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Relevant criteria for testing the quality of turbulence models

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Abstract

Seeking relevant criteria for testing the quality of turbulence models, the scale of turbulence and the gust factor have been estimated from data and compared with predictions from first-order models of these two quantities.

It is found that the mean of the measured length scales is approx. 10% smaller than the IEC model, for wind turbine hub height levels. The mean is only marginally dependent on trends in time series. It is also found that the coefficient of variation of the measured length scales is about 50%.

3sec and 10sec pre-averaging of wind speed data are relevant for MW-size wind turbines when seeking wind characteristics that correspond to one blade and the entire rotor, respectively. For heights exceeding 50-60m the gust factor increases with wind speed. For heights larger than the 60-80m, present assumptions on the value of the gust factor are significantly conservative, both for 3sec and 10sec pre-averages. The usually applied value of $k_p \approx 3$ should probably be reduced.

1 Introduction

The purpose of the paper is to identify relevant measures of the quality of turbulence models for structural response calculations of wind turbines. Thus, the characteristics of the output from a candidate turbulence model should fall within prescribed limits so that the model emulates the natural atmospheric flow satisfactory. Such characteristics are scale of turbulence, standard deviation of wind speed fluctuations, coherence function and gust factor. This paper deals with the scale of turbulence and the gust factor.

The dominant size of turbulent eddies is denominated scale of turbulence, which plays a significant role in structural loading. Thus, if eddies are large and coherent they may act simultaneously at the whole rotor and conversely smaller eddies act locally, but possibly at frequencies close to the structural eigenfrequencies and thereby causing resonance. Specifically, the scale of turbulence appears as a scaling parameter in the power spectrum and in the coherence function of wind speed

fundamentals of the derivation of the quantity are reviewed in the following.

2.1 The gust factor

The task of estimating the largest value of a random process was dealt with by Cartwright and Longuet-Higgins [1] and Davenport [2] and their results summarized in [3]. Thus, assuming the wind speed stationary and normal distributed, the expectation value of the largest gust in the time period T is

$$U_{\max} = U_m + k_p \cdot \sigma_u, \quad (1)$$

where U_m is the mean wind speed, σ_u the standard deviation of wind speed fluctuations and k_p is the largest gust of the normalised wind speed in the considered period, also denominated the gust factor:

$$k_p = \frac{U_{\max} - U_m}{\sigma_u}. \quad (2)$$

Asymptotically, for increasingly larger gusts, through some elaborate calculations, [3], the expectation value of the largest normalised gust is found as

$$k_p \approx \sqrt{2 \ln(\nu_0 T)} + \frac{\gamma}{\sqrt{2 \ln(\nu_0 T)}}, \quad (3)$$

where $\gamma = 0.577$ is Euler's constant and ν_0 is the zero up-crossing rate of the normalised wind speed process. The expected standard deviation of the largest normalised gust is found to be

$$\sigma_{k_p} \approx \frac{\pi}{\sqrt{6} \cdot \sqrt{2 \ln(\nu_0 T)}} \approx \frac{\pi}{\sqrt{6} \cdot k_p}. \quad (4)$$

The wind turbine structure only "senses" wind gusts of a certain spatial size and usually it is assumed that time-averaging of a point measurement is equivalent of spatial averaging, see further on this in Section 2.2. Therefore the wind speed series should be low-pass filtered. Thus, applying a sharp filter¹, the up-crossing frequency is found from

$$\nu_0 = \sqrt{\frac{\lambda_2}{\lambda_0}}, \quad \lambda_n = \int_0^{f_c} f^n S_u(f) df, \quad (5)$$

where S_u is the power spectral density of wind speed fluctuations, f is frequency in Hz and f_c is the cut-off frequency. λ_m are the spectral moments, see the following section.

¹ In practical data analysis block averaging or running average filters are applied.

Based on Kaimal [4], the international wind turbine standard IEC61400-1 [5] applies the following spectrum for wind speed fluctuations:

$$\begin{aligned} S_u(f) &= \frac{4fL_u/U}{\sigma_u^2 (1+6fL_u/U)^{5/3}} \rightarrow \\ S_u(f) &= \sigma_u^2 \frac{4T_u}{(1+6T_u f)^{5/3}} \end{aligned} \quad (6)$$

where U , L_u and $T_u = \frac{L_u}{U}$ are mean wind speed, length scale and time scale of turbulence, respectively. [5] models the length scale as

$$L_u = \begin{cases} 5.67 \cdot z, & z < 60 \text{ m} \\ 340 \text{ m}, & z > 60 \text{ m} \end{cases}, \quad (7)$$

where z is height above terrain. Thus the spectral moment of order n to be calculated is

$$\lambda_n = \int_0^{f_c} f^n S_u df = 4T_u^2 \sigma_u^2 \int_0^{f_c} \frac{f^n}{(1+6T_u f)^{5/3}} df. \quad (8)$$

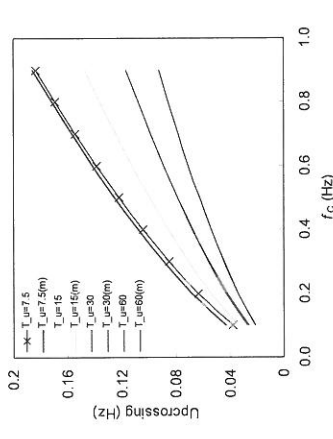


Figure 1. Zero-crossing frequency as function of pre-averaging time for different time scales of turbulence T_u . The lines with (m) correspond to the truncated expressions, (11) and (12).

From (8), the zero and second order moments are found as

$$\lambda_0 = \sigma_u^2 \left(1 - (1+6T_u f_c)^{-2/3} \right) \text{ and} \quad (9)$$

$$\lambda_2 = \frac{1}{54T_u^2} \sigma_u^2 \left(\frac{3}{4} (1+6T_u f_c)^{1/3} - \frac{3}{2} (1+6T_u f_c)^{-2/3} - 6(1+6T_u f_c)^{1/3} + 27 \right) \quad (10)$$

With values of wind speed, length scale and cut-off frequency relevant in the context, these expressions may be approximated as

$$\lambda_0 = \sigma_u^2 \left[- (1 + 6T_u f_c)^{-2/3} \right] \approx 0.95 \cdot \sigma_u^2 \quad (11)$$

$$\lambda_2 \approx \frac{(T_u f_c)^{1/3}}{6.6 \cdot T_u^2} \sigma_u^2 = \frac{\sigma_u^2}{6.6} \cdot \frac{f_c^{4/3}}{T_u^{2/3}} \quad (12)$$

Thus,

$$v_0 = \sqrt{\lambda_0} \approx 0.39 \cdot \frac{f_c^{2/3} \cdot U^{1/3}}{T_u^{1/3}} = 0.39 \cdot \frac{f_c^{2/3} \cdot U^{1/3}}{L_u^{1/3}} \quad (13)$$

This approximation fits well for $T_u > 5 - 10$ s, see Figure 1.

2.2 Emulating rotor cut-off frequency

The wind turbine blade/rotor effectively makes a lateral spatial averaging of the along-wind wind speed. The averaged wind speed cannot in practical terms – or only with great difficulty – be measured. Therefore, we seek alternative ways of reading from a point-measurement of wind speed what is actually “sensed” by the wind turbine blade/rotor. In [5], an option for the cross-wind coherence of the u -component of the wind velocity is offered:

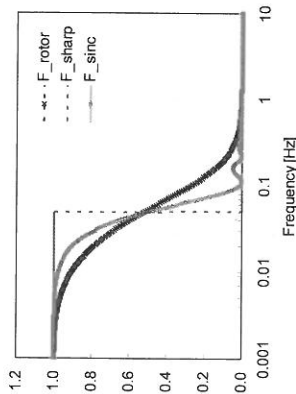


Figure 2. Wind spectrum filter functions.

Coh(r, f) = exp(ar) where

$$a = -b \left[\left(\frac{f}{U} \right)^2 + \left(\frac{0.12}{L_u} \right)^2 \right]^{1/2} \quad (14)$$

and r is the (lateral) separation between the two considered points. The factor b is in [5] assumed to be 12. However, the expression has been analyzed² showing

² Kenneth Thomsen, personal communication.

that a value of 5 makes a better fit to an alternative turbulence model-option in the standard. In homogeneous turbulence, the coherence is the normalized cross-spectrum of wind speed and thus the integral over the rotor disc of the coherence function, normalized so that it approaches unity when $f \rightarrow 0$, could be termed the *rotor filter function*.

$$I(f) = \int_0^{2\pi R} \int_0^{2\pi R} \exp(ar) dr d\theta \Rightarrow \quad (15)$$

$$I(f) = 2\pi \left(R \cdot \frac{1}{a} - \frac{1}{a^2} \right) \exp(aR) + 2\pi \frac{1}{a^2} \Rightarrow \quad (16)$$

$$F_{rotor}(f) = \frac{I(f)}{I(0)} \quad (17)$$

For various reasons, wind speed data are low pass filtered by means of running average and the corresponding filter has the form

$$F_{sync}(f) = \frac{\sin^2(2\pi f \cdot \Delta T / 2)}{(2\pi f \cdot \Delta T / 2)^2} \quad (18)$$

where ΔT is the running pre-averaging time. To obtain a half-value of this filter at the frequency f_c , the running average time must be:

$$\sin^2 x = \frac{1}{2} \Rightarrow \Delta T \approx \frac{0.44}{f_c} \quad (19)$$

Thus, to emulate the rotor filtering with a cut off frequency of f_c , the running average time ΔT calculated from (18) must be applied.

In Figure 2, the rotor filter is shown (for $U = 11$ m/s, $L = 300$ m, $R = 50$ m), where the half-value frequency comes out as $f_c = 0.05$, together with the running-average filter with the averaging time subsequently given by (17).

2.3 Linking gust factor and scale of turbulence

The second term on the right-hand side of (3) accounts for approx. 10% of the value of k_p and it is therefore a fair approximation – in the context of this section – to reformulate (3) as

$$k_p \approx 1.1 \cdot \sqrt{2 \ln(v_0 T)} \quad (20)$$

Inserting the expression for the up-crossing frequency, (13), we get

$$k_p \approx 1.1 \cdot \sqrt{2 \ln \left(\frac{f_c^{2/3} \cdot U^{1/3}}{L_u^{1/3}} \cdot T \right)} \Rightarrow \quad (21)$$

$$k_p = 1.1 \cdot \sqrt{2 \ln \left(\left(\frac{T}{T_c} \right)^{1/3} \cdot \left(\frac{U T_c}{L_u} \right)^{1/3} \right)} \quad (22)$$

Conversely,

$$L_u = 0.06 \cdot \left(\frac{T}{T_c} \right)^3 \cdot (U T_c) \cdot e^{-\frac{2}{3} \ln^2 \left(\frac{k_p}{1.1} \right)} \quad (23)$$

From (20) it is seen that the gust factor is little sensitive to the scale of turbulence.

The expression (20) is similar to what was obtained previously by Kristensen et al [6], except for the constant “0.39”. The discrepancy is ascribed to the difference in definition of cut-off frequency f_c .

In the data analysis in Section 4, the estimation of k_p is done in 2 m/s bin and thus an adjustment of the estimate to the centre of the bin is needed. Rewriting (21), we get

$$k_p = 1.1 \cdot \sqrt{d + \frac{2}{3} \ln(U)} \quad (24)$$

$$d = \frac{2}{3} \ln \left(0.06 \cdot \left(\frac{T}{T_c} \right)^3 \cdot \left(\frac{T_c}{L_u} \right) \right) \quad (25)$$

Thus, for each realization (i) with wind speed U_i , the gust factor is re-calculated to the bin centre as follows:

$$k_p = 1.1 \cdot \sqrt{d - \frac{2}{3} \ln(U_i) + \frac{2}{3} \ln(U_{cr})} \quad (26)$$

where U_{cr} is the bin-centre wind speed.

3 Høvsøre test site

The test site for large wind turbines is located 1.5 km from the North Sea coastline in western Denmark. The site itself and the surroundings in all directions are flat, with no hills or boulders and no significant obstacles, with terrain surface roughness of 2–3 cm. The site has the capacity of testing 5 wind turbines simultaneously and is fully equipped with atmospheric monitoring equipment in nearby met masts, Figure 3.

To the south at the site a 116 m tall mast is erected. The mast is instrument with cup anemometers at 7 levels, 2, 10, 40, 60, 80, 100 and 116 m. The 160 m level is covered by an anemometer at the top of the warning-light mast located between no. 1 and 2 wind turbine counted from south.

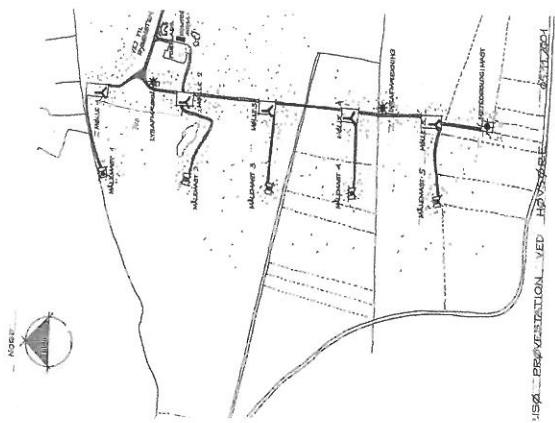


Figure 3 Overview of the test site at Høvsøre. 5 wind turbines may be tested simultaneously, met mast are erected to the west of all machines. South of the most southern wind turbine location the 116 m met tower is located.

4 Measurement results

Wind speed is sampled at 5 Hz and 600 sec. series are stored and statistics calculated.

Measurements have been performed for 4 years and for the present analyses a total of 201 600 sec. time series of wind speed have been included. The selected time series have wind direction from the eastern direction, where the terrain is homogeneous and the data have been filtered so that the atmospheric stratification is near-neutral.

The time series were not de-trended. However, the effect of de-trending was tested and found marginal.

4.1 Estimating turbulence length scale

For all measurement levels the spectra of the 201 sets of wind speed time series were fitted to the Kaiman spectrum (6) and the length scale deduced, Figure 4.

In Figure 5, length scale – mean and standard deviation band – is plotted as function of observation height. From near the ground and up to approx. 40m, L_w increases rapidly with height. After that, the rate of increase levels off. Further, it is seen that the scatter is significant with

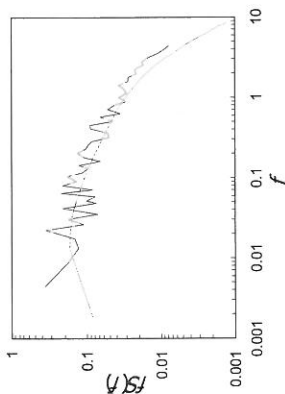


Figure 4 The spectrum of each 10min time series is fitted to the Kaimal spectrum (6) and the scale of turbulence deduced.

the 2σ -band being approximately of the same magnitude as the mean. Also shown is the IEC model (7), which seemingly underestimate L_w significantly close to the ground but otherwise follows the general trend of the measurements.

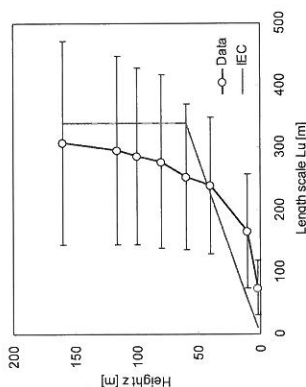


Figure 5 Høvsøre. Scale of turbulence as function of height. Wind speed at $H=100m$: $10m/s < U < 12m/s$. The "error bars" represent the standard deviation of the observations.

Thus, considering the large scatter of the measured length scale, the IEC model fits rather well for heights relevant for MW wind turbines. However, plotting L_w with logarithmic height axis, Figure 6, indicates that $L_w \propto \ln(H)$, which is similar to the wind speed's height-dependency.

4.2 Estimating gust factor

In Figure 8 the measured gust factor is shown as function of pre-averaging time. Also shown is the model (3) calculated with $L_w = 300m$, $U = 11m/s$ and the cut-off frequency determined by means of (18). While the model fits qualitatively well it over-predicts consistently, with 15% at small ΔT s and 10% at high ΔT s.

The data was pre-averaged with two different averaging times, 3 and 10sec. The 3sec averaging time serves as a reference to what is often applied in civil engineering standards as being relevant for typical sizes of buildings.

The pre-averaging period $\Delta T = 10sec$ corresponds to a half-value of the running average filter with $f_c \approx \frac{0.44}{10} = 0.044Hz$. Assuming $L_w = 300m$ and the mean wind speed $U = 11m/s$, then in (16) the rotor radius must be approx. 60m for the rotor filter to take half-value at f_c :

$$F_{rotor}(f_c) \approx \frac{1}{2} \text{ for } f_c = 0.044 \quad (24)$$

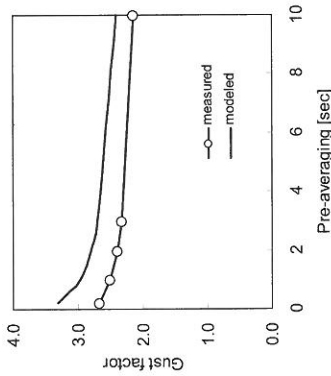


Figure 8. Høvsøre. Gust factor as function of pre-averaging time ΔT measurement height 100m and wind speed $10m/s < U < 12m/s$.

Thus, the 10sec pre-averaging of wind speed measurements corresponds to the filtering effect of a 120m rotor at typical operational wind speeds.

In Figure 9 and Figure 10, the gust factor estimated from measurements is plotted as function of height for different wind speed levels. At wind speeds below approx. 60m, k_p appears independent of wind speed. At larger heights k_p decreases with wind speed. The measured gust factor is approx. 10% lower for the 10sec pre-averaged series than the 3sec averaged series.

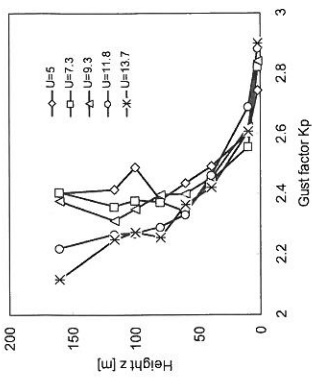


Figure 9 Høvsøre. Gust factor as function of height for different wind speeds. Sinc-filter frequency is 1/3 Hz.

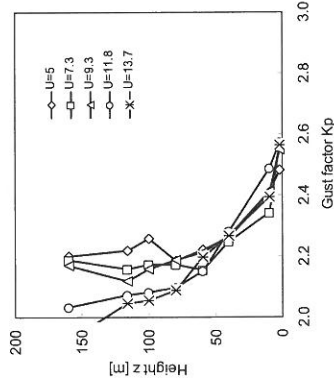


Figure 10 Høvsøre. Gust factor as function of height for different wind speeds. Sinc-filter frequency is 1/10 Hz.

Finally, in Figure 11 the gust factor is plotted for both averaging times together with the model predictions. The predictions (using the measured L_w s) fit well close to the ground. The drop-off of k_p with increasing height is less well predicted and in the range 80-160m the over prediction is approx. 10%.

While the 10sec average corresponds to the wind-integrating effect of the entire rotor, the 3sec corresponds approx. to the filtering effect of one blade.

wind speed is Weibull distributed with parameters $k = 9$ m/s and $k = 2.3$, the annual number of 10 minutes periods in the targeted bin is approximately 7100. The corresponding 1-year return period of k_p becomes 4.6, which is significantly larger than the mean value and reflects that for response quantities dominated by average wind speeds at e.g. 10-12 m/s, the number of 10 minutes periods is very large and therefore k_p with return period e.g. equal to 1 year becomes much larger than the mean k_p value.

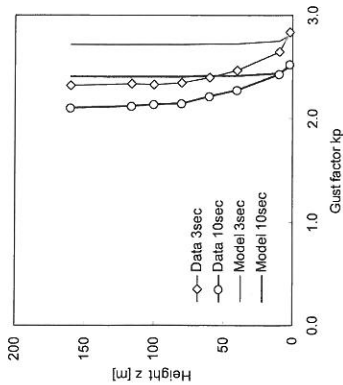


Figure 11 Høvsøre. Measurement and model of normalised wind gust, k_p , as function of height for pre-averaging times of 3 and 10 sec., respectively. 10 m/s $< U < 12$ m/s.

4.3 Extrapolation to rare events

In wind turbine design calculations the distributions of response (conditioned on wind speed) are found by mean of random simulations. A turbulence model gives the input to the simulations and thus the properties of the turbulence model set the lower limit of the uncertainty of the estimates of lifetime extreme response.

Therefore, in this section the measurement results are used to estimate k_p values corresponding to rare events. Using the Maximum Likelihood Method, the 201 representative k_p 80 m height data with 10 sec averaging time is fitted to a Gumbel extreme distribution with scale and shape parameters α and u . The expected value of the Gumbel distribution corresponds approximately to (3). The expected values of the statistical parameters are $\alpha = 3.21$ and $u = 1.97$. The density function (PDF) of k_p is shown in Figure 12. It is seen that the Gumbel distribution fits the data well, especially at higher k_p values.

The standard deviations of the estimates are obtained using that asymptotically the covariance matrix for the statistical parameters become Normal distributed with the covariance matrix determined from the inverse Hessian matrix of the Log-Likelihood function, see [7]: $\sigma_\alpha = 0.18$ and $\sigma_u = 0.023$. The correlation coefficient between the parameters becomes: $\rho_{\alpha,u} = -0.32$.

Assuming that the 10-12 m/s bin is representative for a response quantity under normal operation of a wind turbine and that the long term distribution of the mean

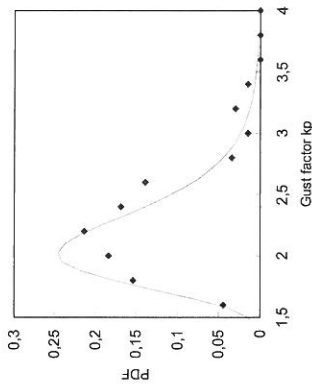


Figure 12 Density function for Gust factor k_p based on 10 sec averaging data for 80m height (201 data points).

A 95% probability interval on the 1-year estimate is [4.3;4.9] and is calculated using the estimated statistical uncertainty and FORM (First Order Reliability Methods), see [3].

If only the 50 largest k_p data values are used to fit the parameters in the Gumbel distribution, the corresponding return periods as function of k_p are shown in Figure 13. The extrapolation is seen to fit the data well. The 1 year k_p -value is estimated to 4.1, i.e. smaller than the estimate based on all 201 data. The 95% probability interval on the 1-year estimate is [3.7;4.6]. It is noted that the probability bounds become larger due to larger statistical uncertainty related to the smaller data sample.

Using instead 3sec pre-averaged data the 95% probability interval on the 1-year estimate is [4.7 ; 5.3] with best estimate equal to 5.0. If only the 50 largest k_p data values are used the 95% probability interval on the 1-year estimate is [4.1 ; 5.0] with best estimate equal to 4.6. As expected larger 1-year k_p estimates are obtained compared to the results obtained using 10 sec averaging.

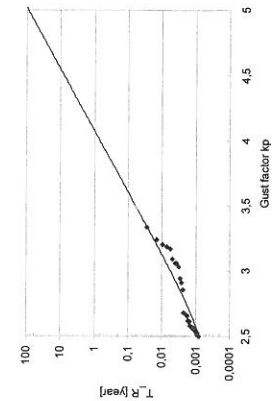


Figure 13 Return period for Gust factor k_p based on 10 sec averaging data for 80m height (50 largest data).

4.4 Scale and gust factor under wake conditions

The previous considerations concern turbulence characteristics in the ambient flow. Without trying to match or verify models, the length scale and gust factor is probed under wake conditions.

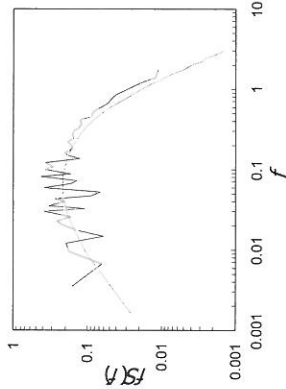


Figure 14 Høvsøre. Fitting of measured spectrum measured at height 100m in signal tower approx 100m from nearest wind turbine; wind direction 355° between tower and wind turbine, which is approx. centre-wake conditions.

Measurements of scale and gust factor made at three levels in the signal tower were analyzed. The distance to the nearest wind turbine is 100m, i.e. the observation points are in the near-wake. Figure 14 shows an example of the fitting of the Kaimal spectrum to the measurements.

In Figure 15, the scale of turbulence is plotted as function of wind direction. For wind directions seemingly corresponding to the edge of the wake, the scale drops

off from approx. 250m (Figure 7) to 25-30% of that value.

Figure 16 shows the gust factor as function of wind direction. k_p appears slightly increased under wake conditions, except for the 160m level, where there is a decrease. These observations confirm similar measurements made under far-wake condition, [8].

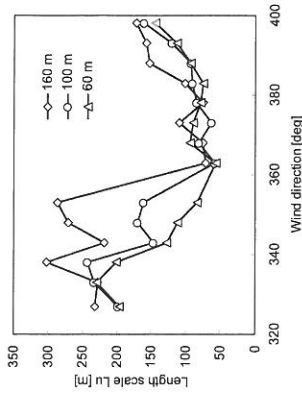


Figure 15 Høvsøre. Turbulence length scale estimated from measurements as function of wind direction. 7m/s $< U < 8$ m/s.

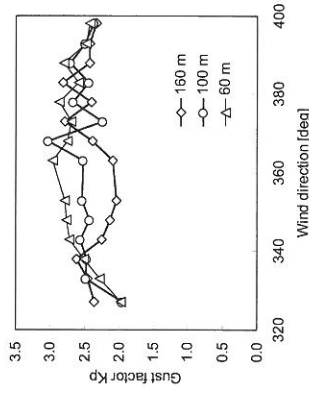


Figure 16 Høvsøre. Gust factor estimated from measurements as function of wind direction. 7m/s $< U < 8$ m/s.

4.5 Mean shear and turbulence profile

The vertical wind shear taken over the utilized data (only near-neutral conditions were considered) shows only marginal deviation from the logarithmic shape.

The measured vertical profile of standard deviation of wind speed fluctuations deviates only marginal from σ_u being height-independent.

5 Conclusions

Seeking relevant criteria for testing the quality of turbulence models, the scale of turbulence and the gust factor have been estimated from data and compared with predictions from models of these quantities.

The main findings are the following:

- The average of the measured length scales at hub height is approx. 10% less than what the IEC model yields. The average of the measured length scales is only marginally dependent on trends in time series
- The COV of the measured length scales is about 50%, only marginally dependent on trends in time series. The measured length scales are approximately Weibull distributed
- The observed length scale appears at measuring height larger than approx. 60-80m to be proportional to wind speed, $L_w \approx 25 \cdot U$
- 3sec and 10sec pre-averaging of wind speed data are relevant for MW-size wind turbines when seeking wind characteristics that correspond to one blade and the entire rotor, respectively
- For heights larger than 50-60m the gust factor decreases with wind speed
- For heights larger the 60-80m present assumptions on the value of the gust factor are significantly conservative, both for 3sec and 10sec pre-averages. The usually applied value of $k_p \approx 3$ should be reduced
- The standard analytical model for the gust factor over-estimates with about 10% relative to the observations
- The one-year 10sec gust factor in the wind speed bin 10-12 m/s is approx. 4.6, assuming the mean wind speed Weibull distributed with $A=9$ m/s and $k=2.3$
- Approx. one rotor diameter downwind the length scale under wake conditions is reduced to 25-30% of ambient level.
- The gust factor under wake conditions is seemingly only slightly affected by the wake.

In addition to specifications of σ_u and L_w , it is suggested to request from turbulence models – which are to be used for structural response simulations – that the mean and COV of gust factor are within specified bounds. Also the

extreme distribution of k_p yielded from the turbulence model should be realistic.

6 Acknowledgements

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