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Enhanced Voltage Control in Distribution Networks: A Data-driven Approach

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Abstract—Traditional voltage/var control(VVC) method assume the prerequisite of an exact distribution network(DN) model that keep unchanged during the whole control period, which may not hold in practice. In this paper, we proposed a data-driven enhanced voltage/var control algorithm(DaE-VVC) to regulate voltage profiles in DN with an incomplete model. The proposed method can estimate the DN topology and line parameter with satisfactory accuracy using the measurements of bus voltages and real/reactive power injections. The proposed algorithm can be integrated into the centralized control center without any additional hardware cost. Case studies on balanced modified IEEE-33 bus system demonstrate the performance of the proposed algorithm.

Index Terms—Voltage/Var control, distribution network, distributed generation, data-driven control

I. INTRODUCTION

VVC is one of the essential tasks to ensure distribution network (DN) operation. The main objective of VVC is to maintain voltage magnitudes of all distribution feeders within the allowed limit. Conventionally, VVC is accomplished by adjusting the control actions of mechanical VVC devices, including transformers with on-load tap changer(OLTC), capacitor banks(CBs) and the fast-responding distributed generations(DGs). To ensure optimality, the control actions of VVC devices are usually optimized in centralized or distributed way [1]–[4]. In [1], the mechanical VVC devices and DGs were scheduled to address voltage problems in scenarios with high penetrated renewable energy. The dispatch of these VVC devices were decomposed into two-stage in [2] considering the characteristics of different voltage regulation equipments. In addition, the real/reactive power dispatch of DGs was determined in distributed way in [3], [4] for the purpose of alleviating the data communication burden in centralized control.

However, the above traditional VVC works all assume the prerequisite of accurate DN model that keeps fixed during the whole control period, which may not hold in reality. Unlike the transmission grid model which are regularly measured and monitored, the DN model may only be available in grid planning files in the early age, such model could be outdated due to network expansion [5]. Furthermore, there

could be changes both in DN topology and parameter due to network reconfiguration during the real-time operation [6]. The traditional VVC methods suffer from shortcomings in such situation.

Fortunately, nowadays the widely deployment of measurement devices in DNs, such as smart meters or micro-phasor measurement unit(μPMU), makes it possible to develop data-driven method to cope with above-mentioned challenges [7]. In this paper, the voltage control problem given an incomplete model is studied and DaE-VVC algorithm is proposed. In the proposed algorithm, the DN model are estimated by measurements of node voltage magnitudes and real/reactive power injections. To this end, firstly, the nonlinear relationship between voltage magnitudes and power injections is approximated by the linear Distflow model(LDF) [8], then the DN model is identified in two steps that can computed in parallel. At last, the real/reactive power output of DG inverters are dispatched to regulate feeder voltages to the allowed range. Compared with existing VVC works in this area, the contributions of this paper are as follows,

- The DaE-VVC algorithm can guarantee the bus voltages within feasible range with an incomplete DN model. As proved later, the algorithm can identify the DN topology and line parameters with satisfactory accuracy.
- The proposed DaE-VVC can be integrated into a centralized VVC control center, thus introducing no additional hardware costs or investment.
- The effectiveness of proposed algorithm are demonstrated by case studies on modified IEEE-33 bus balanced distribution grid.

The reminder of this paper is organized as follows: Section II introduces the modelling of DNs. Section III formulates of the proposed DaE-VVC algorithm. Case studies are presented in Section IV, while conclusions are provided in Section V.

II. PRELIMINARIES

A. Modelling of DNs

A typical DN comprised of $N+1$ buses is shown in Fig. 1. Suppose the topology of DN is represented by $\mathcal{G} = (\mathcal{N}, \mathcal{L})$, let $\mathcal{N} := \mathcal{N}_+ \cup \{0\}$ be the set of buses where bus indexed 0 is the substation bus. For each bus $i \in \mathcal{N}_+$, let p_i, q_i denote the its real/reactive power injection and $\mathcal{C}_i$ denote set of its children bus. Let $\mathcal{L}$ represent the set of lines, for each line $\ell \in \mathcal{L}$ connecting bus (i, j), let r_{ij} and x_{ij} denote its line resistance and reactance. Throughout the rest this paper, we have the following assumptions,
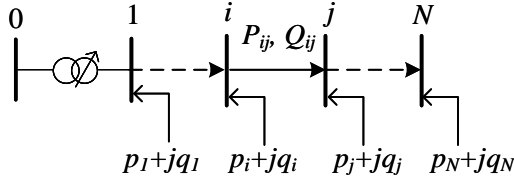
Fig. 1. Topology of DN

C1. Bus 0 is the substation bus and assumed to the slack bus.
C2. The topology of DNs in radial.
C3. The r/x ratio of all branches in DNs is available.

The power flow equation of DN is described by LDF [8],

$$P_{ij} = \sum_{k \in \mathcal{C}_j} P_{jk} - p_j \tag{1a}$$

$$Q_{ij} = \sum_{k \in \mathcal{C}_j} Q_{jk} - q_j \tag{1b}$$

$$v_i - v_j = 2\left(r_{ij}P_{ij} + x_{ij}Q_{ij}\right) \tag{1c}$$

B. Compact DN Model

In this section, we introduce the compact form representation of LDF model, which makes it much easier to develop the proposed algorithm. Let $\overline{\boldsymbol{A}} := \left[\mathbf{a}_0 \ \boldsymbol{A}^\top\right]^\top \in \{0, \pm 1\}^{(N+1) \times N}$ denote the incidence matrix of $\mathcal{G}$, which is defined as: $\overline{\boldsymbol{A}}_{i\ell} = 1$ if branch ℓ starts at bus i whereas $\overline{\boldsymbol{A}}_{i\ell} = -1$ if line ℓ ends at bus i, otherwise $\overline{\boldsymbol{A}}_{i\ell} = 0$ [9]. $\mathbf{a}_0^\top$ is the first row of $\overline{\boldsymbol{A}}$ and $\boldsymbol{A}$ is the reduced incidence matrix. Collect squared voltage magnitude of all buses in set $\mathcal{N}_+$ into $\boldsymbol{v} := [v_1, \cdots, v_N]^\top \in \mathbb{R}^N$, $v_i = V_i^2$, real/reactive power injection in set $\mathcal{N}_+$ into $\boldsymbol{p} := [p_1, \cdots, p_N]^\top$ and $\boldsymbol{q} := [q_1, \cdots, q_N]^\top$, respectively. Collect the resistance and reactance of all branches in set $\mathcal{L}$ into $\boldsymbol{r} = [r_1, \cdots, r_L]^\top$ and $\boldsymbol{x} = [x_1, \cdots, x_L]^\top$, respectively. Then the compact form representation of LDF is given as,

$$\boldsymbol{v} = 2\boldsymbol{A}^{-\top}\boldsymbol{R}\boldsymbol{A}^{-1}\boldsymbol{p} + 2\boldsymbol{A}^{-\top}\boldsymbol{X}\boldsymbol{A}^{-1}\boldsymbol{q} + v_0\mathbf{1}_N \tag{2}$$

where v_0 is the squared voltage magnitude of bus 0, $\boldsymbol{R} := \mathrm{diag}(\boldsymbol{r})$ and $\boldsymbol{X} := \mathrm{diag}(\boldsymbol{x})$ are diagonal matrices with $\boldsymbol{r}$ and $\boldsymbol{x}$ in diagonal.

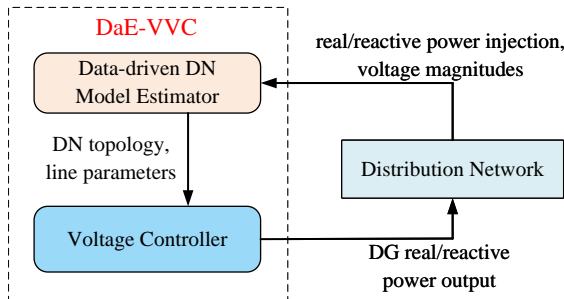## III. FORMULATION OF THE PROPOSED ALGORITHM

Fig. 2. Diagram of the proposed DaE-VVC

In this section, we introduce the details of the proposed DaE-VVC algorithm. The diagram of the proposed algorithm

is illustrated in Fig. 2. There're two functions in the proposed DaE-VVC, the data-driven DN model estimator used to identify the DN model and the voltage controller that dispatching the real/reactive power outputs of DGs to regulate the bus voltages.

A. Data-driven DN Model Estimator

The DNs are usually operated at several possible topology configurations according to different status of section switches. Suppose all possible topologies $\boldsymbol{T}_i$ are collected into set $\mathcal{T}$ and the r/x ratio of branch $j \in \mathcal{L}$ are collected into vector $\boldsymbol{\zeta} = [\zeta_1, \cdots, \zeta_L]^\top$. The purpose of topology identification is to choose the correct topology from set $\mathcal{T}$ while the objective of parameter estimation is to calculate the line resistance and reactance parameters based on given topology configuration.

1) Parameter Estimation

To perform parameter estimation, we reorganize (2) at time instant t into,

$$\tilde{\boldsymbol{v}}(t) = \boldsymbol{\Phi}\boldsymbol{p}(t) + \boldsymbol{\Psi}\boldsymbol{q}(t) \tag{3}$$

where $\boldsymbol{\Phi} := 2\boldsymbol{A}^{-\top}\boldsymbol{R}\boldsymbol{A}^{-1}$, $\boldsymbol{\Psi} := 2\boldsymbol{A}^{-\top}\boldsymbol{X}\boldsymbol{A}^{-1}$ and $\tilde{\boldsymbol{v}}(t) = \boldsymbol{v}(t) - v_0\mathbf{1}_N$.

The line parameter estimation given a certain topology is expressed as,

$$\left(\hat{\boldsymbol{R}}, \hat{\boldsymbol{X}}\right) = \arg\min_{(\boldsymbol{R}, \boldsymbol{X})} \sum_{t \in \mathcal{K}} \|\boldsymbol{\Phi}\boldsymbol{p}(t) + \boldsymbol{\Psi}\boldsymbol{q}(t) - \tilde{\boldsymbol{v}}(t)\|^2 \tag{4}$$

Rewrite the diagonal matrix $\boldsymbol{X}$ into

$$\boldsymbol{X} = \sum_{\ell=1}^{L} x_\ell \boldsymbol{e}_\ell \boldsymbol{e}_\ell^\top \tag{5}$$

where the ℓ_{th} element of $\boldsymbol{e}_\ell$ is 1 while the rest elements are all zeros.

Define $\boldsymbol{\Gamma}_\ell = 2\boldsymbol{A}^{-\top}\sum_{\ell=1}^{L} x_\ell \boldsymbol{e}_\ell \boldsymbol{e}_\ell^\top \boldsymbol{A}^{-1}$, then $\boldsymbol{X}$ can be further rewritten as,

$$\boldsymbol{X} := \sum_{\ell=1}^{L} \boldsymbol{\Gamma}_\ell x_\ell \tag{6}$$

Similarly, $\boldsymbol{R}$ can be reformulated as,

$$\boldsymbol{R} = \sum_{\ell=1}^{L} \boldsymbol{\Gamma}_\ell \zeta_\ell x_\ell \tag{7}$$

Let $\boldsymbol{\phi}(t) = \zeta_\ell \boldsymbol{p}(t) + \boldsymbol{q}(t)$ and define matrix,

$$\boldsymbol{\Theta}(t) = \begin{bmatrix} \boldsymbol{\Gamma}_1\boldsymbol{\phi}_1(t) & \cdots & \boldsymbol{\Gamma}_L\boldsymbol{\phi}_L(t) \\ \vdots & \vdots & \vdots \\ \boldsymbol{\Gamma}_1\boldsymbol{\phi}_1(t-k) & \cdots & \boldsymbol{\Gamma}_L\boldsymbol{\phi}_L(t-k) \end{bmatrix} \tag{8a}$$

$$\boldsymbol{\Xi}(t) = \left[\boldsymbol{v}(t)^\top, \cdots, \boldsymbol{v}(t-k)^\top\right]^\top \tag{8b}$$

The original parameter estimation problem (4) is reformulated as,

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{\Theta}(t)\boldsymbol{x} - \boldsymbol{\Xi}(t)\|^2 \tag{9}$$

Clearly, (9) is in the form of linear regression which can be efficiently solved.

2) Topology Identification

The topology identification equals to estimation of incidence matrix $\boldsymbol{A}$. Suppose the incidence matrix associated with all feasible topology configurations are collected into set $\mathcal{A} := \{\boldsymbol{A}_1, \cdots, \boldsymbol{A}_k\}$. The topology identification problem is equivalent to choose the true incidence matrix in $\mathcal{A}$ given a set of voltage-power injection measurements. Here we define a residual error $\boldsymbol{\varepsilon}_i$ at time instance t for each possible incidence matrix $\boldsymbol{A}_i \in \mathcal{A}$,

$$\boldsymbol{\varepsilon}_i(t) = \hat{\boldsymbol{\Phi}}_i \boldsymbol{p}(t) + \hat{\boldsymbol{\Psi}}_i \boldsymbol{q}(t) - \tilde{\boldsymbol{v}}(t) \tag{10a}$$

where

$$\hat{\boldsymbol{\Phi}}_i = 2\boldsymbol{A}_i^{-\top} \hat{\boldsymbol{R}} \boldsymbol{A}_i^{-1} \tag{10b}$$

$$\hat{\boldsymbol{\Psi}}_i = 2\boldsymbol{A}_i^{-\top} \hat{\boldsymbol{X}} \boldsymbol{A}_i^{-1} \tag{10c}$$

The problem of topology identification is formed as minimizing $\|\boldsymbol{\varepsilon}_i\|$ over the past several time instants $t \in \mathcal{K}$. The algorithm returns the topology configuration with the smallest residual error,

$$\hat{\boldsymbol{A}} = \arg\min_{\boldsymbol{A}_i \in \mathcal{A}} \sum_{t \in \mathcal{K}} \|\boldsymbol{\varepsilon}_i(t)\| \tag{11}$$

Even though the DN topology and line parameters are determined in two steps, they can be computed in parallel. That is, the algorithms compute the residual error and the corresponding line parameters for each $\boldsymbol{A}_i \in \mathcal{A}$ at the same time, then the topology with the smallest residual error and the associated parameters will be identified as the DN model.

3) Voltage Controller

The voltage controller here is similar to that in traditional voltage control. It determines the real/reactive power setpoints for DGs while subjecting to certain requirements on grid codes. In this paper, the DGs are assumed to operate at MPPT mode to ensure maximum active power capture. The objectives of voltage controller is to maintain the bus voltages in the allowed range and minimizing power losses by dispatch the DG reactive power. The formulation of voltage controller is given as,

$$\min_{\boldsymbol{q}_g} \ P_{\text{loss}}(t) \tag{12a}$$

over $\forall (i,j) \in \mathcal{L}, \forall i, j \in \mathcal{N}$
subject to,

$$\boldsymbol{v}(t) = 2\boldsymbol{A}^{-\top}\hat{\boldsymbol{R}}\boldsymbol{A}^{-1}\boldsymbol{p}(t) + 2\boldsymbol{A}^{-\top}\hat{\boldsymbol{X}}\boldsymbol{A}^{-1}\boldsymbol{q}(t) + v_0 \boldsymbol{1}_N \tag{12b}$$

$$P_{\text{loss}}(t) = \sum_{(i,j) \in \mathcal{L}} \hat{r} \frac{P_{ij}^2(t) + Q_{ij}^2(t)}{v_0} \tag{12c}$$

$$\underline{\boldsymbol{v}} \leqslant \boldsymbol{v}(t) \leqslant \overline{\boldsymbol{v}} \tag{12d}$$

$$\underline{\boldsymbol{q}}_g \leqslant \boldsymbol{q}_g \leqslant \overline{\boldsymbol{q}}_g \tag{12e}$$

During the DN operation, the Data-driven model estimator continuously collects the real/reactive power injection and voltage magnitudes measurement from the metering devices, performs the estimation algorithm to obtain the DN model. After that, the DN model will be feed into the voltage controller to get the DG real/reactive power dispatch. The proposed algorithm is summarized in Algorithm 1 for better illustration.

Algorithm 1 The proposed DaE-VVC

> **Input:**
>   The set of incidence matrix $\mathcal{A}$
>   The set of r/x ratio vectors $\boldsymbol{\zeta}$
> **while** t **do**
>   collect real/reactive power and voltage magnitude measurement $\boldsymbol{p}(t)$, $\boldsymbol{q}(t)$ and $\boldsymbol{v}(t)$, $t \in \mathcal{K}$
>   **for** $\boldsymbol{A}_i \in \mathcal{A}$ **do**
>   Compute $\boldsymbol{\varepsilon}_i$ and $\hat{\boldsymbol{\Phi}}_i$, $\hat{\boldsymbol{\Psi}}_i$ according to (10)
>   Return $\hat{\boldsymbol{A}}$ according to (11)
>   Compute the associated line parameters $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{x}}$ according to (9)
>   Voltage controller $\leftarrow \hat{\boldsymbol{A}}, \hat{\boldsymbol{r}}, \hat{\boldsymbol{x}}$
>   **end for**
>   Compute $\boldsymbol{p}(t+1)$, $\boldsymbol{q}(t+1)$ according to (12)
> $t \leftarrow t + 1$
> **end while**

IV. CASE STUDY
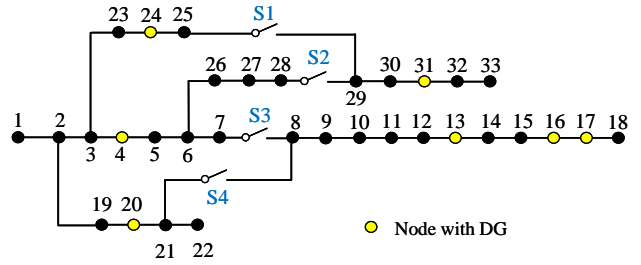
A. System Configuration

Fig. 3. Modified IEEE-33 bus system

The effectiveness of the proposed DaE-VVC algorithm is verified on modified IEEE-33 bus system in this section. The network configuration of the test system is shown in Fig. 3 while the system parameters can be found in [10]. The DGs are located at bus 4, 13, 16, 17, 20, 24, 31, each with the capacity of 500 kVA. There're four switches S1, S2, S3 and S4, the location of which is shown in Fig. 3. The possible topology configurations and the switch status are summarized in Table I. The slack bus voltage are assumed to be constant at 1.0 pu and the upper/lower bound of voltage magnitudes are set as 0.95 pu and 1.05 pu. The normalized load and generation profiles are obtained from Pecan Street [11] and National Renewable Energy Laboratory(NREL) Renewable Resource Data Center [12], as shown in Fig 4 and Fig. 5, respectively. During the simulation, the full nonlinear power flow is solved by OpenDSS software to represent the behaviour of real distribution system.

B. Estimation Accuracy

The model estimation accuracy of the proposed DaE-VVC under noise-free situation and with measurement noise is introduced in this section. The measurement noise is model as normal distributed with zero mean and standard deviation of 0.1. As shown in Fig. 6, the line parameter estimated are
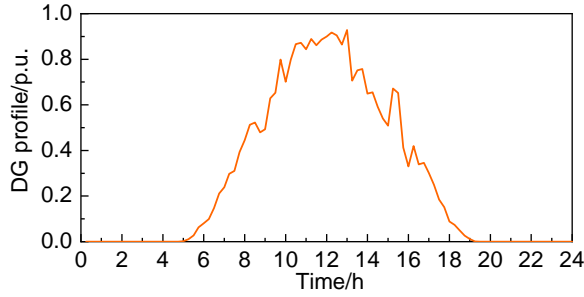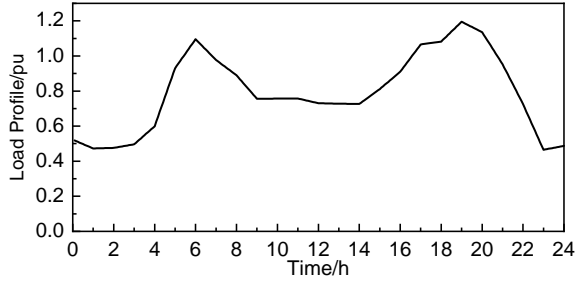
Fig. 4. Generation Profile

Fig. 5. Load Profile

very close to the real value both in situations with or without measurement noise. Here we define mean error factor(MEF) $\frac{1}{L}\sum_{\ell=1}^{L}\left|\frac{\hat{x}_\ell}{x_\ell}-1\right|$ to evaluate the estimation accuracy. The MEF under noise-free situation is 0.75% while the MEF with measurement noise is 2.19%, both of them are small and acceptable.

TABLE I
POSSIBLE TOPOLOGY CONFIGURATIONS

	S1	S2	S3	S4
T1	open	closed	closed	open
T2	closed	open	closed	open
T3	open	closed	open	closed
T4	closed	open	open	closed

C. Performance in Dynamic Simulation

The performance of the proposed DaE-VVC under time-series dynamic simulation is further demonstrated in this section. In the simulation, the DN operate at T1 at the beginning, there is an DN reconfiguration so that the DN change to T3 at 15:00. The bus voltages in traditional VVC and proposed DaE-VVC is shown in Fig. 7 and Fig. 8, respectively. The traditional VVC is unable to detect the change of network reconfiguration, so that the voltage violation happens around 15:15-16:00 with the maximum voltage reaching 1.056 pu. As illustrated in Fig. 8, the feeder voltages are always kept within acceptable range in the proposed DaE-VVC. In addition, the DG reactive power outputs in two methods are shown in Fig. 9 and Fig. 10, respectively. Using the updated DN model, the proposed DaE-VVC avoids sending the incorrect DG reactive power dispatch and the over-voltage buses are regulated into the allowed range. This is mainly achieved by reducing the DG reactive power output at bus 20.
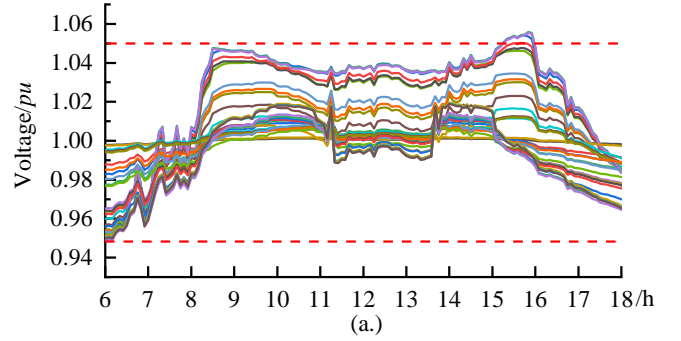

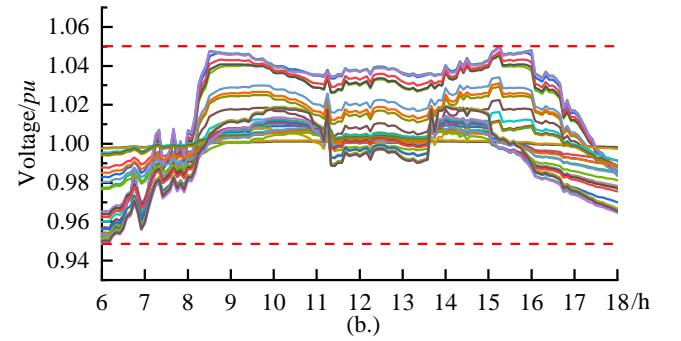Fig. 7. Voltage in traditional VVC
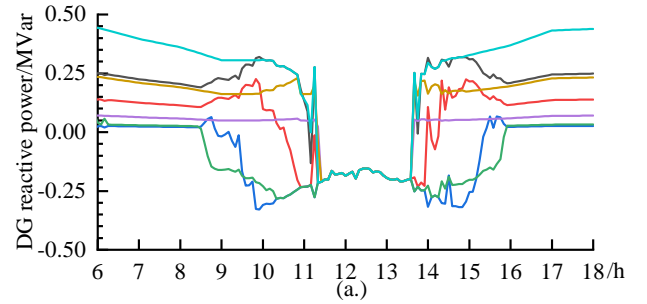

Fig. 8. Voltage in proposed DaE-VVC

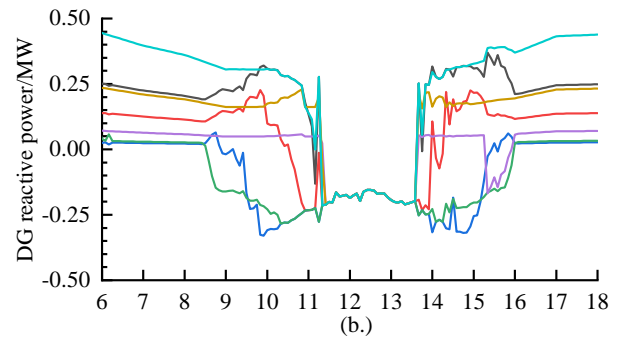Fig. 9. Reactive power output in traditional VVC

Fig. 10. Reactive power output in proposed DaE-VVC

V. CONCLUSION

This paper proposes a data-driven enhanced voltage/var control algorithm to regulate voltage profiles in DN with an
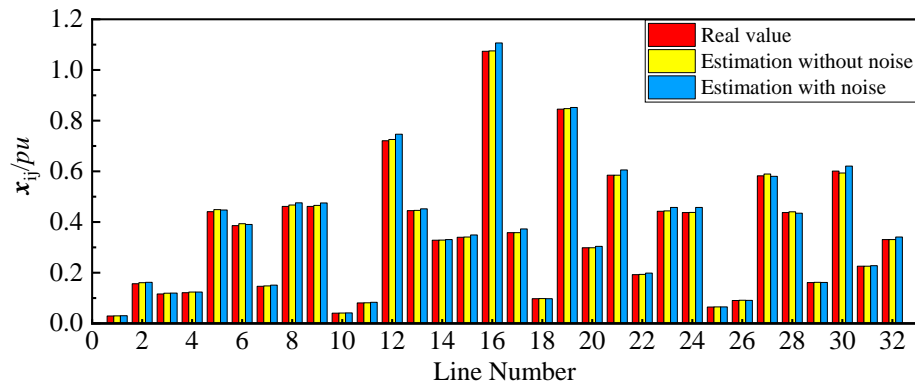
Fig. 6. Line Parameter Estimation

incomplete model. By linearizing the voltage-power injection relationship, the algorithm can identify the model topology and line parameters with accuracy guarantee. The estimation error under noise-free and noise situations are 0.75% and 2.19%, respectively, which is acceptable. The performance and the comparison with traditional VVC methods is illustrated by case studies on balanced IEEE test feeders.

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