

Tight wavelet frames in Lebesgue and Sobolev spaces

We study tight wavelet frame systems in $L_p(\mathbb{R}^d)$ and prove that such systems (under mild hypotheses) give atomic decompositions of $L_p(\mathbb{R}^d)$ for $1 < p < \infty$. We also characterise $L_p(\mathbb{R}^d)$ and Sobolev space norms by the analysis coefficients for the frame. We consider Jackson inequalities for best m -term approximation with the systems in $L_p(\mathbb{R}^d)$ and prove that such inequalities exist. Moreover, it is proved that the approximation rate given by the Jackson inequality can be realized by thresholding the frame coefficients. Finally, we show that in certain restricted cases, the approximation spaces, for best m -term approximation, associated with tight wavelet frames can be characterized in terms of (essentially) Besov spaces.