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THE RETENTION OF KEY DERIVATIVE CONCEPTS BY UNIVERSITY STUDENTS ON CALCULUS COURSES AT A CROATIAN AND A DANISH UNIVERSITY

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This paper reports a part of a larger study investigating the calculus teaching at a Croatian and Danish university. It concerns the students' retention of key procedural and conceptual concepts of derivative two months after having passed similar Calculus 1 courses, and while being taught a Calculus 2 course. The results showed that for both countries, a large portion was forgotten and the passing grades of the Calculus 1 course did not predict the results in the test two months later. In fact, often students with the lowest passing grades had the better results two month later.

INTRODUCTION

Calculus is a central part of not only university mathematics study programmes but also science and engineering study programmes all over the world. This paper therefore focuses on how some groups of students perceive key topics of derivative, which is a central part of any calculus curriculum. However, these groups of students perceive the mathematics differently. For instance when compared to mathematics students, engineering students change their mathematical concepts as they progress through their studies, which mathematics students do not (Maull & Berry, 2001). First-year students in mechanical engineering and mathematics study programmes have in fact different preferences for conception of the derivative, and belonging to a certain department highly affects the students' preferences and views of derivative in term of rate of change (Bingolbali *et al.*, 2007). Hence, we focus on first year students in non-mathematics study programmes taking similar calculus courses.

RETENTION OF PROCEDURAL AND CONCEPTUAL CONCEPTS

One way to describes various kinds of mathematics knowledge is to distinguish between conceptual and procedural knowledge. Conceptual knowledge describes an understanding of the principles and relations between pieces of knowledge in a certain domain whereas procedural knowledge is used to the ability to quickly and efficiently solve problems (Hiebert & Lefevre, 1986). Skemp (1976) divides mathematical knowledge into instrumental and relational understanding. Instrumental knowledge is 'rules without reasons' and relational is 'knowing both what to do and why'. Sfard (1991) sees a duality between structural and operational conceptions, and argues that treating mathematical notions as objects refers to structural notions, while seeing them as processes refers to operational. She argues that these modes of thinking are complementary, but operational conceptions precede structural. We will not go further into a discussion of these notions here but argue that procedural and

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operational knowledge learned without meaning is similar to instrumental understanding and relational, structural, and conceptual understanding have many similarities. We focus on the distinction between conceptual and procedural concepts.

Studies have shown that procedural knowledge is the more easily forgotten (e.g. Cooper & Sweller, 1987). Many forgetting curves have been proposed describing the phenomenon using both a power function and models of exponential decay or flat decay, with the former as the most likely model (Sikström, 1999). Krutetskii (1976, p. 291) showed that generalized relations are retained very well (around 85%) by able students even three months later. Retention is related to a person's limited working memory, while the long-term memory can store very large amounts of information. Sweller (1999) described that new material can be stored in long-term memory permanently in schemas. The number of elements held and processed in the working memory is limited and overloading the working memory with new information that is not integrated into exiting schemas can cause difficulties for understanding and learning. If students, for instance, do not store knowledge about derivatives in an appropriate form in the long-term memory, knowledge retained in their memory two months later might be poor and working memory is not be able to perform the tasks.

This paper therefore examines retained level of conceptual and procedural knowledge of derivatives. The research questions are: (1) Which tasks involving different concepts of derivative do first year students correctly solve two months after passing Calculus 1 in which these concepts were taught? We expect conceptual task to have the higher correct answer rate. (2) Is retention related to the grade or the country?

METHODOLOGY

A survey consisting of tasks on e.g. derivative, questions about study programme, and grades was conducted in 2009 at a Croatian university (spring) and a Danish university (autumn). The Croatian students were from the following study programmes: electrical engineering, civil engineering, food technology, physics, and chemistry; and the Danish students from: biology, chemistry, chemistry & technology, computer science, geology, geotechnology, information technology, molecular biology, medical chemistry, molecular medicine, nanoscience, and physics (there is no engineering programmes). Students in mathematics study programmes also answered our questionnaire, but for this paper, they were excluded. At the Croatian university, each study programmes has its own calculus course but the content and teaching styles are quite similar, in particular in relation to the concepts of derivatives investigated in this paper (Jukić & Brückler, 2010). The Danish university has a joint calculus course which is mandatory for students in all science and mathematics study programmes.

In both countries, the survey was conducted two months after the students had passed the Calculus 1 course that had taught derivatives. Both universities have a Calculus 2 course following a few weeks after the exam of Calculus 1. In Croatia, the questionnaire was distributed at the exercise lessons in Calculus 2 at each study programme. Attendance to the exercises is voluntarily. 227 students participated in the survey. In Denmark the questionnaire was given as part of a lecture during the last week of the Calculus 2 course. The students (750) are divided into three cohorts according to study programmes, but they are allowed to attend any lecture and attendance is not mandatory. We administered the questionnaire to two of the three groups, 193 students. The third group was unfortunately not able to find the time.

In Croatia the Calculus 1 grades taken in written exams were between 1-5, with 2 as minimum passing grade. The Croatian students' grades were distributed as follows with the number of students in brackets: 1 (64), 2 (22), 3 (36), 4 (53), and 5 (36). In Denmark, the students needed at least 7 correct tasks (of 12) to pass in a multiple choice test after Calculus 1. The results were as follows: 7 (3), 8 (6), 9 (15), 10 (21), 11 (44), and 12 (56). Due to a small number of students with 7 and 8, those students are pooled together below. We see that the Croatian students have a more evenly distribution of grades than the Danish students.

The questionnaire was never pre-announced so the participants were those who attended that day. We do not know why some students choose not to attend the lectures, but as seen above, each passing grade is represented by many students. Furthermore, the low number of Danish grade 7 and 8 students also reflects that in fact very few students got this grade in the exam. The students were free to fill out any part of the questionnaire or not at all, hence the number of data varies slightly from task to task. No calculators were used answering, which is what the students are all used to. The students were allowed to use as much time as needed. The response rate was very high; 94% (Croatia) and 97% (Denmark).

The options for answers were based on the first authors' five years experience of teaching and grading calculus and reflect typical misunderstandings. The questionnaire was first given to the Croatian chemistry students as a pilot. The students did not find ambiguities in the questions; hence the questionnaire was given to the other Croatian students. Before being given to the Danish students, we consulted one of the three lecturers and the department head about the relevance of the tasks, formulation, and appropriateness of the options for answers. Only formulation changes were made. The tasks reported in this paper were exactly the same for all students. Due to space, not all derivative questions are shown below.

RESULTS

Geometric interpretation of derivative, Question A (QA)

This question is conceptual and concerned with the geometric interpretation of the derivative of the function $f: R \rightarrow R$ at x_0. The definition of geometric interpretation of derivative is one of the basic definitions and carries the motivation for the definition of derivative. Following options were offered: 1. *maximum/minimum* of the function f at the given point; 2. slope of the *tangent* line to a curve $y = f(x)$ at given point; 3. *continuity* of the function f in given point; and 4. *none* of the above. The first (wrong) answer is closely connected with the application of differentiation when investigating functions, and the third (wrong) answer with limits is related to the definition of derivative of a function at a given point. The distribution of answers is seen below:

	Croatia, N=214	Denmark, N=140
Max/min	16	8
Tangent (correct)	48	66
Continuity	25	20
None	11	6

Table 1: Distribution of answers (%)

We see that half the Croatian students and two thirds of the Danish students answer this correctly. The amount and distribution of incorrect answers appear quite similar in the two countries. The distribution of correct answers is as follows:

Croatia, grades	Tangent	Denmark, grades	Tangent
1 (fail)	40	7-8 (just pass)	44
2 (just pass)	86	9	46
3	60	10	70
4	39	11	70
5 (max grade)	37	12 (max grade)	71

Table 2: Percent distribution of correct answers in relation to grades in Calculus 1

It seems that the best Croatian students in the Calculus 1 test know less than the worst students two months later whereas the opposite seems the case in Denmark.

Differentiation of composite function, Question B (QB)

This question asked the students to differentiate the function $f(x) = sin^2 6x$. This is a procedural task that examined if the students recognize that this is a composition of functions, if they confuse the derivative of functions *sinx* and *cosx*, and if they know how to apply the chain rule for differentiation. Three options were offered:

	Croatia, N=205	Denmark, N=141
2*sin*6*x*	21	14
12*sin*6*x*	14	19
12*sin*6*xcos*6*x* (correct)	66	67

Table 3: Distribution of answers, in percent

Almost the same portion of students in Denmark and Croatia answers this correctly, 2/3. The distribution of correct answers is as such:

Croatia, grades	*12sin6xcos6x*	Denmark, grades	*12sin6xcos6x*
1 (fail)	59	7-8 (just pass)	67
2 (just pass)	77	9	53
3	77	10	67
4	63	11	76
5 (max grade)	61	12 (max grade)	63

Table 4: Percent distribution of correct answers in relation to grades in Calculus 1

It does not appear that in any of the countries, the students with best passing grades also remember the most two months later. The best Croatian students appear to have the worst correct answer rate, but the picture is not as strong as in Table 2. The Danish students appear rather evenly in the distribution of correct answers.

Slope of a tangent at a point, Question C (QC)

This question is related to QA: If given $f(x) = (3x)^2$, then the slope of tangent line to the curve $y = f(x)$ at point $x = 1$ is? It requires a little procedural knowledge but is in fact mostly conceptual since it requires knowledge about the principles of a tangent.

	Croatia, N=217	Denmark, N=142
9	57	35
18 (correct)	16	36
6	27	30

Table 5: Distribution of answers, in percent

Here we see a very poor result for both countries, even though the Danish students do comparably better. In relation to grades obtained, we see the following:

Croatia, grades	18	Denmark, grades	18
1 (fail)	12	7-8 (just pass)	33
2 (just pass)	36	9	13
3	15	10	38
4	4	11	42
5 (max grade)	23	12 (max grade)	37

Table 6: Percent distribution of correct answers in relation to grades in Calculus 1

Again we see that the Croatian students with lowest grades appear to do a little better than those with higher grades, with almost the opposite picture among the Danish students, although it is by no means impressive that only 37% of those with top grades two month later can solve the question correctly.

DISCUSSION AND CONCLUSIONS

Overall, both student groups show a poor result in terms of correct answers two months after passing Calculus 1, which taught these concepts, and at a time when they are on Calculus 2, which builds on Calculus 1. We will now discuss this further.

QA is a solely conceptual question with a correct answer rate of just 48/66% (Croatia/Denmark). QC is closely linked to QA and was correctly answered by the tiny number of 16/36%. Particularly QC answers are very dissatisfactory but QA is also not impressive, particularly the Croatian result. QB is solely procedural and had a correct answer rate of 66/67%, which we find acceptable but not impressive. Three questions are not enough to make any final judgements, but this result corresponds with the other derivative question for both groups (Jukić, 2010). For the questions tested her, it appears firstly, that the students do not have an impressive retention of key concepts; secondly, that the students are better at procedural questions than conceptual, we expected the opposite. However, it may be explained by the fact that the students are first year university students and Kajander & Lovric (2005) showed that students in mathematics courses do not posses routine skills in arithmetic and algebraic computations when they begin first year of university. Dahl Søndergaard (2009) also found that high level mathematics upper secondary school curricula in Denmark (which are pre-requisite for entry to the university mathematics and science programmes) emphasise more conceptual understanding and higher level SOLO competencies whereas university entry courses' curricula are more focused at procedural understanding and lower SOLO competencies. This fits a Swedish study by Brandell (2008, p. 44):

> Routine skills and knowledge of formulas and theorems (procedural knowledge) are considered necessary [at university courses] for the understanding of concepts and theory, as well as important tools in problem solving. This seems to be in sharp contrast with the upper secondary level goals … where calculations and formulas seem to be regarded as difficulties that hinder students from a deeper understanding.

This mismatch of competencies being taught across the upper secondary-tertiary transition may explain some of the result. However, the sample of derivate questions here is small and we will investigate this further.

The results might also indicate a crowded curriculum. Brandell *et al.* (2008) concluded that first year courses have overloaded curricula as a compensation for reductions at secondary level mathematics. Skemp (1976), giving factors that contribute to instrumental understanding, also pointed to an overburden curriculum as a factor among others. Therefore we can speculate that in case of a crowded calculus curriculum, this might lead to emphasis on procedural knowledge. In future studies we will explore this hypothesis. Also, it appears that the better (Croatian) students are often forgetting faster than students with lower grades – or that at least the correct answer rate two months later is seldom predicted by course grade (Denmark). This may also be related with the students' learning mode. Star (2000) refers to planning knowledge, which can be deep but not necessarily conceptual and is related to understanding a procedure. It may also reflect that some students are very good at taking exams and that the exams are not good enough to detect those students who really do have a good understanding. Kajander (2005, p. 157) states: "We have noticed that most high school students adopt a surface-learning attitude (which, by the way, does not prevent them for obtaining very high marks in high school!)". Students being new to university may still use these strategies which (still!) may give them high marks – and thus may not store the information in the long-term memory.

One source of error in the study might be that the students may not have taking much effort into answering our questions correctly since our test was not high stake. On the other hand, they did not feel the pressure of exam which sometime causes students to "go blank", the teachers encouraged the students to think of our test as exam repetition, and they had plenty of time to answer the questions. Hence we would argue that this did not affect the results significantly.

Croatian students appear to be doing worse than the Danish in this study. The teaching style at the two universities is a bit different. Both have traditional lectures for a large number of students. In addition the Croatian students have Exercises in which a Teaching Assistant goes through some problems. The Danish students have Theoretical exercises with a Teaching Assistant which are a combination of problem solving, discussion, teacher presentation and students being at the blackboard. There is also a Mathematics Laboratory which is a home-work café where the students work in groups of 3-4 with some faculty members present. Hence the Danish students are more used to work independently and discussing the mathematics than the Croatian students. One can speculate if such difference in teaching style may explain the different in results, but further study is needed here.

Summing up, our data suggests that for these specific key concepts, the rate of correct answers just two month after passing is quite disappointing in both samples, and perhaps even more interesting, there does not appear to be any strong link between good passing grades and distribution of correct answers two months later.

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