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**Kashif Ali[1]** , **Jianjun Qin[1,2] and Michael Havbro Faber[1]**

## Abstract

Value of information analyses in structural integrity management has gained significant interest over especially the last decade. The concept of value of information analysis provides a methodical framework facilitating for the optimization of strategies for information collection through inspections and structural health monitoring. The information, which is collected, represents indications of the condition and performance of the structure and is generally subject to significant uncertainties. An important part of this uncertainty is directly related to the quality of the techniques that are utilized for collecting information. Whereas such uncertainties are generally appreciated and accounted for in the research literature, it is generally assumed that information of relevance of integrity management, collected over space and time, is unbiased and independent. In the present contribution, we investigate the possible consequences of such assumptions. To this end, we model the value of information associated with information collection and evaluate the effect of introducing biases and dependencies on the value of information. Two different probabilistic models are introduced to represent and study the effect of possible biases corresponding to the case where biases in information collected at different times are independent or fully dependent, respectively. The study is supported by the address of two different integrity management problems considering (1) an oil well tubing system subject to scaling and (2) a welded detail in steel structures subject to fatigue degradation.

## Introduction

Value of information (VoI) in support of risk informed structural integrity management (SIM) has attracted significant interest in the civil engineering research community over the last decade, see, for example, COST Action TU1402.[1]

The theoretical and methodical basis for VoI analysis in the context of risk informed SIM was developed already some 50 years ago through modern methods of structural reliability analysis by Freudenthal[2] and Bayesian decision analysis by Raiffa and Schlaifer.[3] Since then, important further developments of methods, techniques, and tools have been established for application related to especially to civil infrastructures such as offshore structures and bridges.

Original ideas on how to utilize Bayes' theorem as basis for accounting for information collected by inspections in support of integrity management are presented by Tang[4] and Yang and Trapp.[5] These ideas were soon after adopted for applications in the offshore oil and gas industry for optimization of risk informed inspection and maintenance planning. First applications to this end are reported by Skjong[6] and Madsen,[7] considering integrity management of individual welded details in fixed steel jacket structures subject to fatigue crack growth. Adaptation of formulations for systems, considering multiple welded details in structural systems, is developed by Faber et al.,[8] Moan and Song,[9] Faber and Sørensen,[11] and Straub and Faber.[12] In Faber,[13] a comprehensive account is provided on the major developments in the field of risk and reliability informed integrity management for offshore and marine oil and gas facilities.

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A central issue in VoI analysis and system modeling in more general terms concerns the probabilistic representation of the information upon which the analysis and modeling is based.[14] In the context of risk, informed SIM information may be modeled by means of indicators—observable condition states of the structure. Within the framework of Bayesian prior/posterior and pre-posterior decision analysis in the context of SIM, indicators are generally utilized for either (1) updating of probabilistic representations of relevant condition states of the structure or (2) for updating of the probabilistic representation of individual random variables.

In SIM, monitoring and inspections serve to identify possible damage states and to provide information concerning the extent—or size of damages. In the probabilistic modeling of the quality of monitoring and inspections quality of monitoring, these two aspects are addressed through the probability of detection (PoD) and the uncertainty associated with sizing. An overview on probabilistic modeling of the quality of inspections is provided by Straub and Faber.[15] SIM for pipeline systems subject to corrosion-induced damages is developed by Xie and Tian.[16] As a means for detecting and sizing corrosions damages in such systems so-called in-line inspections (ILIs) may be applied. Probabilistic models for errors associated with detections and sizing are reported by, for example, Dann and Maes,[17] Dann and Huyse,[18] and Zhang and Zhou.[19] VoI analysis and associated probabilistic modeling in the context inspection planning for pipeline systems are addressed by Haladuick and Dann.[20]

In the present contribution, closely following the framework for the representation of information in decision analysis,[14] we address the probabilistic modeling of information in the context of SIM with a focus on (1) the significance of biases associated with inspection results and (2) the effect dependencies between results of consecutive inspections—measured in terms of the VoI they facilitate. To this end, two different probabilistic models are introduced to represent and study the effect of possible biases corresponding to the case where biases in information collected at different times are independent or fully dependent, respectively. The study is supported by an example considering two different integrity management problems, that is, (1) an oil well tubing system subject to scaling and (2) a welded detail in steel structures subject to fatigue degradation.

## Methodical framework for VoI-supported SIM

Integrity management of structures aims to maximize the service life benefits and/or to minimize the operational expenditure (OPEX). Information, which may be collected from, for example, inspections and monitoring may adequately be utilized to improve the understanding of the structural condition and general performances—and essentially forms the basis for SIM. However, due to a range of reasons, the collected information is generally not perfect and this may, in turn, result in SIM decisions, which are far less than optimal. Decision-making is fundamentally challenged by the fact that the available decision alternatives are associated with uncertainties. The optimality of SIM decisions critically depends upon the quality of the collected information and the potential and relevance for improving the quality of collected information should ultimately be quantified using VoI analysis.

The introduction to VoI analysis is provided by Raiffa and Schlaifer.[3] VoI analysis in the context of SIM again takes the framework of Bayesian prior/posterior and pre-posterior decision analysis as the basis together with the axioms of utility theory,[21] where the expected value of benefits is the criteria to measure the value of the information. Following the engineering practice of SIM, the idea hereby is to analyze the difference among the expected values of the benefits (and/or reduce the OPEX) over the service life with or without different strategies for collecting additional information, even before it is collected. Uncertainties associated with different perspectives—including the strategy for collecting information, the collected information, actions, and the state of the structure—would be considered in the analysis of the benefits.

The decision event tree considered in SIM is illustrated in Figure 1. The information is collected through different strategies **e** where outcome is represented by information **Z** concerning different states of the structure (performance characteristics), modeled by the random vector **X**. The information **Z** can be modeled probabilistically as the function of the quality of the different strategies **e** (measurement errors). Based on the information **Z**, an action **a** defined as decision rule, taken to change the state of structure **X**. The decision tree used to quantify the particular strategy **e** for the VoI analysis. VoI is the difference between the
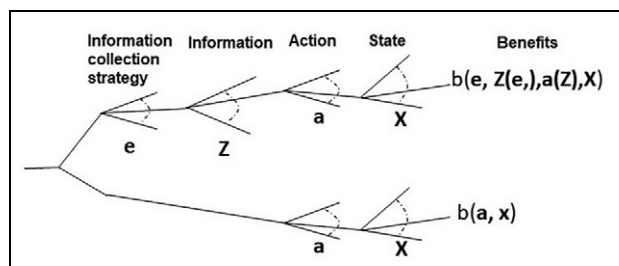


**Figure 1.** Illustration of the decision event tree considered in SIM of engineered structures.

expected value of the benefit associated with the information $\mathbf{Z}$ collected through $\mathbf{e}$, $E_{\mathbf{X},\mathbf{Z}}[b(\mathbf{e}, \mathbf{Z}(\mathbf{e}), \mathbf{a}(\mathbf{Z}), \mathbf{X})]$ and the expected value of the benefit without the collection of information $E_{\mathbf{X}}[b(\mathbf{a}, \mathbf{X})]$

$$
\begin{aligned}
\text{VoI}(\mathbf{e}) &= E_{\mathbf{X},\mathbf{Z}}[b(\mathbf{e}, \mathbf{Z}(\mathbf{e}), \mathbf{a}(\mathbf{Z}), \mathbf{X})] - E_{\mathbf{X}}[b(\mathbf{a}, \mathbf{X})] \\
&= \int_{D_{\mathbf{x}}} \mathbf{x}^T \int_{D_{\mathbf{z}}} \mathbf{z}^T f_{\mathrm{b}} b(\mathbf{e}, \mathbf{z}(\mathbf{e}), \mathbf{a}(\mathbf{z}), \mathbf{x}) d\mathbf{z} d\mathbf{x} \\
&\quad - \int_{D_{\mathbf{x}}} \mathbf{x}^T f_{\mathrm{b}} b(\mathbf{a}, \mathbf{x}) d\mathbf{x}
\end{aligned}
\tag{1}
$$

where $f_{\mathrm{b}}$ represents the probability distribution function of the benefit b in the domain of state of structure ($D_{\mathrm{x}}$) and information ($D_{\mathrm{z}}$). The VoI analysis[13,22,23] as shown in equation (1) ranks the actions (decision alternatives) represented by $\mathbf{a}$ enhanced by means of the information $\mathbf{Z}$ collected through the strategy $\mathbf{e}$. The ranking of $\mathbf{a}$ is informed through the uncertain future realizations of $\mathbf{Z}$ corresponding to the uncertain state $\mathbf{X}$.

VoI analysis provides identification of the steps in the SIM value chain for the uncertainties involved in additional information that has been added across the different types of systems such as structures and wells, and of offshore oil and gas production facilities for the reduction in OPEX. VoI may ensure that the general approach and applied modeling across the different types of systems are consistent with respect to the representation of uncertainties related to information and accounted for the optimization of SIM strategies.

## Information modeling in SIM

The basis for SIM is the information concerning the states of engineered structures and the knowledge synthesized on the basis of the information. Therefore, we need to collect relevant information to update our knowledge about the states of the structures to facilitate SIM. In accordance with Nielsen et al.,[14] five classes to represent the information are proposed as follows:

Class 1: the information is relevant and precise.
Class 2: the information is relevant but imprecise.
Class 3: the information is irrelevant.
Class 4: the information is relevant but incorrect.
Class 5: the flow of information is disrupted or delayed.

In the following, the information $\mathbf{Z}$ collected (by different means of inspection and monitoring) is assumed to be relevant to SIM, and we further assume that the events of disrupted or delayed information are neglected. In some cases, the information collected might be irrelevant to the SIM. An example of this in
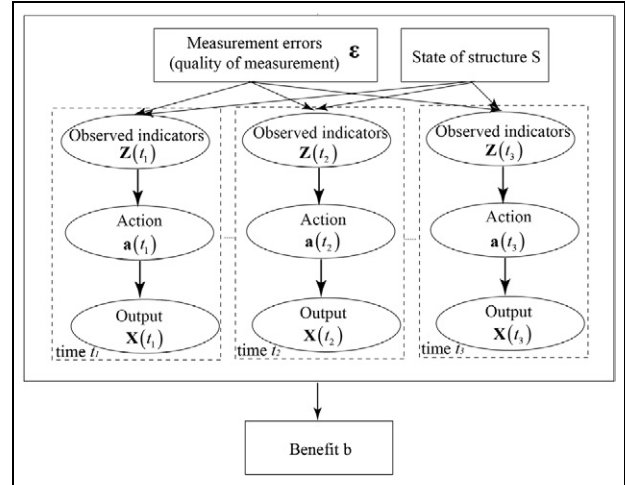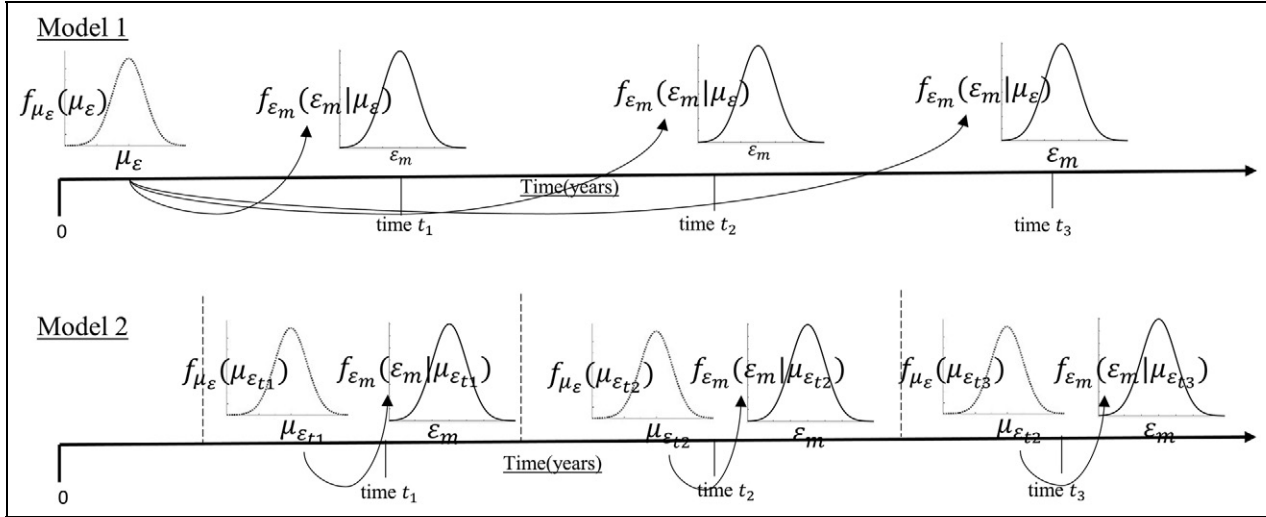


**Figure 2.** Illustration of two types of dependency models of errors in the collected information at different times.

the context of inspection and maintenance planning for structures subject to fatigue crack growth concerns inspection findings of slag inclusions from the welding process, which may be wrongly interpreted as the indications of fatigue crack growth and lead to suboptimal maintenance actions.[14] Such irrelevance is out of the scope of this article. Also in the practice of engineering, the information $\mathbf{Z}$ collected is normally associated with uncertainties that tend to be significant for the ranking of decision alternatives, that is, possible actions and correspondingly the effects of these uncertainties must be accounted for in SIM. In the following discussion, the focus is directed on Classes 2 and 4 as shown above.

There are normally two types of strategies $\mathbf{e}$ applied to collect information relevant to the SIM during the service life of engineered structures, namely, (1) structural health monitoring (SHM) and (2) inspections. The information $\mathbf{Z}$ collected from both of the two types of strategies $\mathbf{e}$ is generally modeled through what is referred to as condition indicators. The information obtained from indicators in the context of SIM is presented for SHM by Moan and Song[9] and for inspections of concrete structures by, for example, Faber and Sørensen[11] and Qin and Faber.[24]

To facilitate SIM, both SHM and inspections may be implemented to observe the indicators relevant to the unknown state $\mathbf{X}$ through the collection of the information at different locations of the structure and at different times during the service life. Despite the different locations and times, there are, however, two sources of important dependencies between the observed indicators, which must be accounted for, see Figure 2 for illustration. On the one side, all the observations (no matter where and when) depend on the state of the

**Figure 3.** Illustration of two sources of dependencies between the observed indicators given the choice e.

structure; on the other side, the observations themselves, for example, through SHM and/or inspections, are associated with uncertainties as presented above. In general, uncertainties are represented by measurement errors (note that here the term error is general and includes bias) and the observed indicators are influenced by common uncertainties, that is, measurement errors. An illustration of the two sources of dependences is provided in Figure 2.

This article presents two different models to describe errors in the collected information **z** (observed indicators) at different times probabilistically taking the dependency between the observations into account, see Figure 3 for illustration. Note that there are two types of errors as introduced by Rothman,[10] that is, random errors and bias (systematic errors), both of which are considered in the two models. The random errors may be reduced by the increasing number of observations. However, increasing the number of observations does not affect the bias. In the proposed probabilistic modeling, the uncertainty associated with measurements is represented by the expected value of $\varepsilon$, that is, $\mu_\varepsilon$ and its standard deviation $\sigma_\varepsilon$. If $\mu_\varepsilon$ is different from zero, there is a bias, and otherwise there is no bias. In Model 1, it is assumed that the measurement error $\varepsilon$ is realized only once and thus identical for all future inspections.

Model 1 might be relevant in the case where a measurement device is installed imperfectly such that measurements are systematically shifted from the true value; an example of this concerns strain gages installed at an incorrect angle. In Model 2, it is assumed that the realizations of the systematic errors might differ at different inspections times. The time indices $t_1, t_2, t_3 \ldots$ shown in Figures 2 and 3 represent the times at which measurements are made. An example where of Model 2

would be relevant concerns inspections of different welded details using one measurement device that is calibrated only one time—and where biased may occur due to differences in actual weld geometries compared to the weld geometry on which the measurement device is calibrated. In engineering practice, the characteristics of the probabilistic models presented here could be identified through statistical analysis of relevant data, and further, statistical tests could be implemented to determine which dependency model (Model 1 or Model 2) is most appropriate.

The VoI analysis for the scenarios with these two different models of measurement errors $\boldsymbol{\varepsilon}$ may be simply expressed as

$$
\begin{aligned}
\mathrm{VoI}_1(\mathbf{e}) &= \mathrm{E}_{\mathbf{X}, \mathbf{Z}}[\mathrm{b}(\mathbf{e}, \mathbf{Z}(\mathbf{e}, \boldsymbol{\varepsilon}(\mu_\varepsilon), \mathbf{X}), \mathbf{a}(\mathbf{Z}), \mathbf{X})] - \mathrm{E}_{\mathbf{X}}[\mathrm{b}(\mathbf{a}, \mathbf{X})] \\
&= \int_{D_\mathbf{x}} \mathbf{x}^T \int_{D_\mathbf{z}} \mathbf{z}^T f_\mathrm{b} \mathrm{b}(\mathbf{e}, \mathbf{z}(\mathbf{e}, \boldsymbol{\varepsilon}(\mu_\varepsilon), \mathbf{x}), \mathbf{a}(\mathbf{z}), \mathbf{x}) d\mathbf{z} d\mathbf{x} \\
&\quad - \int_\mathbf{x} \mathbf{x}^T f_\mathrm{b} \mathrm{b}(\mathbf{a}, \mathbf{x}) d\mathbf{x}
\end{aligned}
$$

(2)

$$
\begin{aligned}
\mathrm{VoI}_2(\mathbf{e}) &= \mathrm{E}_{\mathbf{X}, \mathbf{Z}}[\mathrm{b}(\mathbf{e}, \mathbf{Z}(\mathbf{e}, \boldsymbol{\varepsilon}(\boldsymbol{\mu}_\varepsilon), \mathbf{X}), \mathbf{a}(\mathbf{Z}), \mathbf{X})] - \mathrm{E}_{\mathbf{X}}[\mathrm{b}(\mathbf{a}, \mathbf{X})] \\
&= \int_{D_\mathbf{x}} \mathbf{x}^T \int_{D_\mathbf{z}} \mathbf{z}^T f_\mathrm{b} \mathrm{b}(\mathbf{e}, \mathbf{z}(\mathbf{e}, \boldsymbol{\varepsilon}(\boldsymbol{\mu}_\varepsilon), \mathbf{x}), \mathbf{a}(\mathbf{z}), \mathbf{x}) d\mathbf{z} d\mathbf{x} \\
&\quad - \int_\mathbf{x} \mathbf{x}^T f_\mathrm{b} \mathrm{b}(\mathbf{a}, \mathbf{x}) d\mathbf{x}
\end{aligned}
$$

(3)

Equations (2) and (3) represent VoI analysis for the scenarios with Model 1 and Model 2, respectively. Note that in VoI analysis for the scenarios with Model 1, the

measurement error $\varepsilon$ is a function of the random variable $\mu_\varepsilon$, that is, the uncertain expected value; while in the VoI analysis for the scenarios with Model 2, the measurement errors $\varepsilon$ are the function of all the uncertain expected values included in the vector $\boldsymbol{\mu}_\varepsilon$.

In the next sections, the two dependency models are addressed and investigated in two different contexts of VoI analysis–supported SIM. The first example concerns the modeling and assessment of VoI in the context of integrity management of an oil production well system subject to scale degradation. In the second example, VoI is analyzed considering the life cycle cost minimization for a welded detail in a steel structure subject to fatigue crack growth.

## Example: VoI analysis for measurements of scale degradation of subsurface wells in the context of SIM

### Introduction

In this example, the VoI associated with measurements of scale degradation is investigated in the context of SIM of subsurface wells, accounting for the influence of systematic errors (bias) of the measurements and considering the two different models for the representations of dependencies between those errors at different times. The oil production from subsurface wells during their service life may be reduced or even lost due to a reduction of the inner diameter of the production tubing. The reduction considered here is assumed caused by inorganic depositions of salts—called scaling—including the formation of hard deposits of calcium carbonate ($CaCO_3$) and barium sulfate ($BaSO_4$) as shown in Figure 4. The industrial practice shows that scale such as barium sulfate is formed mainly along the horizontal tubing, at the depth of the reservoir, while calcium carbonate deposits normally are formed at lower depths in the vertical part of the tubing. To gain knowledge about the level of scaling and to facilitate optimization of SIM, inspections are performed to measure the scale propagation using, for example, multi-finger caliper measurements. However, the results from those measurements—at any given location—are generally associated with uncertainties and possible biases. Moreover, measurements at different times are subject to stochastic dependencies. In this section, the value of the information from measurements of scale propagation of one subsurface well is investigated—accounting for the uncertainties and the bias together with the stochastic dependency between the measurement results at different times, to support SIM.

The subsurface well considered here for illustrational purposes is assumed to have a 30-year service life. Inspections are assumed performed with equidistant
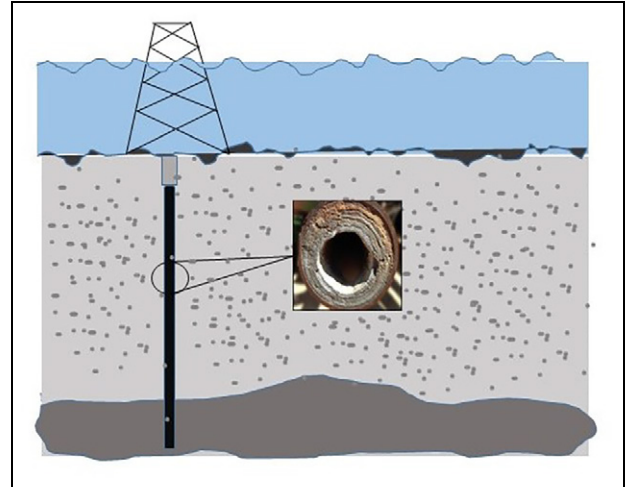


**Figure 4.** Illustration of scale deposits in a subsurface well.

time intervals to support the decision of maintenance actions and further SIM decision optimization. Three different inspection intervals are considered here, that is, 2, 6, and 10 years. The accumulated scale growth at year $t$, $D_0(t)$ is taken as the sum of the annual scale growth rate $S_i$ (mm/year), ($i = 1, 2, \ldots, t$), given that no repair is performed

$$D_0(t) = \sum_{i=1}^{t} S_i \qquad (4)$$

Scale formation is complex and complicated phenomena, and the scale rate varies for different environmental conditions provided. Previous studies[25,26] show that the annual scale growth rate $S_i$ depends upon different factors, that is, pressure, temperature, and solubility of ions. In addition, it is assumed that the random variables $S_i$ representing the scale growth rate at time $t_i$ follow a lognormal distribution.[25]

In addition, measurements from inspections might not necessarily be perfect and could be associated with systematic errors, that is, biases. The biases associated with measurements of accumulated scaling thickness $\varepsilon$ are modeled through random variables with the expected values $\mu_\varepsilon$ and standard deviation $\sigma_\varepsilon$. In the present example, the effect of the two types of temporal dependencies (from inspection to inspection), modeled as shown in Figure 2, is investigated. Four deterministic values of the expected value of $\mu_\varepsilon$ are considered here with the purpose to investigate the influence of the measurement bias on the VoI, that is, $|\mu_\varepsilon| = 0$, 2, 6, 10 mm. In the cases where $\mu_\varepsilon$ attains non-zero expected values, it is assumed that the expected value of $\mu_\varepsilon$ has equal probability to be either positive or negative. In Model 1 (see Figure 2), the outcome of the expected value of $\mu_\varepsilon$ is either positive or negative throughout the entire

**Table 1.** Summary of the definition of the random variables.

| Variable | Distribution | Mean | Standard deviation |
|---|---|---|---|
| **Oil price** ($V_{oil}$) | Deterministic | 65 per bbl | |
| **Production rate** ($P_{rate}$) | Deterministic | 1000 bbl/day | |
| **Radius of the tubing** ($R_t$) | Deterministic | 40 mm | |
| **Annual degradation** ($S_i$), **mm/year** | Lognormal | 1.96 | 0.5 |
| **Bias** ($\varepsilon$) | Normal | $\mu_\varepsilon$ | $0.2\mu_\varepsilon$ |
| **Discount rate** ($r$) | Deterministic | 5% | |
| **Repair cost** ($C_R$) | Deterministic | $1 \times 10^5$ | |
| **Inspection cost** ($C_I$) | Deterministic | $1 \times 10^4$ | |

service life; while for Model 2 (see Figure 2), the realizations of the expected value of $\mu_\varepsilon$ may shift from negative to positive over consecutive inspections. All the possible combinations of $\mu_\varepsilon$ for each inspection interval are considered for Model 2. The summary of the applied probabilistic model of the measurement uncertainties and biases is presented in Table 1.

In the investigations, it is assumed that the characteristics of the probability distribution function of the absolute value of the expect value of $\mu_\varepsilon$ and the deterministic value of standard deviation ($0.2\mu_\varepsilon$) do not change over time. The measurements of accumulated scale growth at year $t$, $D_m(t)$ accounting for the modeled errors is thus

$$D_m(t) = \left( \sum_{i=1}^{t} S_i \right) + \varepsilon \tag{5}$$

The probability density functions for the case of $\varepsilon$ with mean value $|\mu_\varepsilon| = 2$ mm/year and standard deviation (Std) $= 0.2\mu_\varepsilon$ are assumed to follow normal distribution, and it has equal probability to be either positive or negative mean values as illustrated in Figure 5.

The accumulation of scale growth is assumed to reduce oil production due to a decrease in the inner radius of the well tubing. The unit oil price per barrel $V_{oil}$ is assumed constant over time and equal to 65 per bbl. For illustrational purposes, rate is modeled as $P_{rate} = k(R_t - D(t))$, where $k$ is a constant (unit: bbl/day/mm) and calibrated such that $P_{rate} = 1000$ bbl/day for the case of no scale development. In reality, the relationship between the production rate and the inner diameter is more complex due to fact that the produce oil has complex non-Newtonian characteristics.

The annual income or benefit $B(t)$ from the production at year $t$ is calculated as

$$B(t) = V_{oil}k(R_t - D(t)) \frac{1}{(1+r)^t} 365 \tag{6}$$

where $V_{oil}$ is the unit price of the oil, $R_t$ is the radius of the pipeline, $r$ is the annual discount rate, and $D(t)$ is the
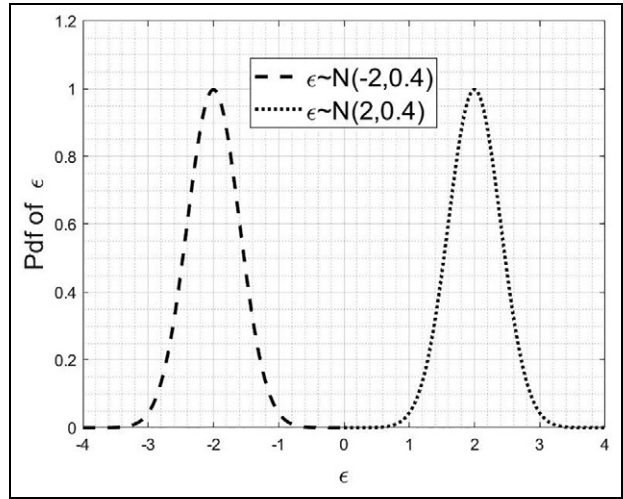


**Figure 5.** Illustration of the probability density functions of normal distribution for measurement errors $\varepsilon$ with $|\mu_\varepsilon| = 2$mm and Std $= 0.2\mu_\varepsilon$.

accumulated scale growth (scale thickness) at year $t$. It is further assumed that "repairs" are performed to remove the scale, when the measured accumulated scale growth $D_m(t)$ exceeds a predefined scale accumulation thickness $D_r$ (repair criteria). Three different values of $D_r$ are considered here to investigate the influence of the repair criteria on the VoI, that is, $D_r = 10$, 15, and 20 mm. Considered repair options can be chemical cleaning or mechanical cleaning, and, in general, the repair cost depends on the options together with the extent of scale propagation. However, here for simplicity, the repair costs are assumed deterministic for each action.

The expected value of total benefit $B_k$ from the production is $E[B_k]$

$$E[B_k] = E\left[ \sum_{t=1}^{30} B(t) \right] \tag{7}$$

In the following investigation, the expected value of the total benefit, the total inspection cost, and the total repair cost for the whole service life together with the VoI are analyzed. The scenarios considered include all

combinations of different inspection intervals (after 2, 6, and 10 years), different absolute values of the expected value of measurement errors (2, 6, and 10 mm), and the three different repair criteria together with the two dependency models. The probabilistic analyses are undertaken through $10^5$ Monte Carlo simulations. The expected value of the net benefit without any inspection and repair, $E[B_0]$ is also estimated.

The expected value of the inspection costs for different time interval–based inspection strategies is estimated as[27]

$$E[C_I] = \sum_{i=1}^{n_{insp}} C_{insp} \frac{1}{(1+r)^{t_i}} \qquad (8)$$

where $C_{insp}$ is the fixed inspection cost, $r$ is the annual discount rate, and $n_{insp}$ is the total number of inspections.

The expected value of the repair cost depending upon the repair criteria is estimated as[27]

$$E[C_R] = \sum_{i=1}^{n_{insp}} C_R p_R(t_i) \frac{1}{(1+r)^{t_i}} \qquad (9)$$

where $C_R$ is the fixed repair cost, and $p_R(t_i)$ is the probability that the repair criteria are reached at year $t_i$.

Given the inspection intervals, the expected values of the total inspection cost, the total repair cost, the total production benefit, and the VoI are analyzed for the scenarios corresponding to the different combinations of absolute values of the expected value of measurement errors and different repair criteria. For example, for the scenario where the expected value of the bias is $\mu_\varepsilon = 6$ mm, the criteria for repair are $D_r \geqslant 10$ mm, and the time interval between inspections is 2 years, the individual components of the service life benefits together with the VoI are shown in Figure 6. The unbiased scenario corresponds to the case $\mu_\varepsilon = 0$ mm, that is, no bias in the measurements.
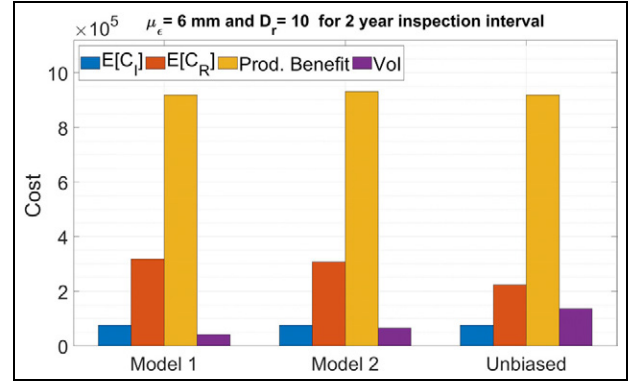
The expected (VoI) is calculated as

$$\text{VoI} = E[B_k] - E[C_I] - E[C_R] - E[B_0] \qquad (10)$$

$$\begin{aligned}
\text{VoI} = E[B_k] &- E\left[\sum_{i=1}^{n_{insp}} C_{insp} \frac{1}{(1+r)^{t_i}}\right] \\
&- E\left[\sum_{i=1}^{n_{insp}} C_{insp} p_R(t_i) \frac{1}{(1+r)^{t_i}}\right] - E[B_0]
\end{aligned} \qquad (11)$$

### Results

The VoI for the scenarios with the combinations of different inspection intervals and different absolute values of the expected value of both measurement errors and different repair criteria together with the two



**Figure 6.** The expected values of the total inspection cost, the total repair cost, the total production benefit, and the VoI given that the inspection interval is 2 years ($\mu_\varepsilon = 6$ mm and $D_r = 10$ mm) for biased (Models 1 and 2) and unbiased measurement ($\mu_\varepsilon = 0$).

dependency models is shown in Figure 7. From Figure 7, it can be seen that the value of the unbiased information is generally higher than that of biased information, while the value of the biased information with the dependency following Model 2 is higher than that of the information with the dependency following Model 1. However, it is not always the case especially when the repair criteria are low (the value of $D_r$ is high) or the inspection interval is long, both of which cases make the influence of bias negligible. Another reason is that biased information might result in preventive repair and such actions sometimes would increase the benefit if the repair is cheap but the loss of production is expensive. The difference between the value of the information from two different dependency models and the value of the unbiased information is small as the bias represented by the value of $|\mu_\varepsilon|$ is small (equal to 2 mm here), that is, the effect of bias is insignificant as $|\mu_\varepsilon|$ is small. If the value of $|\mu_\varepsilon|$ is quite high (equal to 10 mm here) but the inspection interval is short and the repair criteria are strict (the value of $D_r$ is high), the probability to have the irrational decision (unnecessary repair) would become high and the value of such information could even be negative or close to zero.

## Example: VoI of integrity management for steel structures subject to fatigue

### Introduction

In this example, we consider a steel structure with a 20-year service life subject to fatigue crack growth. The information regarding the development of fatigue crack growth is collected during the service life by means of inspections. The VoI associated with inspections is analyzed accounting for possible biases.
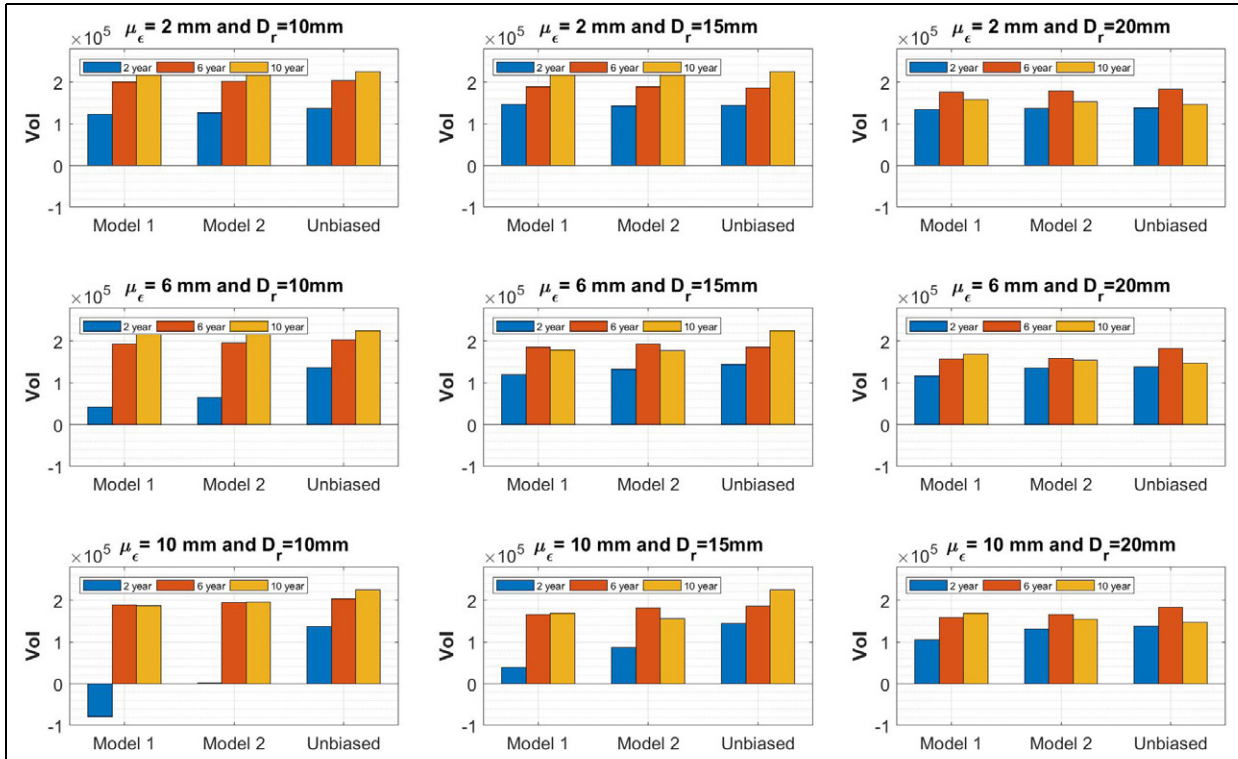
**Figure 7.** The Vol for the scenarios with the combinations of different inspection intervals and different absolute values of the expected value of both measurement errors and different repair criteria together with the two dependency models.

**Table 2.** Definition of parameters relevant to fatigue loading and crack growth parameters.

| Variable | Distribution | Mean | Standard deviation |
|---|---|---|---|
| Stress range ($\sigma_s$) | Normal | 30 MPa | 5 MPa |
| Stress cycle rate ($n$) | Deterministic | $1 \times 10^5 \text{year}^{-1}$ | |
| Initial crack ($a_0$) | Exponential | 1 mm | 1 mm |
| Critical crack ($a_c$) | Deterministic | 40 mm | |
| Material constant ($C$) | Deterministic | $5 \times 10^5$ | |

To model the evolution of crack growth, a simple one-dimensional crack growth model is considered here,[28,29] where the crack growth $a(t)$ at year $t$

$$a(t) = a_0 \exp\left(C \pi \sigma_s^2 n t\right) \quad (12)$$

where $n$ is the number of stress cycles per year, $a_0$ is the initial crack size, $C$ is a material constant, and $\sigma_s$ is a constant (or equivalent constant) stress range representing the characteristics of the fatigue loading. Note that the condition of the structure is assumed to be "as new" after repair action. A summary of the probabilistic model for parameters relevant to fatigue loading and crack growth parameters is given in Table 2.

Fatigue crack growth might result in a structural failure, if the crack size exceeds a critical threshold $a_c$.

The limit state function $g(t)$ representing the fatigue failure at year $t$ can be written as

$$g(t) = a_c - a(t) \quad (13)$$

The crack size $a(t)$ is a function of the realizations of the random variables and the deterministic parameters provided in Table 2. The fatigue performance of the considered structure, represented by $g(t)$, is therefore not deterministic. When $g(t)$ is negative, it is assumed that an event of structural failure occurs. The probability of structural failure in a particular year (annual failure probability $\Delta p_F$) or accumulated up until a given year ($p_F$) can be easily assessed using the Monte Carlo simulation or first-order reliability
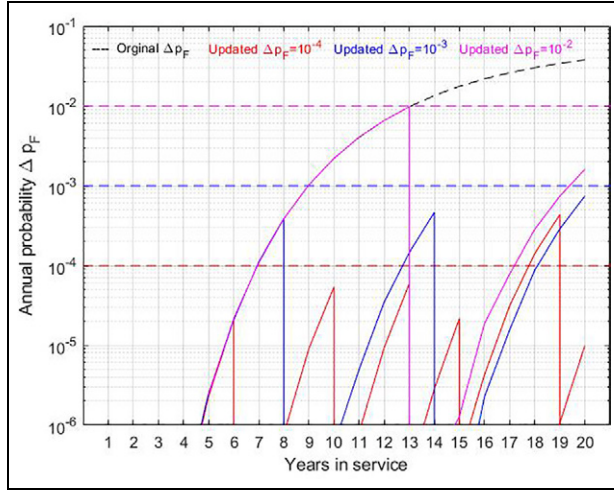
**Figure 8.** Prior and updated $\Delta p_F$ for different thresholds of $\Delta p_F$.



**Figure 9.** Effect of bias and unbiased in PoD on inspection planning.

method (FORM)/second-order reliability method (SORM) analysis.[28]

Depending upon the maximum acceptable threshold for the annual probability of failure, for the considered structure (generally defined by code or regulation), inspections are planned such that they take place at the latest time (year) for which the annual probability of failure is not exceeded. For example, in the case that the maximum acceptable annual probability of failure is set to $10^{-4}$, the first inspection takes place in year 6, see also Figure 8.
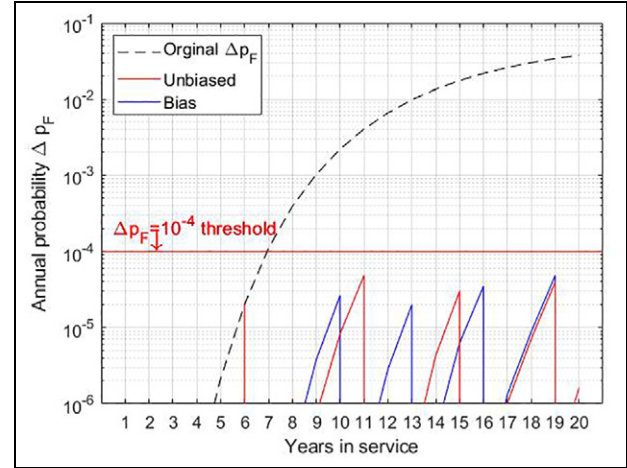
If a crack is detected at an inspection, the structure will be repaired immediately. The probability of detecting a crack is modeled as a function of the size of cracks through the PoD. The PoD is modeled here by an exponentially distributed random variable with expected value $\mu_{PoD} = 2.5$. Given that the structure is inspected at year $t_{insp}$ and no crack is found; the probability of failure $p_F$ at year $t$ may be determined through event updating[28]

$$p_F(t) = P\big(g(t) \leqslant 0 | a(t_{insp}) - \text{PoD} \leqslant 0\big)$$
$$= \frac{P\big(g(t) \leqslant 0 \cap a(t_{insp}) - \text{PoD} \leqslant 0\big)}{P\big(a(t_{insp}) - \text{PoD} \leqslant 0\big)} \quad (14)$$

The annual probability $\Delta p_F$ of failure at year $t$ can be estimated as

$$\Delta p_F(t) = \frac{p_F(t) - p_F(t-1)}{1 - p_F(t-1)} \quad (15)$$

where $\Delta p_F$ is updated assuming that no indication of a crack is identified at the times of the inspections at inspection is estimated using the constant threshold approach.[13] It means that whenever $\Delta p_F$ exceeds, the

threshold and inspection is performed and the annual probability of failure from that point in time is updated on the event of no detection. In Figure 8, the evolution of the annual probability of failure (updated at the time of inspections), together with inspection times, is shown for three different thresholds for the maximum allowable annual probability of failure, that is, $10^{-4}$, $10^{-3}$, and $10^{-2}$.

In the same manner as in the foregoing example, the inspection information is considered associated with errors together with biases. In this example, it is assumed that there are errors (together with biases) $\varepsilon$ associated with the mean value of the PoD, that is, $\mu_{PoD} = 2.5 + \varepsilon$. Four different values of $\varepsilon$ are considered here. If there is no bias, $\varepsilon$ is equal to zero. If there is bias, $\varepsilon$ could be 150%, 200%, and 250% of 2.5. The dependency of the biases at different inspection times follows one of the two models discussed in section "Information modeling in SIM." In the case of Model 1, the non-zero value of $\varepsilon$ will remain the same for all the inspections throughout the service life. In case of Model 2, the non-zero value of $\varepsilon$ will be selected with equal probability among 150%, 200%, and 250% of 2.5 for each inspection during the service life. Note that in either case Model 1 or Model 2, the inspection year and the number of inspections will be different compared to the case of unbiased inspection for the same annual probability of failure threshold as illustrated in Figure 9.

The expected total life cycle cost $E[C_T]$ is considered for VoI analysis that is expressed as the sum of the expected value of failure cost $E[C_F]$, the expected value of inspection cost $E[C_I]$, and the expected value of repair cost $E[C_R]$ defined as[27]

$$E[C_I] = \sum_{i=1}^{n_{insp}} C_{insp} \frac{1}{(1+r)^{t_i}} \qquad (16)$$

where $C_{insp}$ is the fixed inspection cost, $r$ is the annual discount rate, and $n_{insp}$ is the total number of inspections

$$E[C_R] = \sum_{i=1}^{n_{insp}} C_R p_R(t_i) \frac{1}{(1+r)^{t_i}} \qquad (17)$$

where $C_R$ is the fixed repair cost, and $p_R$ is the probability of repair

$$E[C_F] = \sum_{t=1}^{T_{SL}} C_F \Delta p_F \frac{1}{(1+r)^{t}} \qquad (18)$$

where $C_F$ is the fixed failure cost, and $\Delta p_F$ is the annual probability of failure.

Thus, the total cost would be

$$E[C_T] = \sum_{t=1}^{T_{SL}} C_F \Delta p_F \frac{1}{(1+r)^{t}} + \sum_{i=1}^{n_{insp}} C_{insp} \frac{1}{(1+r)^{t_i}} + \sum_{i=1}^{n_{insp}} C_R p_R(t_i) \frac{1}{(1+r)^{t_i}} \qquad (19)$$

The cost terms are defined as $C_{insp} = 1 \times 10^{-3}$, $C_R = 1 \times 10^{-2}$, and $CF = 1$, while the interest rate $r$ is set to 5% and the service life is 20 years. It is assumed that all detected cracks are repaired. The expected value of the total cost $E[C_T]$ is estimated for all the combinations of the three different values of $\varepsilon$ together with the two models of dependency and the six values of thresholds, that is, $\Delta p_F = 5 \times 10^{-5}, 10^{-4}, 5 \times 10^{-4}, 10^{-3}, 5 \times 10^{-3}$ and $10^{-2}$.

For the case of unbiased information, that is, $\varepsilon = 0$ and $\mu_{PoD} = 2.5$, $1 \times 10^8$ Monte Carlo simulations are performed to assess $E[C_T]$ for each value of $\Delta p_F$ threshold. For the case with biased information, $10^3$ random realizations are generated, each of which is simulated by $10^8$ Monte Carlo simulations for each value of $\Delta p_F$ threshold.

## Results

The expected values of the service life cycle costs of integrity management of the structure with unbiased information are presented in Figure 10 as function of the threshold $\Delta p_F$; for the case with biased inspection results, the results resulting from the two bias dependency models are shown in Figure 11 (for Model 1) and Figure 12 (for Model 2). The expected values of both the inspection cost and the repair cost, $E[C_I]$ and $E[C_R]$, gradually decrease with the increase of the $\Delta p_F$ threshold; while the expected value of the failure cost,
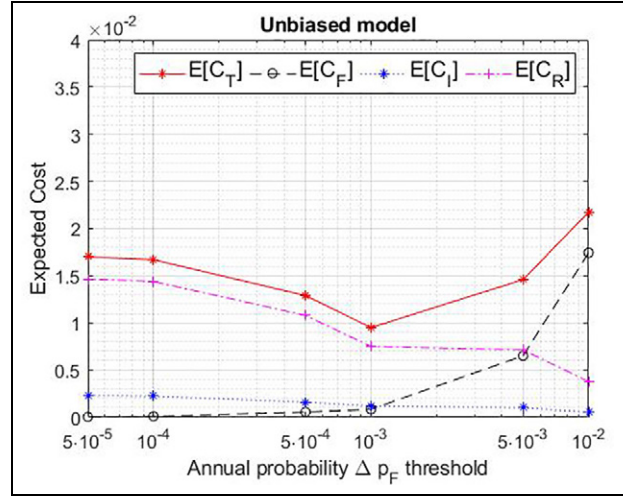


**Figure 10.** Expected life cycle costs with unbiased information.
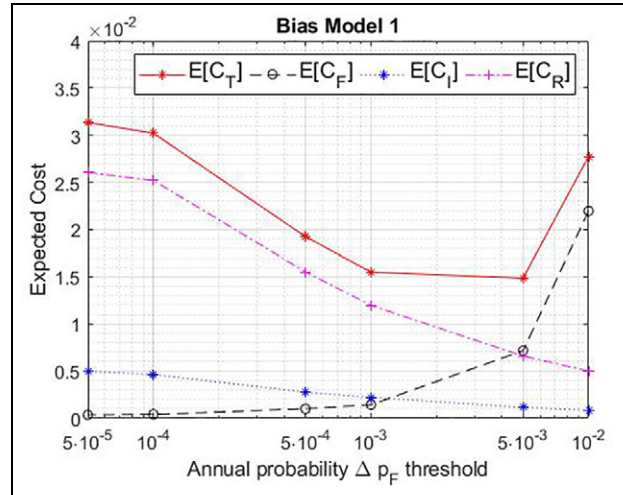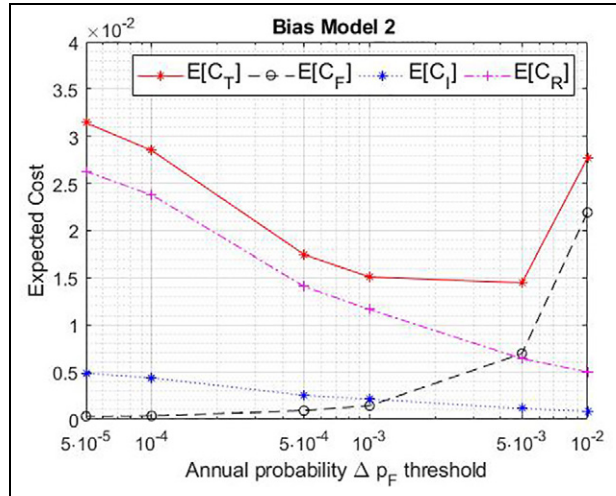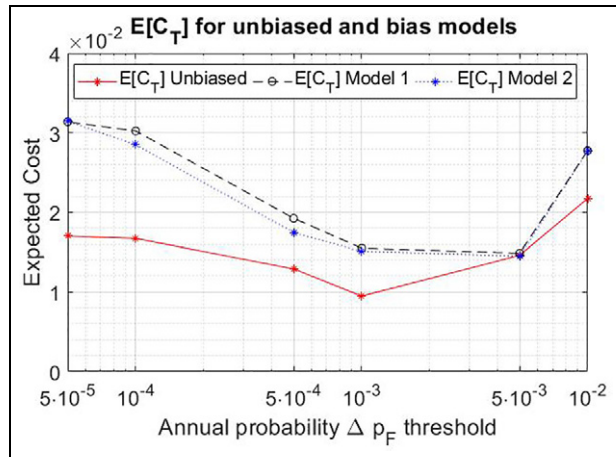


**Figure 11.** Expected life cycle costs with bias information from inspection together following Model 1.

$E[C_F]$, increases simultaneously. When the threshold $\Delta p_F$ is small, the expected value of the repair cost, $E[C_R]$, dominates in the sum of the three cost items; but when $\Delta p_F$ threshold is large, the expected value of the failure cost, $E[C_F]$, becomes dominant instead of $E[C_R]$. As the sum of the expected values of all the three costs, the expected value of the total service life cost, $E[C_T]$, decreases with increasing threshold $\Delta p_F$ but increases again as $\Delta p_F$ threshold becomes large. As shown in Figure 10, the minimal value of $E[C_T]$ in the case of unbiased inspection results corresponds to $\Delta p_F threshold = 1 \times 10^{-3}$; while the minimal value of $E[C_T]$ of the structure for the case of biased inspections corresponds to $\Delta p_F threshold = 5 \times 10^{-3}$ for both Model 1 and Model 2 as shown in Figures 11 and 12,
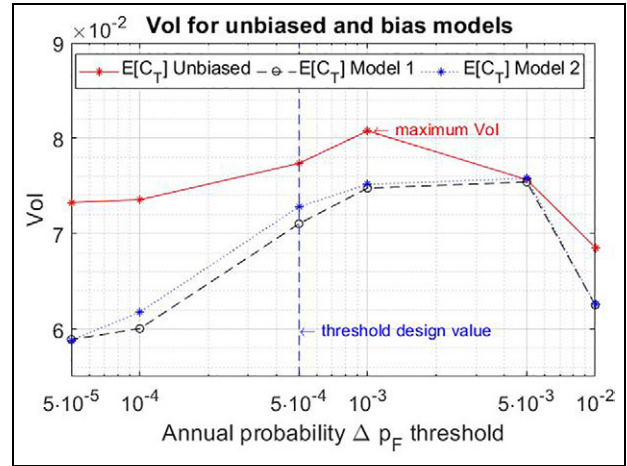
**Figure 12.** Expected value of the life cycle costs with bias information from inspection together following Model 2.



**Figure 13.** Comparison of the expected value of the total life cycle cost considering biased inspection results with two different dependency models as well as for the case of unbiased inspection results.



**Figure 14.** VoI associated with unbiased and biased inspection information in accordance with the two dependency models, as a function of $\Delta p_F$ thresholds.

respectively. The variation of the costs in Figures 11 and 12 are similar to each other, and the effect of the dependency model is insignificant. The service life is set as 20 years for the structure, and there are few inspections during the service life especially when the threshold $\Delta p_F$ is high, which makes the influence of dependency model little.

In Figure 13, a comparison of the expected values $E[C_T]$ corresponding to the two dependency models for the inspection information with biases as well as for the case of no bias, respectively, is provided. The general variation of the expected value of the life cycle costs as function of the threshold value for the annual probability of failure is similar; however, the expected value of the life cycle costs for the case of no bias is the smallest for all the thresholds. As the threshold increases, the difference between the three curves is diminishing. This is due to fact that with higher thresholds, there are fewer inspections, and thus less impact of biased information.

The results of VoI analysis as the function of the threshold $\Delta p_F$ are illustrated in Figure 14. The value of the information without bias is always the highest for all the $\Delta p_F$ thresholds; while the value of the information with bias together with the dependency following two models is close to each other. Only for the case where the threshold is $5 \times 10^3$, the VoI for all the three curves is similar. This is due to fact that fewer inspections due to high threshold level do not make difference in total cost. It is noted that the difference in VoI between the two considered dependency model is insignificant compared to the difference between the cases of no bias and bias.

## Conclusion

In this contribution, we analyze to what extent the value associated with the information collected during the service life of engineered structures, for example, inspections, in the context of integrity management, is affected by not only measurement random errors but also biases (systematic errors), taking the dependency between the collections into account. This is novel since in general the effect of possible biases together with the dependency is not accounted for in integrity management optimization—implicitly assuming that SHM and inspection techniques are always calibrated and free of

bias. To cast light on the effect of possible biases on the VoI, two different models for the representation of the possible dependency in biases from inspection time to inspection time are proposed. Their implications on the VoI are studied by means of two principal examples that are representative for integrity management of well systems subject to scale and fatigue crack growth for welded steel structures, respectively.

The examples show how VoI analysis may be applied accounting for possible biases and provide important insight on the significance of the dependency effect, which facilitate for the identification of measures and strategies to improve the beneficial effect of monitoring and inspections in the context of SIM.

From both of the two examples, it is apparent that when the number of instances over the service life where information is collected, for example, through inspections, the VoI is not significantly affected by biases. However, for the cases where the information collected by means of inspections is essential for the integrity management, the effect of biases is a significant reduction of the VoI, caused by wrong information triggering either unnecessary maintenance and repair or wrong information suggesting that no maintenance or repair is required despite actually needed. Based on the examples, it is seen that biases generally reduce the VoI. Moreover, dependencies in biases associated with information collected for integrity management at different times have a negative effect on the VoI.

The present contribution has put focus on the effects of biases and dependencies associated with information collected in support of integrity management optimization. It has been shown that these effects indeed should be accounted for explicitly, and ignoring them may lead to rather suboptimal integrity management decisions. Future studies should be directed to develop more specific models of uncertainties, biases, and dependencies, for different information collection techniques.

It should be noted here that the two models for the probabilistic representation of the bias presented in section "Information modeling in SIM" are introduced for improving the understanding of the effect of biases associated with measurements, on the VoI offered by inspection. The two models represent the extreme cases where there is either full or no dependency between the realizations of performed measurements. In the examples, and the outlines of the models, we explicitly address collection of information over time. However, it should be noted that dependencies in systematic error (bias) may also have a spatial component. Spatial dependencies may be investigated in principally the same manner as the temporal dependencies investigated within this article. The effects of the temporal and spatial variability should be taken into account in the future study.

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