameter	Value
Line Conductance (p.u.)	G_{12}, G_{13} G_{23}, G_{34}	5.760, 4.800 3.840, 4.608
DGs' Rated Powers (p.u.)	P_{C1}, P_{C2} P_{C3}, P_{C4}	1/3, 1/6 1/3, 1/6
Load Parameters (p.u.)	$G_1^*, G_2^*, G_3^*, G_4^*$ $I_1^*, I_2^*, I_3^*, I_4^*$ $P_1^*, P_2^*, P_3^*, P_4^*$	0.1, 0.0, 0.1, 0.1 0.1, 0.1, 0.0, 0.1 0.0, 0.1, 0.1, 0.1
Communication Network	a_{12}, a_{13}, a_{34}	1

TABLE II
ZIP LOAD PROFILES

Time	Z	I	P	Time	Z	I	P
0.0 – 0.5 s	off	off	off	2.0 – 2.5 s	on	on	off
0.5 – 1.0 s	on	off	off	2.5 – 3.0 s	on	off	on
1.0 – 1.5 s	off	on	off	3.0 – 3.5 s	off	on	on
1.5 – 2.0 s	off	off	on	3.5 – 4.0 s	on	on	on

Finally, by combining (54), (56), and (58), the time derivative of W is given as

$$\dot{W} \leq -\frac{\epsilon k_P}{2} e_P^T P_C^{-1} \langle V \rangle e_P - k_V e_V^2 - \frac{\rho k_V}{2} \hat{e}_V^T \hat{e}_V \quad (59)$$

which is a negative definite function with respect to e_P , e_V , and \hat{e}_V . Thus, the power sharing error e_P , the voltage regulation error e_V , and the estimated voltage regulation error \hat{e}_V will converge to zero asymptotically. The control objectives defined in Section II-C are achieved. ■

V. SIMULATION STUDIES

A detailed switch-level, 4-bus DC microgrid is adopted to test the performance of the proposed control algorithm as illustrated in Fig. 1. The power converters are selected as non-inverting buck-boost ones. The system parameters are listed in Table I, along with the ZIP load profiles listed in Table II. Eight possible load conditions are considered in the simulations to validate the effectiveness of the proposed control algorithm. The control parameters are selected as $k_P = 0.5$ and $k_V = 8$. In this study, the bus voltages are initialized as 0.98 p.u. To test the voltage regulation performance, a step change of the desired weighted geometric mean of the bus voltages, v_m^* , is selected as

$$v_m^* = \begin{cases} 1.00 & , \quad 0 \leq t < 1.7 \\ 1.01 & , \quad t \geq 1.7 \end{cases}. \quad (60)$$

In this test, the proposed control algorithm is compared with the state-of-the-art solution in [31]. The simulation results are given in Figs. 3-8.

From Fig. 3-4, one can see that with the algorithm in [31] and our proposed one, the proportional power sharing is achieved successfully under all load conditions with a power sharing ratio of 2 : 1 : 2 : 1. All DGs' output powers can converge to their equilibria in about 0.2 s.

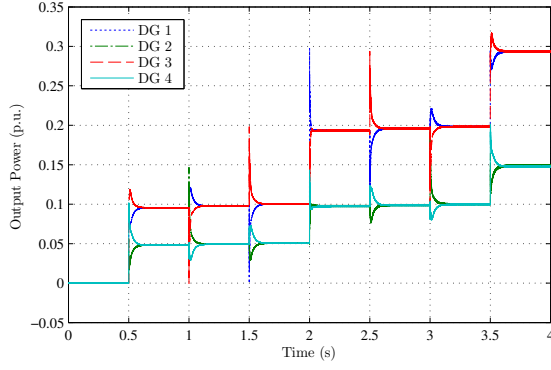


Fig. 3. DGs' output powers with the algorithm in [31].

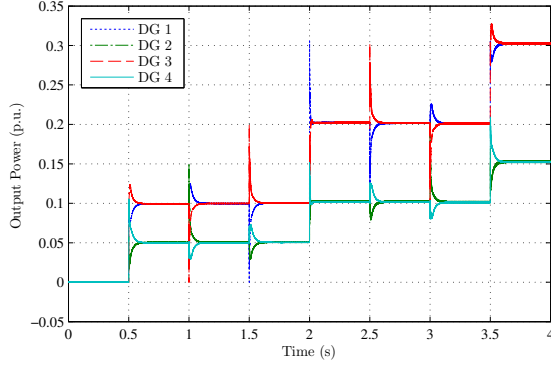


Fig. 4. DGs' output powers with the developed algorithm.

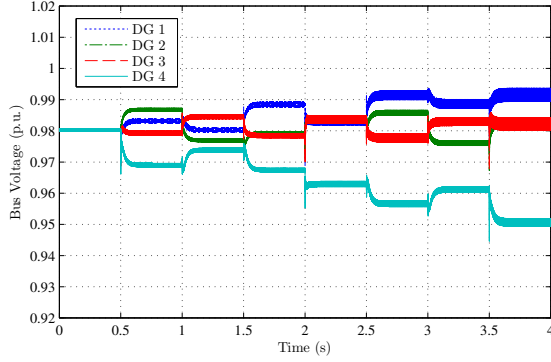


Fig. 5. DGs' bus voltages with the algorithm in [31].

The trajectories of the bus voltages with the control algorithms in [31] and in this study are given in Figs. 5-6, respectively. As presented in Fig. 5, since no active bus voltage regulation strategy is considered in [31], the bus voltages can only be maintained around their initial values, i.e., 0.98 p.u. Compared with the bus voltage performance in Fig. 5, the bus voltages with the developed algorithm in Fig. 6 can be maintained within 0.95 p.u. to 1.05 p.u. for all time. Due to the large changes of the total load at 0.5 s, 2.0 s, and 3.5 s as presented in Table II, the transient performance of the bus voltages at these time instants is slightly degraded. However, notice that a total of 0.3 p.u. load change is not common in practice, which is only simulated to better evaluate the performance of the proposed control algorithm.

In Figs. 7-8, the trajectories of the weighted geometric mean

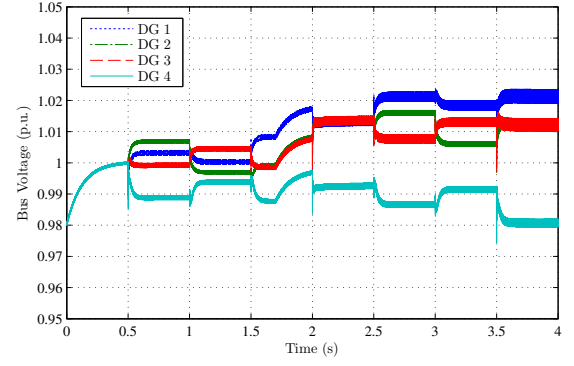


Fig. 6. DGs' bus voltages with the developed algorithm.

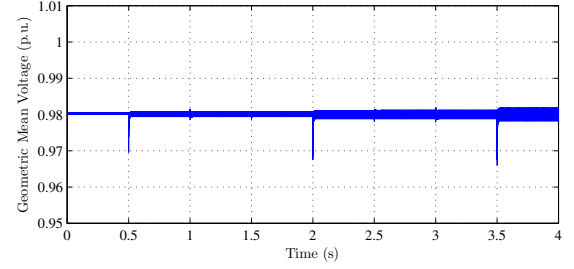


Fig. 7. Weighted geometric mean of the bus voltages with the algorithm in [31].

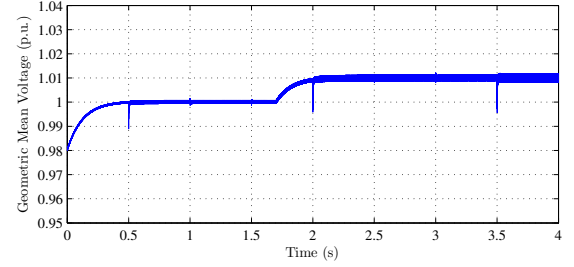


Fig. 8. Weighted geometric mean of the bus voltages with the developed algorithm.

of the bus voltages are given. One can see that the algorithm in [31] fails to regulate the bus voltages with a user-defined voltage reference v_m^* . By comparison, the proposed algorithm can achieve the bus voltage regulation within about 0.5 s. In addition, as illustrated in Fig. 4 and 8, there is no noticeable effect on the power sharing performance during the voltage regulation process.

Thus, with the proposed consensus-based algorithm, the proportional power sharing and the regulation of the weighted geometric mean of the bus voltages can be achieved, simultaneously.

VI. CONCLUSION

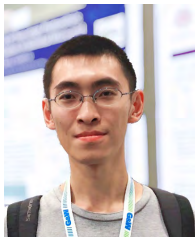
In this paper, a consensus-based power sharing and voltage regulation algorithm is presented for DC microgrids with ZIP loads. With the help of the properties of the Laplacian matrices, a lemma is developed to aid the stability analysis of the system. Through Lyapunov analysis, the power sharing and the voltage regulation errors are proved to converge to zero asymptotically. Proportional power sharing and regulation

of the weighted geometric mean of the bus voltages are achieved. Finally, simulation studies on a 4-bus DC microgrid are conducted to illustrate the merits of the proposed control algorithm.

Further studies, including the determination of $\lambda_2(\mathcal{L}_E)$ in a complex electrical network, and the methods to deal with the uncertainties in communication links, are of interest for DC microgrids.

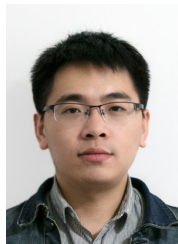
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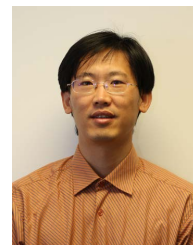
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