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The context of linear algebra problems in university mathematics projects

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Abstract

Aalborg University (AAU) in Denmark has practiced project and problem based learning (PBL) since its start in 1974 on all faculties. A principle behind PBL is that students work with projects where they solve a problem that originates in society. Mathematics, however, is an abstract discipline where the results are sometimes applied in other disciplines but other times, mathematics only deals with abstract objects. Linear algebra is taught as part of most mathematics, engineering, and science university programmes in the world usually during the first year of study. At AAU, mathematics students work in groups on a project in linear algebra during their first semester and this is documented in a joint report. During this semester they have a course in linear algebra. This paper analyses all five student reports done during the autumn 2016. The purpose of the analysis is to answer the following research questions: “To what extent is a concrete problem and its context addressed and acts like the goal of these projects? How do the projects show the theory?”. The analysis is a combination of quantitative and qualitative content analysis of the curriculum and the reports particularly focusing on the problem analysis, context, problem statement, and mathematical content and competencies. The paper includes a discussion about the principles of PBL at AAU in relation to mathematics. The paper concludes that the projects showed the students’ abilities in a range of mathematical competencies. In three of the projects, the outset of the projects was a real problem in society, while in the two others, theory was presented, then a case added to apply the theory. All reports had extensive theory sections and therefore showed both pure and applied mathematics.

Keywords: mathematics, PBL, linear algebra, context, applied mathematics

Type of contribution: research paper.

1 Introduction

When Aalborg University (AAU) in Denmark was established in 1974, the founders decided that the university should be organised around the principles of problem and project based learning (PBL) at all faculties. The structure of the AAU PBL Model has since then undergone several changes (eg. Kolmos et al., 2013 & Dahl et al., 2016) but the AAU PBL principles have remained roughly the same. These principles may not be exactly like PBL principles applied in other institutions but since this study takes place at AAU, it is the AAU PBL principles that will be in focus. The newest formulation (Askehave et al., 2015) states six basic AAU principles:

1. The problem as point of departure
2. Projects organized in groups
3. The project is supported by courses
4. Collaboration – Groups, supervisor, external partners
5. Exemplarity
6. Student responsibility for learning

The AAU PBL principles are therefore based on student centred learning where students work in groups with projects where theory is applied to solve or to explain a problem that originates in society. Particularly at the start of the education, supervisors provide the problems in project catalogue. The problems are therefore contextualised. The problem formulation is developed through a problem analysis of an initiating more loose problem found within a prescribed semester theme, and the problem formulation becomes the guide for the project. The students are responsible for their own learning including project planning, and they collaborate in teams of up to eight students. The student groups receive feedback from other student groups and from supervisors who act like facilitators. The overall structure of a semester allows for half the time to be spent on the project while the other half is reserved to more traditional courses where some of the courses are supposed to support the project, others are more general courses. When appropriate, the projects also have an inter-disciplinary focus. Exemplarity refers to both content and process and implies that the learning outcomes of a project can be transferred to similar problems relevant to the students' future professions and the learning outcomes are therefore applicable beyond the project itself (Kolmos et al., 2004, Kolmos, 2009).

AAU offers a study in mathematics. De Graaff (2016, p. 397) argues that “working in a project is a natural preparation for a professional career in engineering. For other professions such a link to a project is less obvious”. Ravn and Henriksen (2017) argue that university engineering education in the abstract aspects of mathematics can be done in a contextual way. However, this paper focuses on university *mathematics* education. Dahl (2018) discusses that a study in mathematics must include both real (abstract) and applied mathematics in order to fully educate students as mathematicians. This raises the question if PBL is able to provide adequate training in pure mathematics since problems here typically originate in a theoretical realm and PBL might therefore not develop the students' abilities in all the mathematical competencies. These competencies are described by Niss (2015) and they are also part of the OECD PISA framework for comparing school children internationally. In Dahl (2018) it is argued that PBL might also fit the teaching of pure abstract mathematics if the concepts of ‘society’ and ‘context’ are interpreted to also include the ‘society of researchers’ (or community of researchers) indicating that all problems are not necessarily relevant to *everybody* in the society at any given point in time, but that some problems are only relevant for some part of the society, at least for the moment. Abstract mathematics is highly relevant within a context of research mathematics. Abstract mathematics is not always applicable immediately, neither is most basic research, but some basic research becomes relevant after some years - sometimes even several thousand years like was the case with number theory.

This paper focuses on PBL projects in linear algebra and the purpose of the analysis is to answer the following research questions: To what extent is a concrete problem and its context addressed and acts like the goal of these projects? How do the projects show the theory?

2 Mathematics and mathematical competencies

Niss (2015) formulated eight mathematical competencies that are supposed to be taught and learnt throughout the education system from primary through to tertiary in order for students to properly learn *mathematics*. These competencies are: (1) Thinking mathematically, (2) posing and solving mathematical problems, (3) modelling mathematically, (4) reasoning mathematically, (5) representing mathematical entities, (6) handling mathematical symbols and formalisms, (7) communicating in, with, and about mathematics, and (8) use of aids and tools. To possess a competence means to be able to perform certain actions with various mathematical contents. Niss (2015) described three dimensions of competence

possession: *degree of coverage* of the aspects within a competence, *radius of action* which are the various contexts where a person can activate the competence successfully, and *technical level*.

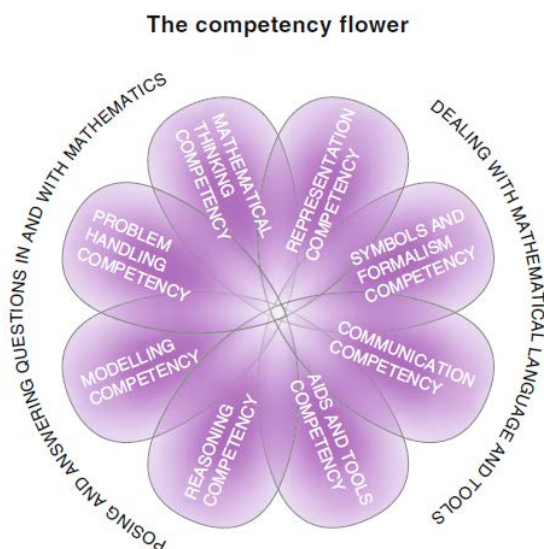


Figure 1: The competency flower of the eight mathematical competencies illustrated by Niss (2015).

Antonsen (2009) argues that the eight competencies are different as four of them may be termed ‘inner’ mathematical competencies (thinking, reasoning, representing, symbols and formalism) as their focus from a mathematical point of view are more relating to pure mathematics. Antonsen (2009) terms the other four ‘outer’ mathematical competencies (modelling, problem handling, communication, aids and tools) as they focus more on the application of mathematics. This division can be discussed as problem handling might also be within abstract mathematics and in order to apply mathematics, symbols and representations are needed. However the prime goal of the competencies differ. This division will be applied in this paper to analyse to what extent the projects are real or applied mathematics, or both.

3 Methods

This paper analyses all project reports at the first semester of mathematics in the autumn of 2016. The groups consisted of between four and seven students, and the projects were supported by a semester course in Linear Algebra. The theme of the projects was: Discrete dynamic systems. The groups had two different supervisors (the author was not one of them).

Bauersfeld (1979) distinguishes between different levels of learning termed ‘matter meant’, ‘matter taught’, and ‘matter learnt’. Matter meant is the official curriculum, matter taught is what actually happens during lecturing and supervision, while matter learnt is what the students by the end of a course or project are able to do. This analysis will be on the first and the third level. The analysis is content analysis (Titscher et al., 2000) and is a combination of quantitative and qualitative content analysis of the curriculum and the reports particularly focusing on the problem analysis, context, problem statement and mathematical content and competencies.

4 Analysis

4.1 Analysis of the curriculum

At Aalborg University each semester consists of usually one project of 15 ECTS and three courses of each 5 ECTS. Students choose their topic of study before entering the university and therefore begin studying mathematics from the first day at the university. Each semester is therefore a “package” of courses and a project to be followed by each student. At the fourth and sixth semester of the three-year bachelor programme, the students have some freedom to choose courses but otherwise the courses are predetermined and each project is within a prescribed semester theme with prescribed learning objectives that are approved by the study board. During the first month at the first semester, the students do a P0 project, which is a small 5 ECTS project where the students work in groups put together at random by the administration. This project is only graded pass/fail. The purpose is that the students should have a first experience with PBL on a smaller scale before starting on the bigger projects. The second project at the first semester is called P1 and is 10 ECTS and graded. This paper analyses the P1 projects. Here the students choose their own group members within some frames such as number of group members. The nine learning objectives for P1 are divided into knowledge, skills, and competencies (Study Board for Mathematics, Physics and Nano Technology, 2015). They are listed in Table 1 and translated from Danish by the author. In brackets each objective is named (eg. K1, Knowledge goal 1) by the author to make later reference easier:

Table 1: Learning objectives of the P1 project with the theme: Discrete Dynamical Systems.

Knowledge	Should have knowledge about models for concrete dynamical systems, eg. for the description of macro economical phenomena (K1)
	Should know about iterative and numerical methods and tools that can be used to simulation of discrete dynamic systems (K2)
	Should know about and have overview over topics and concepts in linear algebra that are relevant to solution and analysis of equilibrium and stability of discrete linear dynamic systems (K3)
Skills	Is able to communicate the relevant abstract mathematical theories and their application in one or more concrete dynamic systems. This communication must be in both writing and oral using correct mathematical concepts, symbols, and rigorous reasoning (S1)
	Is able to perform a concrete analysis of a discrete dynamic system where the analysis includes determining equilibrium points, stability, and perhaps numerical simulation (S2)
	Is able to designate relevant areas of focus for assessing and developing solutions while showing consideration for the societal and humanistic context in which the solutions are be part of (S3)
Competence	Should be based on given conditions be able to reason and argue using mathematical concepts from linear algebra (C1)
	Should develop and strengthen own ability to orally and in writing give a correct and precise mathematical statement (C2)
	Should apply concepts and tools to problem based project management and reflect about problem based learning for groups in a written Process Analysis for P0 and P1 (C3)

It appears that problem based learning is stated directly as part of the competence goals (C3). Here the focus is on project management and the collaboration between group members. The Process Analysis

mentioned is a 5-10 page document that is assessed as part of the project exam where the students should describe and reflect upon their project management, group collaboration, and collaboration with the supervisor(s) (Spliid et al., 2017). The learning objectives do not mention to what extent the project should take an outset in a problem, but the focus is both on abstract and applied mathematics. S3 states that the context of the problem should be considered. Table 2 gives an overview of how the listed knowledge, skills, and competencies in the curriculum fit the competence flower. Given that ‘competence’ may not be used in the same manner in Niss (2015) and the curriculum, the analysis also includes the learning objectives about skills and knowledge.

Table 2: Comparison of learning objectives to the eight mathematical competences.

Real or applied	Competence	Seen in curriculum
Mainly real	Thinking	C2
	Reasoning	S1, C1
	Representing	K1
	Symbols and formalism	S1
Mainly applied	Modelling	K1, K2
	Problem handling	K3, S2, S3
	Communication	S1, C2
	Aids and tools	K2

C3 does not focus on mathematics and is therefore not listed in Table 2. It appears that the learning objectives cover all eight mathematical competencies and that application competencies are covered more often than real competencies. However such a conclusion should be done with caution as the nine learning objectives are not necessarily supposed to be of equal weight. But it is clear that the projects are not intended to be just application projects but they should contain a certain amount of mathematical theory.

4.2 Analysis of the projects as a whole

The five reports are of different length as illustrated in Table 3. The reports are mentioned by the name of the group which refers to a specific room number. Often groups share a room and then one group is denoted ‘a’ the other ‘b’. The capital A in front refers to building name.

Table 3: Overall description of the five project reports: Supervisor and number of pages.

	Group name				
	A217b	A218a	A218b	A219a	A219b
Supervisor	A	A	B	B	B
Introduction section and problem analysis	6	2	6	2	1
Theory section(s)	14	21	22	42	33
Application section(s)	8	20	20	12	26
Discussion & conclusion	4	6	4	3	3
Total number of pages in report	44	56	73	70	68

The number of pages for introduction etc. seen in Table 3 does not add up to the total number of pages in the report as this number also includes preface, abstract, table of contents, bibliography, appendices etc.

Table 4: Percentage (rounded) of introduction/problem analysis, theory and application sections.

	Group name				
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	A217b	A218a	A218b	A219a	A219b
Introduction section and problem analysis	14	4	8	3	1
Theory section(s)	32	38	30	60	49
Application section(s)	18	36	27	17	38

The percentage distribution of pages seen in Table 4 only gives an overall impression of the report as some sections might be written in a more compact ways than others. However based on the distribution it appears that the reports of groups A219a and A219b are quite theoretical, groups A218a and A218b have an even distribution, while A217b is between these two groups as their report appear quite theoretical, but on the other hand, they have a more extensive problem analysis. It also appears that the fact that there was two different supervisors did not influence the balance between theory and application since both of them supervised theory heavy projects and application heavy projects. The analysis below will go deeper into this.

4.3 Analysis of the introduction and context

As stated above, AAU has six basic PBL principles. In terms of Principles 2, 3, 4, and 6, it is a given that these are fulfilled as the projects are evidently organized in groups of between four and seven students, the project is supported by a course in linear algebra, the group had a supervisor, and they were responsible for their own learning. What is more interesting to investigate is the first principle (The problem as point of departure) and fifth principle (Exemplarity), which will be done below.

For all five reports, the problem analysis is done as part of the introduction and these sections are relatively short; i.e. 1-6 pages, or between 1-14% of the report. Table 5 is a first step of the analysis of Principles 1 and shows how the five reports introduced their project, argued for the problem formulation, and presented the real life context.

Table 5: The projects' introduction, problem analysis, and problem statements (translated by the author).

	Introduction and problem analysis	Problem statement
A217b	Begins by describing that most Danes are using the Internet daily to search for information. Then they discuss how it is possible that an Internet search gives useful information and then introduces the search engine Google (½ page). Before the problem statement the group has 1½ pages problem analysis where the PageRank sum formula is presented.	<i>How can the theory of Markov chains be used to describe the Google PageRank algorithm?</i>
A218a	Same as group A217b but 1 page. Before the problem statement the group has a ½ page problem analysis where the PageRank sum formula is presented.	<i>How can PageRank be understood and how is Google, PageRank, and Markov chains connected? Which are the characteristics of Markov chains and how can these be used to make a model of the Internet that makes successful information search possible?</i>
A218b	Begins by referring to the British mathematician Patrick Leslie who in the 1940s investigated the development of biological populations. Then they present the Leslie Model (½ page). The problem limitation (½ page) describes that the goal of the project is to be able to describe how a herd of sheep develops. The rest of the introduction (4 pages) after the problem	<i>How does a population develop and which methods from linear algebra can be used to investigate this?</i>

	statement gives an overall description of population growth and how the Leslie Matrices fit.	
A219a	Begins by stating that probability is a fundamental thing for all humans and that mathematics can create models that predict how problems develop. They then describe their first case which is a model for the woods around Princeton. The second case is 'Snakes and Ladders', which is an old board game. They explain that Markov chains are used in the project on these cases (½ page). Before the problem statement there is ½ page with problem analysis which is basically a description of the structure of the report.	<i>How can mathematical modelling using Markov chains predict how a group of trees will change over time and how can Markov chains be used to calculate the average length of a game of 'Snakes and Ladders'?</i>
A219b	Begins by stating that many problems from the real world can be solved using mathematical methods for instance the distribution of trees in a wood or how an illness develops. Then they describe that Markov chains can be used to analyse the transition in a network of conditions (½ page). Before the problem statement they have ½ page that describes the structure of the report.	<i>How can linear algebra be used to investigate a long term process using Markov chains and how can this knowledge be used to describe a course of a Hepatitis B illness and to calculate the cost of this course?</i>

In terms of the first AAU PBL principle stating that the problem as point of departure, we see that three of the groups (A217b, A218a, A219b) appear to start their report with a problem asking how it is possible that an Internet search gives useful information (groups A217b, A218a) or a little more generally how trees are distributed or an illness develops (A219b). Their problem analysis then introduces the PageRank algorithm or Markov chains and then they formulate the problem statements. Although the problem analyses are relatively short, they nevertheless illustrate how the projects go from a more loosely described (initiating) problem to a specific problem. The two other groups (A218b, A219a) begin by describing a mathematical area and then later introduce a case that fits this piece of mathematics but it does not appear to have been the cases that guided the projects, more the other way around. Particularly for A219a, they choose two unrelated cases. The reason for the differences cannot be determined through a text analysis as they had different supervisors and were in different part of linear algebra. The context of the projects therefore varies as two of them have the Internet and the daily consumer as the context, two have more biological contexts, and a third a game theory context.

In terms of the fifth AAU PBL principle about exemplarity, the projects all worked with the application of mathematics to real problems, which is what many trained mathematicians work with after graduation. However, in most jobs a task begins with a problem to be solved or improved. Usually a job would not involve first to get to know a theory and after that find a way to apply it. Therefore only three of the projects (A217b, A218a, A219b) are truly exemplary.

4.4 Analysis of the theory sections

Table 6 gives an overview of the theoretical chapters in the five reports. Below will be discussed how they relate to the eight mathematical competencies and the learning objectives of P1.

Table 6: Short description of the projects' theoretical chapters.

	Use of definitions, examples, theorems, and proofs
A217b	The 14 pages of theory consists of two chapters. In total seven definitions, eight examples,

	and four theorems with proof. The proofs had length between $\frac{1}{4}$ - $2\frac{1}{4}$ pages.
A218a	The 20 pages of theory consists of several chapters and in between are more application chapters. The chapters are mathematical but seldom uses formal headings to indicate when something is a definition etc. One place has an indication of an example. There is one theorem and the proof is four pages.
A218b	The 22 pages of theory consists of several chapters. In total seven definitions, eight examples, and four theorems without proofs being given. One of the theorems is actually not a theorem but a definition.
A219a	The 42 pages of theory consists of one chapter with several subsections. In total ten definitions, seven examples (as well as other examples not directly with the heading 'Example'), four theorems without proof, and ten theorems with proofs. The proofs had length between $\frac{1}{2}$ - $1\frac{1}{2}$ pages.
A219b	The 33 pages of theory consists of two chapters. The last section in the latter of these explains in two pages how the theory is going to be applied (not counted as theory in Table 3). In total six definitions, 21 examples (usually $\frac{1}{2}$ -1 page, but one examples was $4\frac{1}{2}$ pages), and four theorems without proof.

The purpose is not to grade the projects, and due to the restrictions of page numbers, the analysis does not go in depth with the mathematics. Referring to the mathematical competences (Niss, 2015), the projects show different degrees of coverage of the eight competencies. Mathematical thinking was seen in all reports, but A218b and A219b showed lower level of coverage as they did not provide any proofs. A218b furthermore mixed the concepts of theorem and definition and A219b spent the theory section mostly on long examples. The three other groups had reports that resembled mathematics a lot more. Interestingly, A219b was one of the reports that were initially (Table 3) considered quite theoretical. In relation to mathematical reasoning, the text introducing the definitions and the explanation of the mathematics show a good competence coverage. This is also the case in relation to mathematical representation, symbols and formalism, which is apparent in the reports. All reports had some elements of mathematical modelling and problem solving as the introduction formulated a mathematical problem and the report then used a theory to model a piece of reality. Mathematically written communication is also seen in the reports. Regarding aids and tools, all reports except A217b used MatLab. It therefore appears that with different level of coverage, the reports train the students in all eight mathematical competencies. The curriculum covered all the eight competencies but the aids and tools competence is only asked for in K2, where it required knowledge about numerical methods and tools. Tools might therefore not necessarily be software but also algorithms.

5 Discussion and conclusions

This paper analysed all the project reports of a single semester. Since student groups change year after year, one cannot conclude that project reports would look the same another year. However, both supervisors were experienced supervisors thus leading the projects in a direction that one might anticipate is a shared understanding of what a project report is. In any case, the analysis is not intended to show a generalised picture of what mathematics PBL project *should* look, but how they *can* look regarding how the real world context fits. The research questions asked: To what extent is a concrete problem and its context addressed and acts like the goal of these projects? How do the projects show the theory?

In relation to the first research question, three of the projects appeared to have a problem as a goal of the project while the two others started explaining that some mathematical theory exists and they then later

found a case to apply the theory. On the other hand, writing a project report is a long process where chapters are edited multiple times. Since the students knew that they are studying within a PBL curriculum, it is also possible that all the projects started with theory, then later found a case and explained the context of this, a kind of PBL archaeology digging down to find what might have been a good initiating problem. Nevertheless, the fact that three of the group chose to present their project in a way that fitting the AAU PBL principle, shows that at least they know what PBL is and they are able to make it fit a project in linear algebra.

Regarding the second research question, the projects covered the eight mathematical competencies in various degrees which was also the case with the curriculum. Theory played a central role in the learning objectives and the theoretical chapters took up a larger part even though some of the reports had less mathematics thinking than others. A question could be, why have the theory section? If the problem is truly the point of departure and guide for the project, is it then not enough that students within a few lines refer to existing theory and then go directly on to apply the theory to their case? From a “pure” PBL principle, one may argue that having theory section disrupts the structure of the report and these sections are not necessary for the student to apply the theory. However, given that this is also an *education* in mathematics, and that pure mathematics is an essential part of mathematics, one can argue that it needs to be there.

In terms of the exemplarity principle, only three projects were truly exemplary as they began with a problem, while the two others began with theory. Nevertheless, even these two projects still applied the theory to a concrete case. Given that the learning objectives do not insist on problems being the guide of the projects, the students fulfilled these learning objectives. One may also argue that since all projects applied theory to a real case, they indeed learned to apply theory, which is therefore exemplary. One might also argue that the AAU PBL Principles 1 and 5 to some extent are contradictory as in most jobs, even though a task takes an outset in a problem, it is not always the employee’s problem, but the employer’s problem that the employee is asked to solve. Principle 1 and the general principle about student centred learning stipulates that the students should find a problem that motivates them, but this is not usually how the work market functions.

The projects clearly all fulfil the criteria set up in the curriculum, but a critique might be that the curriculum itself does not explicitly state that the projects should take a point of departure in a problem. However it is clear that the projects should have some part of concrete application. The paper started by asking the question if mathematics fit a PBL curriculum, particularly real mathematics. Based on the finding it appears that a study of linear algebra that aims at developing the students’ abilities in all the eight mathematical competencies are in fact able to do so provided that they take care of leaving a good amount of the report to theoretical chapters. However, a study of mathematics also includes course in mathematical analysis where an application to the real world is less obvious. One solution might be to only have pure mathematics in the courses, but another solution could be to regard the real world as also including the abstract world, which in fact according to Plato *is* the real world.

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