IAHR List of Sea Parameters

an update for multidirectional waves

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IAHR List of Sea State Parameters
– an update for multidirectional waves

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ABSTRACT

A Working Group on multidirectional waves formed by the International Association for Hydraulic Research has proposed an update of the IAHR List of Sea State Parameters from 1986 in the part concerning directional waves. Especially wave structure interactions with reflection of the waves have been treated.
The present paper will mainly discuss the ideas behind the proposed parameters listed in the end of this paper.

1. INTRODUCTION

In 1981 the International Association for Hydraulic Research established a Working Group on Wave Generation and Analysis. This group was chaired by Dr. J. Ploeg from NRC, Canada and presented in 1986 the well known IAHR List of Sea State Parameters.

In the intervening years, this list has served as an inspiration and communication tool among scientists and engineers working within waves. Nevertheless, the growing research through the last years on multidirectional waves and their interaction with structures has now made it necessary to update the chapter on directional waves.

In 1994 the International Association for Hydraulic Research formed a Working

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Group on multidirectional waves. This group was chaired by Dr. Mike Briggs from CERC, USA. The present paper describes one of the outcomes from this new Working Group, namely the update of the List of Sea State Parameters, which hopefully will give a clear definition of the main parameters being used to describe multidirectional waves.

2. A MULTIPEAK MULTIDIRECTIONAL WAVEFIELD

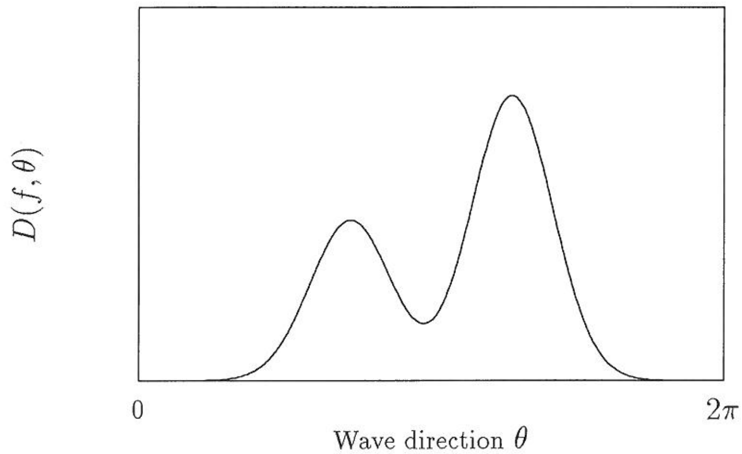

Figure 1: Spreading function of sea with reflection included

The distribution of energy as a function of wave angle is shown in Fig. 1 for one frequency. Parameters like mean direction and spreading of the waves make no straight forward sense in these cases where the directional spreading function has more than one dominant peak. This can be the case in situations with reflection from structures, diffraction around structures or coasts, or swell from a previous storm.

In the presence of one of these phenomena the wave field must be separated into for example incident and reflected waves. For each of the wave field (incident, reflected etc.) the parameters mean direction and spreading must be calculated separately. The separation of the wave field is rather complicated and may give rise to discussions about overlapping of the incident and the reflected waves and how to separate the wave fields. No exact solution to this complex of problems exists but the user should be very careful.

3. MEAN WAVE DIRECTION

Looking at Fig. 1, it is clear that Mean wave direction θ_m is only meaningful with regard to directional spreading functions that do not have more than one peak.

The problem with Mean wave direction is, that in cases where wave directions

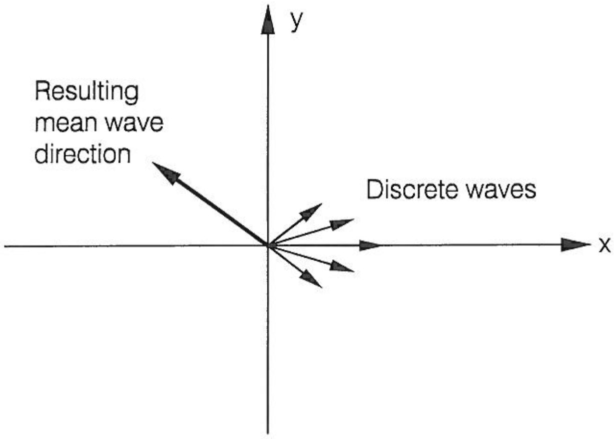

Figure 2: Example of miscalculation of mean wave direction.

are not defined compared to the coordinatesystem then the integration boundaries become very important.
Def. 1 ($\theta_m = arg(c_1)$) from the new List of Sea State Parameters gives an explicit and uniformly valid expression for θ_m, where def. 2 ($\theta_m = \int D_x(f,\theta)\theta d\theta$) does not account for the periodic nature of the problem and may give undesirable results. The example presented in Fig. 2 shows 5 discrete waves and their corresponding mean wave direction calculated using def.2 integrating from 0 to 2π. The result of this calculation will give a mean direction of 144 deg. Because of this problem it is recommended to use def.1 although this definition also needs special care in the calculation of inverse tangets (must be a full four–quadrant inverse tanget routine such as the Fortran ATAN2 function).

4. DIRECTIONAL SPREADING

Generally spreading of the waves has no meaning if more than one dominant peak is present in the directional spreading function.
Problems with calculating the spreading is very similar to problems with calculation of the mean direction. Using def. 2 ($\sigma_\theta^2 = \int D(f,\theta)(\theta - \theta_m)^2 d\theta$) from the new List of Sea State Parameters, the choise of coordinatesystem again becomes very important. Taking the example shown in Fig. 2 the spreading will be a 164 deg. instead of the true value of 11 deg. if the integration is done from 0 to 2π. This problem does not occur if def. 1 ($\sigma_\theta^2 = 2(1 - |c_1|)$) is used or if def. 2 is used with the correct integration limits of $(\theta_m - \pi)$ and $(\theta_m + \pi)$. Nevertheless, the spreadings calculated by def. 1 and def. 2 will be slightly different even when the corect integration limits are used.
The spreading σ_θ has been computed by def. 1 and def. 2 from the the new List of Sea State Parameters for a standard $cos^{2s}(\frac{\theta-\theta_m}{2})$ spreading function and the results are shown in Fig. 3 for s values ranging from 1 to 40.

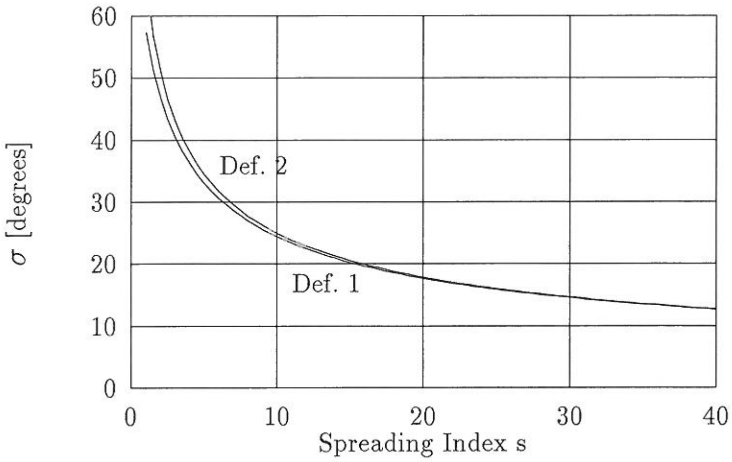

Figure 3: Comparison of calculated spreadings σ using def. 1 and def. 2.

Fig. 3 shows that the calculated values of σ_θ are not identical for the two definitions although only small differences are seen on the plot. The largest differences are seen for small s–values corresponding to a very wide spreading of the waves.

5. DIRECTIONAL REFLECTION

The reflection coefficient C_R is defined as function of the frequency f and the wave direction θ^*. The angle θ^* is the deviation to the head on direction.
Snell's law is assumed to be valid, and the amount of reflected energy are divided by the corresponding incident energy. Then the reflection coefficient is calculated by taking the square root of this ratio $C_R(f, \theta^*) = \sqrt{\frac{S_R(f,\theta_R)}{S_I(f,\theta_I)}}$.
In case of frequency modulation the definition should be used with care.

6. CONCLUSION

A unique definition of multidirectional wave parameters is essential for the communication among scientists and engineers working within multidirectional waves. Such a collection of definitions has been missing. The presented list claims to help on this problem, although updates will be necessary also in the future.

7. REFERENCES

M. Darras: IAHR List of Sea State Parameters: A Presentation. IAHR Seminar on wave generation and analysis in basins, Lausanne 1987.

List of Sea State Parameters. IAHR Seminar on wave generation and analysis in basins, Lausanne 1987.

IAHR List of Sea State Parameters
– 1997 Update for Multidirectional Waves

Computer Notation	Symbol	Description, see also Figs. 1 and 2	Example of Units
	$\boldsymbol{k}$	Wave number vector, defined such that	rad/m
KX	k_x	$k_x = \|\boldsymbol{k}\| \cos\theta, \ \|\boldsymbol{k}\| = k$, (wave number)	rad/m
KY	k_y	$k_y = \|\boldsymbol{k}\| \sin\theta$	rad/m
THETA	θ	Direction of wave propagation (describing direction of $\boldsymbol{k}$). Counter clockwise is positive (right hand rule).	rad
ALPHA	α	Wave direction, expressing where the waves are coming from. Approved by PIANC as the angle between true north and the direction from where the waves are coming. Clockwise is positive to this definition.	rad
DSX	$S_X(f,\theta)$	Directional spectral density, where X may be I for incident spectrum R for reflected spectrum T for total spectrum or omitted if no confusion is possible.	$m^2/(Hz\ rad)$
DWSX	$S_X(k,\theta)$	Directional wave number spectral density, where X is determined as above.	$(m^2/(rad/m))/rad$
CRD	$C_R(f,\theta^*)$	Directional reflection coefficient as may be defined by $C_R(f,\theta^*) = \sqrt{\frac{S_R(f,\theta_R)}{S_I(f,\theta_I)}}$ where $\theta_I = \pi + \theta_s - \theta^*$ $\theta_R = \theta^* + \theta_s$ and where θ^* is the deviation from the head on direction. Further Snell's law is assumed to be valid.	

DX	$D_X(f,\theta)$	Directional spreading function defined as $S_X(f,\theta) = S_X(f)D_X(f,\theta)$ where $\int_0^{2\pi} D_X(f,\theta)d\theta = 1$	rad^{-1}
RXYT	$R_\eta(\boldsymbol{r},\tau)$	Autocorrelation function in space and time, i.e. the normalized autocovariance in space and time domain.	
RXY	$R_\eta(\boldsymbol{r})$	Autocorrelation function in space, i.e. the normalized autocovariance in space domain.	
SETALB	$S_{\eta LB}(f,\theta)$	Directional group bound low frequency spectral density	$m^2/(Hz\ rad)$
SETAHB	$S_{\eta HB}(f,\theta)$	Directional group bound high frequency spectral density	$m^2/(Hz\ rad)$
SKX	$S_X(\boldsymbol{k})$	Wave number vector spectral density, where X is I, R or T.	$m^2/(rad/m)^2$
THETAS	θ_s	Seaward direction of the normal to a reflecting structure.	rad
THETAMX	$\theta_{m,X}(f)$	Mean wave direction as a function of frequency. *Def. 1:* $\theta_{m,X} = \arg(c_1)$ where $a_n = \int_0^{2\pi} D_X(f,\theta)\cos n\theta d\theta$, $b_n = \int_0^{2\pi} D_X(f,\theta)\sin n\theta d\theta$, $c_n = a_n + ib_n$. *Def. 2:* $\theta_{m,X} = \int_{\theta_{m_X}-\pi}^{\theta_{m_X}+\pi} D_X(f,\theta)\theta d\theta$ where X may be I or R. **Caution:** selection of integration limits is crucial. This definition may need iteration, and may converge to $\theta_{m,X}+\pi$. In general situations, Def. 1 should be used instead of Def. 2 to avoid these potential problems.	rad
THETAMOX	$\overline{\theta}_X$	Overall mean wave direction. by *Def. 1:* $\overline{\theta}_X = \arg\left(\int_{f_1}^{f_2} \frac{S_X(f)}{m_{0,X}} \exp(i\theta_{m,X})df\right)$ or by *Def. 2:* $\overline{\theta}_X = \int_{f_1}^{f_2} \frac{S_X(f)\theta_{m,X}(f)}{m_{0,X}} df$ where f_1, f_2, and $m_{0,X}$ are defined as in Chapter C2.	rad

Caution: selection of integration limits is crucial. This definition may need iteration, and may converge to $\overline{\theta}_X + \pi$. In general situations, Def. 1 should be used instead of Def. 2 to avoid these potential problems.

`SIGMA`X $\sigma_{\theta,X}(f)$ Directional spreading (width), describing the directionality of short-crested waves. *rad*

Def. 1: $\sigma^2_{\theta,X} = 2(1 - |c_1|)$

where

$$|c_1|^2 = \left(\int_0^{2\pi} D_X(f,\theta)\sin\theta d\theta\right)^2 + \left(\int_0^{2\pi} D_X(f,\theta)\cos\theta d\theta\right)^2$$

or by

Def. 2: $\sigma^2_{\theta,X} = \int_{\theta_{m,X}-\pi}^{\theta_{m,X}+\pi} D_X(f,\theta)(\theta - \theta_{m,X})^2 d\theta$

`SIGMAO`X $\overline{\sigma}_{\theta,X}$ Overall mean directional spreading. *rad*

example by

$$\overline{\sigma}_{\theta,X} = \int_{f_1}^{f_2} \frac{S_X(f)\sigma_{\theta,X}(f)}{m_{0,X}} df$$

`UI` UI Unidirectivity index. Describing the variation of $\theta_X(f)$. with frequency. If $\theta_X(f)$ is independent of frequency then $UI = 1$. Otherwise $UI < 1$.

$$UI = \mathrm{mod}\left(\int_{f_1}^{f_2} \frac{S_X(f)}{m_{0,X}} \exp(i\theta_{m,X}) df\right)$$

`SKEWD`X $\gamma_X(f)$ Skewness of directional spreading function, where X is I or R,

$$\gamma_X(f) = -n_{2,X}\left(\frac{1 - m_{2,X}}{2}\right)^{-3/2}$$

where

$$m_{2,X}(f) = \int_{-\pi}^{\pi} D_X(f,\theta)\cos(2(\theta - \theta_{m,X}))d\theta$$

$$n_{2,X}(f) = \int_{-\pi}^{\pi} D_X(f,\theta)\sin(2(\theta - \theta_{m,X}))d\theta$$

`KURTD`X $\delta_X(f)$ Kurtosis of directional spreading function, where X is I or R,

$$\delta_X(f) = \frac{6 - 8m_{1,X} + 2m_{2,X}}{4(1 - m_{1,X})^2}$$

where

$$m_{1,X}(f) = \int_{-\pi}^{\pi} D_X(f,\theta)\cos(\theta - \theta_{m,X})d\theta$$

$$m_{2,X}(f) = \int_{-\pi}^{\pi} D_X(f,\theta)\cos(2(\theta - \theta_{m,X}))d\theta$$

`WCW` W_{cw} Largest width of 3D wave crest measured between two zero-crossings.

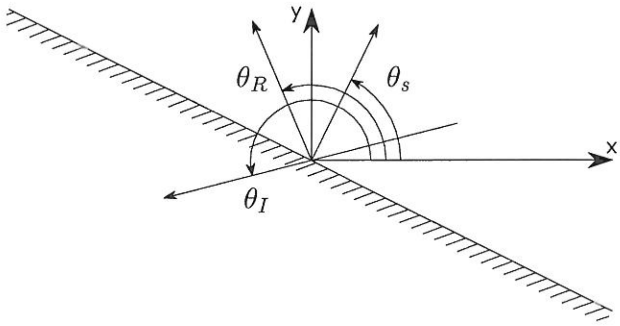

Figure 1: Definition of θ_I, θ_R and θ_s.

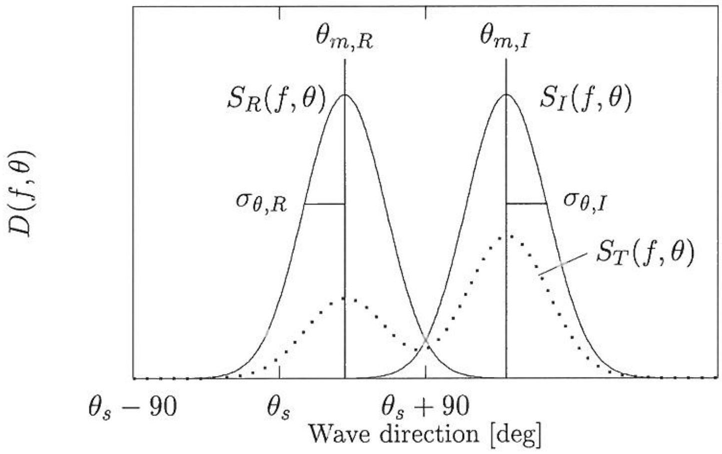

Figure 2: Example of spreading functions with reflection.

Examples of Directional Spreading Functions.

Longuet–Higgins, using s as spreading parameter

$$D(f,\theta) = \frac{2^{2s-1}}{\pi}\frac{\Gamma^2(s+1)}{\Gamma(2s+1)}\cos^{2s}\left(\frac{\theta-\theta_m(f)}{2}\right), \quad -\pi < \theta - \theta_m(f) < \pi$$

Alternatively

$$D(f,\theta) = \frac{1}{\sqrt{\pi}}\frac{\Gamma(s_1+1)}{\Gamma(s_1+1/2)}\cos^{2s_1}(\theta-\theta_m(f)), \quad -\frac{\pi}{2} < \theta - \theta_m(f) < \frac{\pi}{2}$$

Gaussian, using σ_θ as spreading parameter

$$D(f,\theta) = \frac{1}{\sqrt{2\pi}\sigma_\theta}\exp\left(-\frac{(\theta-\theta_m(f))^2}{2\sigma_\theta^2}\right), \quad -\pi < \theta - \theta_m(f) < \pi$$

Mitsuyasu

$$s = \begin{cases} s_p \left(\frac{f}{f_p}\right)^5 & \text{for} \quad f \le f_p \\ s_p \left(\frac{f}{f_p}\right)^{-2.5} & \text{for} \quad f \ge f_p \end{cases}$$

where $s_p = 11.5(2\pi f_p U/g)^{-2.5}$ and U is the wind speed $19.5m$ above the sea surface.

Hasselmann et al. (JONSWAP)

$$s = s_p \left(\frac{f}{f_p}\right)^{\mu}$$

where

$$\left.\begin{aligned} s_p &= 6.97 \\ \mu &= 4.05 \end{aligned}\right\} \quad \text{for} \quad f < 1.05 f_p$$

$$\left.\begin{aligned} s_p &= 9.77 \\ \mu &= -2.33 - 1.45(2\pi f_p U/g - 1.17) \end{aligned}\right\} \quad \text{for} \quad f > 1.05 f_p$$

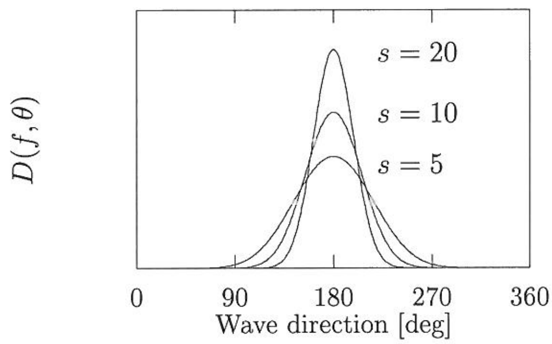

Figure 3: $\cos^{2s}(\frac{\theta-\pi}{2})$ spreading function.

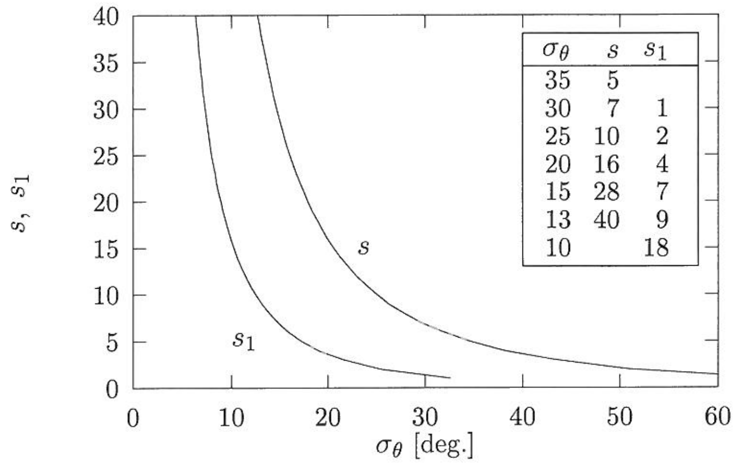

Figure 4: Relation between s, s_1 and σ, using Def. 1.

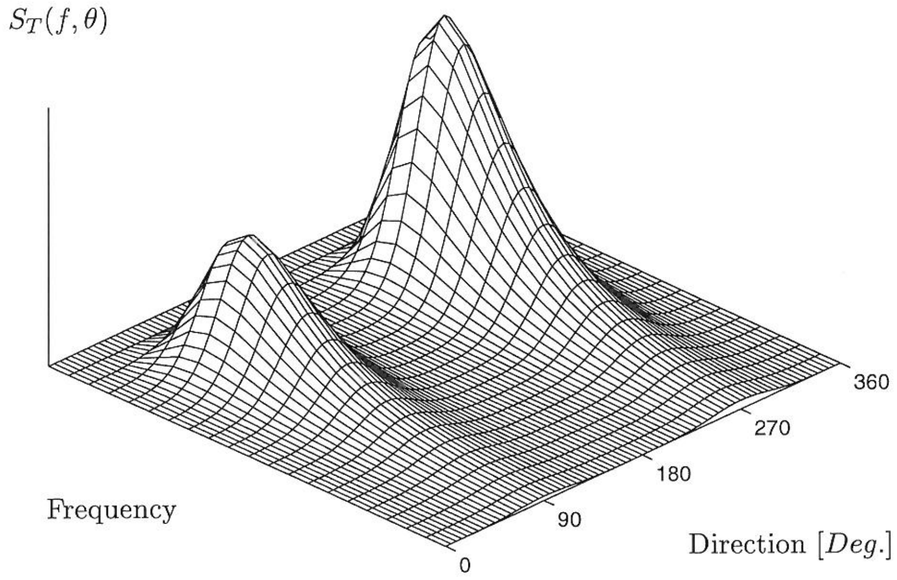

Figure 5: Example of directional spectrum with reflection.