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Optimal Scheduling and Control of a Multi-Pump Boosting System

Zhenyu Yang and Hakon Børsting

Abstract— **An optimal scheduling and control method for a multiple pump system is proposed from the energy efficient point of view. The model-based optimal problem is first formulated and then converted to be a mixed integer nonlinear programming problem. The proposed method provides an optimal solution regarding to how many and which available pumps should be put into operation when the (head) demand to the system and/or system operating condition changes. The running speeds of operating pumps are also derived by the proposed algorithm. A feedback control mechanism is also introduced into the considered framework in order to enhance the system tracking performance and robustness. A nonlinear programming solution is derived and implemented in a testing facility. The experimental results show a clear and huge potential to improve multi-pump system's efficiency using the proposed algorithm and framework.**

I. INTRODUCTION

Pumps have been widely used in our daily life and work, such as for irrigation, water supply, air conditioning systems, refrigeration, sewage treatment, oil and gas pipeline transportation, marine relevant services etc.. Today over 40% industry processes are using pumps/pump systems [4]. The efficiency of pump systems has a significant impact on the energy consumption of entire pump associated systems. For instance, the energy consumption of pumps can account to 21% of the total power load of a central air-conditioning system in a complex building [5].

Traditional pumps are developed with a fixed speed at which the pump runs in a most efficient manner. In order to handle versatile applications, pumps enhanced with variable speed functionality, which are often referred to as Variable Speed Pumps (VSP), have also been developed and commercialized in recent decades. These VSPs are usually integrated with their speed control modules, thereby they are called *E-pumps* [3]. An E-pump can adapt to varying operating requirements/conditions by adjusting its rotating speed through the integrated PI controller. However, sometimes this flexibility may lead to a quite low system efficiency if the pump operates far beyond its most efficient point [9], [11]. In order to keep the flexibility but meanwhile take care of the system efficiency, in most practical situations, a group of pumps are usually employed for large-scale and versatile applications, such as pump groups used for central air-conditioning system [5], [10], and pump stations used for urban water distribution system [12], [14]. Pumps within a group or station can be significantly different in terms of types (fixed speed or variable speeds), physical sizes, capabilities, hydraulic heads and efficiencies etc. As a consequence of that, an interesting and challenging topic has been raised: How to optimally schedule and control a group of pumps in terms of enhancing its operating efficiency, reducing power consumption and maintenance costs, subject to the condition that the system should satisfy the required performances and safety limitations.

Extensive study can be found about optimal scheduling and control of pump systems [4], [10], [11]. An optimal pump scheduling algorithm for water distribution systems is proposed in [14]. The generalized reduced gradient method is used to derive the optimal control of pump flows and valve positions so as to minimize the entire system operation costs. [5] studied energy efficient control of VSPs in central air-conditioning systems for a complex building. Using the efficiency prediction based on VSP's pump characteristics and models of pressure drops over the entire water network, an optimal pump sequence control is proposed to determine both the operating order and point that pumps should be brought online and off-line. The pump scheduling is a typical nonlinear dynamic optimization problem [1], [2], [8], [9]. Some evolutionary algorithms are also been investigated to handle this type of challenge [7], [8], [12]. Normally, a huge amount of extensive experiments and sufficient data are required for using artificial intelligence methods. Besides that, we observed that most of existing work focused on the entire systems including pumps and their associated environment/facilties in analysis and design. Each group of pumps or pump station is simply modeled as a network node with some specific hydraulic and electric constraints. The configuration of pumps within the group/station and the speed control of each running pump are neglected. Moveover, some practical issues and potential risks in serial-/parallel-pump systems, such as potential cavitation and insufficient suction pressure etc [6], [11], are not considered either. Nevertheless, these issues and risks are very crucial for practical implementation and potential commercialization.

Motivated by model-based control approaches and the consideration of potential industrial application, in this paper we focus on a simple multi-pump boosting system. Within the setup, three VSP are configured in parallel and connected with a simple water circular loop and a storage tank. This configuration exhibits many key features of pumping systems without too many trivialities. Our objective is to investigate some optimal algorithm for scheduling and controlling this pump system, so that the controlled system can follow some expected (head) load demands in a best efficient manner, subject to possible changes in operating conditions. Thereby, the following issues are considered: (i) Optimal strategy

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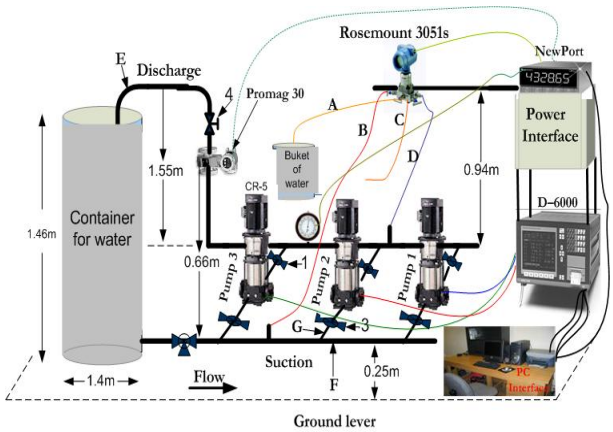

Fig. 1. Schematic diagram of the considered Grundfos Setup

about which available pumps and when they should be put into operation when the demand and/or operating condition possibly changes. This is referred to as a *scheduling problem*; (ii) Determination and control of running speed(s) of operating pump(s). This is referred to as *(real-time) control problem*.

The rest of the paper is followed by a brief introduction of the considered system in Section II; Section III discusses the modeling of pump groups using individual pump's characteristics and experimental verification of these models; Section IV formulates the optimal scheduling and control into a MINLP problem and discusses the CNP solution; Section V discusses the implementation issues and illustrates some experimental results; Finally we conclude the paper in Section VI.

II. Considered System

As shown in Fig.1, the considered system consists of three Grundfos CRE-5 variable speed centrifugal pumps arranged in parallel. The differential pressure over the pump system is measured by a manometer typed of Rosemount 3051s Series. the flow rate of the boosting system is measured by an electromagnetic flow meter typed of Promag 30. The measured pressure and flow rate are transmitted to a NewPort device for display purpose. Moreover, both pressure and flow signals are converted to a Data Acquisition (DAQ) board -NI PCI-6229, which bridges the hardware setup and a PC which has LabVIEW program installed. A power measurement instrument named Power Analyser D-6000 is used to monitor the power consumption of each pump. The measured power consumptions are converted to DAQ board. The scheduling and control algorithms are implemented in LabVIEW environment. The control signal generated by the optimization algorithm is converted by Power Analyzer so as to control motor speeds.

III. Pump Modeling and Validation

The simple static model for pumps is used here. We leave the dynamic modeling as part of future work. The model for a group of pumps is derived based on the Affinity Law and individual pump characteristics. These models are validated by experiments afterwards.

A. Single Variable Pump Model

For a fixed speed pump, the mathematical model can be expressed as set of static relationships among the pump head, flow rate and Brake Horse Power (BHP) [10]. The pump head sometimes is referred to as differential pressure over the pump [5], and it is denoted as H here. The flow rate sometimes is referred to as capability [6], [11], and it is denoted as Q here. In the following we denoted the BHP as P. Then, a single pump model consists of the following two polynomial equations:

$$\begin{aligned} H &= \overline{a}_0 + \overline{a}_1 Q + \overline{a}_2 Q^2, \\ P &= \overline{p}_0 + \overline{p}_1 Q + \overline{p}_2 Q^2 + \overline{p}_3 Q^3. \end{aligned} \tag{1}$$

where system parameters $\overline{a}_i,\ \overline{p}_j$ for $i = 0, 1, 2,\ j = 0, \cdots, 3$ are determined by specific pump characteristics and can be identified by experimental data.

Concerning variable speed pump, the relationship (1) and its parameters will be motor speed dependent. The Affinity Law in pump theory states [10]:

$$\frac{Q(\omega_1)}{Q(\omega_2)} = \frac{\omega_1}{\omega_2},\ \frac{H(\omega_1)}{H(\omega_2)} = \frac{\omega_1^2}{\omega_2^2},\ \frac{P(\omega_1)}{P(\omega_2)} = \frac{\omega_1^3}{\omega_2^3}. \tag{2}$$

where $\omega_1,\ \omega_2$ represent two different operating pump speeds.

Assume the pump model (1) for a VSP at a specific speed $\overline{\omega}_0$ is obtained, and its parameters are $(\overline{a}_0, \overline{a}_1, \overline{a}_2)$ and $(\overline{p}_0, \overline{p}_1, \overline{p}_2, \overline{p}_3)$. According to (2), there is

Lemma 1: The pump model of the considered VSP for any given speed ω has the property:

$$\begin{aligned} H(\omega) &= a_0\omega^2 + a_1\omega Q(\omega) + a_2(Q(\omega))^2, \\ P(\omega) &= p_0\omega^3 + p_1\omega^2 Q(\omega) + p_2\omega(Q(\omega))^2 + p_3(Q(\omega))^3, \end{aligned} \tag{3}$$

where $H(\omega)/Q(\omega)/P(\omega)$ represents the head/flow-rate/BHP of the considered pump at speed ω and

$$\begin{aligned} a_0 &= \tfrac{\overline{a}_0}{\overline{\omega}_0^2},\ a_1 = \tfrac{\overline{a}_1}{\overline{\omega}_0},\ a_2 = \overline{a}_2, \\ p_0 &= \tfrac{\overline{p}_0}{\overline{\omega}_0^3},\ p_1 = \tfrac{\overline{p}_1}{\overline{\omega}_0^2},\ p_2 = \tfrac{\overline{p}_2}{\overline{\omega}_0}, p_3 = \overline{p}_3. \end{aligned}$$

This model (3) is validated through experiments. For instance, the validation of models for 100% and 50% full speeds derived from 75% full speed ($\overline{\omega}_0$) measurement is shown in Fig.2 and Fig.3, respectively. It can be observed that the $H - Q - \omega$ model has a quite good precision, and the $P - Q - \omega$ model also has a reasonable precision even though some slight deviations can be observed.

B. Multi-Identical-Pump Model (at same speeds)

Assume N number of identical pumps arranged in a parallel and they can only run at same speeds, then

Lemma 2: The pump model for N identical pumps in parallel with a common speed ω is

$$\begin{aligned} H_s(\omega) &= a_0^s\omega^2 + a_1^s\omega Q_s(\omega) + a_2^s(Q_s(\omega))^2, \\ P_s(\omega) &= p_0^s\omega^3 + p_1^s\omega^2 Q_s(\omega) + p_2^s\omega(Q_s(\omega))^2 + p_3^s(Q_s(\omega))^3, \end{aligned} \tag{4}$$

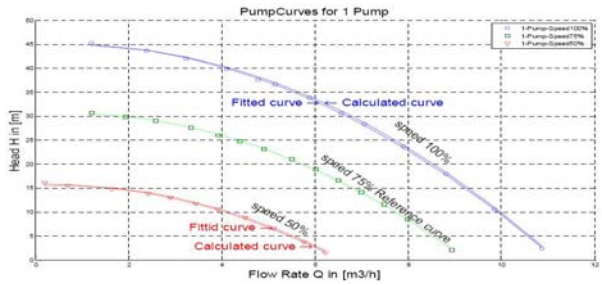

Fig. 2. H-Model Validation for single pump at different speeds

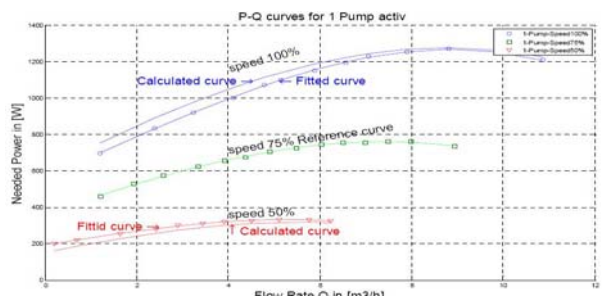

Fig. 3. P-Model Validation for single pump at different speeds

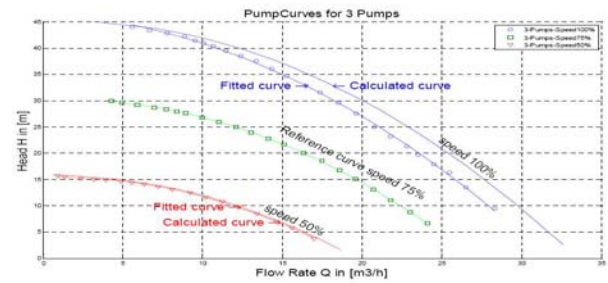

Fig. 4. H-Model Validation for three-pump at Same Speeds

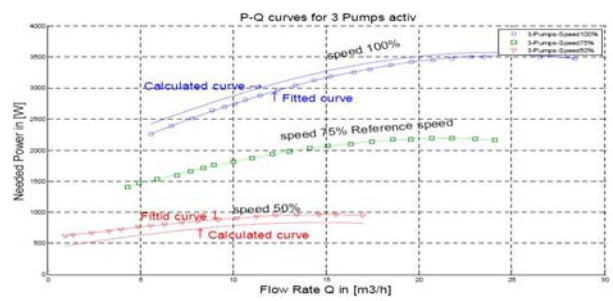

Fig. 5. P-Model Validation for three-pump at same speeds

where $H_s(\omega)/Q_s(\omega)/P_s(\omega)$ represents the head/flow-rate/BHP of the entire pump group at speed ω and system parameters are

$$a_0^s = a_0,\ a_1^s = \tfrac{a_1}{N},\ a_2^s = \tfrac{a_2}{N^2},$$
$$p_0^s = Np_0,\ p_1^s = p_1,\ p_2^s = \tfrac{p_2}{N},\ p_3^s = \tfrac{p_3}{N^2}.$$

Model (4) is further validated through experiments. From Fig.4 it can be observed that the prediction of 50 % full speed is pretty precise, while some deviation can be observed for 100% full speed prediction at high flow rate situation - the model gives higher values than the real system does when the flow rate increases. This could be due to the reduced motor's and driver's efficiencies, which we assume they are 100% efficiency or some fixed values in our prediction model. The comparison of $P-Q-\omega$ models in Fig.5 shows more or less same precision level as for single pump prediction.

C. Multi-Pump General Model

The general multiple (parallel) pump systems could consist of the following scenarios: (i) A group of identical pumps, but they are allowed to run at different speeds; (ii) A group of different pumps. Without loss of generality, in the following we assume there are N pumps with different pump features. $H_i(\omega_i)/Q_i(\omega_i)/P_i(\omega_i)$ represent the ith pump's head/flow-rate/BHP. Instead of using model $H-Q-\omega$, like (1), the $Q-H-\omega$ model is adopted here w.r.t. its better orientation to the following computation. Thereby, the ith pump model is represented by

$$\begin{aligned} Q_i(\omega_i) = &\ \tfrac{1}{\omega_i^3}(b_0^i\omega_i^2 + b_1^i\omega_i H_i(\omega_i) + b_2^i(H_i(\omega_i))^2), \\ P_i(\omega_i) = &\ p_0^i\omega_i^3 + p_1^i\omega_i^2 Q_i(\omega_i) + p_2^i\omega_i(Q_i(\omega_i))^2 \\ &+ p_3^i(Q_i(\omega_i))^3, \end{aligned} \tag{5}$$

A compact formulation of (5) can be derived as

$$\begin{aligned} Q_i &= W_H^{i\,T} B^i W_H^i, \\ P_i &= p_0^i\omega_i^3 + (W_Q^{i\,T} P^i W_Q^i) Q_i, \end{aligned} \tag{6}$$

where

$$W_H^i = [\omega_i \quad H_i]^T,\quad W_Q^i = [\omega_i \quad Q_i]^T,$$
$$B^i = \tfrac{1}{\omega_i^3}\begin{bmatrix} b_0^i & \frac{b_1^i}{2} \\ \frac{b_1^i}{2} & b_2^i \end{bmatrix},\quad P^i = \begin{bmatrix} p_1^i & \frac{p_2^i}{2} \\ \frac{p_2^i}{2} & p_3^i \end{bmatrix},$$

with parameters

$$a_0 = \tfrac{\overline{a}_0}{\overline{\omega}_0^2},\ a_1 = \tfrac{\overline{a}_1}{\overline{\omega}_0},\ a_2 = \overline{a}_2,$$
$$p_0 = \tfrac{\overline{p}_0}{\overline{\omega}_0^3},\ p_1 = \tfrac{\overline{p}_1}{\overline{\omega}_0^2},\ p_2 = \tfrac{\overline{p}_2}{\overline{\omega}_0}, p_3 = \overline{p}_3.$$

Lemma 3: Assume the considered N pumps have the property

$$H_1^{max}(\omega_1) > H_2^{max}(\omega_2) > \cdots > H_N^{max}(\omega_N), \tag{7}$$

this means that shutoff heads of N pumps are different. Then, the model of N pumps in terms of one pump system can be described as

$$\begin{aligned} Q_s(\overline{\omega}) &= \overline{W}_H^T \overline{B}_\omega \overline{W}_H, \\ P_s^N(\overline{\omega}) &= \overline{P}_0\overline{\omega} + \overline{W}_Q^T \overline{P}_Q \overline{W}_Q, \end{aligned} \tag{8}$$

where

$$\begin{aligned} &\overline{\omega} = [\omega_1 \quad \omega_2 \quad \cdots \quad \omega_N]^T, \\ &\overline{P}_0 = [p_0^1 \quad p_0^2 \quad \cdots \quad p_0^N]^T, \\ &\overline{W}_H = [W_H^1 \quad W_H^2 \quad \cdots \quad W_H^N], \\ &\overline{W}_Q = [W_Q^1 \quad W_Q^2 \quad \cdots \quad W_Q^N], \\ &\overline{B}_\omega = diag\{B^i\}_{N\times N}, \\ &\overline{P}_Q = diag\{P^i Q_i\}_{N\times N}. \end{aligned}$$

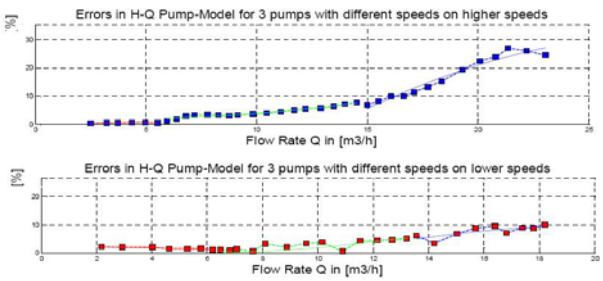

Fig. 6. H-Prediction Errors for Three-pump at Different Speeds

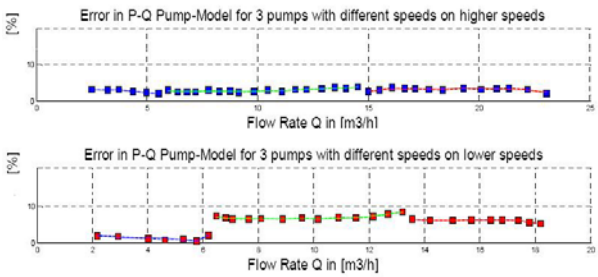

Fig. 7. P-Prediction Errors for Three-pump at Different speeds

Model (8) is further validated based on the available setup. From Fig.6 it can be observed that the $H-Q-\omega$ prediction error for lower speeds is within 10%, while the prediction error for higher speeds starts to over 10% when the flow rate is over $15m/h$. The worst case is about 25% prediction error at very high flow rate. On the contrast, the prediction errors of $P-Q-\omega$ models are successfully controlled within 10%, as shown in Fig.7.

D. System Curve

The system curve needs to be obtained in order to determine pump's/pump system's operating point [10]. The system curve models the terminal impedance that the pump system has to face to and it can be simply modeled as

$$H_{sys} = k_0 + k_1 Q_s, \tag{9}$$

where k_0 is the static head that the pump system needs to lift, e.g., in our setup it is 1.55m. Q_s is the system flow rate and k_1 is the head loss coefficient. The pump system's operating point can be determined by setting the pump system head equal to system curve head H_{sys}. Both of them are functions of Q_s.

E. System Efficiency

The system efficiency η is defined as the ratio of pump hydraulic power to BHP [10] under assumption that the motor's efficiency and motor driver's efficiency are 100 % or fixed values. To extend our proposed method so as to take the motor's and its driver's variable efficiencies into consideration will be part of our future work.

$$\eta_s = \frac{\rho g H_s Q_s}{P_s}. \tag{10}$$

The pump system efficiency can be numerically calculated once the system model is obtained [13].

IV. Optimal Scheduling and Control Algorithm

A. System Constraints

Assume the expected head of the pump system is given as $H^\star$. Once the ith pump is determined to be put into operation with a specific speed ω_i, there should be

$$\omega_i^{max} \geq \omega_i \geq \sqrt{\frac{H^\star}{a_0^i}}. \tag{11}$$

This is due to the fact that $H^i(\omega_i)|_{Q^i=0} \geq H^\star$, i.e., the shut-off head should be over the given head.

If the system curve (9) can be determined beforehand, the system flow rate at the operating point can be estimated as

$$H^\star = k_0 + k_1 Q_s^{\star 2}, \tag{12}$$

this implies to

$$Q_s^\star = \sqrt{\frac{H^\star - k_0}{k_1}}. \tag{13}$$

With respect to the parallel configuration characteristics, i.e., $Q_s = \sum_{i=1}^{N} Q_i(\omega_i)$, there is

$$Q_s^\star(\overline{\omega}) = \sum_{i=1}^{N} \frac{1}{\omega_i^3}(b_0^i \omega_i^2 + b_1^i \omega_i H^\star + b_2^i H^{\star 2}). \tag{14}$$

Similarly, the total system power consumption $P_s(\overline{\omega})$ regarding to this specific head demand $H^\star$ can be estimated by

$$P_s(\overline{\omega}) = \sum_{i=1}^{N} P_i(\omega_i), \tag{15}$$

where $P_i(\omega_i)$ can be obtained from (5) for $i = 1, \cdots, N$.

B. Constraint Optimal Problem

The optimization problem for N available pumps is defined as: For a given head and known system curve coefficients, find a number of available pumps, denoted as $i_1, \cdots, i_k$, which combination leads to

$$\begin{array}{c} \max \\ i_1, \cdots, i_k, \\ i_1, \cdots, i_k \in \{1, 2, \cdots, N\} \\ \omega_{i_1}, \cdots, \omega_{i_k} \\ 0 \leq k \leq N \end{array} \quad \eta_s(\overline{\omega}), \tag{16}$$

subject to constraints (11) and (14) for selected pump $i_1, \cdots, i_k$.

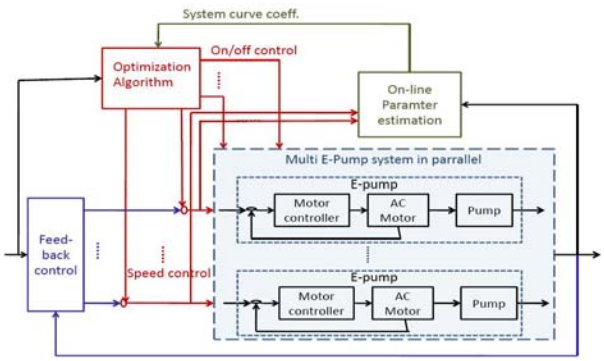

Fig. 8. Combined feed-forward and feedback framework

C. Equivalent MINLP Problem

The system hydraulic power used in (10) is constant for a given system head $H_s^\star$ w.r.t. the relationship (13). Then, the maximal optimization problem (16) can be formulated as a equivalent MINLP minimal problem.

Define a set of binary variable r_i for $i = 1, \cdots, N$. $r_i = 1$ (0) means the ith pump is (not) selected to be put into operation. The equivalent MINLP problem is formulated as

$$\min_{\substack{r_1,\cdots,r_N \in \{0,1\} \\ \omega_1,\cdots,\omega_N}} P_s(\overline{\omega}_N), \tag{17}$$

subject to constraints (11) and (14), where $\overline{\omega}_N = [r_1\omega_1 \quad \cdots \quad r_N\omega_N]^T$.

We refer to [13] for detail discussion of solving this MINLP problem (17). Hereby we report some obtained results by using the Constraint Nonlinear Programming (CNP) in the following.

D. Constraint Nonlinear Programming (CNP) Solution

The feature of the proposed CNP solution lies in the enumeration of all possibilities, such that any potential difficulties due to the mixed integer and real-value optimization can be avoided. The CNP solution consists of the following steps:

1) Check and list all possible pump combinations;
2) Solve a constraint nonlinear programming problem using Lagrangian Multiplier method for each possible combination, i.e.,

$$P_s^{k\star} \hat{=} \min_{\omega_{i_1},\cdots,\omega_{i_k}} P_s^k(\overline{\omega}), \tag{18}$$

subject to constraints (11) and (14).
3) Check the obtained minimal power consumption set $\{P_s^{k\star}\}$ regarding to each possible combination, and finally Pick up the combination which leads to the smallest $P_s^{k\star}$.

E. Complete Scheduling & Control Solution

The CNP solution serves a kind of feed-forward control. In order to get rid of potential offset and enhance the entire system's robustness to disturbance and potential modeling errors, a feedback PI controller is introduced into the framework. The entire control system is illustrated in Fig.8. In case that the system loss coefficient k_1 in (9) is unknown or it could change during the operation. We refer to [13] for details of an estimation algorithm.

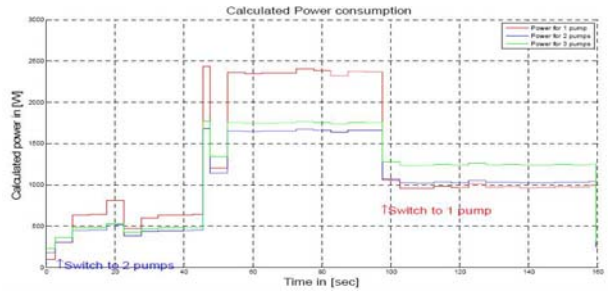

Fig. 9. Identical Pumps with Same Speeds: Power Prediction

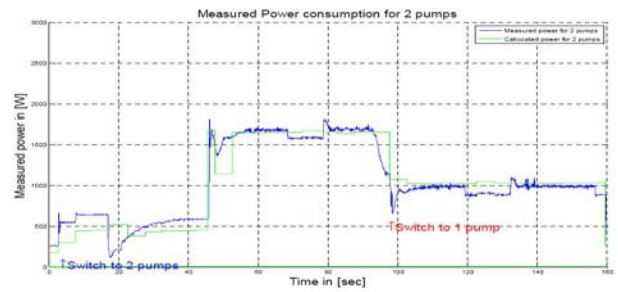

Fig. 10. Identical Pumps with Same Speeds: Measured P- Consumption

V. Implementation and Tests

The CNP solution is implemented in LabVIEW codings. The predicted power consumptions of three possible combinations (identical pumps) at same running speeds are illustrated in Fig.9. at the beginning only one pump is started and the algorithm needs to initialize the process, after a short while, the algorithm decides to switch on second pump until about 97 sec, at that time point the algorithm decides to use only one pump, due to the fact that around 92 sec the system coefficient k_1 is changed. The comparison of measured and calculated power consumptions is illustrated in Fig.10. It is quite clear that after 97 sec., the real power consumption is almost all the way below the predicted 2 pumps power consumption. The dynamic of the system in terms of the pump head response is plotted in Fig.11. In general, the controlled system has a good ability to track given references. It's clear that the optimization algorithm leads to some offset when PI control is switched off. When the system operating condition changes at 92 sec and afterwards one pump is switched off at 97 sec., some small deviations are caused during that period. The PI controller quickly compensates these deviations.

For the situation with different pumps/speeds, the computational load for this CNP optimization is much heavier.

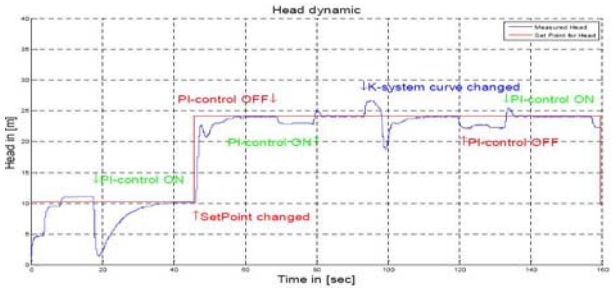

Fig. 11. Identical Pumps with Same Speeds: Head Dynamics

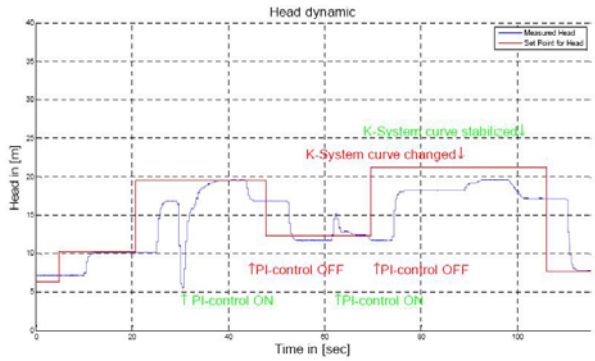

Fig. 12. Different Pumps with Different Speeds: Head Dynamics

The Matlab Optimization Toolbox is employed to solve (18). However, at this moment, we have to manually switch computations between LabVIEW and Matlab environments. This makes the following tests without "real" real-time sense. Nevertheless, these tests still clearly show a consistent functionality of the developed algorithm. The head dynamic for one concerned scenario is shown in Fig.12. During this test, a number of different demanded heads had been arranged. There were always some quite visible time delays in the system head response after the set-point changed. These delays are caused by the waiting time of the LabVIEW program to get the new solution of (18) from Matlab computation. This waiting delays can be clearly observed in Fig.13 as well. Offsets are obviously observed when the feedback control was off.

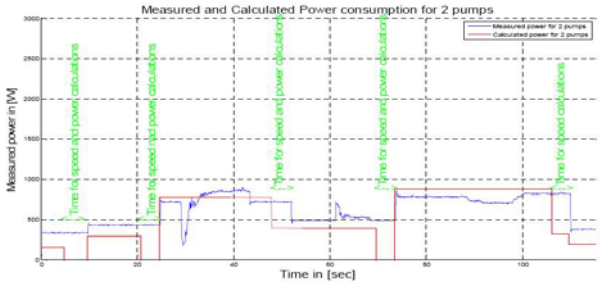

Fig. 13. Different Pumps with Different Speeds: Measured Consumption

VI. Conclusions and Future work

An optimal scheduling and control algorithm for a multi-pump system is proposed from the energy efficient point of view. A set of mathematical models are derived based on the Affinity Law and individual pump's characteristics. The optimal scheduling and control of the considered system is formulated as a MINLP problem and the CNP solution is implemented in the physical setup. The CNP solution serves as a type of feed-forward controller, which determine how many and which pumps should be on and off during the operation, and also provide the recommended speed(s) for running pump(s). A PI type of feedback controller is also introduced into the control framework. The proposed algorithm is tested via extensive experiments. The results show a clear and huge potential to improve multi-pump system's efficiency using the proposed algorithm and framework. From the practical point of view, some tradeoff between computational loads and system performance should be decided before we recommend for extensive industrial application.

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