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Article

Effect of Product Distribution Structures and Government Subsidy Measures on Product Quality and Consumption under Competition

Sani Majumder ^{1,†} , Izabela Nielsen ^{2,†} , Susanta Maity ^{1,†} and Subrata Saha ^{2,*,†} 

¹ Department of Basic & Applied Science, National Institute of Technology, Jote 791113, India; sani81live@gmail.com (S.M.); susanta@nitap.ac.in (S.M.)

² Department of Materials and Production, Aalborg University, 9220 Aalborg, Denmark; izabela@mp.aau.dk

* Correspondence: saha@m-tech.aau.dk; Tel.: +45-9193-9202

† These authors contributed equally to this work.

Abstract: To improve social welfare and the sustainable development index, many governments introduce subsidies to manufacturers. Motivated by a subsidy program, we present a parsimonious analysis to determine the impact of subsidies when two competing manufacturers use different distribution structures under competition to sell their products in a three-echelon distribution setting. The objective is to understand better how distribution structures and social welfare measures affect the government's decision to subsidize. We consider four different distribution structures where the government can provide subsidies to both the manufacturers or one of them. From the perspective of the social welfare optimization goal, we consider two well-established measures to analyze whether those measures impacted the overall dynamics. The two key areas: (i) the effect of distribution structures and (ii) decisions under different social welfare measures are not discussed comprehensively yet. We found that distribution structure significantly impacted product qualities, prices, and amount of government expenditure. The government may need to pay more subsidies in a distribution structure with a two-manufacturers-two-distributors-two-retailers distribution setting, where customers can receive a higher-quality product and pay a higher price. Our analysis reveals that the government's social welfare goal can change the dynamics. Among four distribution structures, none can simultaneously ensure higher quality, product consumption, supply chain profits, and lower prices. The results provide insights for developing practical government subsidy program goals under competition.

Keywords: sustainability; government subsidies; three-level supply chain competition; game theory; green products; social welfare



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1. Introduction

Intermediaries in the supply chain (e.g., dealers, wholesalers, distributors) are indispensable in many pragmatic product distribution structures [1]. In recent times, the wholesale distribution market has shown exponential growth; for example, in the U.S., it reached a record high of USD 6.01 trillion. in 2018 [2]. or in India, USD 1.1 trillion in 2021 [3]. Many academic studies [4,5] also support the notion that intermediaries are the backbone in successful product distribution. However, in the green supply chain management literature, most studies focus on exploring relationships between two-echelon supply chain settings and ignore that manufacturers might deal with intermediaries under competition and operate in different markets with different product distribution structures. For example, most manufacturers use a three-tier selling and distribution structure to ensure a local presence in a country such as India. They often rely on subsidiaries or appoint an agent, representative, or distributor. It is not unlikely that many FMCG companies operating on an all-India basis could have between 40 and 80 distributors to distribute their products to nearly 9 million retail outlets [3]. In the

context of emerging markets, it is important to analyze the dependency and reciprocal actions associated with various types of distribution structures to identify under which the competing manufacturers can achieve competitive advantages.

As stated by Allaoui et al. (2019) [6], “A supply chain is a complex system with many functions, activities, and organizations”, while an empirical study by [7] supported the fact that midtier members can play a pivotal role in improving the product distribution process. Gadde (2004) [8] argued that intermediaries could bring significant benefits to eliminate the disruption in the global retail sector and contribute as a secondary resource layer. Intermediaries might have better insight into consumers’ choices and assist manufacturers in spreading their products in different areas [9]. Intermediaries can significantly influence the whole supply chain in achieving environmental sustainability goals [10].

However, studies on the influence of product distribution structure under competition beyond first-tier dyadic relationships are extremely limited. Specifically, the research has shed relatively little light on the influence of supply chain structure under a government subsidy. To the best of the authors’ knowledge, to date, a comparative analysis of different distribution structures has not been explored at a microlevel. For example, two manufacturers can use two different distributors or the same distributors under competition. To pinpoint the effect of distribution structures on the optimal decisions, this study analyzes the answer to the research question: does the distribution structure impact the sustainability goal of competing manufacturers?

During the past few decades, environmental sustainability has been highlighted as one of the key priorities for consumers. In this regard, government subsidy schemes have become an emerging topic of interest [11–14]. In the literature, researchers have explored different types of subsidy policies in a two-echelon supply chain structure; for example, government organizations can provide subsidies directly to the manufacturers [15,16], consumers [17,18] or the downstream retailer [19]. It is also reported that the government can provide subsidies both to the manufacturer and consumers [20] or manufacturer and retailer [21]. We refer to the systematic literature review by Agi et al. (2021) [22] on the green supply chain management practices for the detailed discussion. However, most studies focused on the two-level supply chain structures without competition. Note that our objective is not to analyze different subsidy policies for different members; we assume only manufacturers receive the subsidy and compare results under different distribution structures. Therefore, we ask: how does government subsidy policy (providing a single manufacturer or both manufacturers) affect green product manufacturing under competition?

Government subsidy is apparent across industries to accomplish higher supply chain performance and social welfare, e.g., the manufacturing industry [23], the automobile industry [24], the agriculture industry [25] and so on. As noted by Zhang and Wang (2017) [26], investment in innovative technology can be improved under the guidance of government policies. For example, the Nordic countries introduced incentives to manufacturers to improve emission reduction technology [27]. Accordingly, Hennes & Mauritz AB clothing company envisions it will be using 100% renewable energy by 2040 in their production model. However, if the government provides a subsidy, then there are also some conflicting issues that must be acknowledged in research. For example, a group of researchers ignored the influence of government social welfare optimization goal [17,19] and another group explored the optimal decision by considering that measure [16,21,28,29]. Moreover, the measure of social welfare from the government’s perspective is also not unique. We summarize some of the social welfare measures explored by the researchers in Table 1.

Table 1. Social welfare measures to determine optimal subsidy rate.

Authors	Social Welfare Function	Government Subsidy	Supply Chain Structure
Hua et al. (2016) [30]	$\pi^c + CS$	YES	FSC
Niu et al. (2017) [31]		YES	FSC
Xiao et al. (2017) [32]	$\pi^c + CS - TS - EI$	YES	FSC
Zhou et al. (2018) [33]	$\pi^c - TS$	YES	FSC
Liu et al. (2019) [19]	$\pi^c + CS - TS$	YES	CLSC
Alizamir et al. (2019) [25]		YES	CLSC
Hong et al. (2019) [34]	$\pi^c + CS + EI$	YES	FSC

π^c —total supply chain profit; EI —environment improvement; CS —consumer surplus; TS —total subsidy.

In practice, supply chain decisions under government subsidy are influenced by several key factors, e.g., quality-improvement measures, increased consumption, distribution structures. Commonly, government organizations measure/monitor the effectiveness of subsidy programs based their on social welfare optimization goals. The government can set social welfare goals by considering the influence of profits for supply chain members, consumer surplus, environmental improvement, how much subsidy they would grant. Further, modeling manufacturers' decisions under subsidy programs without considering optimization goals of the government organizations may lead to a failure to illustrate the effect of the subsidy on profit-maximizing and pricing decisions of the overall system. However, the comparative view of the optimal decision in the presence of the social welfare optimization goal and pricing-quality decision under competition is sparse.

Subsidy schemes are sometimes necessary because, with government support, the domestic manufacturers can remain competitive with foreign manufacturers [35]. Furthermore, subsidies are relevant to those "infant" industries that make a continuous effort towards innovation, but their environmental benefits are not still properly priced by the market, or consumers are not aware of the long-term benefits of consumption of those products. However, additional expenditure might not always be worthwhile for those governments facing tight budget constraints. Moreover, such industrial policies have also been criticized in many countries, and subsidies to domestic producers sometimes face disciplinary action under World Trade Organization agreements [36].

In this study, we assume the products' market demand is influenced by price and quality, which has been shown a growing interest in characteristics of the two-echelon supply chain [17,37–39]. Global manufacturers invest significantly in integrating new technologies to reduce energy consumption or natural resources usage, modify production processes, etc., and improve their products' sustainable attributes. Similar to the present study, under competition, researchers explore how two synonymous green and non-green products influence the pricing decision and profit [40,41], but mostly in a two-echelon supply chain setting and ignoring the influence of subsidy. We explore the decision of the government to provide subsidies to single or both manufacturers under a three-echelon supply chain setting in which two manufacturers are selling their products under different distribution structures. Outcomes of optimal decisions under two social welfare optimization goals are compared to identify which distribution structures lead to higher product quality, product consumption, and lower price. Therefore, the present study contributes to the green supply chain management research by analyzing the effects of distribution structures and government subsidy under competition in achieving sustainability goals.

The rest of the paper is organized as follows. Section 2 presents all necessary assumptions and introduces four supply chain structures considered in this study. Properties of all optimal decisions are discussed in Section 4. By comparing all the decisions, we highlight the possible distribution structure choice that ensures higher benefits for supply chain members, consumers, and government organizations. Finally, we present key insights and future research directions in Section 5.

2. Model Settings

We consider four different distribution structures where two competing manufacturers sell substitutable products and compete in the end-customer market under price- and quality-level-sensitive demand. The distribution structures consist of: (1) two-manufacturers-single-distributor-single-retailer (D_1R_1); (2) two-manufacturers-single-distributor-two-retailers (D_1R_2); (3) two-manufacturers-two-distributors-single-retailer (D_2R_1); and (4) two-manufacturers-two-distributors-two-retailers (D_2R_2), respectively. The overview of distribution structures is presented in Figure 1.

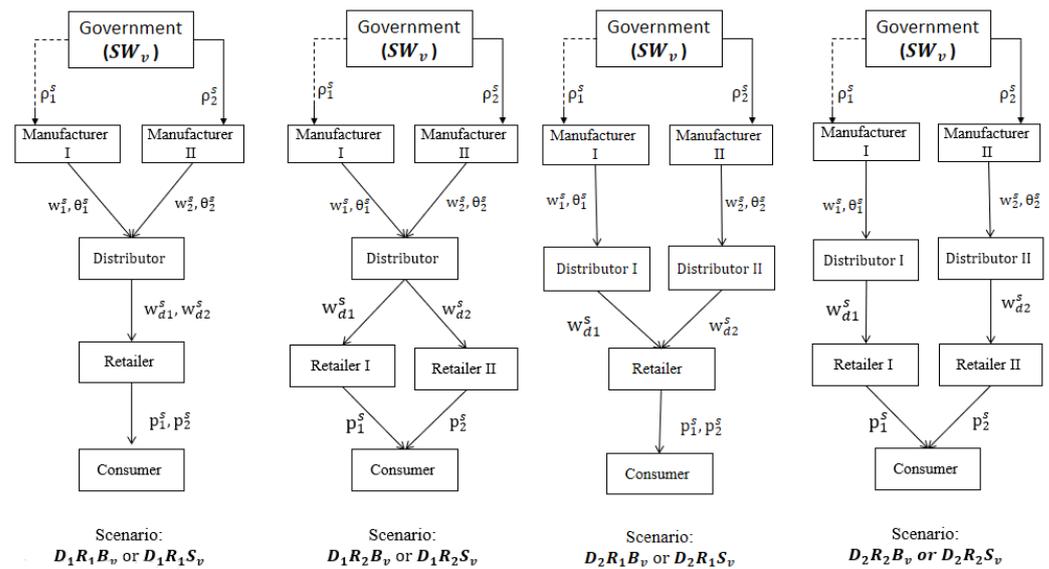


Figure 1. Different distribution structures.

To characterize the demand (q_i^s) for product i , we adopt the following functional forms:

$$q_i^s(p_i^s, \theta_i^s) = a - p_i^s + \beta p_j^s + \gamma \theta_i^s - \delta \theta_j^s, \quad i = 1, 2, \quad j = 3 - i \tag{1}$$

where p_i^s and θ_i^s represents retail price and product quality for i -th product, respectively. Therefore, q_i^s is positively correlated with own product quality (θ_i^s) and retail price of other product (p_j^s); and negatively correlated with other product quality (θ_j^s) and own retail price (p_i^s). The linear demand function is common in the literature [38,42]. Moreover, there are some studies where only quality impact is considered [43] or only retail price is considered [44].

Quadratic investment cost functions $\eta_i \theta_i^{s2}$ are considered to study the impact of product quality improvement, i.e., η_i represent the efficiencies of each manufacturer [45,46]. For simplicity and to keep the focus on analyzing the effect of distribution structures and government subsidy, it is assumed that both manufacturers are equally efficient, i.e., $\eta_1 = \eta_2 = \eta$, [47,48]. For parsimony, we further assume that operational costs for each supply chain are constant and normalized to zero [15].

We assume the government provides the subsidy directly to the manufacturers to improve product quality [15,49,50] and the optimal subsidy rate is determined through optimizing social welfare (SW) functions. Two different social welfare measures are considered in this study are as follows:

$$\left. \begin{aligned} SW_1 &= \langle \text{Total supply chain profits} \rangle + \langle \text{consumer surplus} \rangle - \langle \text{total subsidy} \rangle \\ SW_2 &= \langle \text{Total supply chain profits} \rangle - \langle \text{subsidy} \rangle \end{aligned} \right\} \tag{2}$$

Consequently, we encounter a four-stage game problem. At the first stage, retail prices are determined, prices for distributors or distributors are decided at the second stage, then two manufacturers set their respective product qualities and wholesale prices. Finally,

the government chooses to whether both manufacturers or single manufacturers receive a subsidy and the proportion by optimizing the SW functions.

Note that if the manufacturers are inefficient in quality improvement investment, it is not feasible to produce products. On the other hand, if η is too small, it is also not realistic to make products without any investment. Therefore, we derive the following condition $\eta > \max\left\{\frac{(7-4\beta)\gamma(\gamma-\delta)}{32(2-\beta)(1-\beta)}, \frac{((2-\beta^2)(16-14\beta-6\beta^2+5\beta^3)(\gamma-\delta)^2)}{4(2-\beta)^2(1-\beta)(4-\beta-2\beta^2)^2}\right\}$, which is necessary so that an optimal solution exists in all four supply chain structures. We list notations used to distinguish optimal decisions under different scenarios in Table A1.

3. Model and Decision Making

First, we start with the benchmark single-manufacturer-single-distributor-single-retailer supply chain setting (Scenario-BM), i.e., we present results without competition. The demand function converts into $q_1(p_1, \theta_1) = a - p_1 + \gamma\theta_1$, as cross-price ($\beta = 0$) or cross-quality ($\delta = 0$) are irrelevant. The profit functions for the manufacturer, distributor, and retailer are obtained as follows:

$$\pi_m^{BM} = w_m q_1 - \eta(1 - \rho^s)\theta_1^2; \quad \pi_d^{BM} = (w_{d_1} - w_m)q_1 \quad \pi_r^{BM} = (p_1 - w_{d_1})q_1 \quad (3)$$

One can verify that optimal decision for the supply chain are as follows: $w_{1m}^{BM} = \frac{8a(1-\rho^s)\eta}{16(1-\rho^s)\eta-\gamma^2}$; $\theta^{BM} = \frac{a\gamma}{16(1-\rho^s)\eta-\gamma^2}$; $p_1^{BM} = \frac{14a(1-\rho^s)\eta}{16(1-\rho^s)\eta-\gamma^2}$; $w_{1d}^{BM} = \frac{12a(1-\rho^s)\eta}{16(1-\rho^s)\eta-\gamma^2}$; $\rho^{BM_1} = \frac{3}{7}$; $\rho^{BM_2} = \frac{7}{15}$. Note that the optimal subsidy rates are independent of the parameters, even from the investment efficiency for the manufacturer. Therefore, whether the consumer is willing to buy the product (γ) or price sensitivity (β) does not impact the government decision. Later, we use these results to identify critical differences in subsidy-quality relations under monopoly and competition.

3.1. Optimal Decisions in Supply Chain Structure D_1R_1

In this distribution structure, both manufacturers distribute their products with a single distributor; then, the retailer procures both products from the distributor to sell those to the final customers. Profit functions for the supply chain members in this distribution structure are obtained as follows:

$$\left. \begin{aligned} \pi_{m_1}^{D_1R_1B_v} &= w_{m_1}q_1 - \eta(1 - \rho^s)\theta_1^2; & \pi_{m_2}^{D_1R_1B_v} &= w_{m_2}q_2 - \eta(1 - \rho^s)\theta_2^2 \\ \pi_d^{D_1R_1B_v} &= \sum_{i=1}^2 (w_{d_i} - w_{m_i})q_i \\ \pi_r^{D_1R_1B_v} &= \sum_{i=1}^2 (p_i - w_{d_i})q_i \end{aligned} \right\} \quad (4)$$

We refer to Appendix A for the detailed derivation of the optimal decisions under four different scenarios for the supply chain structure D_1R_1 . Note that when both manufacturers receive a subsidy, the optimal decision is symmetrical. We present the optimal decision under the supply chain structure D_1R_1 in Propositions 1 and 2, respectively:

Proposition 1. Optimal decision in Scenarios- $D_1R_1B_v$ is as follows:

$$\begin{aligned} w_{m_i}^{D_1R_1B_v}(\rho^s) &= \frac{8a(1-\rho^s)\eta}{8(1-\rho^s)(2-\beta)\eta-\gamma(\gamma-\delta)}, \theta_i^{D_1R_1B_v}(\rho^s) = \frac{a\gamma\eta}{8(1-\rho^s)(2-\beta)\eta-\gamma(\gamma-\delta)}, \\ w_{d_i}^{D_1R_1B_v}(\rho^s) &= \frac{4a(1-\rho^s)(3-2\beta)\eta}{(1-\beta)(8(1-\rho^s)(2-\beta)\eta-\gamma(\gamma-\delta))}, p_i^{D_1R_1B_v}(\rho^s) = \frac{2a(1-\rho^s)(7-4\beta)\eta}{(1-\beta)(8(1-\rho^s)(2-\beta)\eta-\gamma(\gamma-\delta))}, \\ \text{where } \rho^{D_1R_1B_1} &= 1 - \frac{2(2-\beta)(1-\beta)\gamma}{(8-5\beta)(\gamma-\delta)}, \rho^{D_1R_1B_2} = 1 - \frac{2(2-\beta)(1-\beta)\gamma}{(7-4\beta)(\gamma-\delta)} \end{aligned}$$

Proposition 2. Optimal decision in Scenarios- $D_1R_1S_v$ is as follows:

$$w_{m_1}^{D_1R_1S_v}(\rho^s) = \frac{8\eta(8(1-\rho^s)(2+\beta)\eta-\gamma(\gamma+\delta))}{\psi_0}, w_{m_2}^{D_1R_1S_v}(\rho^s) = \frac{8a(1-\rho^s)\eta(8(2+\beta)\eta-\gamma(\gamma+\delta))}{\psi_0},$$

$$\theta_1^{D_1 R_1 S_v}(\rho^s) = \frac{a\gamma(8(1-\rho^s)(2+\beta)\eta - \gamma(\gamma+\delta))}{\psi_0}, \theta_2^{D_1 R_1 S_v}(\rho^s) = \frac{a\gamma(8(2+\beta)\eta - \gamma(\gamma+\delta))}{\psi_0},$$

$$w_{d_1}^{D_1 R_1 S_v}(\rho^s) = \frac{4a\eta(8(1-\rho^s)(1+\beta)(2+\beta)(3-2\beta)\eta + (3-2\beta^2 + (1-\rho^s)\beta)\gamma(\gamma+\delta))}{\psi_0},$$

$$w_{d_2}^{D_1 R_1 S_v}(\rho^s) = \frac{4a\eta(8(1-\rho^s)(1+\beta)(2+\beta)(3-2\beta)\eta + ((3-2\beta^2)(1-\rho^s) + \beta)\gamma(\gamma+\delta))}{\psi_0},$$

$$p_1^{D_1 R_1 S_v}(\rho^s) = \frac{2a\eta(8(1-\rho^s)(1+\beta)(2+\beta)(7-4\beta)\eta + (7-4\beta^2 + 3(1-\rho^s)\beta)\gamma(\gamma+\delta))}{\psi_0},$$

$$p_2^{D_1 R_1 S_v}(\rho^s) = \frac{2a\eta(8(1-\rho^s)(1+\beta)(2+\beta)(7-4\beta)\eta + (7-4\beta^2)(1-\rho^s) + 3\beta)\gamma(\gamma+\delta)}{\psi_0},$$

where $\rho^{D_1 R_1 S_1} = \frac{2(1+\beta)(\alpha_2\gamma + (8-5\beta)\delta)(8(2+\beta)\eta - \gamma(\gamma+\delta))^2}{\phi_3},$

$$\rho^{D_1 R_1 S_2} = \frac{(1+\beta)((3+2(1-\beta)\beta)\gamma + (7-4\beta)\delta)(8(2+\beta)\eta - \gamma(\gamma+\delta))^2}{\phi_4}$$

We present all the additional notations used in this study in Appendix D. Unlike the single-manufacturer-single-distributor-single-retailer distribution structure, one can observe that the subsidy rates are dependent on system parameters. Cross-price elasticity or efficiencies for the manufacturers are key factors affecting the consumers' purchase decisions and government subsidy. Therefore, the expression of subsidy rate is more pragmatic under competition. Graphical representation of optimal quality levels, total supply chain profits, and the total amount of subsidy are presented in Figure 2.

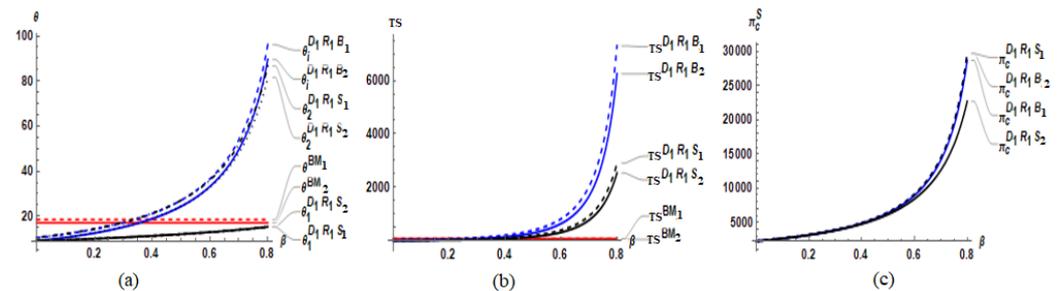


Figure 2. (a) Product quality, (b) total subsidy, and (c) total supply chain profits in distribution structure $D_1 R_1$: $a = 100$; $\delta = 0.3$; $\eta = 0.5$, $\beta \in (0, 0.8)$, $\gamma = 0.5$.

Noticeably, as cross-price sensitivity increases, the total amount of subsidy also increases, i.e., in an intensive competition market, the government needs to pay more contributions. It is also observed that the product quality level is also maximum without competition. The government needs to invest less under such a distribution structure. Therefore, the government needs to decide whether to encourage competition and monitor the distribution structure before providing a subsidy. Next, suppose there are two competing manufacturers. In that case, a subsidy policy to a single manufacturer can lead to significantly lower quality products (the manufacturer who does receive subsidy) but less expenditure. Therefore, it might harm the sustainability goal. Thus, under competition, it might be preferable to subsidize both manufacturers. This is also noticeable from this study.

3.2. Optimal Decisions in Supply Chain Structures $D_1 R_2$ or $D_2 R_1$

In this distribution structure, both manufacturers distribute their products with a single distributor, and finally, the distributor sells two products through two different

retailers. As mentioned earlier, cross-selling is not allowed. Therefore, profit functions for the supply chain members in this distribution structure are obtained as follows:

$$\left. \begin{aligned} \pi_{m_1}^{D_1R_2B_v} &= w_{m_1}q_1 - \eta(1 - \rho^s)\theta_1^2 & \pi_{m_2}^{D_1R_2B_v} &= w_{m_2}q_2 - \eta(1 - \rho^s)\theta_2^2 \\ \pi_d^{D_1R_2B_v} &= \sum_{i=1}^2 (w_{d_i} - w_{m_i})q_i \\ \pi_{r_1}^{D_1R_2B_v} &= (p_1 - w_{d_1})q_1 & \pi_{r_2}^{D_1R_2} &= (p_2 - w_{d_2})q_2 \end{aligned} \right\} \quad (5)$$

Similarly to distribution structure D_2R_1 , both manufacturers distributes their products with their exclusive distributors, and finally, a retailer procures both products from those distributors to sell to the final customers. Therefore, profit functions for the supply chain members in this distribution structure are obtained as follows:

$$\left. \begin{aligned} \pi_{m_1}^{D_2R_1B_v} &= w_{m_1}q_1 - \eta(1 - \rho^s)\theta_1^2 & \pi_{m_2}^{D_2R_1B_v} &= w_{m_2}q_2 - \eta(1 - \rho^s)\theta_2^2 \\ \pi_{d_1}^{D_2R_1B_v} &= (w_{d_1} - w_{m_1})q_1 & \pi_{d_2}^{D_2R_1B_v} &= (w_{d_2} - w_{m_2})q_2 \\ \pi_r^{D_2R_1B_v} &= \sum_{i=1}^2 (p_i - w_{d_i})q_i \end{aligned} \right\} \quad (6)$$

We refer to Appendix B for the detailed derivation process of the optimal decisions for the supply chain structures D_1R_2 and D_2R_1 , respectively. Note that when both manufacturers receive the subsidy, then optimal decisions remain the same in both distribution structures. We present optimal decisions under the supply chain structures D_1R_2/D_2R_1 when both manufacturers receive subsidies in Proposition 3.

Proposition 3. *Optimal decision in Scenarios- $D_1R_2B_v$ is as follows:*

$$\begin{aligned} w_{m_i}^{D_1R_2B_v}(\rho^s) &= \frac{4a(1-\rho^s)(4-\beta^2)\eta}{4(1-\rho^s)(2-\beta)\alpha_3\eta-\alpha_1}, \theta_{m_i}^{D_1R_2B_v}(\rho^s) = \frac{a(2\gamma-\beta\delta)}{4(1-\rho^s)(2-\beta)\alpha_3\eta-\alpha_1}, \\ w_{d_i}^{D_1R_2B_v}(\rho^s) &= \frac{2a(1-\rho^s)(2-\beta)(6-2\beta-3\beta^2)\eta}{(1-\beta)(4(1-\rho^s)(2-\beta)\alpha_3\eta-\alpha_1)}, p_i^{D_1R_2B_v}(\rho^s) = \frac{2a(1-\rho^s)(14-12\beta-5\beta^2+4\beta^3)\eta}{(1-\beta)(4(1-\rho^s)(2-\beta)\alpha_3\eta-\alpha_1)}, \\ \text{where } \rho^{D_1R_2B_1} &= \frac{(16-\beta^2(34-\beta(14+5(2-\beta)\beta)))\gamma+(32-\beta(36+\beta(14-\beta(21+\beta-3\beta^2))))\delta}{(2-\beta^2)(16-\beta(14+\beta(6-5\beta)))(\gamma-\delta)}, \\ \text{or } \rho^{D_1R_2B_2} &= \frac{(12+\beta(4-30\beta+10\beta^2+9\beta^3-4\beta^4))\gamma-(28-32\beta-10\beta^2+17\beta^3-2\beta^5)\delta}{(2-\beta^2)(14-12\beta-5\beta^2+4\beta^3)(\gamma-\delta)}. \end{aligned}$$

Note that different optimal decisions are obtained when one of the manufacturers receives a subsidy for distribution structures D_1R_2 and D_2R_1 , and we present optimal decision in Propositions 4 and 5, respectively:

Proposition 4. *Optimal decision in Scenarios- $D_1R_2S_v$ is as follows:*

$$\begin{aligned} w_{m_1}^{D_1R_2S_v}(\rho^s) &= \frac{4a(4-\beta^2)\eta(4(1-\rho^s)(2+\beta)\alpha_2\eta-\alpha_0)}{\psi_1}, \theta_1^{D_1R_2S_v}(\rho^s) = \frac{a(2\gamma-\beta\delta)(4(1-\rho^s)(2+\beta)\alpha_2\eta-\alpha_0)}{\psi_1}, \\ w_{m_2}^{D_1R_2S_v}(\rho^s) &= \frac{4a(1-\rho^s)(4-\beta^2)\eta(4(2+\beta)\alpha_2\eta-\alpha_0)}{\psi_1}, \theta_2^{D_1R_2S_v}(\rho^s) = \frac{a(2\gamma-\beta\delta)(4(2+\beta)\alpha_2\eta-\alpha_0)}{\psi_1}, \\ w_{d_1}^{D_1R_2S_v}(\rho^s) &= \frac{2a\eta(4(1-\rho^s)(4-\beta^2)(1+\beta)\alpha_2(6-2\beta-3\beta^2)\eta-(12-14\beta^2+3\beta^4+\beta(2-\beta^2)(1-\rho^s))\alpha_0)}{(1-\beta^2)\psi_1}, \\ w_{d_2}^{D_1R_2S_v}(\rho^s) &= \frac{4a\eta(4(1-\rho^s)(4-\beta^2)(1+\beta)\alpha_2(6-2\beta-3\beta^2)\eta-((12-14\beta^2+3\beta^4)(1-\rho^s)+\beta(2-\beta^2))\alpha_0)}{(1-\beta^2)\psi_1}, \\ p_1^{D_1R_2S_v}(\rho^s) &= \frac{2a\eta(4(1-\rho^s)(2+\beta)(1+\beta)\alpha_2(14-12\beta-5\beta^2+4\beta^3)\eta-((14-17\beta^2+4\beta^4)+\beta(2-\beta^2)(1-\rho^s))\alpha_0)}{(1-\beta^2)\psi_1}, \\ p_2^{D_1R_2S_v}(\rho^s) &= \frac{2a\eta(4(1-\rho^s)(2+\beta)(1+\beta)\alpha_2(14-12\beta-5\beta^2+4\beta^3)\eta-((14-17\beta^2+4\beta^4)(1-\rho^s)+\beta(2-\beta^2))\alpha_0)}{(1-\beta^2)\psi_1}, \\ \text{where } \rho^{D_1R_2S_1} &= \frac{2(1+\beta)[((16-34\beta^2+14\beta^3+10\beta^4-5\beta^5)\gamma-(32-36\beta-14\beta^2+21\beta^3+\beta^4-3\beta^5)\delta)(4(2+\beta)\alpha_2\eta-\alpha_0)^2]}{\phi_7}, \end{aligned}$$

$$\rho^{D_1R_2S_2} = \frac{(1-\beta)[((12+4\beta-30\beta^2+10\beta^3+9\beta^4-4\beta^5)\gamma-(28-32\beta-10\beta^2+17\beta^3-2\beta^5)\delta)(4(2+\beta)\alpha_2\eta-\alpha_0)^2]}{\phi_8}$$

Proposition 5. Optimal decision in Scenarios- $D_2R_1S_v$ is as follows:

$$w_{m_1}^{D_2R_1S_v}(\rho^s) = \frac{4a(4-\beta^2)\eta(4(1-\rho^s)(2+\beta)\alpha_2\eta-\alpha_0)}{\psi_1}, \theta_1^{D_2R_1S_v}(\rho^s) = \frac{a(2\gamma-\beta\delta)(4(1-\rho^s)(2+\beta)\alpha_2\eta-\alpha_0)}{\psi_1},$$

$$w_{m_2}^{D_2R_1S_v}(\rho^s) = \frac{4a(1-\rho^s)(4-\beta^2)\eta(4(2+\beta)\alpha_2\eta-\alpha_0)}{\psi_1}, \theta_2^{D_2R_1S_v}(\rho^s) = \frac{a(2\gamma-\beta\delta)(4(2+\beta)\alpha_2\eta-\alpha_0)}{\psi_1},$$

$$w_{d_1}^{D_2R_1S_v}(\rho^s) = \frac{8a(3-\beta^2)\eta(4(1-\rho^s)(2+\beta)\alpha_2\eta-\alpha_0)}{(1-\beta^2)\psi_1}, w_{d_2}^{D_2R_1S_v}(\rho^s) = \frac{8a(1-\rho^s)(3-\beta^2)\eta(4(2+\beta)\alpha_2\eta-\alpha_0)}{(1-\beta^2)\psi_1},$$

$$p_1^{D_2R_1S_v}(\rho^s) = \frac{2a\eta(4(1-x)(2+\beta)(1+\beta)\alpha_2(14-12\beta-5\beta^2+4\beta^3)\eta-((14-17\beta^2+4\beta^4)+\beta(2-\beta^2)(1-\rho^s))\alpha_0)}{(1-\beta^2)\psi_1},$$

$$p_2^{D_2R_1S_v}(\rho^s) = \frac{2a\eta(4(1-\rho^s)(2+\beta)(1+\beta)\alpha_2(14-12\beta-5\beta^2+4\beta^3)\eta-((14-17\beta^2+4\beta^4)(1-\rho^s)+\beta(2-\beta^2))\alpha_0)}{(1-\beta^2)\psi_1},$$

where $\rho^{D_2R_1S_1} = \frac{2(1+\beta)[((16-34\beta^2+14\beta^3+10\beta^4-5\beta^5)\gamma-(32-36\beta-14\beta^2+21\beta^3+\beta^4-3\beta^5)\delta)(4(2+\beta)\alpha_2\eta-\alpha_0)^2]}{\phi_7}$,

$$\rho^{D_2R_1S_2} = \frac{(1-\beta)[((12+4\beta-30\beta^2+10\beta^3+9\beta^4-4\beta^5)\gamma-(28-32\beta-10\beta^2+17\beta^3-2\beta^5)\delta)(4(2+\beta)\alpha_2\eta-\alpha_0)^2]}{\phi_8}.$$

Due to the variation of members in the lower tier, retail pricing and wholesale prices change when one of the manufacturers receives a subsidy; however, the government subsidy rate is not affected. Graphical representation of optimal quality levels, total supply chain profits, and the total amount of subsidy are in supply chain structures D_2R_1 , and D_1R_2 are presented in Figure 3.

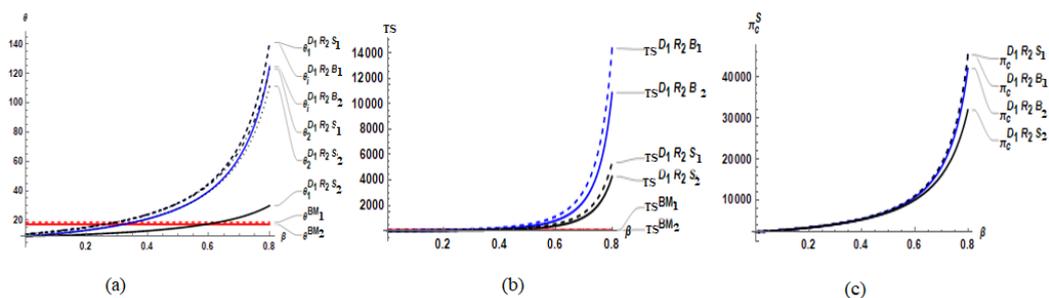


Figure 3. (a) Product quality, (b) total subsidy, and (c) total supply chain profits in distribution structures D_1R_2/D_1R_2 : $a = 100$; $\delta = 0.3$; $\eta = 0.5$, $\beta \in (0, 0.8)$, $\gamma = 0.5$.

Unlike distribution structure, D_1R_1 , product quality and total supply chain profit are maximum in Scenario- $D_1R_2S_2$, but government expenditure reduces. Therefore, distribution structures, as well as SW measures, clearly made an impact on the overall decision. Noticeably, the product quality level is maximum without competition or subsidy to a single manufacturer; therefore, the government does not necessarily encourage competition.

3.3. Optimal Decisions in Supply Chain Structure D_2R_2

Both manufacturers distribute their products with their respective exclusive distributors and retailers in this distribution structure. Therefore, we consider two purely competing supply chains, unlike the three previously discussed structures. Profit functions for the supply chain members in this structure are obtained as follows:

$$\left. \begin{aligned} \pi_{m_1}^{D_2R_2B_v} &= w_{m_1}q_1 - \eta(1-\rho^s)\theta_1^2; & \pi_{m_2}^{D_2R_2B_v} &= w_{m_2}q_2 - \eta(1-\rho^s)\theta_2^2 \\ \pi_{d_1}^{D_2R_2B_v} &= (w_{d_2} - w_{m_1})q_1 & \pi_{d_2}^{D_2R_2B_v} &= (w_{d_1} - w_{m_2})q_2 \\ \pi_{r_1}^{D_2R_2B_v} &= (p_1 - w_{d_1})q_1 & \pi_{r_2}^{D_2R_2B_v} &= (p_2 - w_{d_2})q_2 \end{aligned} \right\} \quad (7)$$

We refer to Appendix C for the detail derivation of the optimal decisions for the supply chain structures D_2R_2 . We present the optimal decision under the supply chain structure D_2R_2 in Propositions 6 and 7, respectively:

Proposition 6. *Optimal decision in Scenarios- $D_2R_2B_v$ is as follows:*

$$w_{m_i}^{D_2R_2B_v}(\rho^s) = \frac{2a(1-\rho^s)(64-84\beta^2+33\beta^4-4\beta^6)\eta}{\psi_3}, \theta_i^{D_2R_2B_v}(\rho^s) = \frac{a(2-\beta^2)\alpha_4}{\psi_3},$$

$$w_{d_i}^{D_2R_2B_v}(\rho^s) = \frac{4a(1-\rho^s)(4-\beta^2)(3-\beta^2)(4-3\beta^2)\eta}{\psi_3}, p_i^{D_2R_2B_v}(\rho^s) = \frac{2a(1-\rho^s)(112-154\beta^2+63\beta^4-8\beta^6)\eta}{\psi_3},$$

$$\text{where } \rho^{D_2R_2B_1} = 1 - \frac{(2-\beta)\alpha_3\alpha_4\alpha_5}{4(8-9\beta^2+2\beta^4)(64-90\beta^2+38\beta^4-5\beta^6)(\gamma-\delta)},$$

$$\rho^{D_2R_2B_2} = 1 - \frac{(2-\beta)\alpha_3\alpha_4\alpha_5}{2(8-9\beta^2+2\beta^4)(112-154\beta^2+63\beta^4-8\beta^6)(\gamma-\delta)}.$$

Proposition 7. *Optimal decision in Scenarios- $D_2R_2S_v$ is as follows:*

$$w_{m_1}^{D_2R_2S_v}(\rho^s) = \frac{2a(4-\beta^2)\alpha_2\alpha_3\eta(2(1-\rho^s)(2+\beta)\alpha_2\alpha_6\eta - (2-\beta^2)(\gamma+\delta)\alpha_4)}{\psi_4},$$

$$\theta_1^{D_2R_2S_v}(\rho^s) = \frac{a(2-\beta^2)\alpha_4(2(2+\beta)(1-\rho^s)\alpha_2\alpha_6\eta - (2-\beta^2)(\gamma+\delta)\alpha_4)}{\psi_4},$$

$$w_{m_2}^{D_2R_2S_v}(\rho^s) = \frac{2a(4-\beta^2)(1-\rho^s)\alpha_2\alpha_3\eta(2(2+\beta)\alpha_2\alpha_6\eta - (2-\beta^2)(\gamma+\delta)\alpha_4)}{\psi_4},$$

$$\theta_2^{D_2R_2S_v}(\rho^s) = \frac{a(2-\beta^2)\alpha_4(2(2+\beta)\alpha_2\alpha_6\eta - (2-\beta^2)(\gamma+\delta)\alpha_4)}{\psi_4},$$

$$w_{d_1}^{D_2R_2S_v}(\rho^s) = \frac{4a(4-\beta^2)(3-\beta^2)(4-\beta^2)(2(1-\rho^s)(2+\beta)\alpha_2\alpha_6\eta - (2-\beta^2)(\gamma+\delta)\alpha_4)\eta}{\psi_4},$$

$$w_{d_2}^{D_2R_2S_v}(\rho^s) = \frac{4a(1-\rho^s)(4-\beta^2)(3-\beta^2)(4-\beta^2)(2(2+\beta)\alpha_2\alpha_6\eta - (2-\beta^2)(\gamma+\delta)\alpha_4)\eta}{\psi_4},$$

$$p_1^{D_2R_2S_v}(\rho^s) = \frac{2a(112-154\beta^2+63\beta^4-8\beta^6)\eta(2(1-\rho^s)(2+\beta)\alpha_2\alpha_6\eta - (2-\beta^2)(\gamma+\delta)\alpha_4)}{\psi_4},$$

$$p_2^{D_2R_2S_v}(\rho^s) = \frac{2a(1-\rho^s)(112-154\beta^2+63\beta^4-8\beta^6)\eta(2(2+\beta)\alpha_2\alpha_6\eta - (2-\beta^2)(\gamma+\delta)\alpha_4)}{\psi_4},$$

$$\text{where } \rho^{D_2R_2S_1} = \frac{(2(2+\beta)\alpha_2\alpha_6\eta - (2-\beta^2)(\gamma+\delta)\alpha_4)^2}{\psi_5} [(1024 + 896\beta - 3360\beta^2 - 1568\beta^3 + 3820\beta^4 + 966\beta^5 - 1940\beta^6 - 253\beta^7 + 454\beta^8 + 24\beta^9 - 40\beta^{10})\gamma - 2(2+\beta)(512 - 448\beta - 904\beta^2 + 786\beta^3 + 562\beta^4 - 485\beta^5 - 148\beta^6 + 127\beta^7 + 14\beta^8 - 12\beta^9)\delta],$$

$$\rho^{D_2R_2S_2} = \frac{(2(2+\beta)\alpha_2\alpha_6\eta - (2-\beta^2)(\gamma+\delta)\alpha_4)^2}{\psi_6} [(768 + 896\beta - 2656\beta^2 - 1568\beta^3 + 3080\beta^4 + 966\beta^5 - 1570\beta^6 - 253\beta^7 + 366\beta^8 + 24\beta^9 - 32\beta^{10})\gamma - 2(896 - 384\beta - 1904\beta^2 + 668\beta^3 + 1540\beta^4 - 408\beta^5 - 596\beta^6 + 106\beta^7 + 111\beta^8 - 10\beta^9 - 8\beta^{10})\delta]$$

Graphical representation of optimal quality levels, the total supply chain profits and total amount of subsidy are presented in Figure 4.

A similar line of observation is obtained as seen in Figures 2 and 3. Summarizing the optimal decision in all four distribution structures, we made the following list of observations.

1. The government needs to monitor the distribution structure for the manufacturers on which they rely. Otherwise, it can increase the expenditure without achieving the sustainability goal;
2. Under competition, it might be possible to provide a subsidy to one of the manufacturers instead of both. Because this scenario can lead to the highest product quality level and supply chain profits, the government also needs to pay less;
3. The SW measure is also critical, and it becomes more apparent when the subsidy is provided to single manufacturers.

The national manufacturer needs to compete with many international manufacturers in many countries. In this scenario, a subsidy to the national manufacturer can contribute to the government’s goal of achieving a higher sustainability development index.

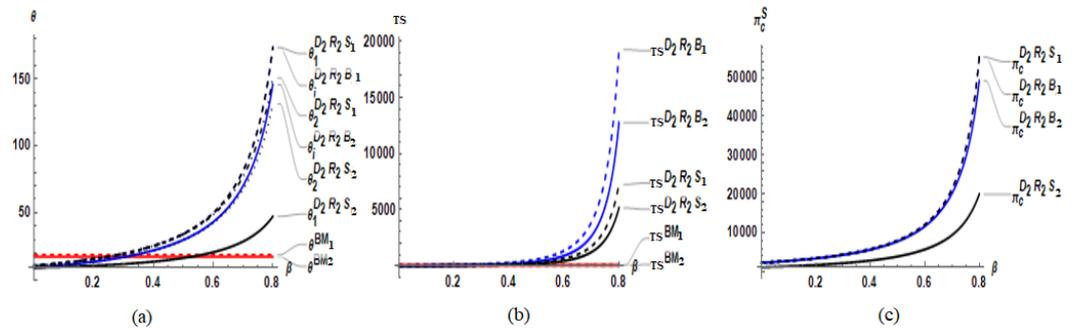


Figure 4. (a) Product quality, (b) total subsidy, and (c) total supply chain profits in distribution structure D_2R_2 : $a = 100; \delta = 0.3; \eta = 0.5, \beta \in (0, 0.8), \gamma = 0.5$.

4. Discussion

In the previous section, we explored optimal decisions under four distribution structures in the presence of government intervention. First, we analyze the characteristics of product qualities in the absence of a government subsidy. In the absence of a subsidy, optimal decisions are symmetrical, and the product qualities in distribution structure D_1R_2 and D_2R_1 remain the same. Therefore, we present graphical representation of the maximum product qualities, maximum and minimum prices among distribution structures D_2R_2 , D_1R_2 , and D_1R_1 in Figure 5.

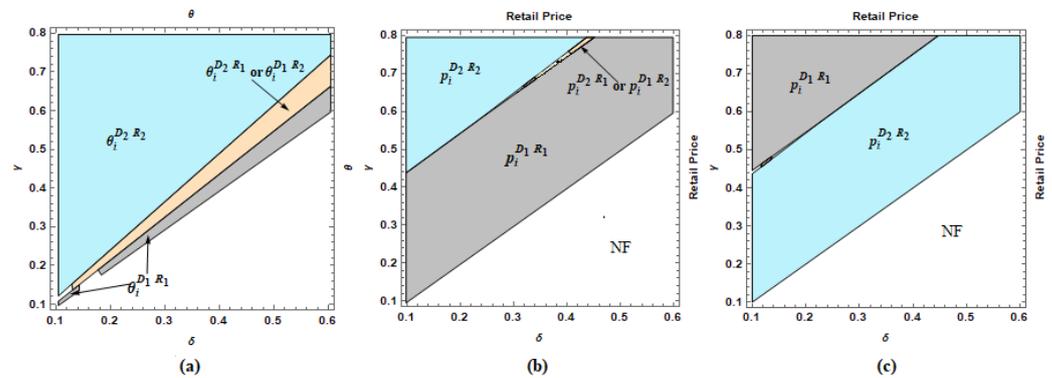


Figure 5. (a) Maximum product qualities; (b) Maximum retail prices; (c) Minimum retail prices in the absence of subsidy: $a = 100; \delta \in (0.1, 0.6); \eta = 0.5, \beta = 0.4, \gamma \in (0, 0.8)$.

From Figure 5, one can see that the product quality can be maximized in all three distribution structures. However, the regions representing retail prices are different. It might be intuitive that maximum retail price can ensure maximum retail prices, but results do not reflect that fact. Different distribution structures can lead to different price–quality pairs. For example, in distribution structure D_2R_2 , price, as well as product qualities, remain higher as well as lower. Hence, customers need to pay more for higher quality products if quality sensitivity is too high. However, the distribution structure D_2R_1 can also lead to a lower price with higher quality. Therefore, we can conclude that the customers will favor the distribution structures D_2R_1/D_1R_2 where they can receive lower-priced products. Overall, the distribution structures impact achieving sustainability goals in the absence of government subsidies.

Now we present the optimal product quality levels in the presence of government subsidies. Note that when the government provides a subsidy to both manufacturers,

the optimal product quality remains the same, but it differs when a subsidy is offered to one manufacturer only.

Therefore, it is clear that a government subsidy and SW measures can change the whole dynamics. If both the manufacturers receive subsidies, the maximum product quality is achieved in the distribution structures D_2R_2 , which is not the case when the subsidy is not provided. Next, the social welfare measure also made an impact. By comparing Figure 6a–f, we can see that under SW_2 , the product quality be might higher in Scenario D_2R_2 or D_1R_1 . Next, we explore the nature of retail prices.

It is clear that the government subsidy and its social welfare optimization objective significantly influence the customers. Product quality might be higher in D_2R_2 . If both manufacturers try to follow the other distribution structures, then the customers never receive higher quality products, but they still need to pay more. Previously, we saw that government need to invest more under the SW_2 measures (Figures 2–4). Additionally, we found from Figure 7 that retailers can charge more for the product when the manufacturer does not receive the subsidy. Therefore, there is a need to unify social welfare measures in the supply chain literature. Next, we explore the investment ratio for each manufacturer $\left(IR = \frac{\text{Product quality}}{\text{Total subsidy form the government}} \right)$ efficiencies in different distribution structures.

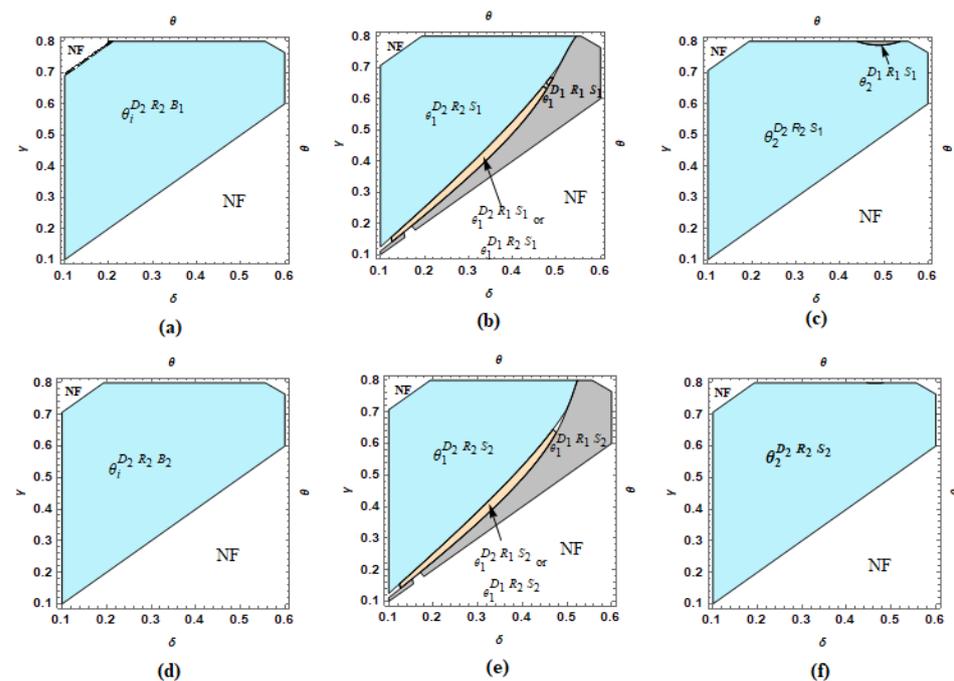


Figure 6. Maximum product qualities of (a) both products when both the manufacturers receive a subsidy under SW_1 ; (b) first product when the second manufacturer receives a subsidy under SW_1 ; (c) second product when the second manufacturer receives a subsidy under SW_1 ; (d) both products when both when both the manufacturers receive a subsidy under SW_2 ; (e) first product when the second manufacturer receives a subsidy under SW_2 ; (f) second product when the second manufacturer receives a subsidy under SW_2 . in different distribution structures: $a = 100; \delta \in (0.1, 0.6); \eta = 0.5, \beta = 0.4, \gamma \in (0, 0.8)$.

Figure 8 also demonstrates the variation in effectiveness in government expenditure in various distribution structures. Note that we also explore the characteristics of supply chain profits, subsidy, and quantity (Figures A1–A3), and similar variations are observed.

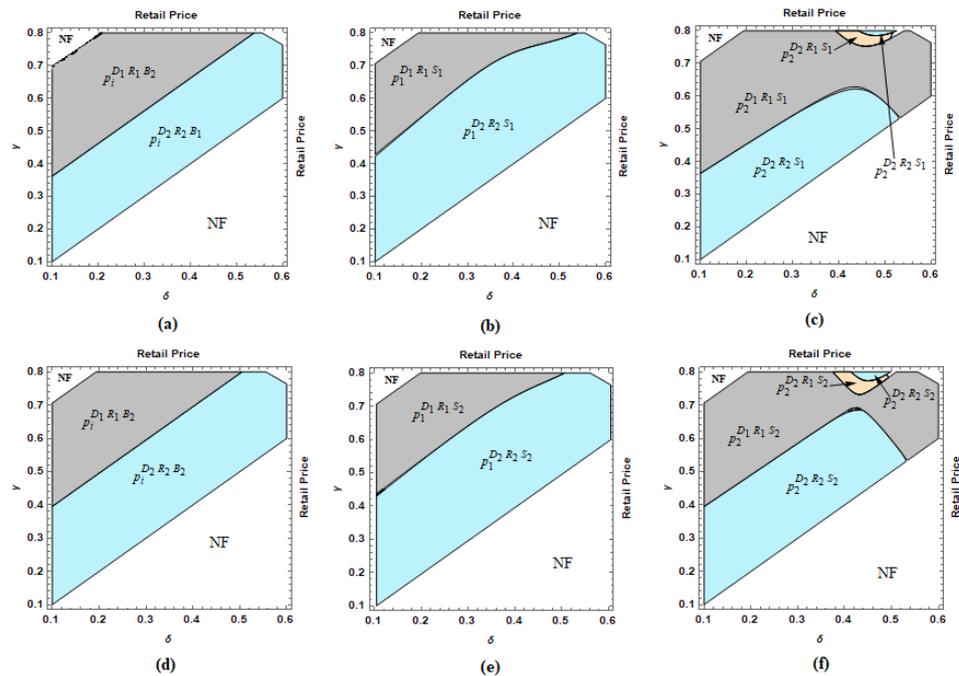


Figure 7. Minimum retail prices of (a) each products when both the manufacturers receive a subsidy under SW_1 ; (b) first product when the second manufacturer receives a subsidy under SW_1 ; (c) second product when the second manufacturer receives a subsidy under SW_1 ; (d) each products when both the manufacturers receive a subsidy under SW_2 ; (e) first product when the second manufacturer receives a subsidy under SW_2 ; (f) second product when the second manufacturer receives a subsidy under SW_2 ; in different distribution structures: $a = 100; \delta \in (0.1, 0.6); \eta = 0.5, \beta = 0.4, \gamma \in (0, 0.8)$.

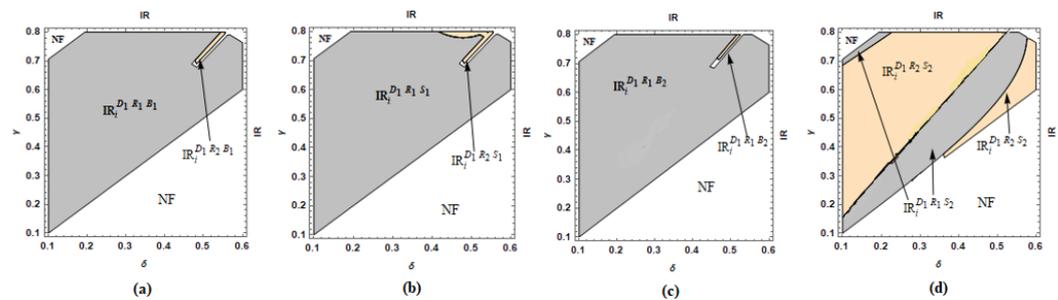


Figure 8. Maximum investment efficiencies of (a) Both the manufacturers receive a subsidy under SW_1 ; (b) Second manufacturer receives a subsidy under SW_1 ; (c) Both manufacturers receive a subsidy under SW_2 ; (d) Second manufacturer receives a subsidy under SW_2 ; in different distribution structures: $a = 100; \delta \in (0.1, 0.6); \eta = 0.5, \beta = 0.4, \gamma \in (0, 0.8)$.

Managerial Implications

Previously, we mainly focus on exploring whether the government encourages monopoly or competition [51]. We found that subsidy under both scenarios can lead to higher product quality. In this section, we focus mainly on exploring the variation of outcomes in different distribution structures and social welfare measures. In this direction, we also found that the outcomes are not unique. Although we conduct our analysis with respect to quality sensitivity (γ) and cross-quality sensitivity (δ), one can also observe similar variations for other parameters.

Due to increased environmental awareness among the stakeholders, establishing sustainable products is one of the main goals of contemporary organizations. In a recent review article, ref. [52] refer to the “green” concept as *doing actions while integrating environmental or ecological concern* and identify key barriers such as financial and cost associated with ecode-

sign, fear of failure, market competition and uncertainty, poor commitment from partners, and others. In this direction, refs. [53,54] also conducted a literature review to highlight current practices in several countries, product types, modeling approaches, and drew a list of factors that are the key barriers. Of those review articles, none highlight how (i) product distribution structures and (ii) social welfare measures can affect sustainability goals.

Three-level distribution under competition is not completely rare in practice [55]. This study is perhaps the first to clearly show that the distribution structures can make an enormous impact on the effectiveness of the government subsidy program and product quality. Results suggest that as a strategic decision, manufacturers use different distribution structures, or sometimes they are bound to use, which can significantly impact product quality. The government should encourage competition to ensure consumers can receive the product at the highest quality, depending on how manufacturers design their distribution channels. Regarding what is listed in the literature review, a second issue that we explore was also overlooked: how can social welfare function be set in the context of green supply chain practice? We have found that there is a need for standardization; some key insights highlighted so far, both in forward and closed-loop supply chains, might change if different social welfare measures were considered while determining the subsidy rate.

5. Conclusions and Future Research Direction

We have studied unified models to examine a situation where the government uses a subsidy program by capturing the strategic interactions under competition in a three-echelon supply chain setting. We first analyzed the base case when the subsidy program is intended to improve social welfare for a supply chain consisting of a single-manufacturer-single-distributor-single-retailer. We then examine four possible distribution structures of the base model: (1) when the subsidy program involves two competing manufacturers; and (2) when the subsidy program is intended for one of the manufacturers. Throughout the study, we have considered two social welfare optimization goals.

This study mainly focuses on exploring the effect of two relatively unexplored issues in the green supply chain literature: (i) the impact of distribution structures in a three-echelon supply chain under competition and (ii) social welfare measures while the government decides subsidy. Our study reveals that when the government's goal is to optimize social welfare, the performance differs significantly. Out of four distribution structures, it is difficult to identify one particular structure that ensures optimal product quality, lower retail prices, and higher product consumption under competition. The efficiency of subsidy in improving product quality and total supply chain profit might not be concurrent in a particular structure. The relative emphasis that the government places on defining social welfare also comes into play. No definite one ensures lower subsidies and the highest product quality among the four distribution structures. Whether government should invite multiple competing manufacturers to participate in the subsidy to ensure higher product quality levels is also responsible for the variation. Therefore, the government must enhance the supervision of manufacturers' distribution structures before introducing a subsidy program. In the literature, several empirical research has been conducted under a multicriteria decision-making framework by incorporating several factors affecting sustainability [53]; however, the two factors highlighted in this study are not explored yet. Our research can provide a guideline for the government to design an effective subsidy program that depends on its relative emphasis placed on product distribution structures and social welfare optimization goals.

Our study reveals that when the government's goal is to optimize social welfare, it is difficult to achieve optimal product quality that ensures lower prices and higher product consumption under competition. The efficiency of subsidy expenditure and total supply chain profit optimization goal might not be concurrent. The relative emphasis that the government places on defining social welfare comes into play. Among the four distribution structures, there is no definite one in which the government cares a lot about social welfare that ensures lower subsidies and the highest product quality. Therefore, the gov-

ernment should enhance the manufacturers' distribution structures before introducing a subsidy program. The government should invite multiple competing manufacturers to participate in the subsidy to ensure higher product quality levels, but not always. However, several empirical research has been conducted under a multicriteria decision-making framework [53], two factors highlighted in this study are not explored yet. Our research has provided a guideline for the government to design an effective subsidy program that depends on its relative emphasis placed on product distribution structures and social welfare optimization goals.

This study assumes that both manufacturers offer green products; therefore, one can compare four different outcomes by taking one manufacturer offers green products and the other offers nongreen products. We consider the influence of two social warfare measures. However, it will also be a crucial work to compare the result for different social welfare measures to construct a generalized measure (Table 1). We highlight the variation of social welfare measures on the forward supply chain. Therefore, it will also be interesting to study various social welfare measures on the closed-loop or dual-channel supply chain [56].

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Appendix A. Derivation of Optimal Decision for Supply Chain Structure D_1R_1

First, we derive optimal decision in Scenarios D_1R_1 . Solving first-order conditions for the retailer's profit function, $\frac{\partial \pi_r}{\partial p_1} = 0$ and $\frac{\partial \pi_r}{\partial p_2} = 0$ simultaneously, one can obtain the quantities as follows:

$$p_i = \frac{a(1 + \beta) + (1 - \beta^2)w_{d_i} + (\gamma - \beta\delta)\theta_i + (\beta\gamma - \delta)\theta_j}{2(1 - \beta^2)}, \quad i = 1, 2, \quad j = 3 - i \quad (A1)$$

Because $\frac{\partial^2 \pi_r}{\partial p_1^2} \cdot \frac{\partial^2 \pi_r}{\partial p_2^2} - \left(\frac{\partial^2 \pi_r}{\partial p_1 \partial p_2}\right)^2 = 4(1 - \beta^2) > 0$ and $\frac{\partial^2 \pi_r}{\partial p_1^2} = \frac{\partial^2 \pi_r}{\partial p_2^2} = -2 < 0$; π_r is jointly concave in p_i 's. Substituting retailer's response in Equation (4), profit function for the distributor is obtained as follows:

$$\pi_d(w_{d_i}) = \sum_{i=1}^2 \frac{(w_{d_i} - w_{m_i})(a - w_{d_i} + \beta w_{d_j} + \gamma\theta_i - \delta\theta_j)}{2}; \quad i = 1, 2, \quad j = 3 - i \quad (A2)$$

Solving first order conditions for the distributor's profit function given in Equation (A2), $\left(\frac{\partial \pi_d}{\partial w_{d_1}} = 0 \text{ and } \frac{\partial \pi_d}{\partial w_{d_2}} = 0\right)$ simultaneously, one can obtain the response for the distributor as follows:

$$w_{d_i} = \frac{a(1 + \beta) + (1 - \beta^2)w_{m_i} + (\gamma - \beta\delta)\theta_i + (\beta\gamma - \delta)\theta_j}{2(1 - \beta^2)} \quad (A3)$$

π_d is jointly concave in w_{d_i} , because $\frac{\partial^2 \pi_d}{\partial w_{d_1}^2} \cdot \frac{\partial^2 \pi_d}{\partial w_{d_2}^2} - \left(\frac{\partial^2 \pi_d}{\partial w_{d_1} \partial w_{d_2}} \right)^2 = 1 - \beta^2 > 0$ and $\frac{\partial^2 \pi_d}{\partial w_{d_i}^2} = -1 < 0$. Using distributor’s response, profit function for two manufacturers are obtained as follows:

$$\pi_{m_i}(w_{m_i}, \theta_i) = \frac{(a - w_{m_i}) - w_{m_i}^2 + w_{m_i}(\beta w_{m_j} + \gamma \theta_i - \delta \theta_j) - 4(1 - \rho^s)\eta \theta_i^2}{4}, \quad i = 1, 2, \quad j = 3 - i \tag{A4}$$

Profit function in Equation (A4) for each manufacturer is concave because, $\frac{\partial^2 \pi_{m_i}}{\partial w_{m_i}^2} \cdot \frac{\partial^2 \pi_{m_i}}{\partial \theta_i^2} - \left(\frac{\partial^2 \pi_{m_i}}{\partial w_{m_i} \partial \theta_i} \right)^2 = \frac{16(1 - \rho^s)\eta - \gamma^2}{16} > 0$; $\frac{\partial^2 \pi_{m_i}}{\partial w_{m_i}^2} = -\frac{1}{2} < 0$, and $\frac{\partial^2 \pi_{m_i}}{\partial \theta_i^2} = -2(1 - \rho^s)\eta < 0$. Solving the first order conditions of Equation (A4) ($\frac{\partial \pi_{m_i}}{\partial w_{m_i}} = 0$ and $\frac{\partial \pi_{m_i}}{\partial \theta_i} = 0$) for each manufacturer simultaneously, the responses for each manufacturer is obtained as follows:

$$\left. \begin{aligned} w_{m_i} &= \frac{8a(1 - \rho^s)\eta}{8(1 - \rho^s)(2 - \beta)\eta - \gamma(\gamma - \delta)} \\ \theta_i &= \frac{a\gamma\eta}{8(1 - \rho^s)(2 - \beta)\eta - \gamma(\gamma - \delta)} \end{aligned} \right\} \tag{A5}$$

Using the responses for the supply chain members, social welfare functions are obtained as follows:

$$SW_1(\rho^s) = TS + CS - \rho^s \eta (\theta_1^2 + \theta_2^2) = \frac{2a^2 \eta (4(1 - \rho^s)^2 (8 - 5\beta)\eta - (1 - \beta)\gamma^2)}{(1 - \beta)(8(1 - \rho^s)(2 - \beta)\eta - \gamma(\gamma - \delta))^2} \tag{A6}$$

$$SW_2(\rho^s) = TS - \rho^s \eta (\theta_1^2 + \theta_2^2) = \frac{2a^2 \eta (4(1 - \rho^s)^2 (7 - 4\beta)\eta - (1 - \beta)\gamma^2)}{(1 - \beta)(8(1 - \rho^s)(2 - \beta)\eta - \gamma(\gamma - \delta))^2} \tag{A7}$$

Therefore, optimal subsidies are obtained by solving $\frac{dSW_i}{d\rho^s} = 0$ as follows:

$$\rho^{D_1 R_1 B_1} = 1 - \frac{2(2 - \beta)(1 - \beta)\gamma}{(8 - 5\beta)(\gamma - \delta)} \tag{A8}$$

$$\rho^{D_1 R_1 B_2} = 1 - \frac{2(2 - \beta)(1 - \beta)\gamma}{(7 - 4\beta)(\gamma - \delta)} \tag{A9}$$

Note that each $SW_i(\rho^s)$ is concave as $\frac{d^2 SW_1}{d\rho^s^2} \Big|_{\rho^{D_1 R_1 B_1}} = \frac{16a^2(8 - 5\beta)^4(\gamma - \delta)^4\eta^2}{(1 - \beta)\gamma^2\phi_2^3} < 0$, and $\frac{d^2 SW_2}{d\rho^s^2} \Big|_{\rho^{D_1 R_1 B_2}} = \frac{16a^2(7 - 4\beta)^4(\gamma - \delta)^4\eta^2}{(1 - \beta)\gamma^2\phi_1^3} < 0$, respectively, Substituting the response of Equation (A8) and Equation (A9), we can obtain the product qualities, wholesale prices for the manufacturers and distributor, and retail prices of products are as presented in Proposition 1.

Appendix A.1. Derivation of Optimal Decision for Supply Chain Structure D₁R₁S_j

This subsection presents an optimal decision where one of the manufacturers receives a subsidy. In this scenario, the response of the retailer and the distributor are remain the same as given in Equations (A1) and (A3), respectively. Therefore, the profit functions for the manufacturers based on the responses of the retailer and distributor are obtained as follows:

$$\pi_{m_1}(w_{m_1}, \theta_1) = \frac{w_{m_1}(a - w_{m_1} + w_{m_2}\beta + \gamma\theta_1 - \delta\theta_2) - 4\eta\theta_1^2}{4} \tag{A10}$$

$$\pi_{m_2}(w_{m_2}, \theta_2) = \frac{w_{m_2}(a - w_{m_2} + w_{m_1}\beta - \delta\theta_1 + \gamma\theta_2) - 4(1 - \rho^s)\eta\theta_2^2}{4} \tag{A11}$$

As we assume, the second manufacturer receives a subsidy; therefore, the profit function for the first manufacturer is independent of the subsidy rate. By solving first-order

conditions of Equations (A10) and (A11) with respect to w_{m_i} & θ_i , we obtain the responses for the manufacturers as follows:

$$\left. \begin{aligned} w_{m_1} &= \frac{8\eta(8(1-\rho^s)(2+\beta)\eta - \gamma(\gamma+\delta))}{64(1-\rho^s)(4-\beta^2)\eta^2 - 8(2-\rho^s)\gamma(2\gamma-\beta\delta)\eta + \gamma^2(\gamma^2-\delta^2)} \\ \theta_1 &= \frac{a\gamma(8(1-\rho^s)(2+\beta)\eta - \gamma(\gamma+\delta))}{64(1-\rho^s)(4-\beta^2)\eta^2 - 8(2-\rho^s)\gamma(2\gamma-\beta\delta)\eta + \gamma^2(\gamma^2-\delta^2)} \end{aligned} \right\} \quad (A12)$$

$$\left. \begin{aligned} w_{m_2} &= \frac{8a(1-\rho^s)\eta(8(2+\beta)\eta - \gamma(\gamma+\delta))}{64(1-\rho^s)(4-\beta^2)\eta^2 - 8(2-\rho^s)\gamma(2\gamma-\beta\delta)\eta + \gamma^2(\gamma^2-\delta^2)} \\ \theta_2 &= \frac{a\gamma(8(2+\beta)\eta - \gamma(\gamma+\delta))}{64(1-\rho^s)(4-\beta^2)\eta^2 - 8(2-\rho^s)\gamma(2\gamma-\beta\delta)\eta + \gamma^2(\gamma^2-\delta^2)} \end{aligned} \right\} \quad (A13)$$

Finally, substituting response for the supply chain members, social welfare functions are obtained as follows:

$$\begin{aligned} SW_1(\rho^s) &= TS + CS - \rho^s \eta \theta_2^2 \\ &= \frac{2a^2\eta}{(1-\beta^2)(64(1-\rho^s)(4-\beta^2)\eta^2 - 8(2-\rho^s)\gamma(2\gamma-\beta\delta)\eta + \gamma^2(\gamma^2-\delta^2))} \\ &\quad [((1-\beta^2)\gamma^4(\gamma+\delta)^2 + \gamma^2(\gamma+\delta)((\rho^{s^2}(15-9\beta^2) - 4\rho^s(1+\beta)(12-\beta(7+2\beta))) \\ &\quad + 4(1+\beta)(16-\beta(9+4\beta)))\gamma + (32+4(3-5\beta)\beta + 4\rho^s(1+\beta)(8-5\beta) + \rho^{s^2}(15-9\beta^2))\delta)\eta \\ &\quad - 32(1+\beta)(2+\beta)\gamma((20-2\beta(6+\beta) + \rho^{s^2}(10-\beta(6+\beta)) + \rho^s(28-\beta(17+2\beta)))\gamma \\ &\quad - (2-\rho^s)(1-\rho^s)(8-5\beta)\delta)\eta^2 - 256(1-\rho^s)^2(1+\beta)(2+\beta)^2(8-5\beta)\eta^3)] \end{aligned} \quad (A14)$$

$$\begin{aligned} SW_2(\rho^s) &= TS - \rho^s \eta \theta_2^2 \\ &= \frac{2a^2\eta}{(1-\beta^2)(64(1-\rho^s)(4-\beta^2)\eta^2 - 8(2-\rho^s)\gamma(2\gamma-\beta\delta)\eta + \gamma^2(\gamma^2-\delta^2))} \\ &\quad [32(8(1-\rho^s)(2+\beta)\eta - (2-\rho^s)\gamma\delta)(1-\rho^s)(1+\beta)(2+\beta)(7-4\beta)\eta^2 \\ &\quad + 4(22+10\beta-4\beta^2(4+\beta) + \rho^{s^2}(7-4\beta^2) - 2\rho^s(1+\beta)(9-5\beta-\beta^2))\gamma^3\delta\eta \\ &\quad + 2\gamma^2\eta((14-7(2-\rho^s)\rho^s + 6(1-\rho^s)\beta - 4(2-2\rho^s + \rho^{s^2})\beta^2)\delta^2 \\ &\quad + 16(1+\beta)(2+\beta)(\rho^s(25-2\beta(7+\beta)) - (2+\rho^{s^2})(9-\beta(5+\beta)))\eta) \\ &\quad + \gamma^4(2(30-22\rho^s + 7\rho^{s^2})\eta + 4\beta(7-2\rho^{s^2}\beta - 4\beta(3+\beta) - \rho^s(5-2\beta(4+\beta)))\eta \\ &\quad - (1-\beta^2)\delta^2) - (1-\beta^2)\gamma^5(\gamma+2\delta)] \end{aligned} \quad (A15)$$

Solving first-order condition of Equations (A14) and (A15) with respect to ρ^s , we obtain subsidy rate as follows:

$$\rho^{D_1R_1S_1} = \frac{2(1+\beta)(\alpha_2\gamma + (8-5\beta)\delta)(8(2+\beta)\eta - \gamma(\gamma+\delta))^2}{\phi_3} \quad (A16)$$

$$\rho^{D_1R_1S_2} = \frac{(1+\beta)((3+2(1-\beta)\beta)\gamma + (7-4\beta)\delta)(8(2+\beta)\eta - \gamma(\gamma+\delta))^2}{\phi_4} \quad (A17)$$

Similar to the previous section, one can find that $\frac{d^2SW_1}{d\rho^{s^2}}|_{\rho^{D_1R_1S_1}} < 0$, and $\frac{d^2SW_2}{d\rho^{s^2}}|_{\rho^{D_1R_1S_2}} < 0$ and therefore each $SW_i(\rho^s)$ is concave in ρ^s . Upon simplification, we present the optimal decision in Proposition 2.

Appendix B. Derivation of Optimal Decision for Supply Chain Structures D_1R_2

First, we derive optimal decision for Scenario D_1R_2 . Solving first order conditions for the retailer's profit function ($\pi_{r_i} = (p_i - w_{d_i})q_i \quad i = 1, 2$), $\frac{d\pi_{r_i}}{dp_i} = 0 \quad i = 1, 2$ simultaneously, one can obtain retail prices as follows:

$$p_i = \frac{2w_{d_i} + w_{d_j}\beta + a(2+\beta) + (2\gamma - \beta\delta)\theta_i + (\beta\gamma - 2\delta)\theta_j}{4-\beta^2} \quad i = 1, 2; j = 3 - i \quad (A18)$$

Because $\frac{d^2\pi_{r_i}}{dp_i^2} = -2 < 0$; i.e., π_{r_i} are concave functions in p_i . Substituting responses for two retailers, profit function for the distributor is obtained as follows:

$$\pi_d(w_{d_i}, w_{d_j}) = \sum_{i=1}^2 \frac{(w_{d_i} - w_{m_i})(a(2 + \beta) + w_{d_i}\beta - w_{d_i}(2 - \beta^2) + (2\gamma - \beta\delta)\theta_i + (\beta\gamma - 2\delta)\theta_j)}{4 - \beta^2} \quad j = 3 - i \quad (A19)$$

The profit function of the distributor given in Equation (A19) is jointly concave in w_{d_i} , because $\frac{\partial^2 \pi_d}{\partial w_{d_1}^2} \cdot \frac{\partial^2 \pi_d}{\partial w_{d_2}^2} - \left(\frac{\partial^2 \pi_d}{\partial w_{d_1} \partial w_{d_2}}\right)^2 = \frac{4(1-\beta^2)}{4-\beta^2} > 0$ and $\frac{\partial^2 \pi_d}{\partial w_{d_1}^2} = -\frac{2(2-\beta^2)}{4-\beta^2} < 0$. Solving the first-order conditions of Equation (A19) with respect to w_{d_i} , the response for the distributor is obtained as follows:

$$w_{d_i} = \frac{a(1 + \beta) + w_{m_i}(1 - \beta^2) + (\gamma - \beta\delta)\theta_i + (\beta\gamma - \delta)\theta_j}{2(1 - \beta^2)} \quad i = 1, 2; j = 3 - i \quad (A20)$$

Based on the responses, profit functions for two manufacturers are obtained as follows:

$$\pi_{m_i}(w_{m_i}, \theta_i) = \frac{w_{m_i}(a(2 + \beta) - (2 - \beta^2)w_{m_i} + \beta w_{m_i} + (2\gamma - \beta\delta)\theta_i + (\beta\gamma - 2\delta)\theta_j) - 2(1 - \rho^s)(4 - \beta^2)\eta\theta_i^2}{2(4 - \beta^2)} \quad (A21)$$

The profit function for each manufacturers (π_{m_i}) given in Equation (A21) are jointly concave in w_{m_i} and θ_i because $\frac{\partial^2 \pi_{m_i}}{\partial w_{m_i}^2} \cdot \frac{\partial^2 \pi_{m_i}}{\partial \theta_i^2} - \left(\frac{\partial^2 \pi_{m_i}}{\partial w_{m_i} \partial \theta_i}\right)^2 = \frac{8(1-\rho^s)(2-\beta^2)(4-\beta^2)\eta - (2\gamma-\beta\delta)^2}{4(4-\beta^2)^2} > 0$ and $\frac{\partial^2 \pi_{m_i}}{\partial w_{m_i}^2} = -\frac{2-\beta^2}{4-\beta^2} < 0$, and $\frac{\partial^2 \pi_{m_i}}{\partial \theta_i^2} = -2(1 - \rho^s)\eta < 0$. Solving first-order conditions, product qualities, and wholesale prices for two manufacturers are obtained as follows:

$$\left. \begin{aligned} w_{m_i} &= \frac{4a(1 - \rho^s)(4 - \beta^2)\eta}{4(1 - \rho^s)(2 - \beta)\alpha_3\eta - \alpha_1} \\ \theta_{m_i} &= \frac{a(2\gamma - \beta\delta)}{4(1 - \rho^s)(2 - \beta)\alpha_3\eta - \alpha_1} \end{aligned} \right\} \quad (A22)$$

Finally, we compute the social welfare measures as given below:

$$\begin{aligned} SW_1(\rho^s) &= TS + CS - \rho^s \eta \sum_{i=1}^2 \theta_i^2 \\ &= \frac{2a^2\eta(4(1-\rho^s)^2(2-\beta^2)(16-14\beta-6\beta^2+5\beta^3)\eta - (1-\beta)(2\gamma-\beta\delta)^2)}{(1-\beta)(4(1-\rho^s)(2-\beta)\alpha_3\eta - \alpha_1)^2} \end{aligned} \quad (A23)$$

$$\begin{aligned} SW_2(\rho^s) &= TS - \rho^s \eta \sum_{i=1}^2 \theta_i^2 \\ &= \frac{2a^2\eta(4(1-\rho^s)^2(2-\beta^2)(14-12\beta-5\beta^2+4\beta^3)\eta - (1-\beta)(2\gamma-\beta\delta)^2)}{(1-\beta)(4(1-\rho^s)(2-\beta)\alpha_3\eta - \alpha_1)^2} \end{aligned} \quad (A24)$$

Solving firstorder conditions of Equations (A23) and (A24) with respect to ρ^s , we obtain subsidy rate as follows:

$$\rho^{D_1R_2B_1} = \frac{(16 - \beta^2(34 - \beta(14 + 5(2 - \beta)\beta)))\gamma + (32 - \beta(36 + \beta(14 - \beta(21 + \beta - 3\beta^2))))\delta}{(2 - \beta^2)(16 - \beta(14 + \beta(6 - 5\beta)))(\gamma - \delta)} \quad (A25)$$

$$\rho^{D_1R_2B_2} = \frac{(12 + \beta(4 - 30\beta + 10\beta^2 + 9\beta^3 - 4\beta^4))\gamma - (28 - 32\beta - 10\beta^2 + 17\beta^3 - 2\beta^5)\delta}{(2 - \beta^2)(14 - 12\beta - 5\beta^2 + 4\beta^3)(\gamma - \delta)} \quad (A26)$$

Because $\frac{d^2 SW_1}{d\rho^s{}^2} \Big|_{\rho^{D_1R_2B_1}} = -\frac{16a^2(2-\beta^2)^4(16-14\beta-6\beta^2+5\beta^3)^4(\gamma-\delta)^4\eta^2}{(1-\beta)(2\gamma-\beta\delta)^2\phi_5^3} < 0$ and $\frac{d^2 SW_2}{d\rho^s{}^2} \Big|_{\rho^{D_1R_2B_1}} = -\frac{16a^2(2-\beta^2)^4(14-12\beta-5\beta^2+4\beta^3)^4(\gamma-\delta)^4\eta^2}{(1-\beta)(2\gamma-\beta\delta)^2\phi_6^3} < 0$, the SW function given in Equations (A23) and (A24) are concave in ρ^s . We present optimal decision in this distribution structure in Proposition 3.

Derivation of Optimal Decision for Supply Chain Structure $D_1R_2S_j$

This subsection presents the detailed derivation of an optimal decision where one manufacturer receives a subsidy. In this scenario, the responses of the retailers and the distributor are the same as given in Equations (A18) and (A20). Therefore, the profit

functions for the manufacturers based on the responses of the retailers and distributor are obtained as follows:

$$\pi_{m_1}(w_{m_1}, \theta_1) = \frac{w_{m_1}(a(2 + \beta) - w_{m_1}(2 - \beta^2)) + w_{m_2}\beta + (2\gamma - \beta\delta)\theta_1 + (\beta\gamma - 2\delta)\theta_2 - 2(4 - \beta^2)\eta\theta_1^2}{2(4 - \beta^2)} \quad (\text{A27})$$

$$\pi_{m_2}(w_{m_2}, \theta_2) = \frac{w_{m_2}(a(2 + \beta) - w_{m_2}(2 - \beta^2)) + w_{m_1}\beta + (2\gamma - \beta\delta)\theta_2 + (\beta\gamma - 2\delta)\theta_1 - 2(4 - \beta^2)(1 - \rho^s)\eta\theta_2^2}{2(4 - \beta^2)} \quad (\text{A28})$$

As we assume, the second manufacturers receives subsidy, therefore profit function for first manufacturers independent for the subsidy rate. By solving first order conditions of Equation (A27) with respect to w_{m_1} & θ_1 and Equation (A28) with respect to w_{m_2} & θ_2 , we obtain the responses for the manufacturers as follows:

$$\left. \begin{aligned} w_{m_1} &= \frac{4a(4 - \beta^2)\eta(4(1 - \rho^s)(2 + \beta)\alpha_2\eta - \alpha_0)}{\psi_1} \\ \theta_1 &= \frac{a(2\gamma - \beta\delta)(4(1 - \rho^s)(2 + \beta)\alpha_2\eta - \alpha_0)}{\psi_1} \end{aligned} \right\} \quad (\text{A29})$$

$$\left. \begin{aligned} w_{m_2} &= \frac{4a(1 - \rho^s)(4 - \beta^2)\eta(4(2 + \beta)\alpha_2\eta - \alpha_0)}{\psi_1} \\ \theta_2 &= \frac{a(2\gamma - \beta\delta)(4(2 + \beta)\alpha_2\eta - \alpha_0)}{\psi_1} \end{aligned} \right\} \quad (\text{A30})$$

where $\psi_1 = 16(1 - \rho^s)(4 - \beta^2)\alpha_2\alpha_3\eta^2 - 4(2 - \rho^s)(2\gamma - \beta\delta)\alpha_4\eta + (\gamma^2 - \delta^2)(2\gamma - \beta\delta)^2$.

Finally, similar to the previous Section (Section A.1), substituting, response for the supply chain members, one can obtain the social welfare functions, and by solving first-order conditions with respect to ρ^s , we obtain subsidy rate as follows:

$$\rho^{D_1R_2S_1} = \frac{2(1+\beta)[((16-34\beta^2+14\beta^3+10\beta^4-5\beta^5)\gamma - (32-36\beta-14\beta^2+21\beta^3+\beta^4-3\beta^5)\delta)(4(2+\beta)\alpha_2\eta - \alpha_0)^2]}{\phi_7} \quad (\text{A31})$$

$$\rho^{D_1R_2S_2} = \frac{(1-\beta)[((12+4\beta-30\beta^2+10\beta^3+9\beta^4-4\beta^5)\gamma - (28-32\beta-10\beta^2+17\beta^3-2\beta^5)\delta)(4(2+\beta)\alpha_2\eta - \alpha_0)^2]}{\phi_8} \quad (\text{A32})$$

One can verify that SW functions are concave also. We present optimal decision in this distribution structure in Proposition 4. Note that the derivation of optimal decision in distribution structure D_2R_1 is similar with D_1R_2 , we omitted the proof.

Appendix C. Derivation of Optimal Decision for Supply Chain Structure D_2R_2

The optimal responses of the retailers remain the same as Equation (A18), for this scenario. Therefore, substituting retailer's response, profit function for each distributor are obtained as follows:

$$\pi_{d_i}(w_{d_i}) = \frac{(w_{d_i} - w_{m_i})(a - w_{d_i} + \beta w_{d_j} + \gamma\theta_i - \delta\theta_j)}{2}; \quad i = 1, 2; j = 3 - i \quad (\text{A33})$$

Solving first-order conditions for the profit functions of each distributors given in Equation (A33), ($\frac{\partial \pi_{d_i}}{\partial w_{d_i}} = 0 \quad i = 1, 2$) simultaneously, one can obtain the responses of each distributor as follows:

$$w_{d_i} = \frac{(a + \gamma\theta_j - \delta\theta_i)(2 + \beta)\alpha_2 + (2w_{m_i}(2 - \beta^2) + w_{m_j}\beta)(2 - \beta^2) + (8 - 3\beta^2)(\gamma + \delta)(\theta_i - \theta_j)}{16 - 17\beta^2 + 4\beta^4}; \quad (\text{A34})$$

Each π_{d_i} is concave in w_{d_i} , because $\frac{\partial^2 \pi_{d_i}}{\partial w_{d_i}^2} = -\frac{2(2 - \beta^2)}{4 + \beta^2} < 0$. Substituting distributor's response, profit function for two manufacturers are obtained as follows:

$$\pi_{m_i}(w_{m_i}, \theta_i) = \frac{1}{(4 - \beta^2)(16 - 17\beta^2 + 4\beta^4)} [(2 - \beta^2)w_{m_i}(a(2 + \beta)\alpha_2 + w_{m_j}\beta(2 - \beta^2) - w_{m_i}(8 - 9\beta^2 + 2\beta^4) + (8 - 3\beta^2)(\gamma\theta_i - \delta\theta_j) - 2\beta(3 - \beta^2)(\delta\theta_i - \gamma\theta_j)) - (1 - \rho^s)(4 - \beta^2)\alpha_2\alpha_3\eta\theta_i^2] \quad (\text{A35})$$

Profit function for each manufacturer is concave given in Equation (A35) because, $\frac{\partial^2 \pi_{m_i}}{\partial w_{m_i}^2} \cdot \frac{\partial^2 \pi_{m_i}}{\partial \theta_i^2} - \left(\frac{\partial^2 \pi_{m_i}}{\partial w_{m_i} \partial \theta_i} \right)^2 = \frac{(2 - \beta^2)\psi_2}{(64 - 84\beta^2 + 33\beta^4 - 4\beta^6)^2} > 0$; if $\psi_2 = 4(1 - \rho^s)(4 - \beta^2)(8 - 9\beta^2 + 2\beta^4)$
 $\alpha_2\alpha_3\eta - (2 - \beta^2)\alpha_4^2 > 0$ and $\frac{\partial^2 \pi_{m_i}}{\partial w_{m_i}^2} = -\frac{2(2 - \beta^2)(8 - 9\beta^2 + 2\beta^4)}{(4 - \beta^2)\alpha_2\alpha_3} < 0$, and $\frac{\partial^2 \pi_{m_i}}{\partial \theta_i^2} = -2(1 - x)\eta < 0$. Solving the first-order conditions of Equation (A35) ($\frac{\partial \pi_{m_i}}{\partial w_{m_i}} = 0$ and $\frac{\partial \pi_{m_i}}{\partial \theta_i} = 0$) for each manufacturer simultaneously, the responses for each manufacturer is obtained as follows:

$$\left. \begin{aligned} w_{m_i} &= \frac{2a(1 - \rho^s)(4 - \beta^2)\alpha_2\alpha_3\eta}{\psi_3} \\ \theta_i &= \frac{a(2 - \beta^2)\alpha_4}{\psi_3} \end{aligned} \right\} \quad (\text{A36})$$

where $\psi_3 = 2(1 - \rho^s)(2 - \beta)\alpha_5\alpha_3\eta - (2 - \beta^2)\alpha_4(\gamma - \delta)$. Finally, we obtain the social welfare functions for this distribution structures as follows:

$$\begin{aligned} SW_1(\rho^s) &= TS + CS - \rho^s\eta(\theta_1^2 + \theta_2^2) \\ &= \frac{1}{\psi_3^2} [2a^2(2 - \beta^2)\eta(8(1 - \rho^s)^2(8 - 9\beta^2 + 2\beta^4)(64 - 90\beta^2 + 38\beta^4 - 5\beta^6)\eta - (2 - \beta^2)\alpha_4^2)] \end{aligned} \quad (\text{A37})$$

$$\begin{aligned} SW_2(\rho^s) &= TS - \rho^s\eta(\theta_1^2 + \theta_2^2) \\ &= \frac{1}{\psi_3^2} [2a^2(2 - \beta^2)\eta(4(1 - \rho^s)^2(8 - 9\beta^2 + 2\beta^4)(112 - 154\beta^2 + 63\beta^4 - 8\beta^6)\eta - (2 - \beta^2)\alpha_4^2)] \end{aligned} \quad (\text{A38})$$

Therefore, solving the first-order condition of Equations (A37) and (A38) w.r.t. ρ^s , the optimal subsidies are obtained as follows:

$$\rho^{D_2R_2B_1} = 1 - \frac{(2 - \beta)\alpha_3\alpha_4\alpha_5}{4(8 - 9\beta^2 + 2\beta^4)(64 - 90\beta^2 + 38\beta^4 - 5\beta^6)(\gamma - \delta)} \quad (\text{A39})$$

$$\rho^{D_2R_2B_2} = 1 - \frac{(2 - \beta)\alpha_3\alpha_4\alpha_5}{2(8 - 9\beta^2 + 2\beta^4)(112 - 154\beta^2 + 63\beta^4 - 8\beta^6)(\gamma - \delta)} \quad (\text{A40})$$

Note that SW_i 's are concave in ρ^s because: $\frac{d^2 SW_1}{d\rho^s^2} \Big|_{\rho^{D_2R_2B_1}} = -\frac{256a^2(2 - \beta^2)^2(8 - 9\beta^2 + 2\beta^4)^4(64 - 90\beta^2 + 38\beta^4 - 5\beta^6)^4(\gamma - \delta)^4\eta^2}{\alpha_4^2\phi_9^3} < 0$, and $\frac{d^2 SW_2}{d\rho^s^2} \Big|_{\rho^{D_2R_2B_2}} = -\frac{16a^2(2 - \beta^2)^2(8 - 9\beta^2 + 2\beta^4)^4(112 - 154\beta^2 + 63\beta^4 - 8\beta^6)^4(\gamma - \delta)^4\eta^2}{\alpha_4^2\phi_{10}^3} < 0$. We present optimal decision in this distribution structure in Proposition 6.

Derivation of Optimal Decision for Supply Chain Structure $D_2R_2S_j$

This subsection presents an optimal decision where one of the manufacturers receives a subsidy. In this scenario, the response of the retailer and the distributor are same as given in Equations (A18) and (A34). Therefore, the profit functions for each manufacturer based on the responses of the retailers and distributors are obtained as follows:

$$\pi_{m_1} = \frac{1}{(4 - \beta^2)(16 - 17\beta^2 + 4\beta^4)} [w_{m_1}(2 - \beta^2)(a(2 + \beta)\alpha_2 - w_{m_1}(8 - 9\beta^2 + 2\beta^4) + w_{m_2}\beta(2 - \beta^2) + (8 - 3\beta^2)(\gamma\theta_1 - \delta\theta_2) - 2(3 - \beta^2)\beta(\delta\theta_1 - \gamma\theta_2)) - (4 - \beta^2)\alpha_2\alpha_3\eta\theta_1^2] \quad (\text{A41})$$

$$\begin{aligned} \pi_{m_2} &= \frac{w_{m_2}(a - w_{m_2} + w_{m_1}\beta - \delta\theta_1 + \gamma\theta_2) - 4(1 - x)\eta\theta_2^2}{4} [w_{m_2}(2 - \beta^2)(a(2 + \beta)\alpha_2 - w_{m_2}(8 - 9\beta^2 + 2\beta^4) \\ &+ w_{m_1}\beta(2 - \beta^2) + (8 - 3\beta^2)(\gamma\theta_2 - \delta\theta_1) - 2(3 - \beta^2)\beta(\delta\theta_2 - \gamma\theta_1)) - (1 - \rho^s)(4 - \beta^2)\alpha_2\alpha_3\eta\theta_2^2] \end{aligned} \quad (\text{A42})$$

As we assume, the second manufacturers receive a subsidy, therefore profit function for the first manufacturers is independent for the subsidy rate. By solving first-order conditions of Equation (A41) with respect to w_{m_1} & θ_1 and Equation (A42) with respect to w_{m_2} & θ_2 , we obtain the responses for the manufacturers as follows:

$$\left. \begin{aligned} w_{m_1} &= \frac{2a(4 - \beta^2)\alpha_2\alpha_3\eta(2(1 - \rho^s)(2 + \beta)\alpha_2\alpha_6\eta - (2 - \beta^2)(\gamma + \delta)\alpha_4)}{\psi_4}; \\ \theta_1 &= \frac{a(2 - \beta^2)\alpha_4(2(2 + \beta)(1 - \rho^s)\alpha_2\alpha_6\eta - (2 - \beta^2)(\gamma + \delta)\alpha_4)}{\psi_4} \end{aligned} \right\} \quad (A43)$$

$$\left. \begin{aligned} w_{m_2}(\rho^s) &= \frac{2a(4 - \beta^2)(1 - \rho^s)\alpha_2\alpha_3\eta(2(2 + \beta)\alpha_2\alpha_6\eta - (2 - \beta^2)(\gamma + \delta)\alpha_4)}{\psi_4} \\ \theta_2(\rho^s) &= \frac{a(2 - \beta^2)\alpha_4(2(2 + \beta)\alpha_2\alpha_6\eta - (2 - \beta^2)(\gamma + \delta)\alpha_4)}{\psi_4} \end{aligned} \right\} \quad (A44)$$

where $\psi_4 = 4(1 - \rho^s)(4 - \beta^2)\alpha_6\alpha_5\alpha_2\alpha_3\eta^2 - (2 - \beta^2)\alpha_4(2(2 - \rho^s)(2(64 - 90\beta^2 + 38\beta^4 - 5\beta^6)\gamma - \beta(112 - 154\beta^2 + 63\beta^4 - 8\beta^6)\delta)\eta - (2 - \beta^2)(\gamma^2 - \delta^2)\alpha_4)$. Finally, similar to the previous sections, by substituting responses for the supply chain members, one can obtain the social welfare functions, and by solving the first-order condition of SW_i 's with respect to ρ^s , we obtain subsidy rate as follows:

$$\rho^{D_2R_2S_1} = \frac{(2(2+\beta)\alpha_2\alpha_6\eta - (2-\beta^2)(\gamma+\delta)\alpha_4)^2}{\psi_5} \frac{[(1024 + 896\beta - 3360\beta^2 - 1568\beta^3 + 3820\beta^4 + 966\beta^5 - 1940\beta^6 - 253\beta^7 + 454\beta^8 + 24\beta^9 - 40\beta^{10})\gamma - 2(2 + \beta)(512 - 448\beta - 904\beta^2 + 786\beta^3 + 562\beta^4 - 485\beta^5 - 148\beta^6 + 127\beta^7 + 14\beta^8 - 12\beta^9)\delta]}{\quad} \quad (A45)$$

$$\rho^{D_2R_2S_2} = \frac{(2(2 + \beta)\alpha_2\alpha_6\eta - (2 - \beta^2)(\gamma + \delta)\alpha_4)^2}{\psi_6} [(768 + 896\beta - 2656\beta^2 - 1568\beta^3 + 3080\beta^4 + 966\beta^5 - 1570\beta^6 - 253\beta^7 + 366\beta^8 + 24\beta^9 - 32\beta^{10})\gamma - 2(896 - 384\beta - 1904\beta^2 + 668\beta^3 + 1540\beta^4 - 408\beta^5 - 596\beta^6 + 106\beta^7 + 111\beta^8 - 10\beta^9 - 8\beta^{10})\delta] \quad (A46)$$

where

$$\begin{aligned} \psi_5 &= \alpha_7\alpha_8((16 - 14\beta^2 + 3\beta^4)^2\gamma^5 + (2 - \beta^2)^2(8 - 3\beta^2)(8 - 12\beta - 3\beta^2 + 4\beta^3)\gamma^4\delta - 4(6\beta - 5\beta^3 + \beta^5)^2\delta^5) - 4\beta(3 - \beta^2)(2 - \beta^2)(278528 + 346112\beta - 1186304\beta^2 - 1423616\beta^3 + 2113024\beta^4 + 2460544\beta^5 - 2060280\beta^6 - 2335516\beta^7 + 1206292\beta^8 \\ &+ 1334072\beta^9 - 435944\beta^{10} - 471057\beta^{11} + 95342\beta^{12} + 100764\beta^{13} - 11580\beta^{14} - 11980\beta^{15} + 600\beta^{16} + 608\beta^{17})\delta^3\eta - 16(2 + \beta)^2\alpha_7(64 - 90\beta^2 + 38\beta^4 - 5\beta^6)\alpha_2^2\alpha_3^2\delta\eta^2 - (2 - \beta^2)\gamma^3((2 - \beta^2)\alpha_7\alpha_8(64 + 96\beta - 84\beta^2 - 68\beta^3 + 33\beta^4 + 12\beta^5 - 4\beta^6)\delta^2 + 2(8 - 3\beta^2)(507904 + 358400\beta - 1895936\beta^2 - 1317120\beta^3 + 3015296\beta^4 + 2061472\beta^5 - 2668472\beta^6 - 1795116\beta^7 + 1438376\beta^8 + 952108\beta^9 - 484292\beta^{10} - 315431\beta^{11} + 99651\beta^{12} + 63858\beta^{13} - 11480\beta^{14} - 7236\beta^{15} + 568\beta^{16} + 352\beta^{17})\eta) - (2 - \beta^2)\gamma^2\delta((2 - \beta^2)\alpha_7\alpha_8(64 - 96\beta - 84\beta^2 + 68\beta^3 + 33\beta^4 - 12\beta^5 - 4\beta^6)\delta^2 + 4(2 - \beta)(196608 - 1294336\beta - 2386944\beta^2 + 4269824\beta^3 + 7110144\beta^4 - 5750720\beta^5 - 10145904\beta^6 + 3969020\beta^7 + 8350348\beta^8 - 1355256\beta^9 - 4259864\beta^{10} + 81525\beta^{11} + 1372253\beta^{12} + 107131\beta^{13} - 272213\beta^{14} - 40258\beta^{15} + 30400\beta^{16} + 6084\beta^{17} - 1464\beta^{18} - 352\beta^{19})\eta) + 2\gamma(2\beta(3 - \beta^2)(2 - \beta^2)^2\alpha_7\alpha_8(8 - 3\beta - 3\beta^2 + \beta^3)\delta^4 + (2 - \beta^2)(2228224 + 3358720\beta - 12218368\beta^2 - 16009216\beta^3 + 27480576\beta^4 + 32275200\beta^5 - 34059840\beta^6 - 36325440\beta^7 + 26046456\beta^8 + 25278804\beta^9 - 12923204\beta^{10} - 11332768\beta^{11} + 4225400\beta^{12} + 3286883\beta^{13} - 898554\beta^{14} - 596900\beta^{15} + 118364\beta^{16} + 61780\beta^{17} - 8616\beta^{18} - 2784\beta^{19} + 256\beta^{20})\delta^2\eta + 8(2 + \beta)^2\alpha_7(64 - 90\beta^2 + 38\beta^4 - 5\beta^6)(\alpha_2)^2(\alpha_6)^2\eta^2) \\ \psi_6 &= (8 - 3\beta^2)^2(2 - \beta^2)^2\alpha_7\alpha_9\gamma^5 + (2 - \beta^2)^2(8 - 3\beta^2)\alpha_7\alpha_9(8 - 12\beta - 3\beta^2 + 4\beta^3)\gamma^4\delta - 4\alpha_7(6\beta - 5\beta^3 + \beta^5)^2\alpha_9\delta^5 - 8\beta(3 - \beta^2)(2 - \beta^2)(57344 + 75776\beta - 245504\beta^2 - 311552\beta^3 + 437920\beta^4 + 537352\beta^5 - 426336\beta^6 - 508204\beta^7 + 248644\beta^8 + 288860\beta^9 - 89342\beta^{10} - 101385\beta^{11} + 19400\beta^{12} + 21540\beta^{13} - 2337\beta^{14} - 2542\beta^{15} + 120\beta^{16} + 128\beta^{17})\delta^3\eta - 4(2 + \beta)^2\alpha_2^2\alpha_3^2\alpha_7\alpha_9\delta\eta^2 - (2 - \beta^2)\alpha_9\gamma^3((2 - \beta^2)\alpha_7(64 + 96\beta - 84\beta^2 - 68\beta^3 + 33\beta^4 + 12\beta^5 - 4\beta^6)\delta^2 + (8 - 3\beta^2)(4096 + 2816\beta - 9472\beta^2 - 6304\beta^3 + 8368\beta^4 + 5376\beta^5 - 3530\beta^6 - 2186\beta^7 + 715\beta^8 + 426\beta^9 - 56\beta^{10} - 32\beta^{11})\eta) - (2 - \beta^2)\gamma^2\delta((2 - \beta^2)\alpha_7\alpha_9(64 - 96\beta - 84\beta^2 + 68\beta^3 + 33\beta^4 - 12\beta^5 - 4\beta^6)\delta^2 + 2(393216 - 2637824\beta - 3350528\beta^2 + 10264576\beta^3 + 9375232\beta^4 - 17323264\beta^5 - 13430272\beta^6 + 16657792\beta^7 + 11512440\beta^8 - 10070776\beta^9 - 6325876\beta^{10} + 3975470\beta^{11} + 2287798\beta^{12} - 1026189\beta^{13} - 542228\beta^{14} + 167259\beta^{15} + 81018\beta^{16} - 15640\beta^{17} - 6912\beta^{18} + 640\beta^{19} + 256\beta^{20})\eta) + 4\gamma(\beta(3 - \beta^2)(2 - \beta^2)^2\alpha_7\alpha_9(8 - 3\beta - 3\beta^2 + \beta^3)\delta^4 + (2 - \beta^2)(458752 + 753664\beta - 2609152\beta^2 - 3615232\beta^3 + 5994240\beta^4 + 7313472\beta^5 - 7534080\beta^6 - 8242608\beta^7 + 5821980\beta^8 + 5735412\beta^9 - 2914862\beta^{10} - 2568184\beta^{11} + 961934\beta^{12} + 743375\beta^{13} - 206868\beta^{14} - 134648\beta^{15} + 27677\beta^{16} + 13894\beta^{17} - 2064\beta^{18} - 624\beta^{19} + 64\beta^{20})\delta^2\eta + (2 + \beta)^2\alpha_2^2\alpha_3^2\alpha_7\alpha_9\eta^2). \end{aligned}$$

We present optimal decision in this distribution structure in Proposition 7.

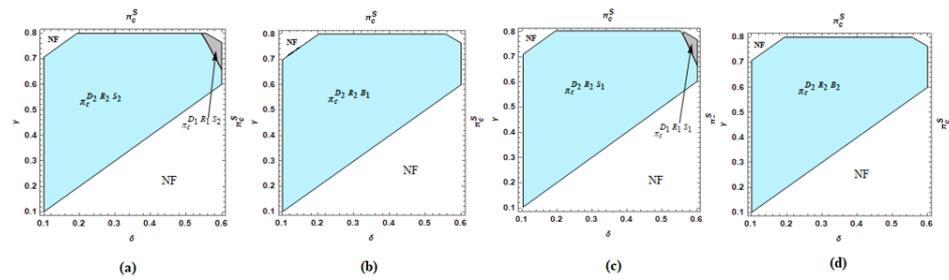


Figure A1. Maximum supply chain profit when (a) both manufacturers receive a subsidy under SW_1 ; (b) second manufacturer receives a subsidy under SW_1 ; (c) both manufacturers receive a subsidy under SW_2 ; (d) second manufacturer receives a subsidy under SW_2 ; in different distribution structures: $a = 100; \delta \in (0.1, 0.6); \eta = 0.5, \beta = 0.4, \gamma \in (0, 0.8)$.

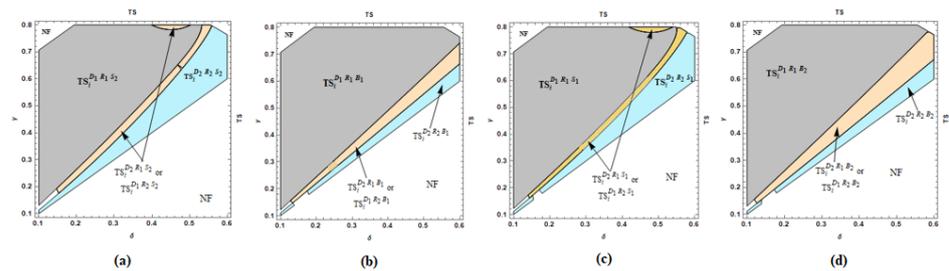


Figure A2. Minimum subsidy when (a) both the manufacturers receive a subsidy under SW_1 ; (b) second manufacturer receives a subsidy under SW_1 ; (c) both manufacturers receive a subsidy under SW_2 ; (d) second manufacturer receives a subsidy under SW_2 ; in different supply chain structures: $a = 100; \delta \in (0.1, 0.6); \eta = 0.5, \beta = 0.4, \gamma \in (0, 0.8)$.

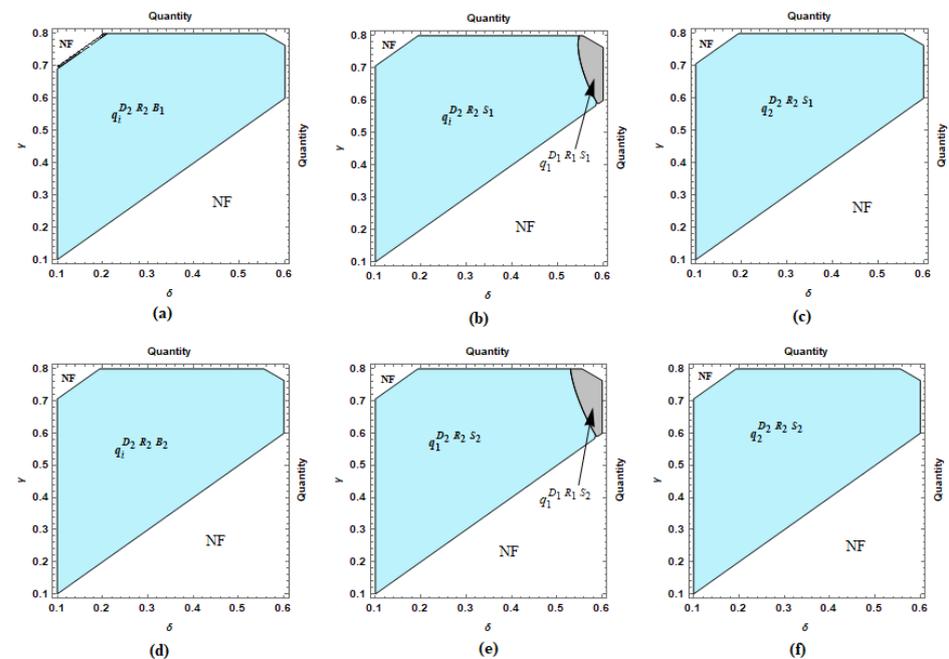


Figure A3. Maximum selling quantity of (a) each product when both the manufacturers receive a subsidy under SW_1 ; (b) first product when the second manufacturer receives a subsidy under SW_1 ; (c) second product when the second manufacturer receives a subsidy under SW_1 ; (d) each product when both the manufacturers receive a subsidy under SW_2 ; (e) first product when the second manufacturer receives a subsidy under SW_2 ; (f) second product when the second manufacturer receives a subsidy under SW_2 ; in different supply chain structures: $a = 100; \delta \in (0.1, 0.6); \eta = 0.5, \beta = 0.4, \gamma \in (0, 0.8)$.

Appendix D. List of Symbols

Table A1. Notations.

	Descriptions
Indices	
u	subsidy provided to single manufacturer (S) or both manufacturers (B), $u \in \{S, B\}$
v	social welfare measures, $v \in \{1, 2\}$
s	index for scenario, $s \in \{D_1R_1u_v, D_1R_2u_v, D_2R_1u_v, D_2R_2u_v\}$
Parameters	
a	market potential for each product
β	the cross-price sensitivity between two products, $\beta \in [0, 1)$
γ	product quality level sensitivity with own product, $\gamma > \delta$
δ	cross-quality sensitivity between two products, $0 \leq \delta < \gamma$
η	investment efficiency for each manufacturer, $\eta > 0$
Variables	
p_i^s	market price per unit of ith product
w_{di}^s	wholesale price per unit for distributor of ith product
w_{mi}^s	wholesale price per unit for manufacturer of ith product
θ_i^s	quality level of ith product
ρ^s	subsidy rate in Scenarion s
π_{ri}^s	retailer's profit while selling ith product
π_{di}^s	distributor's profit while selling ith product
π_{mi}^s	manufacturer's profit while selling ith product
π_c^s	total supply chain profits
SW^s	social welfare
TS^s	total government subsidy
q_i^s	sales volume of ith product

Appendix E. List of Additional Symbol

$$\alpha_0 = (2\gamma - \beta\delta)(\gamma + \delta)$$

$$\alpha_1 = (2\gamma - \beta\delta)(\gamma - \delta)$$

$$\alpha_2 = 4 + \beta - 2\beta^2$$

$$\alpha_3 = 4 - \beta - 2\beta^2$$

$$\alpha_4 = (8 - 3\beta^2)\gamma - 2\beta(3 - \beta^2)\delta$$

$$\alpha_5 = 16 - 2\beta - 18\beta^2 + \beta^3 + 4\beta^4$$

$$\alpha_6 = 16 + 2\beta - 18\beta^2 - \beta^3 + 4\beta^4$$

$$\alpha_7 = (8 - 9\beta^2 + 2\beta^4)$$

$$\alpha_8 = (240 - 334\beta^2 + 139\beta^4 - 18\beta^6)$$

$$\alpha_9 = 112 - 154\beta^2 + 63\beta^4 - 8\beta^6$$

$$\psi_0 = 64(1 - \rho^s)(4 - \beta^2)\eta^2 - 8(2 - \rho^s)\gamma(2\gamma - \beta\delta)\eta + \gamma^2(\gamma^2 - \delta^2)$$

$$\phi_1 = 16(2 - \beta)^2(1 - \beta)\eta - (8 - 5\beta)(\gamma - \delta)^2$$

$$\phi_2 = 16(2 - \beta)^2(1 - \beta)\eta - (7 - 4\beta)(\gamma - \delta)^2$$

$$\phi_3 = 3(5 - 3\beta^2)\gamma^2(\gamma - \delta)(\gamma + \delta)^2 - 8\gamma((62 + \beta(37 - \beta(2 + \beta)(11 + 4\beta)))\gamma^2 + 2(1 - \beta)(6 - \beta(11 + 8\beta))\gamma\delta + (34 + \beta(55 - \beta(16 + 19\beta)))\delta^2)\eta + 128(1 + \beta)(2 + \beta)^2(8 - 5\beta)(\gamma - \delta)\eta^2)$$

$$\phi_4 = 64(1 + \beta)(2 + \beta)^2(7 - 4\beta)(\gamma - \delta)\eta^2 - 8\gamma((28 + \beta(17 - 2\beta(2 + \beta)^2))\gamma^2 + (1 - \beta)(6 - \beta(11 + 8\beta))\gamma\delta - (14 + \beta(26 - \beta(5 + 8\beta)))\delta^2)\eta + (7 - 4\beta^2)\gamma^2(\gamma - \delta)(\gamma + \delta)^2$$

$$\phi_5 = 4(2 - \beta)^2(1 - \beta)\alpha_3^2\eta - (2 - \beta^2)(16 - 14\beta - 6\beta^2 + 5\beta^3)(\gamma - \delta)^2$$

$$\phi_6 = 4(2 - \beta)^2(1 - \beta)\alpha_3^2\eta - (2 - \beta^2)(14 - 12\beta - 5\beta^2 + 4\beta^3)(\gamma - \delta)^2$$

$$\phi_7 = 32(1 + \beta)(2 + \beta)^2(2 - \beta^2)\alpha_2^2(16 - 14\beta - 6\beta^2 + 5\beta^3)(\gamma - \delta)\eta^2 - 4(2\gamma - \beta\delta)((992 + 664\beta - 1860\beta^2 - 1272\beta^3 + 1200\beta^4 + 830\beta^5 - 329\beta^6 - 226\beta^7 + 33\beta^8 + 22\beta^9)\gamma^2 + 2(1 - \beta)(96 - 200\beta - 468\beta^2 + 48\beta^3 + 328\beta^4 + 42\beta^5 - 73\beta^6 - 12\beta^7 + 4\beta^8)\gamma\delta - (544 + 744\beta - 1212\beta^2 - 1496\beta^3 + 980\beta^4 + 1088\beta^5 - 335\beta^6 - 338\beta^7 + 41\beta^8 + 38\beta^9)\delta^2)\eta + (3 - \beta^2)(2 - \beta^2)(10 - 9\beta^2)(\gamma - \delta)\alpha_0^2$$

$$\phi_8 = 16(1 + \beta)(2 + \beta)^2(2 - \beta^2)\alpha_2^2(14 - 12\beta - 5\beta^2 + 4\beta^3)(\gamma - \delta)\eta^2 - 4(2\gamma - \beta\delta)((448 + 296\beta - 816\beta^2 - 552\beta^3 + 504\beta^4 + 346\beta^5 - 130\beta^6 - 89\beta^7 + 12\beta^8 + 8\beta^9)\gamma^2 + (1 - \beta)(96 -$$

$$200\beta - 468\beta^2 + 48\beta^3 + 328\beta^4 + 42\beta^5 - 73\beta^6 - 12\beta^7 + 4\beta^8)\gamma\delta - (224 + 336\beta - 492\beta^2 - 664\beta^3 + 394\beta^4 + 475\beta^5 - 133\beta^6 - 145\beta^7 + 16\beta^8 + 16\beta^9)\delta^2)\eta + (2 - \beta^2)(14 - 17\beta^2 + 4\beta^4)(\gamma - \delta)\alpha_0^2$$

$$\phi_9 = (2 - \beta)^2\alpha_3^2\alpha_5^2\eta - 2(2 - \beta^2)(8 - 9\beta^2 + 2\beta^4)(64 - 90\beta^2 + 38\beta^4 - 5\beta^6)(\gamma - \delta)^2;$$

$$\phi_{10} = (2 - \beta)^2\alpha_3^2\alpha_5^2\eta - (2 - \beta^2)(8 - 9\beta^2 + 2\beta^4)(112 - 154\beta^2 + 63\beta^4 - 8\beta^6)(\gamma - \delta)^2$$

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