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Research Article

Quantifying the Bullwhip Effect in a Reverse Supply Chain: The Impact of Different Forecasting Methods

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The reason for this study is that the bullwhip effect can pose very serious consequences for enterprises, such as increased production costs, additional manufacturing costs, excessive inventory levels, excess storage costs, large capital overstocking, and excessive transportation costs. Thus, the problem for this study is that quantifying the bullwhip effect in a reverse supply chain and comparing the impact of different forecasting methods on it. The objective of this paper is the bullwhip effect (BE) in a reverse supply chain (RSC). In particular, this study proposes a quantitative expression of the BE in a RSC, that is, $BE_R = Var(q_t)/Var(r_t)$, and analyzes the impact of different forecasting methods (e.g., the moving average technique (MA), the exponential smoothing technique (ES), and the minimum mean square error forecasting technique (MMSE)) on the bullwhip effect. We evaluate the conditions under which the collector should select different forecasting methods based on the BE. We use simulation data and get some conclusions that, in some cases when using the MMSE method, the BE does not exist in a RSC. This finding is significantly different from the results on the BE in a forward supply chain. Moreover, the MMSE method can reduce the lead-time demand forecast error to the greatest possible extent, which allows the BE to reach the lowest level.

1. Introduction

With increasing awareness of environmental protection issues, many countries and firms pay greater attention to activities related to product recycling. Reverse logistics is an emerging research field, and it involves the process of moving goods from their final destination for the purpose of properly disposing of the goods or capturing value and profiting from them. Many scholars have proposed a definition of reverse logistics. For example, reverse logistics describes the recycling and remanufacturing activities undertaken by an enterprise that collects used products from the consumer in supply chain management. This definition implies that reverse logistics involves saving raw materials in the production process, recycling, reuse and recovery of used products, reusing packaging, and dismantling or repairing defective products. From a wider societal perspective, the implementation of reverse logistics can effectively protect the environment, reduce the consumption of energy and

nonrenewable resources, improve the utilization rate of resources, and realize the sustainable development strategy. For enterprises, effective reverse logistics operations can not only reduce material costs and increase an enterprise's income, but also enhance their corporate image by improving customer satisfaction, increase the information exchange at each node of the supply chain (SC), improve market share, and establish a competitive advantage. Thus, the reverse logistics networks, structure performance, and operations modeling have become hot issues in academic research.

The bullwhip effect (BE) is a phenomenon that is observed when there is an amplification of demand fluctuation up the supply. As demand information becomes distorted, an enlargement can occur in the process of the transmission of demand information from downstream to upstream. Moreover, the fluctuation in demand in upstream enterprises is greater than that observed in downstream enterprises, which can cause the bullwhip effect (BE). The BE is the most important performance indicator in the SC

structure, and it is also the most important performance index in the SC operation. The BE can pose extremely serious consequences for enterprises, such as increased production costs, additional manufacturing costs, excessive inventory levels, excess storage costs, large capital overstocking, and excessive transportation costs. A substantial body of literature has examined the BE. Forrester [1] first identified the existence of the BE. Thereafter, many scholars studied the existence of the BE, aiming to identify its causes and assessing how it may be reduced. Although many scholars have carried out a lot of research to examine the strategic, tactical, and operational levels of reverse logistics, few scholars discussed the quantification of the BE and its impact on reverse logistics. Therefore, this paper develops a quantitative expression of the BE in a reverse supply chain (RSC). The impact of different forecasting techniques on the BE in an RSC is also compared. Finally, the relevant measures to reduce the BE in a RSC are proposed.

Of course, many firms have also found the bullwhip effect in their operational practices. For example, in the middle of 1990s, when workers at Procter & Gamble were examining order patterns for their best-selling baby diapers, they notice a strange phenomenon that the selling quantity of this product is fairly stable and does not fluctuate much, but when examining orders in the distribution center, we were surprised to find that the fluctuation increased significantly. The distribution center said that it placed orders based on aggregate demand for orders from vendors. Further study found that retailers are often based on historical sales volume and real sales forecast and determine the quantity of a more objective, but in order to guarantee that the quantity of goods is available on time, as well as the ability to adapt to incremental changes in customer requirements, they will usually request that the forecast order quantity must be enlarged to wholesalers. As a result, consumer demand has been amplified.

The main purpose of this paper is that we find and derive the mathematical expression of bullwhip effect in reverse supply chain in theoretically. This is the biggest difference comparing the previous studies. The mathematical expression of the bullwhip effect that we derive does not change depending on the forecasting techniques used. Based on this, we analyze the different influence of different forecasting techniques on bullwhip effect in reverse supply chain. In other words, the main contributions of this paper are as follows. First, we come up with the quantitative expression of the BE in a RSC by using different forecasting techniques (i.e., the moving average technique (MA), the exponential smoothing technique (ES), and the minimum mean square error forecasting technique (MMSE)). Second, we compare the influence of different forecasting techniques on the BE in a RSC. Third, this paper proposes relevant measures that can be used to reduce the BE in a RSC, and we can obtain some important managerial insights.

The remainder of this paper is organized as follows. Section 2 reviews relevant literature. Section 3 presents a description of the problem and describes the modeling. Section 4 determines the BE using different forecasting methods, that is, the moving average technique (MA), the

exponential smoothing technique (ES), and the minimum mean square error forecasting technique (MMSE). The simulation and results of the analysis are introduced in Section 5. The final section presents the conclusion and direction for future research.

2. Literature Review

In traditional forward SCs, the BE refers to the phenomenon of amplification of demand variability from the point of final demand to the point of origin [2]. This phenomenon can lead to substantial problems that affect SC performance, such as superfluous inventory, erroneous product forecasts, and high costs for correction [3]. The BE has become one of the main obstacles affecting the efficiency of the SC. Thus, it has attracted the attention of numerous administrators and scholars.

Early studies mainly focused on the existence of the BE and recognized its causes in traditional forward SCs (Forrester [1]; Lee et al. [2]; Lee et al. [3]). Lee et al. [4] discussed the BE in a two-level SC under an AR(1) demand process. Alwan et al. [5] studied the BE under the ARMA(1,1) demand process. In addition, the ARMA(p,q) demand process was discussed by Gaalman and Disney [6] and Gaalman et al. [7]. Michael [8] analyzed the bullwhip effect problem from the carriers' viewpoint under the AR(1) demand process. Moreover, many scholars discussed the BE in a two-stage forward SC using control approaches. Holt et al. [9] developed the HMMS control model in a two-stage forward SC and pointed out that this model could effectively balance the relationship between ordering from the retailer and ordering from the supplier. Blinder et al. [10] proposed that it could reduce the BE by using the (S,S) ordering strategy. Jose and Rafael [11] used a web-based supply chain simulator to demonstrate the potential benefits of using Electronic Data Interchange (EDI) in supply chain management. Moreover, they pointed out that it could reduce the bullwhip effect by using EDI technique. Baganha et al. [12] designed a particular inventory control strategy and highlighted that it could reduce the fluctuation of demand information. In addition, more scholars studied the BE using the discrete control theory, H_∞ control theory, control-based forecasting technique, and O-S feedback control method in two-stage forward SCs (e.g., Towill [13]; Huang et al. [14]; Disney et al. [15]; Rong et al. [16]).

With economic and societal development, more and more companies are involved in the SC, and many scholars attempted to analyze the BE in a multistage forward SC using different approaches. Li et al. [17] derived closed-form formula to analytically describe how the bullwhip and antibullwhip effects originated initially and then evolved over time and spaced in the supply chain. Vicente et al. [18] measured the bullwhip effect through four metrics: the echelon average inventory, the echelon inventory variance ratio, the echelon average order, and the echelon order rate variance ratio. Zhu et al. [19] investigated the factors that impacted the bullwhip effect in the oil and gas supply chain using case study evidence from six companies in North America, which cover refining and marketing, exploration

and production, integrated oil and gas, and drilling. Yao et al. [20] estimated the product level bullwhip effect using various methods, analyzed consequences of its different measurements and aggregations, and examined its impact on supply chain performance in terms of inventory ratio and stock outs. Yin [21] have considered the market competition among retailers and measured the bullwhip effect, in which multiple retailers exhibit AR(1) demand processes, and the degree of market competition was captured with copula. Lu et al. [22] have adopted regression models to test the proposed model and conducted a series of robustness tests by using moving average forecasting methods. Costantino et al. [22] studied the influence of demand sharing on order strategy using the simulation method. Hossein et al. [23] quantified the BE, order rate variance ratio (OVR), and inventory variance ratio (IV) in a three-stage forward SC with multiple retailers. Marieh et al. [24] investigated the measure of the BE in a three-stage forward SC. Alexandre et al. [25] performed a simulation-based study to investigate the interrelations of the structural and operational dynamics in the forward SC. Moreover, they pointed out that the forward SC managers need to take into account the risk of bullwhip effect during the capacity disruption and recovery periods. Ki and Jae [26] built a four-echelon supply chain simulation model where each echelon shares some of the customer demand forecast information with a retailer, the lowest echelon. What is more, they analyzed the impact of information sharing on the bullwhip effect. Bray et al. [27] modeled a single-supplier, 73-store forward supply chain as a dynamic discrete choice problem. They estimate the model with transaction-level data, spanning 3251 products and 1370 days. Ojha et al. [28] used simulation to investigate the impact of information sharing on both the bullwhip effect (BWE) and the order-fulfillment performance (OFP) in multiechelon forward supply chain system. Li [29] researched the bullwhip effect in a two-echelon forward supply chain consisting of one single supplier and multiple retailers, and the vertical and horizontal cooperation game for carbon emission reduction was analyzed under carbon tax scheme. Yao et al. [20] estimated the product level bullwhip effect using various methods, analyzed consequences of its different measurements and aggregations, and examined its impact on supply chain performance in terms of inventory ratio and stockouts. Yin [21] measured the bullwhip effect in a two-stage supply chain with one supplier and multiple retailers, in which multiple retailers exhibit AR(1) demand processes, and the degree of market competition was captured with copula.

The simple two-stage forward SC modeling assumption has been widely used to study the BE. However, due to the complexity of the forward SC network's structure, the simple two-stage forward SC modeling assumption is outdated. Recently, some scholars discussed the BE in a two-stage forward SC network (Zhang and Zhao [30]; Zhang and Yuan [31]; Yuan and Zhu [32]). Nonetheless, most of these studies assumed that customer demand follows the AR(1) autoregressive process, and they mainly discussed SC coordination, and the BE could be reduced. Yuan and Zhu [32] provided three quantitative models of the BE in a two-stage forward SC network.

It is known that a great number of scholars studied the BE in two-stage or multistage forward SCs. On the contrary, fewer scholars discussed the reverse BE in a forward SC. The term "reverse bullwhip effect" (RBE) was first proposed by Svensson [33], who analyzed the RBE in intraorganizational echelons. He pointed out that the RBE occurs when there is a high degree of postponement in inbound logistics flows, and a high degree of speculation in outbound logistics flows. Ozelkan et al. [34] analyzed the impact of procurement price variability from upstream to downstream in a forward SC. Ozelkan and Cakanyildirim [35] investigated the conditions that lead to the amplification of price variations moving from upstream to downstream in a SC, which is referred to as the "reverse bullwhip effect in pricing" (RBEP). Ozelkan et al. [36] investigated the RBEP conditions for SCs, in which joint replenishment and pricing decisions are made.

The main difference between this paper and the existing literature lies in the following aspects: (1) as discussed in the literature review above, most papers mainly analyze the bullwhip effect in forward supply chain including two-level or multilevel supply chain. However, we give the quantitative expression of the bullwhip effect in reverse supply chain. (2) Most of the above papers discuss the impact of three different forecasting techniques on the bullwhip effect in forward supply chain; on the contrary, in this paper, we discuss the impact of different forecasting techniques on the bullwhip effect in reverse supply chain and propose relevant measures that can be used to reduce the bullwhip in reverse supply chain.

3. Problem Description and Modeling

3.1. Problem Description. We consider a two-stage RSC consisting of one collector and one remanufacturer (see Figure 1). The collector is the only supplier of used products to the remanufacturer. The trading activities occur over an infinitely discrete period t , where $t \in (-\infty, 0, +\infty)$. At the end of period t , according to the past data on the supply of used products, the collector estimates the quantity of used products in period $t + 1$ by using different forecasting methods. Moreover, according to a certain inventory strategy, the collector determines the quantity of used products that should be transferred to the remanufacturer. The collector's lead time is a constant that can be expressed as l , and the remanufacturer can receive the used products at the beginning of the period $t + l + 1$.

3.2. Description of Parameters. The relevant symbols that will be used throughout the paper are described and explained as follows:

- (i) r_t : the supply quantity of used products from customers to the collector;
- (ii) q_t : the quantity of used products transferred from the collector to the remanufacturer at the end of period t ;
- (iii) ρ : autocorrelation coefficient;
- (iv) μ : nonnegative constant;

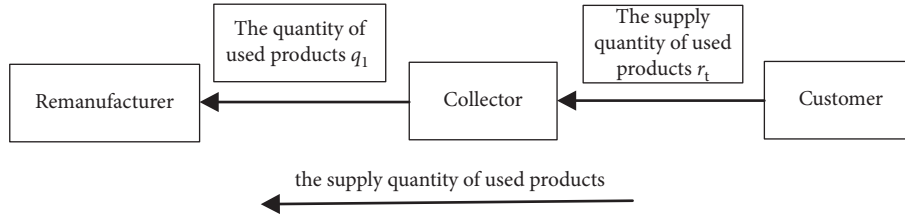


FIGURE 1: The conception model in revers supply chain.

- (v) ε_t : independent identically distributed random variable;
- (vi) S_t : the collector's highest inventory quantity at the end of period t ;
- (vii) S_{t-1} : the collector's highest inventory quantity at the end of period $t - 1$;
- (viii) \hat{r}_t^l : the estimate of the leading-time supply quantity using different forecasting methods;
- (ix) z : the constant to achieve a desired service level;
- (x) $\hat{\delta}_t^l$: the estimate of the standard deviation of the l period forecasting error of collector;
- (xi) p : the moving average period;
- (xii) l : the collector's leading time;
- (xiii) α : the collector's smoothing constant.

3.3. Modeling Assumption. In order to make the quantitative expression of the BE more meaningful in practical terms in the case of the RSC, we made the following assumptions:

- (1) The collector only collects one kind of used products;
- (2) The quantity of used products from customers to the collector r_t belongs to an AR(1) autocorrelation process;
- (3) We only focus on the quantitative expression of the BE for the collector. Thus, we assume that the collector can forecast the supply of used products from customers but will not share the information with the remanufacturer.
- (4) Assume that the quantity of used products from customers to the collector r_t belongs to an AR(1) autocorrelation process:

$$r_t = \mu + \rho r_{t-1} + \varepsilon_t, \quad (1)$$

where μ is a nonnegative constant, ρ is the autocorrelation coefficient $|\rho| < 1$, and ε_t is an independent identically distributed random variable with a zero mean and variance δ^2 . Thus, for any period t , it can be written as

$$\begin{aligned} E(r_t) &= \frac{\mu}{1 - \rho}, \\ \text{Var}(r_t) &= \frac{\delta^2}{1 - \rho^2}. \end{aligned} \quad (2)$$

- (5) Assume that the quantity of used products transferred from the collector to the remanufacturer is q_t ,

which can be expressed relatively to the supply quantity of used products from customers to the collector r_t as

$$q_t = r_t + (S_{t-1} - S_t), \quad (3)$$

where S_t is the collector's highest inventory quantity at the end of period t , and S_{t-1} is the collector's highest inventory quantity at the end of period $t - 1$, which is estimated from the observed the supply quantity of used products from customers to the collector as equation (4).

$$S_t = \hat{r}_t^l + z \hat{\delta}_t^l, \quad (4)$$

where \hat{r}_t^l are estimates of the lead-time supply quantity using different forecasting methods, z is a constant to achieve a desired service level, and $\hat{\delta}_t^l$ is the estimate of the standard deviation of the l period forecasting error of the recycler. Thus, the quantity of used products transferred from the collector to the remanufacturer q_t can be calculated relative to the estimates of the lead-time supply quantity \hat{r}_t^l as

$$q_t = (\hat{r}_{t-1}^l - \hat{r}_t^l) + r_t + z (\hat{\delta}_{t-1}^l - \hat{\delta}_t^l). \quad (5)$$

4. The BE in a RSC Using Different Forecasting Methods

Many scholars have discussed the influence of different forecasting methods on the BE in the forward SC. For example, using the ES technique, Holland et al. [37] highlighted that price fluctuations could cause the BE. Zhang [30] analyzed the BE using the MMSE method and compared the results with those obtained by the MA and ES techniques. Liu et al. [38] formulated a mathematical model of the BE in a multistage SC using the MMSE method. Liu et al. [38] also analyzed the BE in a multistage SC using the ES and MMSE techniques. On the contrary, in this subsection, we discuss the impact of different forecasting methods on the BE in a RSC.

Before deriving the quantitative expression of the BE in a RSC, it is necessary to define it first. It is widely accepted that, in a traditional forward SC, the BE is defined as the phenomenon of amplification of demand variability from downstream to upstream (Lee et al. [2]). Similarly, in a RSC, the BE refers to the amplification of supply variability of used products as one moves upstream in the RSC, that is, from customers to the remanufacturer. It is expressed as

$BE_R = Var(q_t)/Var(r_t)$. This means that the amplification in the quantity of used products transferred from the collector to the remanufacturer is greater than that of the supply quantity of used products from customers to the collector. We can then formulate the quantitative expression of the BE in the RSC under different forecasting methods, that is, the MA technique, the SE technique, and the MMSE technique.

4.1. The BE in a RSC Using the MA Method. Based on equation (5), we can determine the mean lead-time supply quantity \hat{r}_t^l using the MA method:

$$\hat{r}_t^l = l \left(\frac{\sum_{i=1}^p r_{t-i}}{p} \right), \quad (6)$$

where p is the moving average period.

Then, the quantity of used products transferred from the collector to the remanufacturer q_t in equation (5) can be formulated as follows:

$$\begin{aligned} q_t &= l \left(\frac{\sum_{i=1}^p r_{t-1-i}}{p} - \frac{\sum_{i=1}^p r_{t-i}}{p} \right) + r_t + z \left(\hat{\delta}_{t-1}^l - \hat{\delta}_t^l \right) \\ &= \left(\frac{l}{p} \right) r_{t-p-1} - \left(\frac{l}{p} \right) r_{t-1} + r_t + z \left(\hat{\delta}_{t-1}^l - \hat{\delta}_t^l \right). \end{aligned} \quad (7)$$

Lemma 1. Using the MA forecasting method, the estimate of the standard deviation of the l period forecast error of collector $\hat{\delta}_t^l$ is a constant and can be expressed as follows:

$$\hat{\delta}_t^l = \sqrt{l\delta^2 + \frac{\delta^2}{(1-\rho)^2} \left[l + \frac{\rho(1-\rho^l)(\rho^{l+1}-\rho-2)}{1-\rho^2} \right]}. \quad (8)$$

Proof: Using the MA forecasting method, the estimate of the standard deviation of the l period forecast error of collector $\hat{\delta}_t^l$ can be expressed as follows:

$$\begin{aligned} Var(\hat{\delta}_t^l) &= Var(\hat{r}_t^l - r_t^l) \\ &= Var \left(\sum_{i=0}^{l-1} (\mu + \rho r_{t+i} + \varepsilon_{t+i}) - \sum_{i=0}^{l-1} (\mu + \rho \hat{r}_{t+i}) \right) \\ &= Var \left(- \sum_{i=0}^{l-1} \sum_{j=0}^N (\rho^{i-j} r_{t+j}) + \sum_{i=0}^{l-1} \varepsilon_{t+i} \right) \\ &= Var \left(\sum_{i=0}^{l-1} \varepsilon_{t+i} \right) + Var \left(\sum_{i=0}^{l-1} \sum_{j=0}^N \rho^{i-j} r_{t+j} \right) \\ &= l\delta^2 + \frac{\delta^2}{(1-\rho)^2} \left[l + \frac{\rho(1-\rho^l)(\rho^{l+1}-\rho-2)}{1-\rho^2} \right]. \end{aligned} \quad (9)$$

where $r_t = \mu + \rho r_{t-1} + \varepsilon_t$.

Then, the quantity of used products transferred from the collector to the remanufacturer q_t in (7) can be formulated as follows:

$$q_t = \left(\frac{l}{p} \right) r_{t-p-1} - \left(\frac{l}{p} \right) r_{t-1} + r_t. \quad (10)$$

Thus, the variance of the quantity of used products q_t in equation (10) can be derived as follows:

$$\begin{aligned} Var(q_t) &= Var \left(\left(\frac{l}{p} \right) r_{t-p-1} - \left(\frac{l}{p} \right) r_{t-1} + r_t \right) \\ &= \left(\frac{l}{p} \right)^2 Var(r_{t-p-1}) + \left(\frac{l}{p} \right)^2 Var(r_{t-1}) + Var(r_t) \\ &\quad - 2 \left(\frac{l}{p} \right)^2 Cov(r_{t-p-1}, r_{t-1}) + 2 \left(\frac{l}{p} \right) Cov(r_{t-p-1}, r_t) \\ &\quad - 2 \left(\frac{l}{p} \right) Cov(r_{t-1}, r_t) \\ &= \left(2 \left(\frac{l}{p} \right)^2 + 1 \right) Var(r_t) + 2 \left(\frac{l}{p} \right) Cov(r_{t-p-1}, r_t) \\ &\quad - 2 \left(\frac{l}{p} \right) Cov(r_{t-1}, r_t) - 2 \left(\frac{l}{p} \right)^2 Cov(r_{t-p-1}, r_{t-1}). \end{aligned} \quad (11)$$

It needs to be pointed out that, in equation (11), it can be proved that

$$\begin{aligned} Cov(r_{t-1}, r_t) &= \rho Var(r_t), \\ Cov(r_{t-1}, r_{t-p-1}) &= \rho^p Var(r_t), \\ Cov(r_t, r_{t-p-1}) &= \rho^{p+1} Var(r_t). \end{aligned} \quad (12)$$

Theorem 1. In two-level reverse SCs, when the collector estimates the supply quantity of used products by using MA forecasting technique, the quantitative expression of the BE is as follows:

$$BE_R^{MA} = \frac{Var(q_t)}{Var(r_t)} = 2\Lambda(1-\rho^p)(\Lambda-\rho) + 1, \quad (13)$$

where $\Lambda = l/p$.

4.2. The BE in a RSC Using the ES Technique. Similarly, based on equation (5), we can determine the mean lead-time supply quantity \hat{r}_t^l using the ES technique:

$$\hat{r}_t^l = l[\alpha r_{t-1} + (1-\alpha)\hat{r}_{t-1}^l], \quad (14)$$

where α ($0 < \alpha < 1$) is a smoothing constant for the collector.

Then, the transfer quantity of the used products q_t in equation (5) can be formulated as follows:

$$q_t = r_t + l\alpha(r_{t-2} - r_{t-1}) + l(1 - \alpha)(\widehat{r}_{t-2} - \widehat{r}_{t-1}) + z(\widehat{\delta}_{t-1}^l - \widehat{\delta}_t^l). \quad (15)$$

Lemma 2. Using the ES forecasting technique, the estimate of the standard deviation of the l period forecast error of collector $\widehat{\delta}_t^l$ is a constant and can be expressed as follows:

$$\widehat{\delta}_t^l = \sqrt{l^2\delta^2 + \frac{\delta^2}{1-\rho^2} + l^2\delta^2\left(\frac{\alpha}{2-\alpha} + \frac{2(1-\alpha)\rho}{(2-\alpha)(1-(1-\alpha)\rho)}\right) - 2\frac{l\alpha\rho(1-\rho^l)}{(1-\rho)(1-(1-\alpha)\rho_1)}\delta^2}. \quad (16)$$

Proof: Using the ES forecasting technique, the estimate of the standard deviation of the l period forecast error of recycler $\widehat{\delta}_t^l$ can be expressed as follows:

$$\widehat{\delta}_t^l = \sqrt{\text{Var}(\widehat{r}_t^l - r_t^l)} = \sqrt{\text{Var}(r_t^l) - 2\text{Cov}(r_t^l, \widehat{r}_t^l) + \text{Var}(\widehat{r}_t^l)}. \quad (17)$$

where $\text{Var}(r_t^l) = \text{Var}(l^2(\mu + \rho r_{t-1} + \varepsilon_t)) = l^2\delta^2 + \delta^2/1 - \rho^2$.

$$\begin{aligned} \text{Var}(\widehat{r}_t^l) &= l^2\text{Var}(\widehat{r}_t) \\ &= l^2\delta^2\left(\frac{\alpha}{2-\alpha}\text{Var}(r_t) + \frac{2(1-\alpha)}{2-\alpha}\text{Cov}(r_{t-1}, \widehat{r}_t)\right) \\ &= l^2\delta^2\left(\frac{\alpha}{2-\alpha} + \frac{2(1-\alpha)\rho}{(2-\alpha)(1-(1-\alpha)\rho)}\right), \end{aligned}$$

$$\begin{aligned} \text{Cov}(r_t^l, \widehat{r}_t^l) &= \text{Cov}\left(\sum_{i=0}^{l-1} (r_{t+i}, l\widehat{r}_t)\right) \\ &= l \sum_{i=0}^{l-1} \text{Cov}(r_{t+i}, \widehat{r}_t) \\ &= l\alpha \sum_{i=0}^{l-1} \sum_{j=1}^{\infty} (1-\alpha)^{j-1} \rho^{i+j} \delta^2 \\ &= \frac{l\alpha\rho(1-\rho^l)}{(1-\rho)(1-(1-\alpha)\rho_1)}\delta^2. \end{aligned} \quad (18)$$

Then, the supply quantity of the waste product q_t in equation (15) can be formulated as follows:

$$q_t = r_t + l\alpha(r_{t-2} - r_{t-1}) + l(1 - \alpha)(\widehat{r}_{t-2} - \widehat{r}_{t-1}). \quad (19)$$

Thus, the variance of the transfer quantity of used products q_t in equation (19) can be derived as follows:

$$\begin{aligned} \text{Var}(q_t) &= \text{Var}(r_t + l\alpha(r_{t-2} - r_{t-1}) + l(1 - \alpha)(\widehat{r}_{t-2} - \widehat{r}_{t-1})) \\ &= \text{Var}(r_t) + l^2\alpha^2\text{Var}(r_t) + l^2\alpha^2\text{Var}(r_t) \\ &\quad + 2l^2(1-\alpha)^2\frac{1+\rho(1-\alpha)}{1-\rho(1-\alpha)}\text{Var}(r_t) \\ &\quad + 2l\alpha\frac{\alpha\rho^2}{1-(1-\alpha)\rho}\text{Var}(r_t) \\ &\quad - 2l\alpha\frac{\alpha\rho}{1-(1-\alpha)\rho}\text{Var}(r_t) \\ &\quad + 2l(1-\alpha)\frac{\alpha\rho^2}{1-(1-\alpha)\rho}\text{Var}(r_t) \\ &\quad - 2l(1-\alpha)\frac{\alpha\rho}{1-(1-\alpha)\rho}\text{Var}(r_t) \\ &\quad - 2l^2\alpha^2\frac{\alpha\rho}{1-(1-\alpha)\rho}\text{Var}(r_t) \\ &\quad + 2l^2\alpha(1-\alpha)\frac{\alpha}{1-(1-\alpha)\rho}\text{Var}(r_t) \\ &\quad - 2l^2\alpha(1-\alpha)\frac{\alpha\rho}{1-(1-\alpha)\rho}\text{Var}(r_t) \\ &\quad - 2l^2\alpha(1-\alpha)\frac{\alpha\rho}{1-(1-\alpha)\rho}\text{Var}(r_t) \\ &\quad + 2l^2\alpha(1-\alpha)\frac{\alpha}{1-(1-\alpha)\rho}\text{Var}(r_t) \\ &\quad - 2l^2(1-\alpha)^2\frac{\alpha\rho}{1-(1-\alpha)\rho}\text{Var}(r_t). \end{aligned} \quad (20)$$

It needs to be pointed out that, in equation (20), it is easy to prove that

$$\begin{aligned}
 \text{Var}(\hat{r}_{t-1}) &= \text{Var}(\hat{r}_{t-2}) \\
 &= \frac{1 + \rho(1 - \alpha)}{1 - \rho(1 - \alpha)} \times \frac{\alpha}{2 - \alpha} \text{Var}(r_t), \\
 \text{Cov}(r_t, \hat{r}_{t-2}) &= \frac{\alpha\rho^2}{1 - (1 - \alpha)\rho} \text{Var}(r_t), \\
 \text{Cov}(r_t, \hat{r}_{t-1}) &= \frac{\alpha\rho}{1 - (1 - \alpha)\rho} \text{Var}(r_t), \\
 \text{Cov}(r_{t-2}, \hat{r}_{t-2}) &= \frac{\alpha}{1 - (1 - \alpha)\rho} \text{Var}(r_t), \\
 \text{Cov}(r_{t-2}, \hat{r}_{t-1}) &= \frac{\alpha\rho}{1 - (1 - \alpha)\rho} \text{Var}(r_t), \\
 \text{Cov}(r_{t-1}, \hat{r}_{t-2}) &= \frac{\alpha\rho}{1 - (1 - \alpha)\rho} \text{Var}(r_t), \\
 \text{Cov}(r_{t-1}, \hat{r}_{t-1}) &= \frac{\alpha}{1 - (1 - \alpha)\rho} \text{Var}(r_t), \\
 \text{Cov}(\hat{r}_{t-2}, \hat{r}_{t-1}) &= \frac{\alpha\rho}{1 - (1 - \alpha)\rho} \text{Var}(r_t).
 \end{aligned} \tag{21}$$

Theorem 2. In two-stage reverse SCs, when the collector estimates the supply quantity of used products by using the ES forecasting technique, the quantitative expression of the BE is as follows:

$$\begin{aligned}
 BE_R^{ES} &= \frac{\text{Var}(q_t)}{\text{Var}(r_t)} \\
 &= 1 + 2l^2\alpha^2 + 2l^2(1 - \alpha)^2 \frac{1 + \rho(1 - \alpha)}{1 - \rho(1 - \alpha)} \\
 &\quad + 2l\alpha\Omega(\rho^2 - \rho) + 2l(1 - \alpha)\Omega\rho^2(\rho^2 - \rho) \\
 &\quad - 2l^2\alpha^2\Omega\rho + 2l^2\alpha(1 - \alpha)\Omega(2 - 3\rho).
 \end{aligned} \tag{22}$$

where $\Omega = \alpha/1 - (1 - \alpha)\rho$.

4.3. The BE in a RSC Using the MMSE Method. Box and Jenkins have pointed out that the demand forecasts value \hat{d}_{t+i} is the conditional expectation of historical demand information for the period $t + i$ ($i = 0, 1, 2, \dots$), i.e., $\hat{d}_{t+i} = E(d_{t+i}|d_{t-1}, d_{t-2}, \dots)$.

Particularly for the AR(1) process, $\hat{d}_{t+i} = E(d_{t+i}|d_{t-1})$. In this paper, the supply quantity of used products from the customer to collector at the end of period t follows the AR(1) process, such that

$$\hat{r}_{t+i} = E(r_{t+i}|r_{t-1}) = \frac{1 - \rho^{i+1}}{\rho} \mu + \rho^{i+1} r_{t-1}. \tag{23}$$

Then, the transfer quantity of used product q_t in equation (5) can be formulated as follows:

$$q_t = \frac{(1 - \rho^{l-1})\mu}{1 - \rho} r_{t-1} - \frac{(1 - \rho^l)\mu}{1 - \rho} r_t + r_t + z(\hat{\delta}_{t-1}^l - \hat{\delta}_t^l). \tag{24}$$

Lemma 3. Using the MMSE forecasting technique, the estimate of the standard deviation of the l period forecast error of collector $\hat{\delta}_t^l$ can be expressed as follows:

$$\hat{\delta}_t^l = \sqrt{l\sigma^2 + \left(l + \frac{\rho(1 - \rho^l)(\rho^{l+1} - \rho - 2)}{1 - \rho^2} \right) \delta_1^2}. \tag{25}$$

Proof: Using the MMSE forecasting method, the estimate of the standard deviation of the l period forecast error of collector $\hat{\delta}_t^l$ can be expressed as follows:

$$\begin{aligned}
 \hat{\delta}_t^l &= \sqrt{\text{Var}(\hat{r}_t^l - r_t)} = \sqrt{\text{Var}\left(\sum_{i=0}^{l-1} \left(\sum_{j=0}^N \rho^{i-j} \varepsilon_{t+j} + \varepsilon_{t+i}\right)\right)} \\
 &= \sqrt{\text{Var}\left(\sum_{i=0}^{l-1} \varepsilon_{t+i}\right) + \text{Var}\left(\sum_{i=0}^{l-1} \sum_{j=0}^N \rho^{i-j} \varepsilon_{t+j}\right)} \\
 &= \sqrt{l\sigma^2 + \text{Var}\left(\sum_{i=0}^{l-1} \varepsilon_{t+i} \sum_{j=0}^{l-1-i} \rho^j\right)} \\
 &= \sqrt{l\sigma^2 + \left(l + \frac{\rho(1 - \rho^l)(\rho^{l+1} - \rho - 2)}{1 - \rho^2} \right) \delta_1^2}.
 \end{aligned} \tag{26}$$

where $r_t = \mu + \rho r_{t-1} + \varepsilon_t$.

Then, the transfer quantity of used products q_t in equation (24) can be formulated as follows:

$$q_t = \frac{(1 - \rho^{l-1})\mu}{1 - \rho} r_{t-1} - \frac{(1 - \rho^l)\mu}{1 - \rho} r_t + r_t. \tag{27}$$

Thus, the variance of the transfer quantity of used products q_t in equation (19) can be derived as follows:

$$\begin{aligned}
 \text{Var}(q_t) &= \text{Var}\left(\frac{(1 - \rho^{l-1})\mu}{1 - \rho} r_{t-1} - \frac{(1 - \rho^l)\mu}{1 - \rho} r_t + r_t\right) \\
 &= \left(\frac{(1 - \rho^{l-1})\mu}{1 - \rho}\right)^2 \text{Var}(r_{t-1}) + \left(\frac{(1 - \rho^l)\mu}{1 - \rho}\right)^2 \text{Var}(r_t) \\
 &\quad + \text{Var}(r_t) - 2\left(\frac{(1 - \rho^l)\mu}{1 - \rho}\right) \text{Var}(r_t) \\
 &\quad - 2(1 - \rho^{l-1})(1 - \rho^l) \left(\frac{\mu}{1 - \rho}\right) \text{Cov}(r_{t-1}, r_t) \\
 &\quad + 2\left(\frac{(1 - \rho^{l-1})\mu}{1 - \rho}\right) \text{Cov}(r_{t-1}, r_t).
 \end{aligned} \tag{28}$$

TABLE 1: The quantitative expression of the BE in a RSC.

| Forecasting techniques | Quantitative expression of the BE | The relevant condition |
|------------------------|--|--|
| MA | $BE_R^{MA} = 2\Lambda(1 - \rho^l)(\Lambda - \rho) + 1$ | $\Lambda = l/p$ |
| ES | $BE_R^{ES} = 1 + 2l^2\alpha^2 + 2l^2(1 - \alpha)^2 + \rho(1 - \alpha)/1 - \rho(1 - \alpha) + 2l\alpha\Omega(\rho^2 - \rho) + 2l(1 - \alpha)\Omega\rho^2(\rho^2 - \rho) - 2l^2\alpha^2\Omega\rho + 2l^2\alpha(1 - \alpha)\Omega(2 - 3\rho)$ | $\Omega = \alpha/1 - (1 - \alpha)\rho$ |
| MMSE | $BE_R^{MMSE} = (\Theta(1 - \rho^{l-1}))^2 + (\Theta(1 - \rho^l))^2 + 1 - 2(\Theta(1 - \rho^l)) - 2\rho\Theta(1 - \rho^{l-1})(1 - \rho^l) + 2\rho\Theta(1 - \rho^{l-1})$ | $\Theta = \mu/1 - \rho$ |

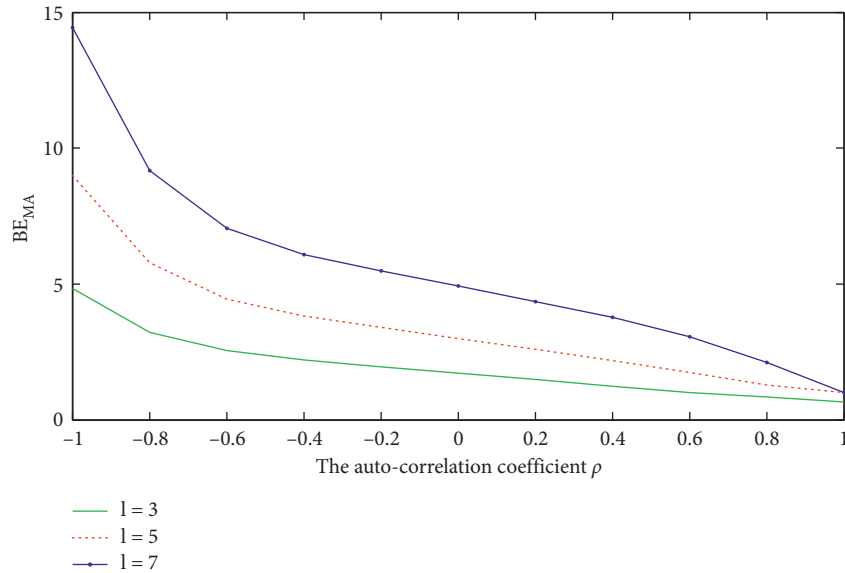


FIGURE 2: The influence of ρ on the BE using the MA method for different l .

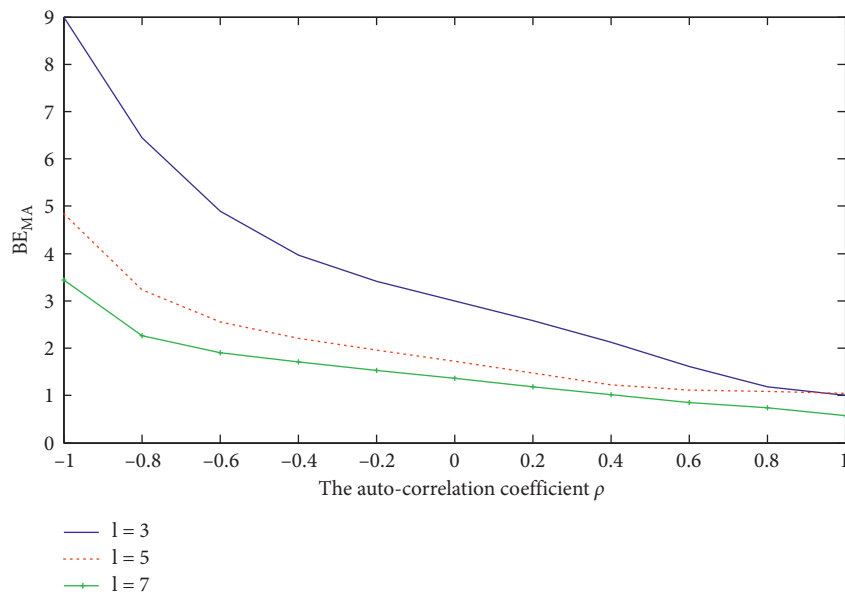


FIGURE 3: The influence of ρ on the BE using the MA method for different p .

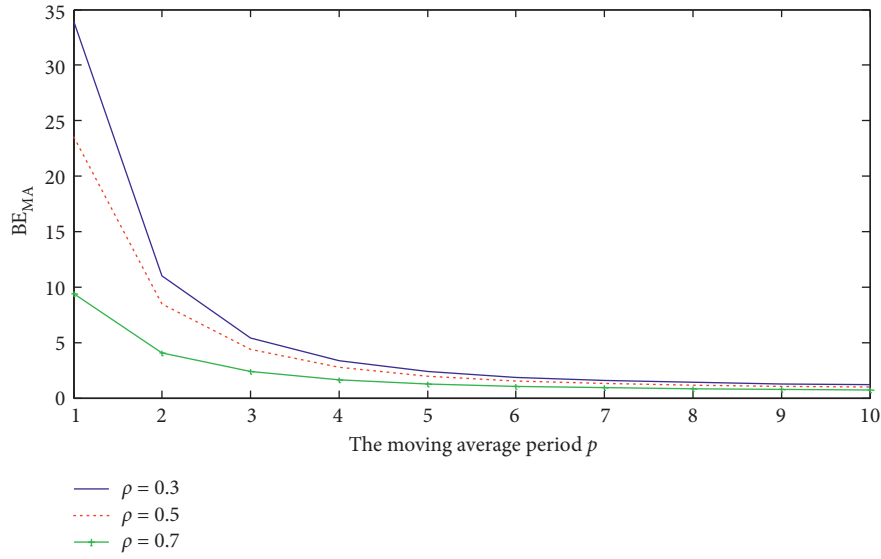


FIGURE 4: The influence of p on the BE using the MA forecasting method for different ρ .

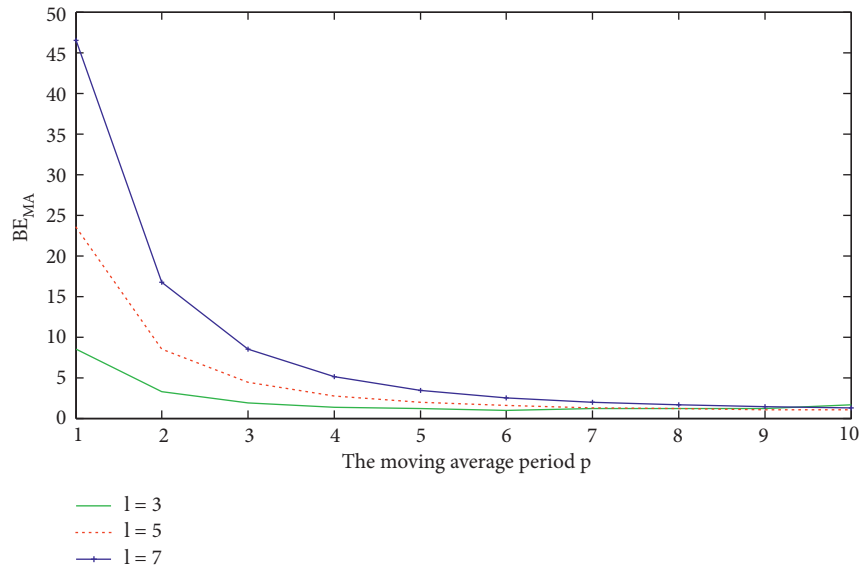


FIGURE 5: The influence of p on the BE using the MA forecasting method for different l .

It needs to be pointed out that, in equation (28), it is easy to prove that

$$Cov(r_{t-1}, r_t) = Cov(\mu + \rho r_{t-1} + \varepsilon_t, r_{t-1}) = \rho Var(r_{t-1}). \tag{29}$$

Theorem 3. In two-stage reverse SCs, when the collector estimates the supply quantity of used product by using MMSE forecasting method, the quantitative expression of the BE is as follows:

$$BE_R^{MMSE} = \frac{Var(q_t)}{Var(r_t)} = (\Theta(1 - \rho^{l-1}))^2 + (\Theta(1 - \rho^l))^2 + 1 - 2(\Theta(1 - \rho^l)) - 2\rho\Theta(1 - \rho^{l-1})(1 - \rho^l) + 2\rho\Theta(1 - \rho^{l-1}). \tag{30}$$

where $\Theta = \mu / (1 - \rho)$.

The above results are summarized in Table 1.

5. Numerical Examples and Results

As can be seen from the quantitative expression of the BE in a RSC using three types of forecasting methods, the main factors affecting the BE include the autocorrelation coefficient, the moving average period, the collector's lead-time, and the smoothing constant for the collector. In order to analyze the influence of different factors on the BE, the simulation analysis is divided into two steps. First, we analyze the influence of the relevant factors on the BE when using different forecasting methods. We then compare the influence of different forecasting methods on the BE.

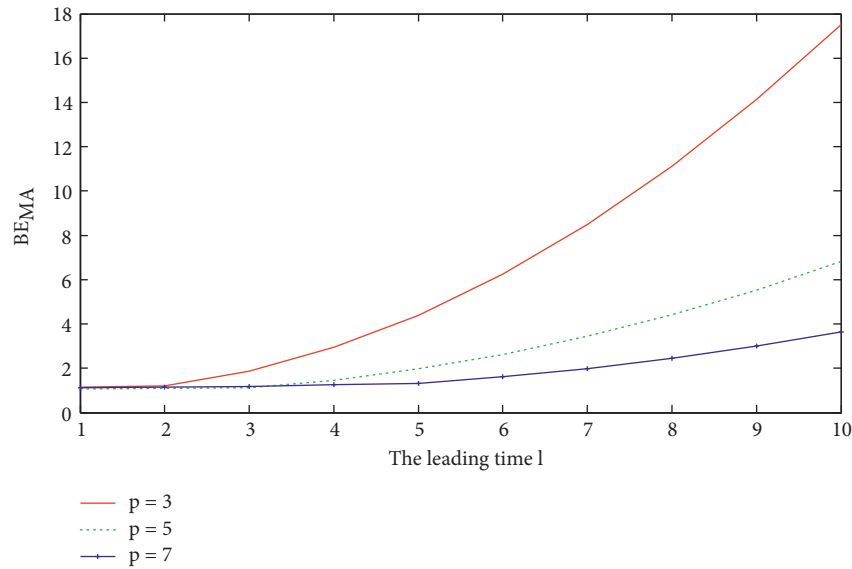


FIGURE 6: The influence of l on the BE using the MA forecasting method for different p .

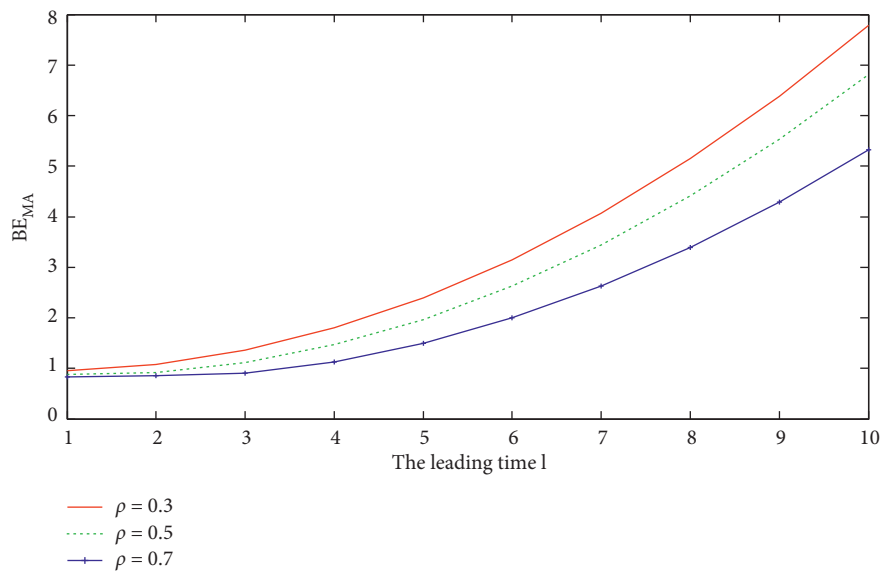


FIGURE 7: The influence of l on the BE using the MA forecasting method for different ρ .

5.1. The Influence of Relevant Factors on the BE Using the MA Forecasting Method. Figures 2–7 depict the influence of relevant factors on the BE using the MA forecasting method. Figure 2 illustrates that the BE in an RSC decreases quickly as the autocorrelation coefficient ρ varies from -1 to -0.4 . But when ρ is greater than -0.4 , the BE shows a steady state. Additionally, the BE increases with the increase of leading time l . As shown in Figure 3, we can see that the influence of ρ on the BE when the moving average period p takes different values, which is similar to that when l takes different values. However, as p increases, the BE will decrease. That is, it is possible to reduce the BE in an RSC by increasing the moving average

period, which is consistent with the result in a forward SC (Lee et al. [2]).

As can be seen in Figure 4, the BE in an RSC decreases as p varies from 1 to 4. But when p is greater than 4, the BE shows a steady state. Moreover, the BE decreases with the increase of ρ . As shown in Figure 5, the BE in a RSC decreases quickly when p is less than 4. After that, the BE tends to be stable, which is also consistent with the result in a forward SC (see Lee et al. [2]).

As shown in Figures 6 and 7, when l is less than 4, the BE is relatively stable. But when l is greater than 4, the BE increases quickly. In other words, l has a positive effect on the BE in an RSC.

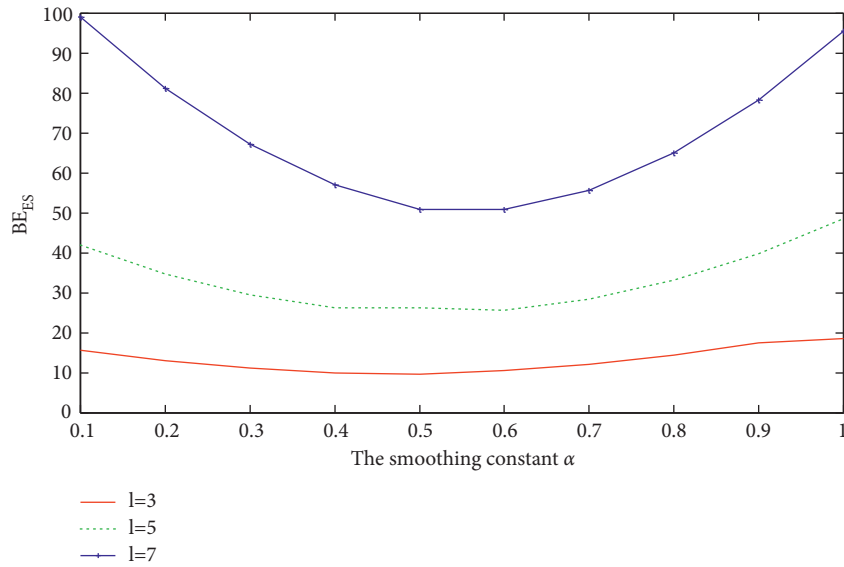


FIGURE 8: The influence of α on the BE using the ES forecasting technique for different l .

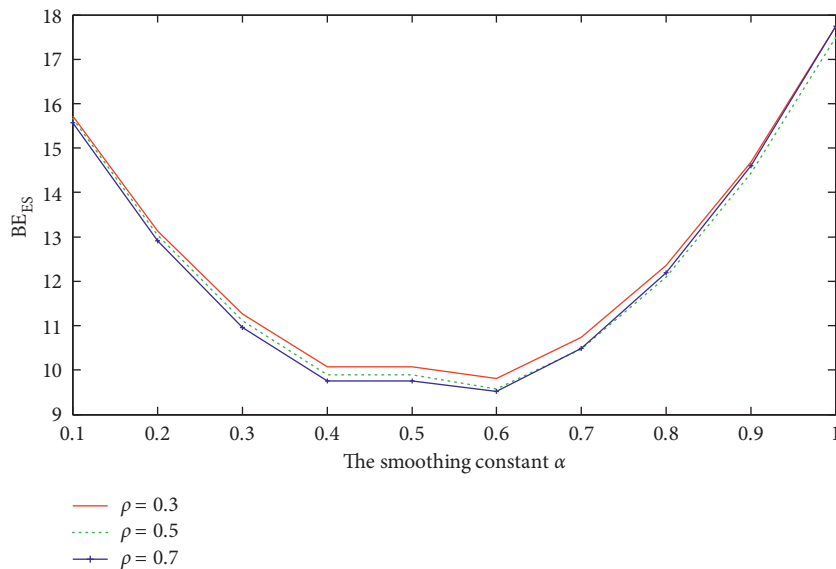


FIGURE 9: The influence of α on the BE using the ES forecasting method for different ρ .

5.2. *The Influence of Relevant Factors on Bullwhip Effect Using the ES Forecasting Method.* Figures 8–13 illustrate the influence of relevant factors on the BE using the ES method. Figures 8 and 9 show that the BE in an RSC decreases as α varies from 0.1 to 0.4. When the α value varies from 0.4 to 0.6, the BE is relatively stable. However, when the α value is greater than 0.6, the BE increases quickly. In contrast, l has a positive effect on the BE, while a smaller BE is associated with a greater ρ . It is worth noting that the smoothing constant α and the lead time l have similar effects on the BE in a forward SC (Alwan et al. [5]).

Figures 10 and 11 also illustrate that ρ has a negative effect on the BE in a RSC, while Figures 12 and 13 also

illustrate that the lead time l has a positive effect on the BE. We can see that when using the ES method, the lead time l and the autocorrelation coefficient ρ also have opposite effects on the BE.

5.3. *The Influence of Relevant Factors on the BE Using the MMSE Method.* Figures 14 and 15 depict the influence of relevant factors on the BE using the MMSE method. As shown in Figure 14, when ρ is relatively small, the BE decreases sharply as l varies from 0 to 1, but it increases quickly as l is greater than 1. However, when ρ is large, the BE first increases slowly as l varies from 0 to 3, and after that, it increases rapidly.

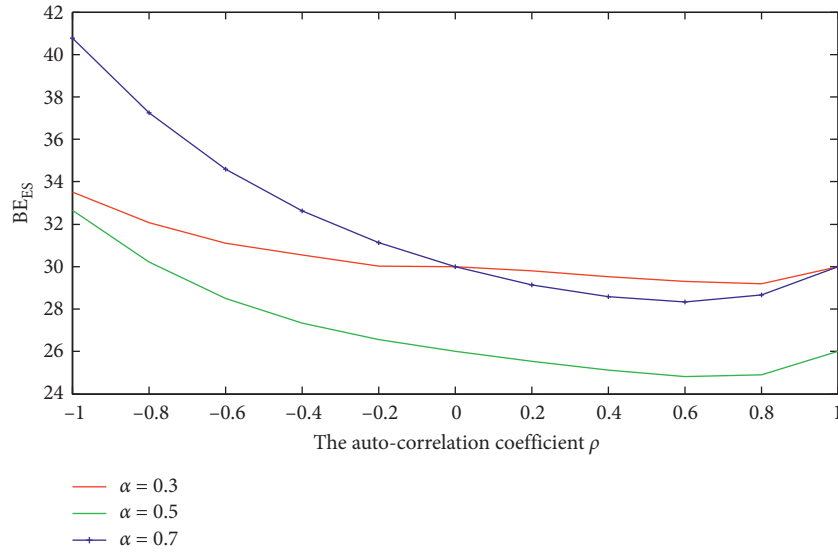


FIGURE 10: The influence of ρ on the be using the ES forecasting method for different α .

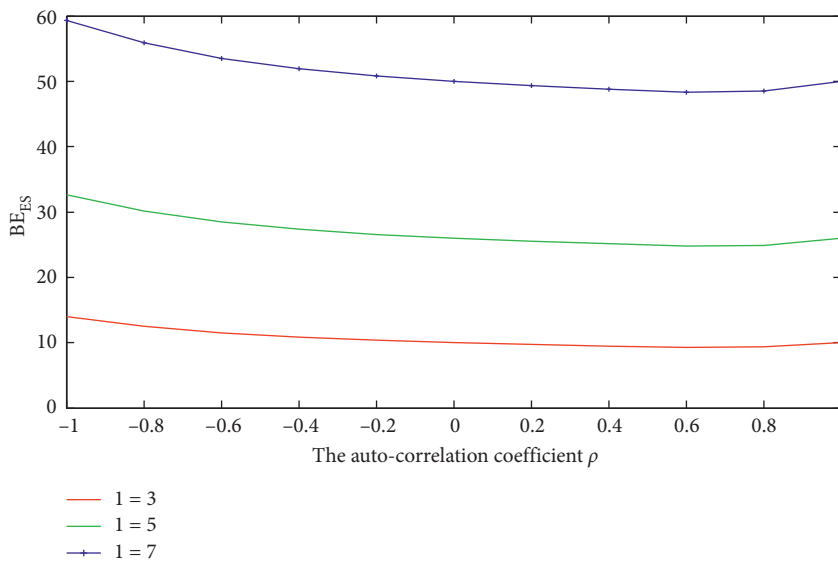


FIGURE 11: The influence of ρ on the BE using the ES forecasting method for different l .

This is different from the results obtained using the previous two forecasting methods. As can be seen in Figure 15, the BE in a RSC is relatively stable when the ρ value varies from -1 to 0.2. After that, the BE increases quickly. This differs from the results obtained using the previous two forecasting methods. Furthermore, in some cases, when ρ varies from -1 to 0.2, the BE does not exist in a RSC, which is significantly different from the results on the BE in a forward SC.

5.4. Comparing the Influence of Different Forecasting Methods on the Bullwhip Effect. From the above analysis, it is evident that the autocorrelation coefficient ρ and the lead time l have an important influence on the BE under three kinds of forecasting methods. Next, we will compare the influence of

different forecasting techniques on the BE. Assume that $\alpha = 2/(p + 1)$, $p = 4$, $\alpha = 0.4$. Figure 16 illustrates the impact of ρ and l on the BE in a RSC under three kinds of forecasting techniques.

In Figure 16, we can see that the BE in a RSC reaches the largest level under the ES method, which is comparable to that in a forward SC. In addition, when the autocorrelation coefficient is less than 0.6, the BE is at the lowest level under the MMSE method; when the autocorrelation coefficient is greater than 0.6, the BE is at the lowest level.

Theorem 4. When the lead time l and the autocorrelation coefficient ρ satisfy the condition that $(\Theta(1 - \rho^{l-1}))^2 + (\Theta(1 - \rho^l))^2 + 1 - 2(\Theta(1 - \rho^l)) + 2\rho\Theta(1 - \rho^{l-1}) \leq p/l + p^2/l$, the collector should select the MMSE

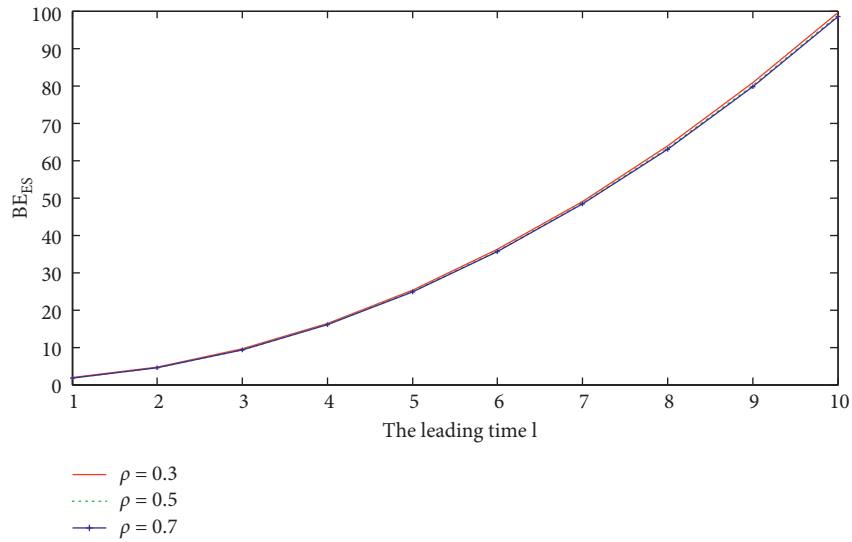


FIGURE 12: The influence of l on the BE using the ES forecasting method for different ρ .

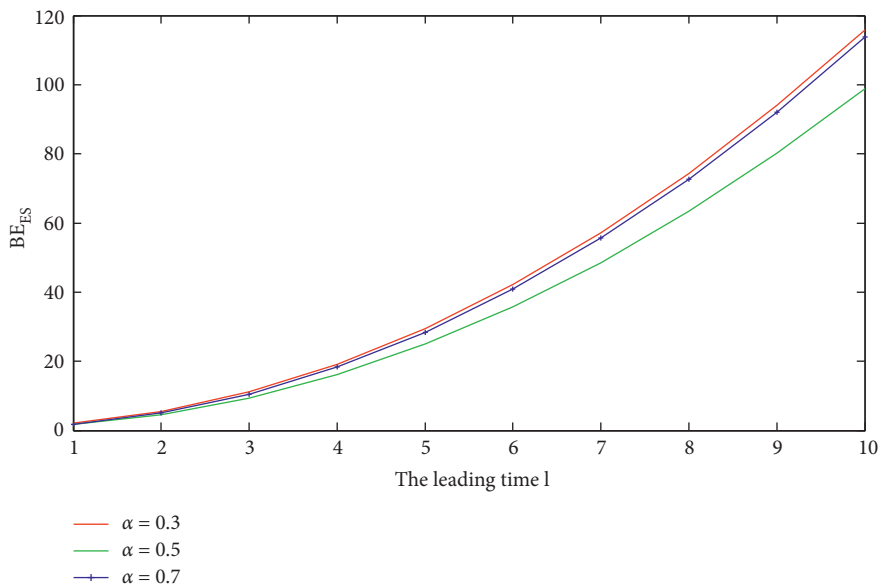


FIGURE 13: The influence of l on the BE using the ES forecasting method for different α .

method to reduce the BE; otherwise, we can select the MA method.

To sum up, we can obtain the following managerial insights:

- (i) Managerial insight 1: when the collector predicts the supply of used products from customers using the MA method, both the autocorrelation coefficient ρ and the moving average period p have a negative effect on the BE, while the lead time l has a positive effect on the BE. Thus, the collector could reduce the BE by increasing the autocorrelation coefficient and

the moving average period, or by reducing the lead time.

- (ii) Managerial insight 2: when the collector predicts the supply of used products from customers using the ES method, the BE is at the lowest level when the smoothing constant α varies from 0.4 to 0.6. Thus, the collector should choose an appropriate smoothing constant to reduce the BE.

Managerial insight 3: when the autocorrelation coefficient and the lead time satisfy Theorem 4, the collector should use the MMSE method to predict the supply of used products from customers so as to

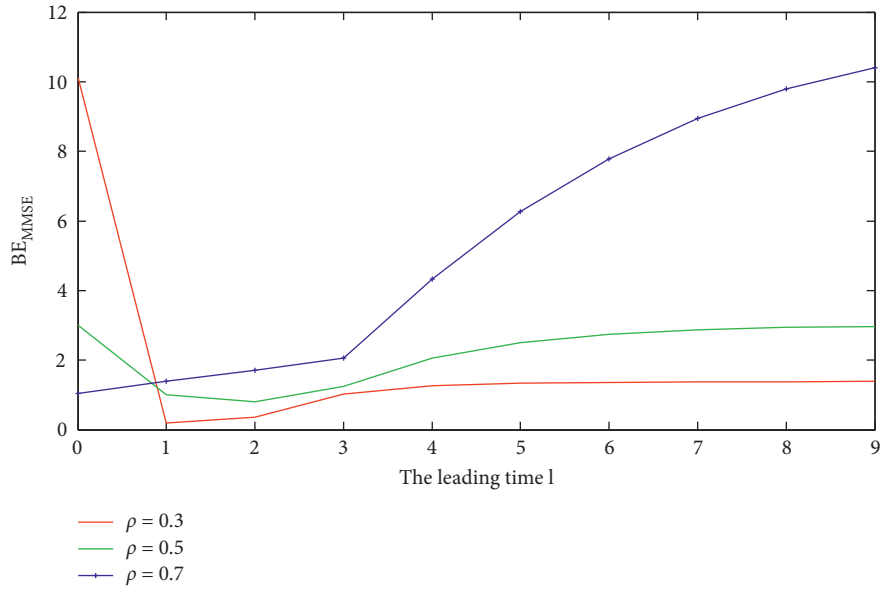


FIGURE 14: The influence of l on the BE using the MMSE forecasting method for different ρ .

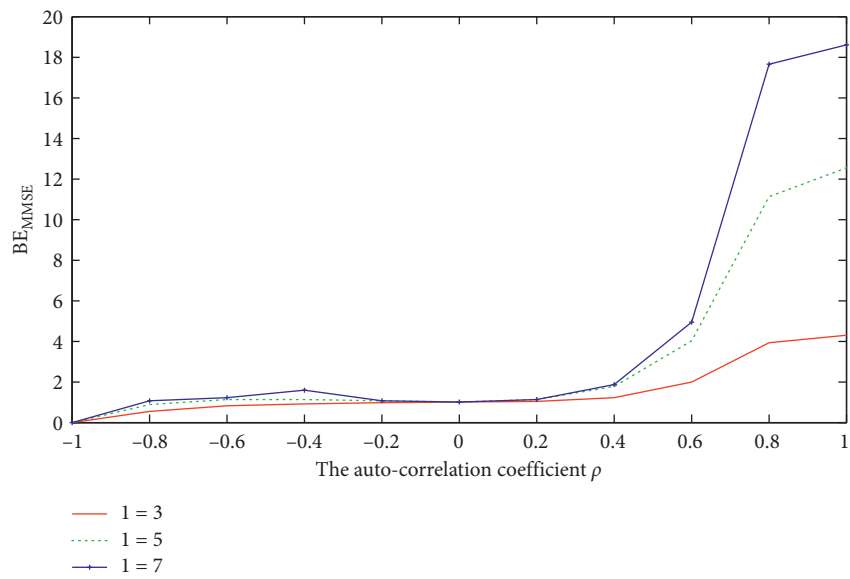


FIGURE 15: The influence of ρ on the BE using the ES forecasting method for different l .

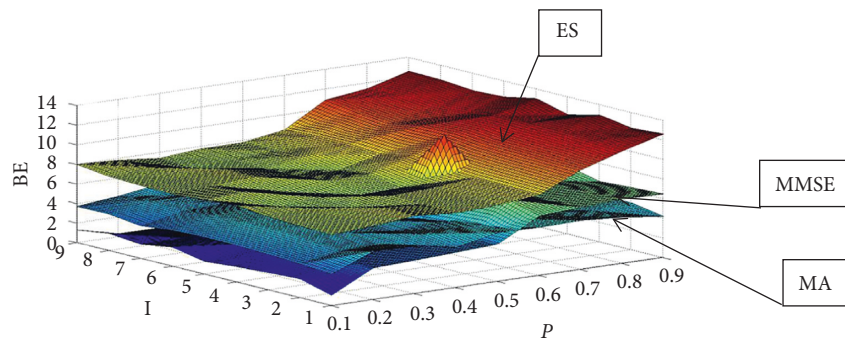


FIGURE 16: The influence of different forecasting methods on the BE in a RSC.

reduce the BE. Otherwise, the collector should use the MA method.

6. Conclusion

In this paper, we examine the influence of different forecasting methods on the BE in a two-stage RSC that consists of one collector and one remanufacturer. First, we develop a quantitative expression of the BE in a RSC. We then analyze the influence of the autocorrelation coefficient, the lead-time, the moving average period, and other factors on the BE in a RSC. Finally, we analyze the conditions under which using different forecasting methods can reduce the BE. We reached the following conclusions:

- (1) When the collector predicts the supply of used products from customers using the MA method, both the autocorrelation coefficient ρ and the moving average period p have a negative effect on the BE, while the lead-time l has a positive effect on the BE.
- (2) In some cases, when ρ varies from -1 to 0.2 and using the MMSE method, the BE does not exist in a RSC, which is significantly different from the result on the BE in a forward SC.
- (3) To reduce the BE in a RSC, when the autocorrelation coefficient and the lead-time satisfy Theorem 4, the collector should use the MMSE method to predict the supply of used products from customers. Otherwise, the collector should use the MA method.

In this paper, we only study the impact of different forecasting methods on the BE in a two-stage RSC. In the future, we could discuss the BE in two competitive RSCs, each of which consists of one collector and one remanufacturer.

Data Availability

The data used to support the finds of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Y. X. G. and Z. X. Q. conceptualized the study and wrote the original study. W. M. and Z. D. L. reviewed and edited the article.

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