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Lightning surges in hybrid cable-overhead lines. Part II: Voltage estimation for strikes to shield wire

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Abstract

This paper (Part II) continues the study from Part I by presenting formulas able to do a fast estimation of the voltage at an underground cable sheath for lightning surges striking an earth wire, with and without back-flashover. Similar to Part I, these formulas do not require an Electromagnetic Transients (EMT) software and can be implemented as scrip for a fast screening of potential issues, requiring solely the geometric data of the cable and the overhead line. The potential impact and limitations of the simplifications are evaluated.

Keywords: Lightning; HVAC cables; Insulation coordination; Screening tool

I. Introduction

Part I of this work demonstrated that lightning in hybrid cable-Overhead Lines (OHL) may lead to overvoltage in the cable section of the line and it proposed formulas for screening. Part II presents formulas able to estimate the peak overvoltage at the sheath of an undergrounded cable originating from a lightning surge on the shield wire of an OHL. The proposed formulas are easier to implement than those proposed for shielding failure in Part I and the cable length only impacts the voltage magnitude for very short cables, no longer than dozens of meters.
Section II develops formulas for a lightning surge on the earth wire without back-flashover and Section III the formulas with back-flashover. Section IV presents a discussion and Section V concludes the paper.

II. Lightning striking earth wire without Back-flashover

The most common scenario for a lightning surge on an OHL is the lightning surge striking an earth-wire and no back-flashover. When the surge occurs near a cable-OHL transition point, part of the energy propagates into the cable sheaths. As standard procedure, the cable sheaths, the earth wire(s) and the ground short-circuit, resulting in a discontinuity. The relation between voltages and currents at the transition point is given by (1), where the injected voltage at the sheath (V_S) is equal to the injected (V_{EW_I}) and reflected (V_{EW_R}) voltages at the earth wire. The summation of the injected (I_{EW_I}) and reflected (I_{EW_R}) currents in the earth wire is equal to the summation of the sheath currents (I_{S1}, I_{S2}, I_{S3}) plus the current into the ground (I_G).

$$\begin{cases} V_{EW\_I} + V_{EW\_R} = V_S \\ I_{EW\_I} + I_{EW\_R} = I_{S1} + I_{S2} + I_{S3} + I_G \end{cases} \tag{1}$$

The currents are expressed using the voltage and the impedance (2), which allows to relate the sheath voltage (V_S) with the injected voltage (V_{EW_I}), the earth wire impedance (Z_{EW}), the grounding impedance (Z_G) and the sheath impedances (Z_{Sx}) (3).

$$\begin{cases} I_{EW\_I} = \dfrac{V_{EW\_I}}{Z_{EW}} \\ I_{EW\_R} = -\dfrac{V_{EW\_R}}{Z_{EW}} \\ I_G = \dfrac{V_S}{Z_G} \\ I_{Sx} = \dfrac{V_S}{Z_{Sx}} \end{cases} \tag{2}$$

$$V_S\left(\frac{1}{Z_{S1}}+\frac{1}{Z_{S2}}+\frac{1}{Z_{S3}}+\frac{1}{Z_G}+\frac{1}{Z_{EW}}\right)=\frac{2V_{EW\_I}}{Z_{EW}} \tag{3}$$

There is no back-flashover and thus, (4) is valid and (3) simplifies into (5), with I_L being the lightning current peak magnitude, which is an input parameter.

$$\frac{2V_{EW\_I}}{Z_{EW}} = I_L \tag{4}$$

$$V_S = \frac{I_L}{\dfrac{1}{Z_{S1}}+\dfrac{1}{Z_{S2}}+\dfrac{1}{Z_{S3}}+\dfrac{1}{Z_G}+\dfrac{1}{Z_{EW}}} \tag{5}$$

The current I_L divides into two currents, half propagating in the cable direction and half in the opposite direction. This simplification considers that lightning strikes the earth-wire along the span, not at the tower. Additionally, the propagation of the energy into the tower is neglected. This approach is chosen, because it leads to maximum current entering the cable and a worst-case scenario, which is appropriate for a screening tool.

The impedance of the sheaths (Z_{Sx}) is approximated by calculating the zero-sequence impedance of the sheath while neglecting the core (6). The reasoning is that the sheaths are short-circuited and an equal current is injected in all three, if that cable has a symmetrical configuration. In flat formation, the variable $2L_{S,M}$ becomes $L_{S,M\_d1}+L_{S,M\_d2}$ for the two outer sheaths, but the impact in the value of Z_S is very small for typical dimensions; where L_{S,M_d1} and L_{S,M_d2} are the mutual coupling between the sheaths of the centre and outer cables, and between the sheaths of the outer cables, respectively.

$$Z_S = \sqrt{\frac{L_{S,Self}+2L_{S,M}}{C_S}} \tag{6}$$

According to [1], $L_{S,Self}$ can be written as (7), which can simplify into (8) for typical cable dimensions, where $L_{S,M}$ is given by (9), ρ_e is the ground resistivity, f is the target frequency, d_{xy} is the distance between two conductors and R_1, R_2 and R_3 are the radiuses of the core, insulation and sheath, respectively. The formulas are frequency dependent, but a double exponential includes a wide spectrum of frequencies. In practice, the sheath inductance and surge impedance (6) change very slowly at higher frequencies, resulting in a small error if a reasonable frequency value is chosen, i.e., a frequency in the high frequency range associated to the lightning impulse. Additionally, the impact of not using the most suitable frequency in (5) is lessened, because Z_G is lower than Z_S for typical grounding impedances. To show this, the value of 50 kHz is chosen as a generic value and used in this paper, but a frequency of 60 kHz gave the best match for the 1.2/50 μs waveform and the test case of this paper. The error in the estimated surge impedance from using 50 kHz instead of 60 kHz is less than 1%, being even less for the estimated sheath voltage.

$$L_{S,Self} = \frac{\mu_0\left(R_3-R_2\right)}{6\pi}\left(\frac{1}{R_3}+\frac{1}{R_2+R_3}\right)+\frac{\mu_0}{2\pi}\ln\left(\frac{659\sqrt{\dfrac{\rho_e}{f}}}{R_3}\right) \tag{7}$$

$$L_{S,Self} = \frac{\mu_0}{2\pi}\ln\left(\frac{659\sqrt{\frac{\rho_e}{f}}}{R_3}\right) \quad (8)$$

$$L_{S,M} = \frac{\mu_0}{2\pi}\ln\left(\frac{659\sqrt{\frac{\rho_e}{f}}}{d_{xy}}\right) \quad (9)$$

In case of no back-flashover, the maximum overvoltage at the sheath is expected to occur not at a reflection, but at the instant the surge arrives to the transition point. This is due to dominant ground propagation mode. This propagation mode is slower and with larger attenuation than the other propagation modes, resulting in a larger damping along the cable. Additionally, the sheaths are also grounded at the other end and part of the energy flows to the ground, instead of being reflected. Therefore, it should not be necessary to perform iterations and it is sufficient to use (5).

If the cable is short enough that reflections increase the peak voltage, the iterative method of Part I can be applied. However, for the cable used in this example open at receiving end and a sheath grounding impedance of 10**Ω**, the cable would have to be shorter than 50 m for the peak overvoltage to be at one of the reflections. For lower grounding impedances, the critical length would be shorter.

Figure 1 shows the sheath voltage obtained via simulation and using (5), for a 10 kA lightning impulse. The estimations are made using (7) and (8). The differences between the estimation and the simulation are virtually inexistent when using (7) with the two plot lines overlapping, being negligible when using (8), demonstrating the accuracy of the proposed method. The same level of precision is obtained for the sheath currents.

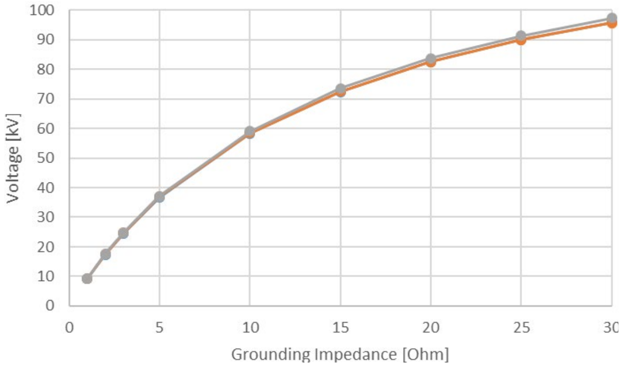

Figure 1 – Sheath Voltage in function of grounding impedance. Orange: Simulation overlaps estimation using (7). Grey: Estimation using (8)

III. Lightning striking earth wire and Back-flashover

III.1. Estimation of the back-flashover voltage

In a back-flashover, part of the lightning current flows to the OHL phase conductor and consequently, into the cable core. However, it is not relevant to estimate this core voltage. Reference [2] shows that the ground mode is the main propagation mode in a cable in the event of a back-flashover and this mode does not depend on the voltage at the cable core. The low magnitude of the coaxial propagation modes when compared with the ground mode means that the core and sheath have approximate the same voltage and the main insulation is not particularly stressed. As a result, only the sheath voltage requires estimation.

A challenge in case of back-flashover is the estimation of the respective voltage. This voltage depends on the current waveshape, the tower's grounding impedance, the insulator length, etc… A methodology for estimating the critical back-flashover current, i.e., the current at which back-flashover starts occurring, is presented in [3]. However, this procedure may be considered too complicated for a screening tool. A simpler approach is to consider the voltage equal to 1.2xCFO [4], where CFO is the Critical Flashover Overvoltage of the insulator. This simple approach estimates the maximum amplitude of the surge for a standard waveform and it is a suitable for a screening tool, as it is the one proposed in this paper.

In the following validations, the voltage was obtained via a single simulation for a better comparison between simulations and formulas. For practical screening applications, the suggestion is to consider it 1.2xCFO.

III.2. Towers with one earth wire

The voltage at the cable sheath is calculated considering the reflection and refractions at the transition point between the earth wire, the ground and the three sheaths. The voltage at the sheath (V_S) is equal to the injected (V_{EW_I}) plus the reflected voltages (V_{EW_R}) at the earth wire. The summation of the injected (I_{EW_I}) and reflected (I_{EW_R}) currents in the earth wire is equal to the summation of the current flowing into the ground (I_G), with the currents in the sheaths of the sound phases (I_S) and with the current in the sheath of the phase with back-flashover (I_{S_Back}) (10). A schematic showing these currents and voltages is presented in Appendix for an easier understanding.

$$\begin{cases} V_{EW\_I} + V_{EW\_R} = V_S \\ I_{EW\_I} + I_{EW\_R} = 2I_S + I_{S\_Back} + I_G \end{cases} \tag{10}$$

Given the high frequency of the phenomenon, the calculation of surge impedances neglects the resistance, as explained in Part I. In this case, the estimation of the inductance of the OHL phase conductor or earth wire is given by (11).

$$L_{OHL} = \frac{\mu_0}{2\pi} \ln\left(\frac{2h}{GMR}\right) \tag{11}$$

The current in the sheath of the phase with back-flashover is calculated by (12). The equation is developed based on Figure 2 and it neglects couplings between the cable phases, because the currents in the sound phases have a small impact in this sheath current. The current in the cable core closes in the sheath inner surface, represented by the value '1' in (12) . This current flows to the earth via the grounding resistance (Z_G) and the outer surface of the sheath, which is represented by the surge impedance of the sheath (Z_S). A voltage divider is applied and the current in the outer surface of the sheath subtracts to the current in the inner surface, represented by the second term inside the parentheses in (12).

$$I_{S\_Back} = -I_{C\_Back}\left(1 - \frac{Z_G}{Z_S + Z_G}\right) \tag{12}$$

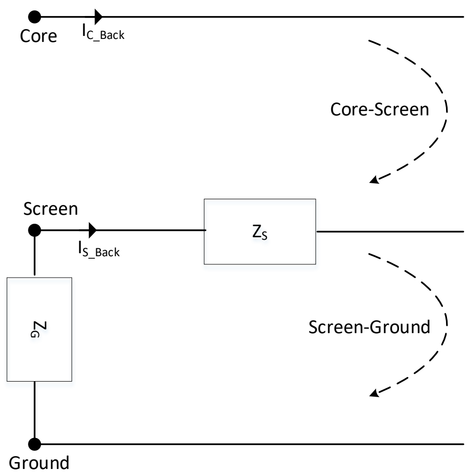

Figure 2 – Current loops (dashed lines) for defining the current in the sheath of the phase with back-flashover

The current (I_{C_Back}) is defined using the current in the OHL phase with back-flashover (I_{P_Back}), but replacing it by the injected voltage (V_{EW_I}) (13). Equation (13) considers that the current in the OHL phase with back-flashover refracts into the cable core, with the former calculated by dividing the back-flashover voltage by the surge impedance of the phase (Z_P).

$$\begin{aligned} & I_{C\_Back} = I_{P\_Back}\frac{2Z_P}{Z_C+Z_P} \Leftrightarrow I_{C\_Back} = \frac{V_{EW\_I}}{Z_P}\cdot\frac{2Z_P}{Z_C+Z_P} \\ & \Leftrightarrow I_{C\_Back} = \frac{2V_{EW\_I}}{Z_C+Z_P} \end{aligned} \tag{13}$$

The current in the sheaths of the sound phases (I_S) depends also on the grounding impedance and it is given by (14), where, Z_G is the grounding impedance.

$$I_S = \frac{V_S}{Z_S}\left(\frac{Z_S}{Z_G+Z_S}\right) \tag{14}$$

All currents from (10) are replaced by the voltage at the sheaths and earth-wire (15).

$$\begin{cases} I_{EW\_I} = \dfrac{V_{EW\_I}}{Z_{EW}} \\ I_{EW\_R} = -\dfrac{V_{EW\_R}}{Z_{EW}} \\ I_G = \dfrac{V_S}{Z_G} \\ I_S = \dfrac{V_S}{Z_S}\left(\dfrac{Z_S}{Z_G+Z_S}\right) \\ I_{S\_Back} = -\dfrac{2V_{EW\_I}}{Z_C+Z_P}\left(\dfrac{Z_S}{Z_G+Z_S}\right) \end{cases} \tag{15}$$

The combination of (10) with (15) results in (16) for the calculation of the sheath voltage at the transition point. The appendix shows the mathematical development.

$$V_S = V_{EW\_I}\frac{2Z_G\left(Z_C+Z_P\right)\left(Z_G+Z_S\right)+2Z_{EW}Z_SZ_G}{\left(2Z_{EW}Z_G+Z_{EW}\left(Z_G+Z_S\right)+Z_G\left(Z_G+Z_S\right)\right)\left(Z_C+Z_P\right)} \tag{16}$$

Equation (16) is validated by comparing the voltage at the sheath for different grounding resistances. The 1.2/50 µs waveform with a peak magnitude of 10 kA is used.

To perform a detailed simulation of a back-flashover is time consuming and to use the exact back-flashover voltage is not relevant for this validation. Instead, in the simulation, the earth-wire and one of the phases are short-circuited at 100 m from the junction point. Then, the injected voltage (V_{EW_I}) is obtained from the simulation and applied as input parameter in the formula. For the validation, the behaviour after the back-flashover is the relevant factor, not the behaviour leading to it, allowing this approach. For practical applications of the formula, the voltage can be approximated for a worst-case by considering it 1.2 times the CFO, as proposed at the start of Section III. The appendix presents a schematic of the circuit for this simulation.

Figure 3 shows the voltage and current at the sheaths obtained via simulations and the proposed method up to a grounding impedance of 30 Ω. The difference is small for the voltage, which is the parameter of interest, a maximum of 9% at Z_G equal to 10 Ω.

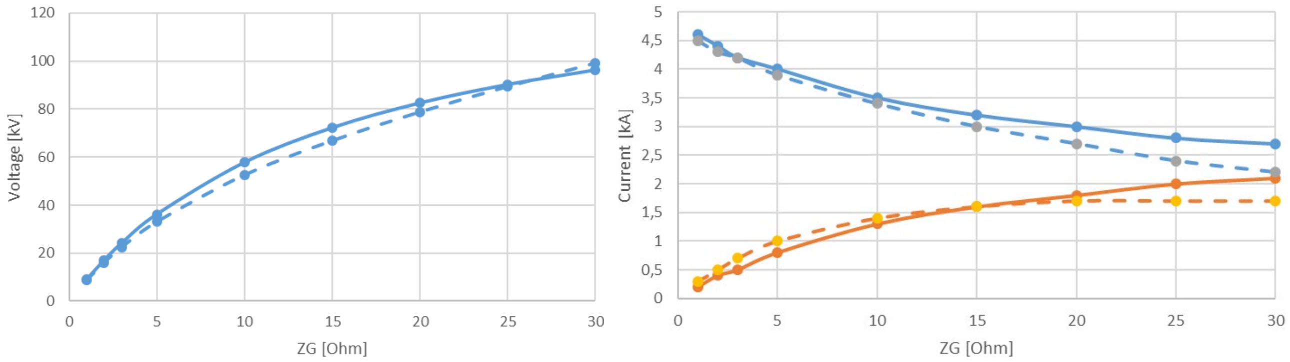

Figure 3 – Left: Voltage at the sheath; Right: Current in the sheath connected to the phase with back-flashover (blue) and in the sheath of one of the other two phases (orange). Solid line: Simulation; Dashed line: Calculation

III.3. Towers with two earth wires

The voltage at the sheath for the case with two earth wires can be obtained using (16), by correcting the earth wire surge impedance (Z_{EW}). Ideally, by neglecting coupling, it would be sufficient to divide the previous Z_{EW} by two. However, the error by doing this may be considerable for larger grounding resistance values (orange plot in Figure 4).

An alternative approach that might be used in some cases is to have the value available from other studies. System operators often use the same design in multiple locations and the value of the impedance may be available from other studies. The black plot in Figure 4 shows the estimate voltage when using a more precise value obtained from the simulation model and it shows a good match with the simulation results (blue plot).

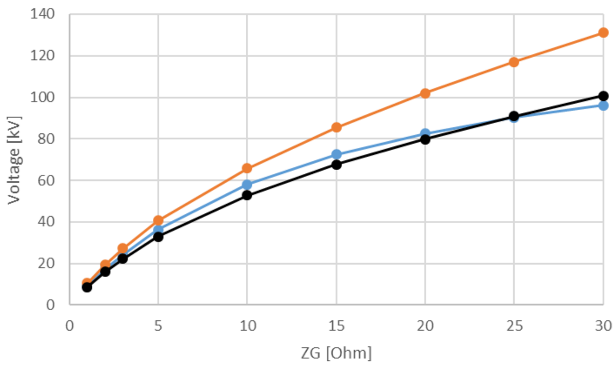

Figure 4 - Voltage at the sheath: Simulation (Blue); Calculation via division of surge impedance (Orange); Calculation using simulation value of the surge impedance (Black)

IV. Discussion

The research work presented in Part I and Part II has two main objectives:

- To develop screening formulas for estimating both the voltage due to lightning surges at a OHL-cable junction point and the energy absorbed in a surge arrester, if installed;
- To discuss the impact of different parameters on the voltage magnitude, to improve planning and designing of hybrid lines.

Different formulas were presented for estimating the voltage at the core and at the sheath of a cable, covering a range of potential cases: lightning at phase conductor (shielding failure), with and without surge arrester at the transition point, and lightning at earth wire, with and without back-flashover. In general, the formulas show good agreements with EMT-type simulations for different cable lengths, lightning impulses and soil resistivity.

The data required for the formulas are the OHL and cable geometric parameters (thickness, resistivity, etc…) available on a datasheet. The remaining input parameters are test parameters: lightning impulse waveform and magnitude, length of the cable, ground resistivity and instantaneous phase voltage. A script using these input parameters and the proposed

formulas can estimate the voltage at the cable core or sheath, allowing a fast screening of potential issues via a sensitivity analyses of different parameters.
Several parts of the discussion done in part I, as the choice of lightning waveshape and the impact of the ground impedance are valid in part II. Additional topics for discussion are:

IV.1. Voltage in the cable sheath:

In the case of lightning surge striking on the earth wire, the main concern is the cable sheaths. The cable length does not affect the voltage magnitude unless the cable is very short (below 50 m for the case tested in this paper), because of the dominance of the ground propagation mode. This propagation mode is slower than the coaxial modes resulting in a large attenuation between reflections. As expected, the grounding impedance is the parameter with the largest impact on the magnitude of the sheath voltage. Equation (5) demonstrates it: as the value of the sheath impedances (Z_{SX}) is some dozens of Ohm (around 43 Ω in this case), the voltage increases almost linearly for low grounding impedances and the formula is simple.
The formula to estimate the sheath voltages changes in case of back-flashover (16). The core voltage is impacted by the back-flashover, but to estimate its value is of little relevance, because the ground mode is still the main propagation mode. This means that the voltage difference between the cable core and sheath is not particularly affected by the back-flashover and the main insulation is not particularly stressed. As a result, the discussion in the previous paragraph is applicable for back-flashover.

IV.2. Ground resistivity:

For lightning striking the earth wire, the ground resistivity affects the value of the sheath impedance and it is considered in the equations. A simulation for the case with backflashover is done considering a ground resistivity of 10000 Ω.m and a sheath grounding resistance of 30 Ω, the grounding resistance leading to the largest difference in Figure 1. The error of the proposed formula for the sheath overvoltage is 2.6%, corresponding to a difference of 2.8 kV for a simulated value of 107.8 kV.

IV.3. Two earth wires and backflashover:

The estimation of the surge impedance for an OHL with two earth wires has an increasing error when the sheath grounding resistance increases (Figure 4). A typical grounding resistance at a transition point is between 8-10 Ω [5] and as an example, in Denmark all grounding resistances must be below 10 Ω [6]. As a result, this error is not considerable for typical grounding resistances, as can be observed in Figure 4.
Furthermore, the high magnitude of the lightning current flowing into the ground will result in soil ionisation [7], reducing the value of the grounding impedance when compared with the respective DC impedance, which further reduces the risk of underestimating the overvoltage.

V. Conclusions

The impact of lightning surges on hybrid cable-OHL becomes more relevant, as these lines become more common. The estimation of the overvoltage for short cable sections is of special interest, because of the numerous reflections that occur. This paper developed formulas for a fast estimation of the voltage at the cable sheath, when the earth wire attracts the lightning surge.
Alike Part I, the formulas in Part II can be implemented for a fast screening of potential problems with minimum data requirements. Additionally, the development of these equations allowed a better understanding of the phenomenon.

The two papers showed that the cable length might have a large impact in the overvoltage and energy absorbed by the surge arrester in case of shielding failure, but little if the lightning strikes the earth wire, even for back-flashover. For the latter, the grounding impedance is the key factor and special care should be taken to assure that it is as low as possible, for a better protection of the cable

For lightning striking on the earth wire and no black-flashover, the formulas are very simple and precise.

For lightning striking on the earth wire, the main limitation is the estimation of the back-flashover voltage, which is an input parameter. The simplest solution is to consider this voltage equal to 1.2 times the critical flashover overvoltage of the insulator [4].

VI. References

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VII. Appendix

All simulations were done in PSCAD/EMTDC. The cables and OHLs were modelled using geometric frequency-dependent models, with the data from Table A.1 and Table A.2, respectively. Figure A.1 shows the position of the phase conductors and earth wires for the OHL, as well as the thickness and position of the core, insulation, sheath and outer insulation for the cable. Figure A.2 shows the schematic of the simulated cases, where C_2 is without backflashover and $C_{2.2}$ is with backflashover.

Table A.1 – Cable data

Layer	Radius (mm)	Properties
Conductor	27.7	ρ=3.5917x10^{-8} [Ω.m]
Insulation	58.2	ε_r=2.87
Sheath	59.0	ρ=2.826x10^{-8} [Ω.m]
Outer Insulation	65.0	ε_r=2.3
Ground		ρ=100 [Ω.m]

Table A.2 – OHL data

Layer	Radius (mm)	Properties
Phase	18.085	DC_Res=0.04277 [Ω/km]
Bundle	3 conductors	d=0.4 [m]
Earth wire	8	DC_Res=0.299 [Ω/km]
Dist. phases		d=13.1 [m]
Ground		ρ=100 [Ω.m]

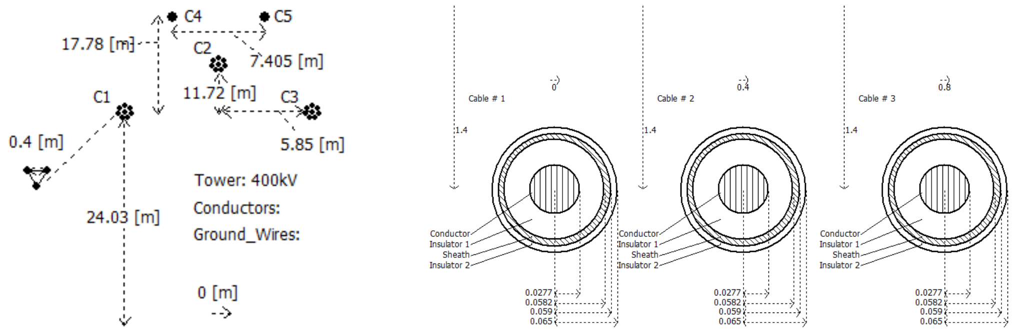

Figure A.1 – Geometric data on OHL (left) and cable (right)

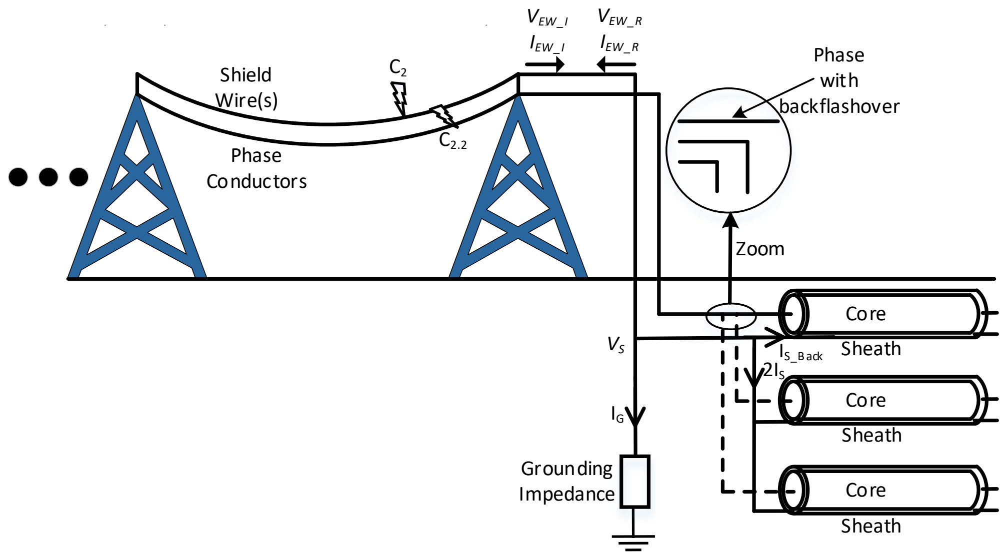

Figure A.2 – Schematic for the simulations: C_2 corresponds to striking on earth wire and no back-flashover; $C_{2.2}$ to striking on earth wire and back-flashover. The three phases of the OHL are represented as one line, with the back-flashover being to only one phase. The current variables are the ones used for the case with backflashover (10)

Equation (17) shows the calculation of the sheath voltage at the transition point by combining (10) with (15).

$$
\begin{aligned}
&\frac{V_{EW\_I}}{Z_{EW}} - \frac{V_S - V_{EW\_I}}{Z_{EW}} = 2\frac{V_S}{Z_S}\left(\frac{Z_S}{Z_G + Z_S}\right) + \frac{V_S}{Z_G} - \frac{2V_{EW\_I}}{Z_C + Z_P}\left(\frac{Z_S}{Z_G + Z_S}\right) \\
\Leftrightarrow\ &\frac{2V_{EW\_I}}{Z_{EW}} + \frac{2V_{EW\_I}}{Z_C + Z_P}\left(\frac{Z_S}{Z_G + Z_S}\right) = 2\frac{V_S}{Z_S}\left(\frac{Z_S}{Z_G + Z_S}\right) + \frac{V_S}{Z_G} + \frac{V_S}{Z_{EW}} \\
\Leftrightarrow\ &\frac{2V_{EW\_I} Z_S Z_G}{Z_{EW} Z_S Z_G} + \frac{2V_{EW\_I}}{Z_C + Z_P}\left(\frac{Z_S}{Z_G + Z_S}\right) = \frac{V_S}{Z_{EW} Z_S Z_G}\left(2Z_{EW} Z_G\left(\frac{Z_S}{Z_G + Z_S}\right) + Z_{EW} Z_S + Z_S Z_G\right) \\
\Leftrightarrow\ &\frac{2V_{EW\_I} Z_S Z_G}{Z_{EW} Z_S Z_G} + \frac{2V_{EW\_I}}{Z_C + Z_P}\left(\frac{Z_S}{Z_G + Z_S}\right) = \frac{V_S}{Z_{EW} Z_S Z_G (Z_G + Z_S)}\left(2Z_{EW} Z_G Z_S + Z_{EW} Z_S (Z_G + Z_S) + Z_S Z_G (Z_G + Z_S)\right) \\
\Leftrightarrow\ &\frac{2V_{EW\_I} Z_G}{Z_{EW} Z_G} + \frac{2V_{EW\_I}}{Z_C + Z_P}\left(\frac{Z_S}{Z_G + Z_S}\right) = \frac{V_S}{Z_{EW} Z_G (Z_G + Z_S)}\left(2Z_{EW} Z_G + Z_{EW} (Z_G + Z_S) + Z_G (Z_G + Z_S)\right) \\
\Leftrightarrow\ &2V_{EW\_I} Z_G (Z_C + Z_P)(Z_G + Z_S) + 2V_{EW\_I} Z_{EW} Z_S Z_G = V_S\left(2Z_{EW} Z_G + Z_{EW}(Z_G + Z_S) + Z_G(Z_G + Z_S)\right)(Z_C + Z_P) \\
\Leftrightarrow\ &V_S = V_{EW\_I}\frac{2Z_G (Z_C + Z_P)(Z_G + Z_S) + 2Z_{EW} Z_S Z_G}{\left(2Z_{EW} Z_G + Z_{EW}(Z_G + Z_S) + Z_G(Z_G + Z_S)\right)(Z_C + Z_P)}
\end{aligned}
$$

(17)