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Shape Grammars for Innovative Hybrid Typological Design

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Abstract. *This paper describes a new methodology of deriving innovative hybrid designs using shape grammars of heterogeneous designs. The method is detailed within three phases of shape grammars: analysis, synthesis and evaluation. In the analysis phase, the research suggests that original rules of each design component are grouped in subclass rule sets to facilitate rule choices. Additionally, adding new hybrid rules to original rules expands the options available to the grammar user. In the synthesis phase, the research adopts state labels and markers to drive the design generation. The former is implemented with a user guide grammar to ensure hybridity in the generated design, while the latter aims to ensure feasible designs. Lastly evaluation criteria are added to measure the degree of innovation of the hybrid designs. This paper describes the derivation of hybrid minaret designs from a corpus of heterogeneous traditional minaret designs.*
Keywords. *Shape grammar; parallel grammar, hybrid design; typology.*

Shape grammar techniques for innovative hybrid design

Innovation techniques in shape grammars such as transformation and substitution are well established. On the other hand, hybrid adaption in which "multiple precedents are incorporated into a new design" (Oxman & Oxman, 1992) has received less attention. Hybridization is a process of cross-breeding between heterogeneous designs in the corpus of antecedents. There are few practices of hybridization in shape grammars (Chase & Ahmad, 2005). One example, generating cross-over vehicles in product design, depends on mixing modified rules which embody different features than the original car brands using parametric curve modification rules (Orsborn, Cagan, Pawlicki, & Smith, 2006). However, in architecture, parametric variation of universal rules is not enough to represent the diverse characters of heterogeneous classes of designs. This paper describes an attempt to identify a shape grammar process for deriving innovative hybrid designs.

Shape grammars are used to describe typologies of past and present designs and to generate existing designs and new ones. There are two main stages in a shape grammar process: analyzing the corpus of antecedents to extract the rules that govern their compositions; and synthesizing grammar rules to derive existing and new designs. In some cases, an evaluation phase is added depending on the problem at hand. In this study, the three stages are put forward to evolve innovative hybrid designs. The analysis phase is defined by the author of the grammar, and the synthesis and evaluation phases are proposed to be done by a grammar user.

Analysis phase: Identifying rules of a heterogeneous corpus of antecedents

This study considers that a heterogeneous class of building type consists of designs having the same components but different configurations. A hybrid design results from mixing components from different designs. In this case, there are multi-choice rules in each step of rule application which need to be organized and increased as follows:

Arranging rules in subclass rule sets

In terms of organization, the grammar user should be able to compare the available choices easily and to select and apply one of them. To enable this, the varied rules of each component can be grouped in a subclass rule set which contains more than one rule having the same shapes, spatial relations and main markers in their left hand side (LHS), and different shapes, spatial relations and (or) markers in their right hand side (RHS). Figure 1 clarifies how most grammar rules for interior layout of Palladian villas written by Stiny and Mitchell (1978) (Figure 1 left) can be re-formed in a subclass rule set (Figure 1 right) to facilitate rule comparison, selection and application simultaneously.

Adding new hybrid rules

To increase available choices in each subclass rule set, new hybrid rules are added to the original grammar rules. The hybrid rule combines two original rules (parents) by keeping the same LHS rule of one of them, and merging all or part of shapes, spatial relations and (or) markers of both their RHS rules. Figure 2 shows the addition of two new hybrid rules to original rules of Palladian villa grammars written by Stiny and Mitchell (1978).

The synthesis phase: applying rules to derive a hybrid design

The study analyzed a heterogeneous corpus of 12 traditional minaret designs. The sample is drawn from different geographical regions and times. The

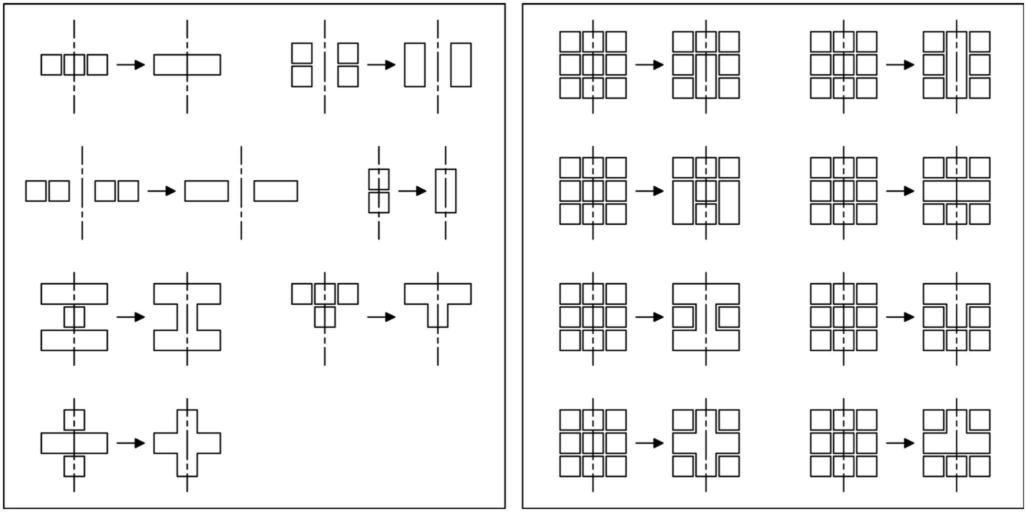


Figure 1
Left: Grammar rules for interior layout of Palladian Villas (Stiny & Mitchell, 1978)
Right: Subclass rule set for some interior layout rules of Palladian Villas

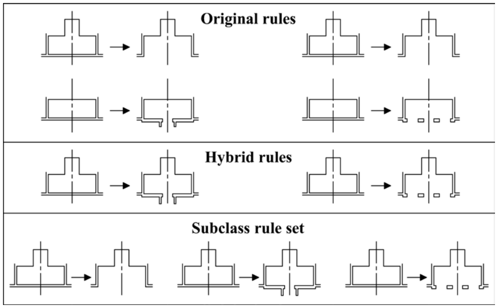


Figure 2
Hybrid rule derived from merging two original rules

formal composition of antecedents differs in shape, number and sequence of their components. The grammar defines 54 original rules that embody six subclass rule sets of minaret base, body, joint, balcony, lantern and head. In addition, 90 hybrid rules have been identified. Each one is derived by merging two original rules from the same subclass rule set.

To derive hybrid designs, the grammar constrains rule application to limit the ways that rules apply. Both state labels and spatial labels (markers) (Knight, 1994) are used to obtain feasible hybrid designs. State labels are alphanumeric characters which are attached to rules to ensure hybridity by mixing rules derived from varied designs. Markers, on the other hand, are symbols that are attached to shapes to ensure valid designs.

State labels of shape grammar for hybrid designs

Each rule in subclass rule set has both constant and variable state labels. Constant state labels are attached to left hand side (LHS) of each rule to indicate the antecedent(s) in the corpus from which the rule is derived. For example, in the shape grammar for hybrid minaret design, there are 12 antecedents in corpus *n*; each one is symbolized as follows:

n={d1, d2, d3,....d12}

On the other hand, variable state labels are attached to the right hand side (RHS) of each rule in a subclass rule set. In shape grammars for hybrid minaret designs, the variable *n*1 is attached to all rules of the first subclass rule set: minaret base; and *nx* is attached to all rules of the other subclass rule sets: body, joint...etc. This is due to the variation in both the number and the sequence of minaret components in the analyzed antecedents. Therefore the variable *x* is an ascending integer starting from 2 to *y*, and *y* is a variable to represent the total number of rules needed to derive the minaret which varies from 3 to 12 in the analyzed corpus of antecedents.

The values of *n*1 and *nx* are specified by a user guide grammar for hybrid design as a parallel grammar which is “a network of two or more grammars that operate simultaneously” (Knight, 2003). The user guide grammar is added to each rule to define the set of possible LHS labels for the next stage of application. In this case, the set of labels *n*1 excludes the LHS labels of the current first rule from the set *n*; while the set *nx* of other rules excludes the LHS labels of the current rule from the set of state labels of the previous rule (*nx*-1).

*n*1= {*n*\LHS labels of the current rule}

nx= {*n*(*x*-1)\LHS labels of the current rule}

Examples of these state labels are shown in Figure 3. The upper row is one of the rules of a subclass rule set of minaret bases that initiates the design derivation while the lower row is one of the rules of a subclass rule set of minaret bodies which can be added at subsequent steps of design derivation.

However, there are two cases in which the value of *nx* can be {Ø}. The first case results from a

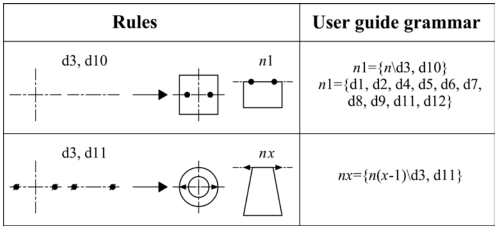


Figure 3
Examples of rules of shape grammar for hybrid minaret designs

non-matching condition between *nx* values and LHS labels of the next subclass rule set. The other case happens when state labels of all antecedents are exhausted in the previous steps of a design derivation. In both cases the value of *nx* is replaced by nx^* which excludes the current LHS labels from the set *m*. The set *m* contains labels of antecedents that have the least number of applied rules in the previous steps of design derivation. The least number is one rule in the case of hybrid minarets derived from less than 12 original rules or less than 6 hybrid rules; and one and two rules in the case of designs derived from 6 to 12 hybrid rules.

$nx^*=\{m\backslash\text{LHS labels of the current rule}\}$

Additionally, if nx^* is {Ø}, then nx^* is replaced by nx^{**} which is all the state labels of *n* that are not in *nx* and nx^*, as follows:

$nx^{**}=\{n\backslash(nx+nx^*)\}$

To ensure adding the minaret head in the last step of a grammar, the symbol (‘) is added to the state labels of the LHS of all rules that add head

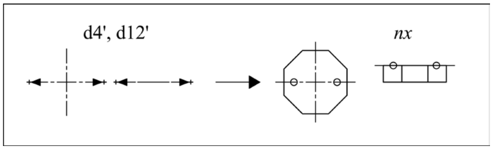


Figure 4
Adding the symbol (‘) to LHS rules that add head markers

markers on the right hand side at the penultimate stage of rule application, such as the following rule in figure 4:

In this case, if the total number of rules needed to generate the minaret is (*y*), then the user’s guide grammar of $nx=n(y\text{-}2)$ should add the symbol (‘) to a set of antecedent labels at that stage, such as

$n(y\text{-}2)=\{d5', d8', d9'\}$

Therefore, the penultimate rule to be applied must add head markers to the generated design in order to end with adding a head to the generated shape. In addition, the state labels of RHS of adding the head rule is defined in the user guide grammar as zero, as follows:

If $nx=ny$, then $ny=0$

The only rule in the grammar that has state labels (0) in its LHS is the termination rule that delete the centre lines from top and front views of the generated design.

Markers of shape grammar for hybrid designs

On the other hand, markers play an important role in deriving acceptable designs. Main markers and secondary markers are used in the grammar for hybrid minaret designs to ensure that each component is generated and placed in a reasonable relationship with other components.

Main markers symbolize component types such as body (●), joint (◀), balcony (■), lantern (□) and head (○). Secondary markers, on the other hand, are part of the main markers which aim to limit the rule application within specific constraints derived from the antecedents. The constraints using grammar markers have two functions: the first concerns the logical sequence of component types while the second concerns the proper sequence of component shapes, as follows:

Firstly, markers keep the valid sequence of minaret components which starts, in most cases, with a minaret base and finishes, in all cases, with a minaret head. In addition, the secondary markers of joint and balcony components prevent forbidden cases such as adding either a joint followed by a lantern or a balcony followed by a lantern above the minaret base directly; or adding either joint followed by body or balcony followed by body above the lantern. Details of constraints on minaret components sequence using joint markers and balcony markers are clarified in Table 1.

The second type of constraint in the antecedents keeps the proper relations between the boundaries of minaret components. If the minaret components share the same boundary diameter, then the added shapes must be inscribed in the boundary of the component underneath it, except the lobular and stellar shapes that can be circumscribed by octagonal and circular shapes. Examples in Figure 5 clarify adding an octagon or circle (dashed line) above a square (continuous line) or adding a circle (dashed line) above an octagon (continuous line) which can share the same diameter. However, other cases are avoided such as adding square shapes above octagons or circles, or adding octagons above circles.

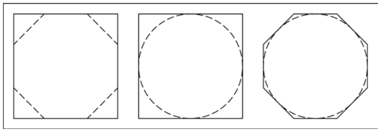


Figure 5
Constraints on the sequence of shapes of minaret components

Table 1
Functions of markers in shape grammars for hybrid minaret designs

	Markers	Functions
Body Markers	●	**Adding any body shape**
	●	**Adding octagonal, circulrar, lobular or stellar shapes**
	●	**Adding circulrar, lobular or stellar shapes**
Joint Markers	◀	**Adding any component above joint**
	●◀	**Adding a body or a balcony followed by a body**
	+◀	**Adding a head or a balcony followed by a head**
Balcony Markers	■	**Adding any balcony shape**
	■	**Adding an octagonal balcony shape**
	■	**Adding a circular balcony shape**
	●■	**Adding a balcony followed by a body**
	○■	**Adding a balcony followed by a lantern or a head**
Lantern Markers	□	**Adding any lantern shape**
	⧄	**Adding a circular lantern shape**
Head Markers	○	**Adding any head shape**
	∅	**Adding a circular head shape**

Table 1 defines the function of the main and secondary markers in shape grammars of hybrid minaret designs.

The evaluation phase: assessing innovation in hybrid designs using shape grammars

Applying shape grammars for hybrid design is associated with evaluation descriptions that follow compositions and drive the design derivation. The proposed system works on two levels: rule evaluation and grammar evaluation, as parallel grammars (Knight, 2003; Stiny, 1981) that compute with text and numbers. By linking evaluation criteria to each rule, two types of feedback are provided to the user. They are:

1. Rule evaluation values feedback before choosing the rule. It has default values added to each rule in grammar.
2. Grammar evaluation values feedback after applying the rule. Its values are computed and triggered automatically at shape grammar runtime depending on the choice of shape rules.

The feedback obtained from the rules' evaluation values helps the grammar user to control grammar evaluation values which in turn can be used to indicate preferences for the next rule choices. Evaluation criteria in both levels use metrics of the degree of innovation in hybrid design. The grammar proposes that the generated design with best hybridity is the more differentiated and varied than the antecedents in the corpus. This hypothetical definition of best hybridity in minaret designs using shape grammars can be measured via criteria of rule assessment such as rule prevalence, rule geometrical difference and rule sequential difference, as follows:

Rule prevalence = the number of antecedents that the current rule is derived from / the total number of antecedents in corpus

Rule geometrical difference = 1- (the number of antecedents in the same subclass rule set having similar geometry / the total number of antecedents in corpus)

Rule sequential difference = 1- (the number of antecedents in the same subclass rule set having a similar sequence / the total number of antecedents in corpus)

On the other hand, innovative hybridity of the generated design can be quantified via concepts such as diversity, abundance, matching degree, geometrical difference and sequential difference. These criteria are based on comparisons between the generated designs and the antecedents in the corpus. For example, diversity measures the percentage of antecedents in the generated design as follows:

Design diversity = the number of antecedents that the generated design is derived from / the total number of designs in corpus

Abundance measures the average number of antecedents in each rule of the generated design, as follows:

Design abundance = the sum of the number of antecedents in the applied rules / the number of applied rules

Matching degree measures the highest percentage of similarity between one antecedent in the corpus and the generated design, as follows:

Matching degree = the highest number of rules derived from one antecedents / the number of applied rules

In the case of designs derived from hybrid rules only or a combination of hybrid and original rules, each hybrid rule is multiplied by (0.5) for matching degree calculations.

Both geometrical difference and sequential difference measures the average of these metrics in the applied rules, as follows:

Design geometrical difference = the total of rule geometrical difference values of the applied rules / the number of applied rules

Design sequential difference = the total of rule sequential difference values of the applied rules / the number of applied rules

Implementation of shape grammars for hybrid minaret design

The study applied the process above to derive hybrid minaret designs using parallel grammars which consist of a parametric shape grammar, a user guide grammar and an evaluation grammar. Twelve antecedents are analyzed by means of labeled 2D shape grammars to represent their top and front views. The aim of the application is to derive new hybrid designs with best innovation measures using original rules and hybrid rules. Even though it is possible to derive new hybrid designs from a mixture of original and hybrid rules, the study has concentrated on deriving hybrid designs from original rules and hybrid rules separately. The reason is to explain the differences in innovation criteria values between hybrid designs generated by original rules and those generated by hybrid rules. In addition, the study has sought to track the impact of using different numbers of rules on innovation values of hybrid minaret designs derived from original or hybrid rules.

The initial shape of this grammar consists of centre lines of minaret top and front views with a state label *n* to identify its antecedents. The initial shape exists in the LHS of all rules of the base subclass rule

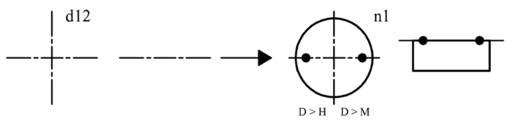
Rule no.	Example: Hybrid design derivation - 6 Original rules	User guide grammar for hybid design	Evaluation of rule	Evaluation of grammar
OR 7a		n1={n\d12} n1={d1, d2, d3, d4, d5, d6, d7, d8, d9, d10, d11}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= 0.083 Design abundance= 1.0 Matching degree= 1.0 Geometrical difference= 0.916 Sequential difference= 0.5

Figure 6
Applying a rule from base subclass rule set

Figure 7
Applying a rule from body subclass rule set

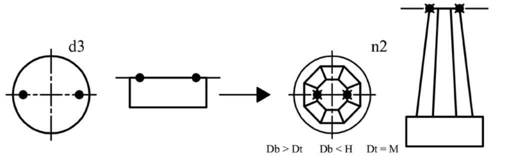
Rule no.	Example: Hybrid design derivation - 6 Original rules	User guide grammar for hybid design	Evaluation of rule	Evaluation of grammar
OR 4b		n2={n1\d3} n2={d1, d2, d4, d5, d6, d7, d8, d9, d10, d11}	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.5	Design diversity= 0.166 Design abundance= 1.0 Matching degree= 0.5 Geometrical difference= 0.916 Sequential difference= 0.5

Figure 8
Applying a rule from body subclass rule set

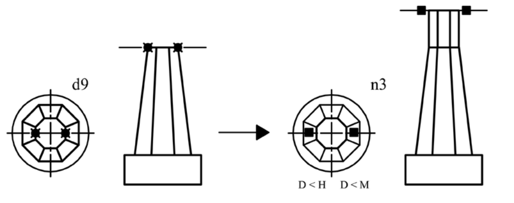
Rule no.	Example: Hybrid design derivation - 6 Original rules	User guide grammar for hybid design	Evaluation of rule	Evaluation of grammar
OR 12b		n3={n2\d9} n3={d1, d2, d4, d5, d6, d7, d8, d10, d11}	Rule Prevalence=0.083 Geometrical difference=0.583 Sequential difference=0.916	Design diversity= 0.25 Design abundance= 1.0 Matching degree= 0.333 Geometrical difference= 0.805 Sequential difference= 0.638

Figure 9
Applying a rule from joint subclass rule set

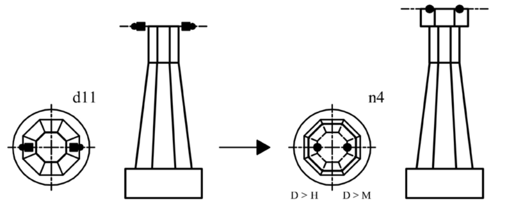
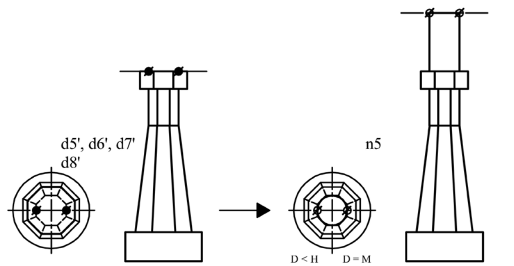
Rule no.	Example: Hybrid design derivation - 6 Original rules	User guide grammar for hybid design	Evaluation of rule	Evaluation of grammar
OR 5d		n4={n3\d11} n4={d1', d2', d4', d5', d6', d7', d8', d10'}	Rule Prevalence=0.083 Geometrical difference=0. 583 Sequential difference=0.5	Design diversity= 0.333 Design abundance= 1.0 Matching degree= 0.25 Geometrical difference= 0.749 Sequential difference= 0.603

Figure 10
Applying a rule from lantern subclass rule set

Rule no.	Example: Hybrid design derivation - 6 Original rules	User guide grammar for hybid design	Evaluation of rule	Evaluation of grammar
OR 8b		n5={n4\d5, d6, d7, d8} n5={d1, d2, d4, d10}	Rule Prevalence=0.333 Geometrical difference=0. 416 Sequential difference=0. 583	Design diversity= 0.666 Design abundance= 1.6 Matching degree= 0.2 Geometrical difference= 0.682 Sequential difference= 0.599

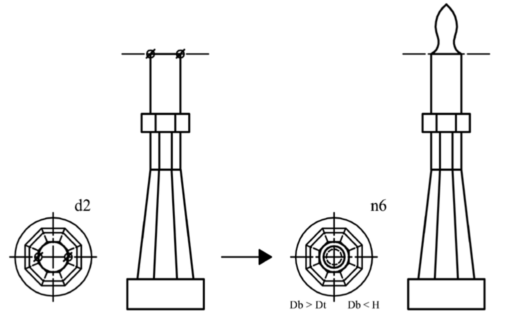
Rule no.	Example: Hybrid design derivation - 6 Original rules	User guide grammar for hybid design	Evaluation of rule	Evaluation of grammar
OR 2f		n6=0	Rule Prevalence=0.083 Geometrical difference=0.916 Sequential difference=0.0	Design diversity= 0.75 Design abundance= 1.5 Matching degree= 0.166 Geometrical difference= 0.721 Sequential difference= 0.499

Figure 11
Applying a rule from head subclass rule set

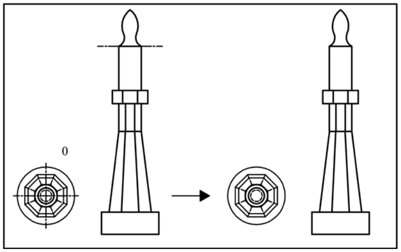


Figure 12
Termination rule

set that have at their RHS either a base and markers or only markers if there is no base in the antecedents. Figures 6-12 show an example of deriving hybrid minaret design using six original rules. The first rule adds a cylindrical base with body markers to the initial shape as shown in Figure 6.

Because the markers in the RHS of the applied rule are body main markers, the second rule is chosen from the body subclass rule set as shown in Figure 7.

The markers in RHS of the applied rule in Figure 7 are secondary markers of body which allows the addition of octagonal, circular or other shapes. The third rule thus adds an octagonal body component with balcony markers as shown in Figure 8.

The fourth rule adds a balcony component with body markers as shown in Figure 9. In this example, the number of rules needed to derive the hybrid minaret is six. Therefore, the set $n4$ as $n(y\text{-}2)$ adds the symbol (') to its state labels to constrain the next rule choice with rules that add only head markers.

Accordingly, the fifth rule adds head markers above the body as shown in Figure 10. The following rule adds the head component without any marker. Because the sixth rule is the last rule in the design derivation, the value of $nx=ny=n6$ is 0 in the user guide grammar. In addition, the innovation values of the generated design are the final values, as shown in Figure 11.

Finally, the last rule is the termination rule with the state label *0* in its LHS. This rule deletes the centre lines of the top and front views of the generated design (Figure 12).

Conclusions

The paper has shown the practical applicability of shape grammars to generate innovative hybrid designs. The user is able to explore a variety of hybrid design alternatives in a short time compared to traditional design methods. Results of innovation measures of 40 examples derived using (6 and 10) original rules and (6 and 10) hybrid rules are analyzed. Comparison between the hybrid designs generated by original rules and the ones generated by hybrid rules showed that the latter is more efficient in innovation criteria such as diversity, abundance and geometrical difference. On the other hand, distinctions between hybrid designs composed of 6 and 10 rules revealed that minarets composed of 10 rules have, in most cases, higher diversity values than minarets composed of 6 rules. However, higher values of the other innovation measures fluctuate between the hybrid designs consisting of 6 and 10 rules.

By associating innovation criteria values with rules and grammar, users can identify rule changes that have positive and negative effects on the design innovation values. The grammar users will be aware of how their decisions of rule choice affect the degree of innovation in hybrid minaret designs.

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