



**AALBORG UNIVERSITY**  
DENMARK

**Aalborg Universitet**

## **All-dielectric one-dimensional gratings exhibiting Fano resonances in the terahertz region**

Westerkam, Anders M.; Sonne, Jesper L.W.; Danielsen, Karl G.; Skovsen, Esben;  
Søndergaard, Thomas M.

*Published in:*  
Journal of the Optical Society of America B: Optical Physics

*DOI (link to publication from Publisher):*  
[10.1364/JOSAB.462062](https://doi.org/10.1364/JOSAB.462062)

*Publication date:*  
2022

*Document Version*  
Accepted author manuscript, peer reviewed version

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Westerkam, A. M., Sonne, J. L. W., Danielsen, K. G., Skovsen, E., & Søndergaard, T. M. (2022). All-dielectric one-dimensional gratings exhibiting Fano resonances in the terahertz region. *Journal of the Optical Society of America B: Optical Physics*, 39(7), 1723-1729. <https://doi.org/10.1364/JOSAB.462062>

### **General rights**

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal -

### **Take down policy**

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.

# All-dielectric one-dimensional gratings exhibiting Fano resonances in the terahertz region

ANDERS M. WESTERKAM,<sup>1</sup> JESPER L. W. SONNE,<sup>1</sup> KARL G. DANIELSEN,<sup>1</sup> ESSEN SKOVSEN,<sup>1</sup> AND THOMAS M. SØNDERGAARD<sup>1,\*</sup>

<sup>1</sup>Department of Materials and Production, Aalborg University, DK-9220 Aalborg Øst, Denmark  
\*ts@mp.aau.dk

**Abstract:** Transmittance spectra of an all-dielectric sub-wavelength grating exhibiting Fano resonances are studied both experimentally and theoretically for a frequency interval spanning several terahertz using both S- and P-polarized radiation. Modelled spectra based on the Rigorous Coupled Wave Analysis are shown to be in good agreement with the Fourier transform of measured time-domain transmission spectra for normal and angled incidence. Finally, a model is presented that explains Fano resonance frequencies by matching the propagation constant of a discrete set of guided modes to wave-numbers that both match diffraction orders in the grating and are in a range that allows propagation in the dielectric medium.

© 2023 Optica Publishing Group

## 1. Introduction

The application of terahertz (THz) radiation has shown promising results in numerous fields, e.g., spectroscopy and imaging which can be utilised in the public safety and defense sector [1,2], as non-destructive quality control [3–6], medical diagnostics [3, 7, 8], and research in biochemistry [8, 9]. Furthermore, it is proposed that the THz region will be fundamental for the next generation of telecommunication since following the current trend of the technology it is expected that the 6G-bands will extend into the THz region [10–12].

Many of the components commonly used to manipulate optical and microwave radiation are not readily available in the THz range [13], thus components such as optical filters, switches, and reflectors [14, 15] enabling the manipulation of THz waves are of interest [12]. These components would, depending on the application, benefit from having high quality-factors (Q-factors), making them very frequency selective [14, 15].

A phenomenon which can be exploited to achieve high Q-factors is Fano resonances [14–19]. They are well established in the literature [15–22], first described by Fano as a means to explain the asymmetric spectra from helium atoms subjected to inelastic scattering from electrons [23]. Fano resonances occur when a discrete and continuous set of states are available at the same time resulting in an asymmetric spectral line-shape, known as a Fano resonance. These resonances may exhibit complete destructive interference that can be exploited to achieve zero transmission at certain frequencies [15, 23, 24].

In the literature Fano resonances have been found for a multitude of different structures, or metamaterials (MMs) [21]. The complexity in the designs of the MMs ranges from one-dimensional gratings and structures such as split-ring resonators, to more intricate three-dimensional structures [12, 14, 16, 17, 19, 20, 22]. Furthermore, the Fano resonances of the MMs have in different studies been shown to be highly tunable, hence enabling the possibility to tailor the MMs depending on the task [14, 17, 19–21, 25]. Sub-wavelength structures exploiting Fano resonances has been used for devices such as high-Q-factor optical filters [26, 27] and single-material high-reflectivity mirrors [28].

We present here an all-dielectric structure consisting of a sub-wavelength one-dimensional grating etched in a thin high-resistivity silicon wafer. The wafer was obtained from *SILICON MATERIALS, INC.* with a thickness of  $100 \pm 20 \mu\text{m}$ , and a resistivity of  $\rho > 5000 \Omega\cdot\text{cm}$ . Similar

46 structures have been reported earlier in [14,25], and for other frequency ranges [29–31]. However,  
47 we present exceptional agreement between experimentally and theoretically obtained data over a  
48 broad frequency range in the THz region. The agreement is shown to hold for both normal and  
49 angled incidence, and for both polarisations. The transmittance spectrum of the structure was  
50 calculated using the rigorous coupled wave analysis, also known as the Fourier Modal Method  
51 (FMM) [32].

52 Furthermore, a novel model for predicting the existence and center frequencies of Fano  
53 resonances in structures similar to the structure presented here is proposed and validated  
54 numerically. The advantages of this model are that it gives an intuitive understanding of the  
55 underlying physical mechanisms, and that the prediction is done at a low computational cost.

## 56 2. Method

57 The fabricated structure and a schematic of the design are seen in Fig. 1(a). The structure is  
58 fabricated using standard photolithography procedures and reactive ion etching (RIE) [33–36],  
59 and has the following dimensions: Strip height,  $t = 12.9 \mu\text{m}$ , period,  $\Lambda = 99.7 \mu\text{m}$ , strip width  
60  $w_t = 32.5 \mu\text{m}$  and wafer thickness,  $h = 105 \mu\text{m}$  (Fig. 1). The RIE procedure produces sidewalls  
61 with slopes  $\Phi = 55^\circ$ , with respect to the base of the strip. In the FMM the refractive index of  
62 silicon is set to  $\varepsilon = 3.4^2$ , and the structure is divided into layers in which the permittivity profile  
63 is layerwise constant in the  $z$ -direction. Thus the sloping sidewalls can in the FMM either be  
64 accounted for by performing an area preserving averaging resulting in  $w_{t,avg} = 41.5 \mu\text{m}$ , or by  
65 adding layers to the FMM model and gradually widening the strip width at the top,  $w_t = 32.5$   
66  $\mu\text{m}$ , towards the base width,  $w_{t,base} = 50.5 \mu\text{m}$  (Fig. 1(b)) [32]. In Fig. 1(c) the FMM model is  
67 plotted for S-polarised light using the averaged strip width, and for strips discretised with three to  
68 seven steps. It is clear that there is no significant difference between the approaches, why the  
69 averaged strip method is used for further spectra.

70 The time-domain spectrometer (TDS), a *TeraSmart, Compact THz-TDS* from *Menlo Systems*  
71 using a scan range of 420 picoseconds, giving a spectral resolution of  $> 2.4 \text{ GHz}$ , is used to  
72 measure time-domain transmittance spectra. Detailed descriptions of the optical and nitrogen  
73 purge setup can be found in Ref. [37]. The sample is placed at the collimated part of the beam  
74 using an alumina sample holder to block any part of the beam not passing through a  $45^\circ$  tapered  
75 circular hole with a minimum diameter of 13 mm in the holder where the sample is placed (Fig.  
76 2). The area of the grated region is  $\sim 280 \text{ mm}^2$ , while the area of the tapered hole is  $\sim 133 \text{ mm}^2$ ,  
77 and consequently the beam does not hit the sample outside of the micro-structured region. The  
78 holder can have angles up to  $10^\circ$  without letting any radiation bypass the tapered hole. Fourier  
79 transforms are performed in accordance with the method outlined in Ref. [38].

## 80 3. Measurements

81 Fig. 3 shows the measured and modelled transmittance spectra for both polarisations, which are  
82 seen to be in good agreement, e.g., the first three measured resonances for both polarisations  
83 are found within 9 GHz or less than what is predicted by the model. Furthermore, all measured  
84 resonances are also present in the modelled spectra, though there are a few very narrow resonances  
85 predicted by the model which are hardly or not visible at all in the measurement, e.g., at 1.82  
86 THz for P-polarisation ( $\mathbf{H}$  along  $\hat{\mathbf{y}}$ ) and at 1.94 THz for S-polarisation ( $\mathbf{E}$  along  $\hat{\mathbf{y}}$ ). This may in  
87 part be due to the finite resolution of the spectrometer. Additionally, the FMM predicts that Fano  
88 resonances shift on the order of hundreds of MHz when parameters such as  $\Lambda$  and  $w_t$  change by  
89 tenths of micrometers, which is approximately the fabrication uncertainty for these parameters.  
90 Thus the very narrow resonances could simply be smeared out due to structure variations.

91 Fano resonances for S-polarised light at 1.40 and 1.73 THz, have a width at 50%-transmittance  
92 of 43 and 74 GHz, respectively. This corresponds to Q-factors of 32.55 and 23.38, hence both

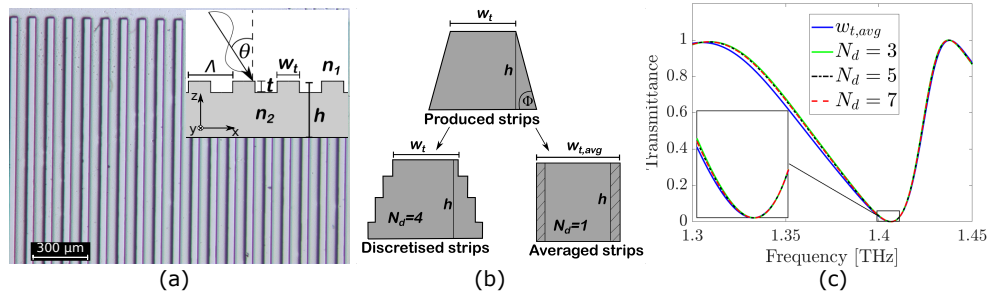


Fig. 1. (a) Light-microscopy image of the fabricated sample and an insert of the modelled structure. Structure parameters: Strip height,  $t = 12.9 \mu\text{m}$ , period,  $\Lambda = 99.7 \mu\text{m}$ , strip width  $w_t = 32.5 \mu\text{m}$  and wafer thickness,  $h = 105 \mu\text{m}$  (b) Cross-section sketch of the produced, discretised and averaged strips. (c) FMM model using the averaged and discretised strips with 3 to 7 discretisations.

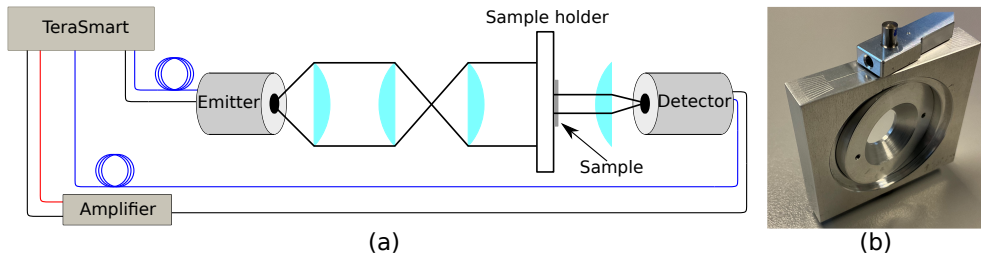


Fig. 2. (a) Schematic of the experimental setup. (b) Image of the front side of the sample holder, in which the taper can be seen.

93 resonances exceed the Q-factor measured for a similar structure shown in Ref. [25]. The upper  
 94 frequency limit, considered in Fig. 3, is determined by the spectrometer signal-to-noise ratio.

95 To further test the correspondence between model and measurements, the angle of incidence  
 96 is varied between  $0^\circ$  and  $10^\circ$  in steps of  $1^\circ$ , the result of which is presented in Fig. 4. The  
 97 correspondence between model and measurement are for most resonances striking, both with  
 98 regards to frequency position and broadness. Only the sharpest resonances in the model are hard  
 99 to see in the measurements, e.g., the one seen at  $\sim 1.8 \text{ THz}$  for the P-polarisation, which is barely  
 100 visible between  $0^\circ$  and  $2^\circ$  but then fades.

101 It can be seen that many resonances at  $\theta = 0^\circ$  splits into two resonances, which move in  
 102 opposite directions as the angle of incidence is increased. Using the clearest five resonances for  
 103 each polarisation, the average change of the Fano resonances between  $\theta = 0^\circ$  and  $10^\circ$  is tabulated  
 104 in Table 1, showing that on average, the change in center frequency of a resonance predicted  
 105 by the model is within 8 GHz of the measured change, which is the same uncertainty observed  
 106 between model and measurements at normal incidence. The table also shows that for the sample  
 107 presented here, the frequency with the lowest transmittance in the resonance can be tuned on  
 108 the order of tens of GHz simply by changing the angle of incidence. The angle which has to be  
 109 changed is shown in Fig. 1(a), corresponding to a rotation around the y-axis. Rotating around  
 110 the x-axis does not appreciably change the spectra, this fact is also reported in [25]. Thus the  
 111 structure can be used as a spectral specific mirror for a large set of incidence angles.

112 The discrepancy between the change seen in the measurements and the FMM model for all  
 113 angles of incidence can be attributed to two factors. The first is that the spectral resolution of the  
 114 model is much higher than the measurements, making it easier to determine the center frequency



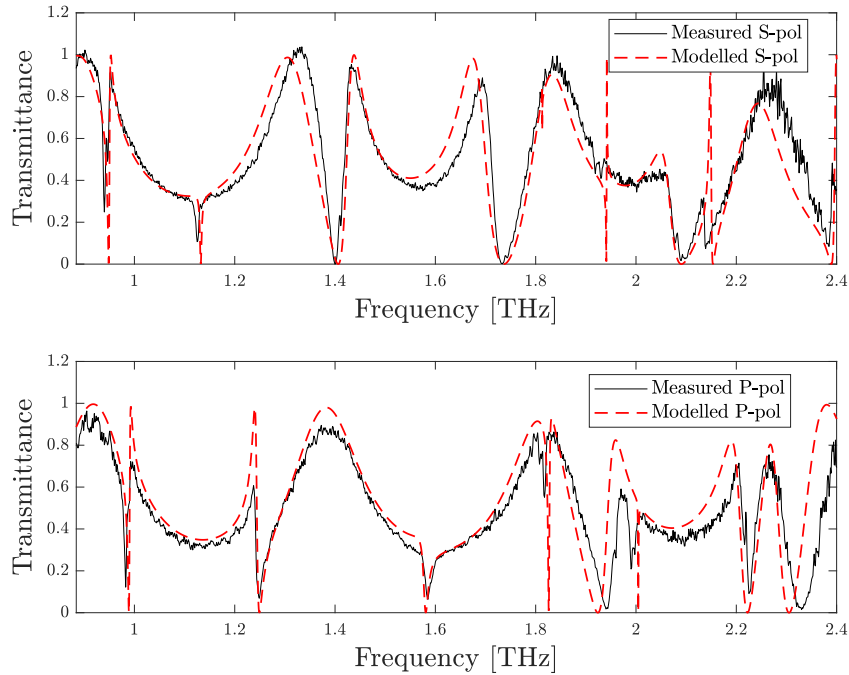


Fig. 3. Fourier transformed time-domain measurements and the transmittance found using the FMM model. The FMM settings were  $t = 12.89\mu\text{m}$ ,  $\Lambda = 99.74\mu\text{m}$ ,  $w_t = 41.5\mu\text{m}$ ,  $h = 105\mu\text{m}$ , and spectral resolution of 0.42 GHz

115 of the resonance, and secondly because the angle of incidence on the measured spectra has an  
 116 uncertainty of  $\pm 0.25^\circ$ .

	Increase [GHz]	Decrease [GHz]
$\Delta\nu_{S,measured}$	63.3	47.3
$\Delta\nu_{S,modelled}$	65.2	55.2
$\Delta\nu_{P,measured}$	55.5	47.5
$\Delta\nu_{P,modelled}$	60.9	51.4

Table 1. Lists the average increase and decrease of the center frequency of five Fano resonances found using the FMM model and the measured spectra when the angle of incidence is changed from  $\theta = 0^\circ$  to  $\theta = 10^\circ$ .

#### 117 4. One-dimensional propagation constant matching model

118 The rigorous FMM model clearly predicts the center frequency of the Fano resonances with high  
 119 accuracy. However, it is a 2D model which is somewhat computationally expensive, and as such  
 120 it does not lend itself well to the design of structures with Fano resonances at select frequencies.  
 121 Furthermore, the FMM does not provide an intuition as to what is causing the Fano resonances,  
 122 an insight that may further improve the ability to exploit the phenomenon in optical devices.

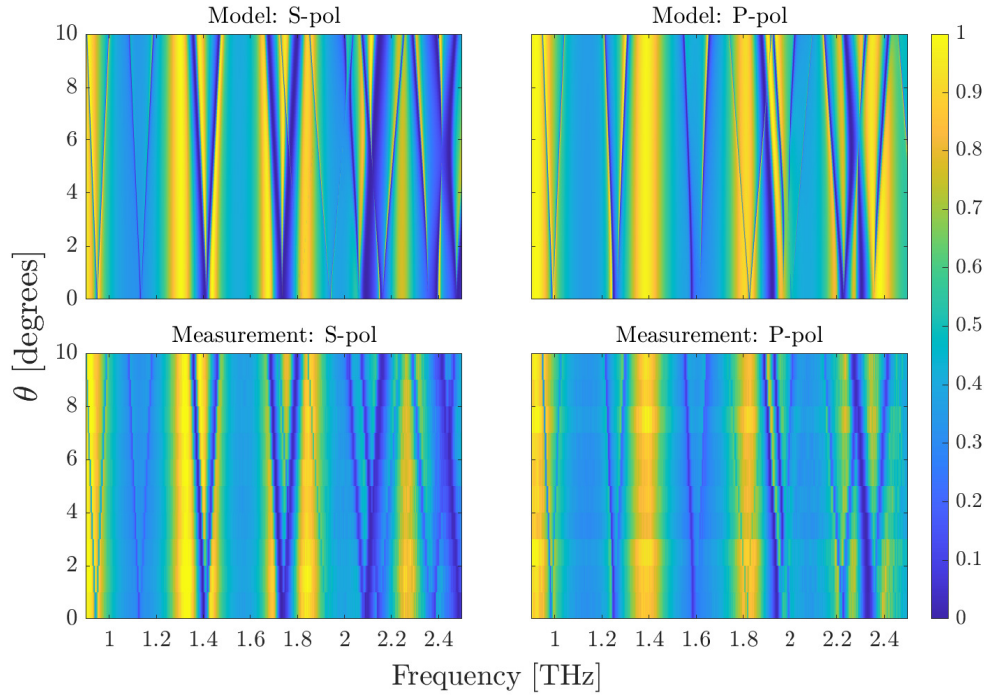


Fig. 4. The Fourier transformed time-domain measurements and the transmittance found using the FMM model for both polarisations with angles of incidence between zero and ten degrees.

123 Here, we present a novel one-dimensional propagation constant matching model (PMM), which  
 124 solves both issues.

125 Assume an incident field on the form of Eq. 1,

$$\mathbf{A} = A_0 e^{-ik_z z} e^{ik_x x} \hat{\mathbf{y}}, \quad (1)$$

126 with  $\mathbf{A}$  denoting either an electric or magnetic field based on the polarisation of interest. The  
 127 orientation of the field in relation to the structure can be identified from Fig. 5. According  
 128 to the Helmholtz equation for regions of homogeneous dielectric constant,  $\varepsilon_i$ , the wave-vector  
 129 component perpendicular to the surfaces is given as

$$k_z = \sqrt{k_0^2 \varepsilon_i - k_x^2}. \quad (2)$$

130 If the structured layer is then viewed as a one-dimensional photonic crystal the resulting in-plane  
 131 components of the wave vector has to satisfy Bloch's theorem as [39]

$$k_{x,m} = k_x + \frac{2\pi}{\Lambda} m, \quad m = 0, \pm 1, \pm 2, \pm 3, \dots \quad (3)$$

132 When the in-plane component of a resulting wave vector matches the propagation constant of a  
 133 quasi-bound mode, it is possible to excite the mode in question. This is the core of the PMM. In  
 134 other words there are both a discrete and a continuous set for the light to couple to, and hence  
 135 Fano resonances are possible. In this context the discrete set is the bound modes accessed by the  
 136 diffracted light and the continuous set is the otherwise reflected and transmitted light.

137 In the following discussion we limit ourselves to the case  $k_x = 0$ , though the method does, in  
 138 theory, work for any angle of incidence. By inspecting Eq. 2 and Eq. 3 limits for the existence  
 139 of Fano resonances can be extracted. When  $k_0 n_{\text{Si}} < \frac{2\pi}{\Lambda}$  propagating solutions with a non-zero  
 140  $\mathbf{k}$  projection along  $\hat{\mathbf{x}}$  does not exist in any regions and hence coupling to the discrete set of  
 141 bound modes is impossible. This presents a hard boundary for when Fano resonances can occur,  
 142 and they are indeed not observed for frequencies below this limit in the case of the structure  
 143 presented in this article. Furthermore, a soft boundary can be introduced by requiring that no  
 144 propagating diffraction orders exist in the air regions. The frequency region of interest would  
 145 then be  $\frac{c}{\Lambda n_{\text{Si}}} < \nu < \frac{c}{\Lambda}$ , corresponding to  $0.88 < \nu < 3.01$  THz for the structure presented in Fig.  
 146 1(a).

147 To calculate the propagation constants of quasi-bound modes a one-dimensional finite element  
 148 method (FEM) for a slab waveguide is employed. In this model bound solutions on the form

$$\mathbf{f}(x, z) = f(z)e^{i\beta_n x} \hat{\mathbf{y}}, \quad (4)$$

149 are sought after. In Eq. 4  $f(z)$  denotes the field distribution along  $z$  and  $\beta_n$  is the propagation  
 150 constant along  $\hat{\mathbf{x}}$  with the subscript  $n$  denoting the mode number. It is this propagation constant  
 151 which is matched with the diffraction orders to predict the center frequency of the Fano resonances.  
 152 For normal incidence a Fano resonance is then expected whenever Eq. 5 is satisfied:

$$\beta_n = \frac{2\pi}{\Lambda} m. \quad (5)$$

153 In the FEM model the dielectric constant of the photonic crystal layer is approximated by an  
 effective dielectric constant calculated as the geometric average within a unit cell. Using the

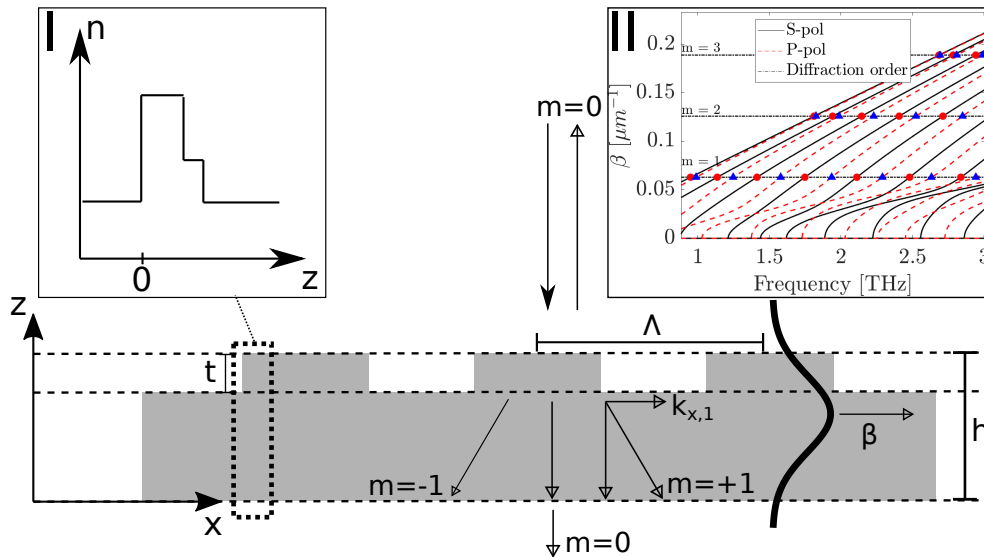


Fig. 5. Sketch of the structure and the diffraction orders for reflected and transmitted light. Insert I: Sketch of the effective-refractive-index in the one-dimensional slab waveguide. Insert II: Matching of the propagation constant with the diffraction orders. Circles: S-polarisation. Triangles: P-polarisation.

154 geometric average for the dielectric constant does naturally neglect some effects, namely that  
 155 high-frequency radiation tends to accumulate in the high-refractive-index regions and hence the  
 156 effective refractive index is expected to increase for higher frequencies [39]. Furthermore, the  
 157

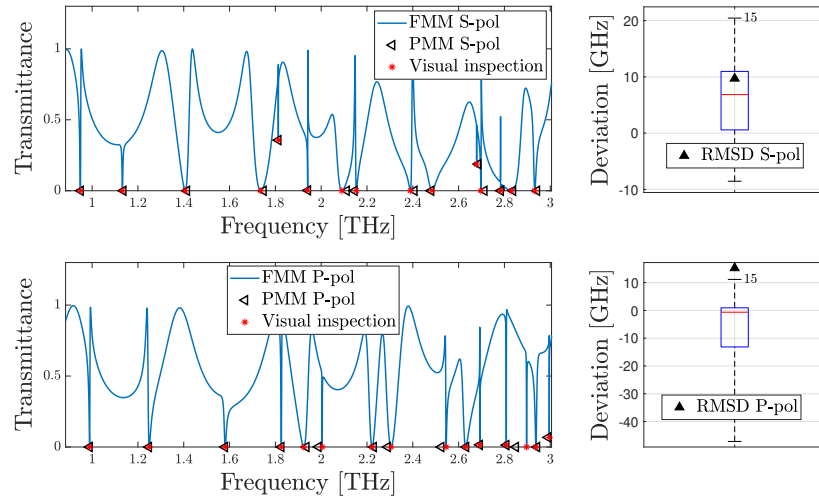


Fig. 6. Comparison of the PMM predictions and visual inspection of the FMM spectra for the fabricated structure studied in this article. The FMM settings were  $t = 12.89\mu\text{m}$ ,  $\Lambda = 99.74\mu\text{m}$ ,  $w_t = 41.5\mu\text{m}$ ,  $h = 105\mu\text{m}$ , and the calculation had a spectral resolution of 0.1 GHz. The graphs to the left show the spectra calculated with FMM, overlaid with the PMM predictions for the center frequencies. The graphs to the right are box plots of the deviation between the PMM predictions and the visual inspection. Tilted triangles: PMM predictions of Fano center frequencies. Asterisks: Fano center frequencies found from visual inspection of the spectra. Black triangles: Root mean square deviation.

158 region is a photonic crystal in the  $x$ -direction, and band gaps are therefore expected for select  
 159 frequency regions. The effect of both these considerations were tested by calculating a refractive  
 160 index based on the method presented in Ref. [40]. Using this method the effect of propagation at  
 161 an angle with respect to the  $x$ -axis in the layer was also tested. However, it was found that it had  
 162 little effect, hence to preserve the computational speed the geometric average was chosen instead.

163 To test the validity of the PMM it is compared to the results obtained using the FMM model.  
 164 To do this, the effective refractive index is first calculated for the photonic crystal layer of the  
 165 structure. Based on this refractive index, the propagation constants are calculated for a slab  
 166 waveguide of equal thickness,  $h$ , to that of the structure, for both S- and P-polarised light, after  
 167 which the propagation constants,  $\beta_n$ , of any mode can be matched to any  $k_{x,m}$  in accordance  
 168 with Eq. 5. The intersections of these correspond to frequencies where a discrete and continuous  
 169 set of modes are accessible, for which Fano resonances are expected. The procedure is illustrated  
 170 in the second insert of Fig. 5, where the circles and triangles denote frequencies for which Fano  
 171 resonances are expected for S- and P-polarised light, respectively. Fig. 6 shows the result of  
 172 using the PMM and the FMM on a structure with the same parameters as the structure fabricated  
 173 in this article. The location of the resonances predicted with PMM are denoted by triangles  
 174 plotted at the same height as the resonances, asterisks, found from visual inspection of the FMM.  
 175 Furthermore to condense and quantify the data, boxplots has also been made over the differences  
 176 in the placement of the resonances between the PMM and FMM, calculated from Eq. 6,

$$\Delta\nu = \nu_{PMM} - \nu_{FMM}. \quad (6)$$

177 In a box plot, a box of arbitrary width contains the second and third quartile of the data set,  
 178 separated by the median value, marked with a horizontal line. Each quartile contains a quarter

179 of the data points. At the bottom and top of the box a dashed line goes to the minimum  
180 and maximum of the data set, marked with horizontal lines, containing the first and fourth  
181 quartile, respectively. The number of Fano resonances are shown over each box plot, and the  
182 root-mean-square-difference (RMSD) between the two models are also calculated, marked with a  
183 triangle. Generally it can be seen that there is great agreement between the two models, though  
184 slightly better for the S-polarisation. Half of the resonances found with PMM are within  $\sim 13$   
185 GHz of those found using FMM, while the entire data set is within 55 GHz. The RMSD are  
186 9.7 and 15.4 GHz for S- and P- polarisation respectively. Besides the advantage of reduced  
187 computational speed the PMM also find the resonance location without having to sort out other  
188 local minima that may appear in the spectrum. Furthermore, the frequency resolution of the  
189 PMM need not be as strict as the FMM to find the very narrow Fano resonances, such as the one  
190 located around 1.8 THz for the S-polarisation in Fig. 6. Hence the PMM presents advantages  
191 over the rigorous FMM while still showing great agreement.

## 192 5. Comparison of FMM and PMM

193 To further confirm the validity of the model a more quantitative analysis of the PMM is conducted.  
194 In this comparison the correspondence between the PMM and the FMM was analysed by  
195 examining the 36 different possible settings from combinations of the following:  $\varepsilon = \{3.4^2\}$ ,  
196  $\Lambda = \{100, 125\} \mu\text{m}$ ,  $w_t = \{21, 54\} \mu\text{m}$ ,  $t = \{9, 15, 22\} \mu\text{m}$ , and  $h = \{101, 149, 201\} \mu\text{m}$ . The  
197 FMM spectra are calculated with a spectral resolution of 0.1 GHz, and the Fano resonances  
198 are then found by visual inspection of the calculated spectra. The deviation between each peak  
199 calculated by the PMM and FMM is found from Eq. 6. For each setting a box plot was made over  
200 the calculated deviations, and the result is presented in Fig. 7. The average of the RMSD values  
201 are 20.14 GHz for the S-polarisation and 13.72 GHz for P-polarisation. With the largest RMSD  
202 value being 62.42 and 31.37 for S- and P-polarisation, respectively. It can also be seen in Fig. 7  
203 that the extremes for S-polarised radiation is  $-67.4$  GHz (setting 21) and 161.7 GHz (setting 12),  
204 where for P-polarised radiation the extremes are  $-73.8$  GHz (setting 30) and 50.1 GHz (setting  
205 34). Furthermore, for the settings examined here, the average absolute median deviation for the  
206 S-polarisation is 4.38 GHz and 2.40 GHz for the P-polarisation. If these values are compared  
207 with the box plot shown in Fig. 6 it can be seen that the general agreement shown in this figure  
208 is not an isolated incidence. In fact the plot shown in Fig. 6 represents the standard prediction  
209 accuracy of the PMM as the RMSD for this setting is close to the middle of the range of RMSD  
210 calculated for the 36 settings, as can be seen in Fig. 7. Hence, it is expected that the PMM will  
211 show just as good agreement for a wide range of settings as was the case for the fabricated  
212 sample shown in Fig. 6, despite the numerous approximations.

213 To determine the location of the Fano resonances a visual inspection was conducted of the FMM  
214 calculated transmission spectra. The FMM calculations were only done within the frequency  
215 range described in section 4, while the PMM was calculated 20% past the upper limit, to ensure  
216 resonances found with FMM right at the upper limit could always be matched with a prediction  
217 made by the PMM. Hence if multiple peaks were found in this extended region only the closest  
218 one were retained.

219 It is worth noting that besides the 1387 Fano Resonances found there are possible two additional  
220 resonances. For both cases the extra resonance either predicted by the PMM or found from  
221 the visual inspection of the FMM, was excluded. Common for both, and the reasoning for the  
222 exclusion, being that the modelled transmittance spectra were difficult to interpret around the  
223 discrepancies. Hence the number of peaks is predicted with very high accuracy with the PMM.

224 Fig. 7 also shows the great agreement between the PMM and the rigorous FMM as the  
225 observed trend is that for the majority of the settings the median is found rather close to zero  
226 deviation in the entire range of data points. Generally it is also observed that the spread of the two  
227 first quartiles is small compared to the spread of the whole data set. Hence the average RMSD

228 is also much lower than the extremes of the data set which shows that the general accuracy is  
 229 high. The precision of the PMM is furthermore well within the prediction accuracy of other  
 230 reported models, namely the 3D FEM model presented in Ref. [25], which shows a precision of  
 231 prediction in the same range, i.e., within 59 and 98 GHz for S- and P-polarisation respectively.  
 232 Given the correspondence between the model and the visual inspection presented above and in  
 233 Fig. 7, we consider this model to be useful and reliable, when determining the placement of  
 234 Fano resonances. The usefulness really comes in fruition as it is able to find very sharp peaks,  
 235 which would otherwise require a high spectral resolution in the calculation as well as the reduced  
 236 computational time.

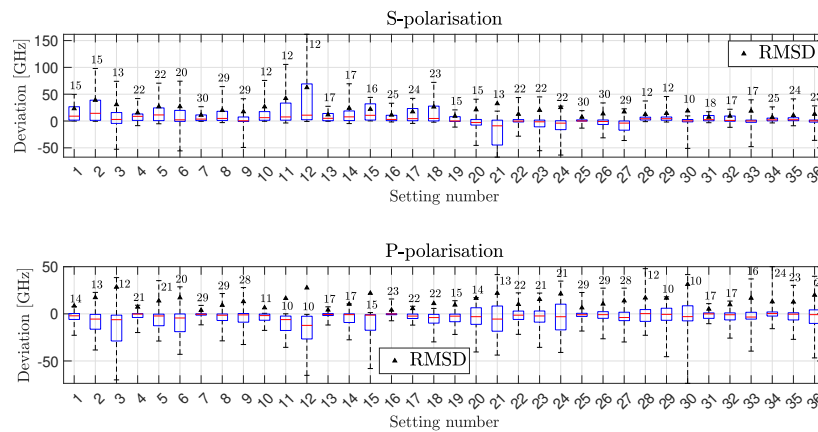


Fig. 7. Box plot of the deviation between the frequency locations of the Fano resonances for both polarisations. The average absolute median deviation for the S-polarisation is 4.38 GHz, and 2.40 GHz for the P-polarisation. The average RMSD is 20.14 and 13.72 GHz for the S- and P-polarisation, respectively.

## 237 6. Conclusion

238 We have studied transmission of THz radiation (0.8 – 2.4 THz) through an all-dielectric  
 239 periodic sub-wavelength high-resistivity silicon grating and shown excellent agreement,  $\pm 8$   
 240 GHz, between measured and modelled Fano resonances for angles of incidence between  $0^\circ$  and  
 241  $10^\circ$ . Furthermore, a novel 1D propagation-constant-matching model was shown to predict the  
 242 frequencies of resonances in excellent agreement with the Fourier Modal Method, having a root  
 243 mean square deviation within 25 GHz for a wide range of modelled structures.

244 **Acknowledgement.** A special thanks to Mathias Hedegaard Kristensen who supplied a MATLAB program  
 245 capable of performing the Fourier transforms.

246 **Disclosures.** The authors declare no conflicts of interest.

247 **Data availability.** Data underlying the results presented in this paper are available on request to the  
 248 corresponding author.

## 249 References

- 250 1. A. G. Davies, A. D. Burnett, W. Fan, E. H. Linfield, and J. E. Cunningham, "Terahertz spectroscopy of explosives  
 251 and drugs," *Mater. Today* **11**, 18–26 (2008).
- 252 2. J. F. Federici, B. Schulkin, F. Huang, D. Gary, R. Barat, F. Oliveira, and D. Zimdars, "THz imaging and sensing for  
 253 security applications—explosives, weapons and drugs," *Semicond. Sci. Technol.* **20**, S266–S280 (2005).

- 254 3. S. S. Dhillon, M. S. Vitiello, E. H. Linfield, A. G. Davies, M. C. Hoffmann, J. Booske, C. Paoloni, M. Gensch,  
255 P. Weightman, G. P. Williams, E. Castro-Camus, D. R. S. Cumming, F. Simoens, I. Escorcia-Carranza, J. Grant,  
256 S. Lucyszyn, M. Kuwata-Gonokami, K. Konishi, M. Koch, C. A. Schmuttenmaer, T. L. Cocker, R. Huber, A. G.  
257 Markelz, Z. D. Taylor, V. P. Wallace, J. A. Zeitler, J. Sibik, T. M. Korter, B. Ellison, S. Rea, P. Goldsmith, K. B.  
258 Cooper, R. Appleby, D. Pardo, P. G. Huggard, V. Krozer, H. Shams, M. Fice, C. Renaud, A. Seeds, A. Stöhr,  
259 M. Naftaly, N. Ridler, R. Clarke, J. E. Cunningham, and M. B. Johnston, "The 2017 terahertz science and technology  
260 roadmap," *J. Phys. D: Appl. Phys.* **50**, 043001 (2017).
- 261 4. C. Jansen, S. Wietzke, O. Peters, M. Scheller, N. Vieweg, M. Salhi, N. Krumbholz, C. Jördens, T. Hochrein, and  
262 M. Koch, "Terahertz imaging: applications and perspectives," *Appl. Opt.* **49**, E48–E57 (2010).
- 263 5. M. Tonouchi, "Cutting-edge terahertz technology," *Nat. Photonics* **1**, 97–105 (2007).
- 264 6. S. Wietzke, C. Jansen, N. Krumbholz, O. Peters, N. Vieweg, C. Jördens, M. Scheller, D. Romeike, T. Jung, M. Reuter,  
265 S. Chatterjee, M. Koch, F. Physik, B. Baudrit, T. Zentgraf, T. Hochrein, and M. Bastian, "Terahertz spectroscopy: A  
266 powerful tool for the characterization of plastic materials," in *2010 10th IEEE International Conference on Solid  
267 Dielectrics*, (2010), pp. 1–4.
- 268 7. G. G. Hernandez-Cardoso, L. F. Amador-Medina, G. Gutierrez-Torres, E. S. Reyes-Reyes, C. A. Benavides Martínez,  
269 C. Cardona Espinoza, J. Arce Cruz, I. Salas-Gutierrez, B. O. Murillo-Ortiz, and E. Castro-Camus, "Terahertz imaging  
270 demonstrates its diagnostic potential and reveals a relationship between cutaneous dehydration and neuropathy for  
271 diabetic foot syndrome patients," *Sci. Reports* **12**, 3110 (2022).
- 272 8. I. V. Il'ina, D. S. Sitnikov, and M. B. Agranat, "State-of-the-art of studies of the effect of terahertz radiation on living  
273 biological systems," *High Temp.* **56**, 789–810 (2018).
- 274 9. B. Ferguson and X.-C. Zhang, "Materials for terahertz science and technology," *Nat. Mater.* **1**, 26–33 (2002).
- 275 10. H. Viswanathan and P. E. Mogensen, "Communications in the 6g era," *IEEE Access* **8**, 57063–57074 (2020).
- 276 11. S. Dang, O. Amin, B. Shihada, and M.-S. Alouini, "What should 6g be?" *Nat. Electron.* **3**, 20–29 (2020).
- 277 12. C. Xu, Z. Ren, J. Wei, and C. Lee, "Reconfigurable terahertz metamaterials: From fundamental principles to advanced  
278 6g applications," *iScience* **25**, 103799 (2022).
- 279 13. A. Kaur, J. C. Myers, M. I. M. Ghazali, J. Byford, and P. Chahal, "Affordable terahertz components using 3d printing,"  
280 in *2015 IEEE 65th Electronic Components and Technology Conference (ECTC)*, (2015), pp. 2071–2076.
- 281 14. X. Cui, H. Tian, Y. Du, G. Shi, and Z. Zhou, "Normal incidence filters using symmetry-protected modes in dielectric  
282 subwavelength gratings," *Sci. Reports* **6**, 36066 (2016).
- 283 15. M. F. Limonov, M. V. Rybin, A. N. Poddubny, and Y. S. Kivshar, "Fano resonances in photonics," *Nat. Photonics* **11**,  
284 543–554 (2017).
- 285 16. Q. Xie, G.-X. Dong, B.-X. Wang, and W.-Q. Huang, "High-q fano resonance in terahertz frequency based on an  
286 asymmetric metamaterial resonator," *Nanoscale Res. Lett.* **13**, 294 (2018).
- 287 17. C. Wu, N. Arju, G. Kelp, J. A. Fan, J. Dominguez, E. Gonzales, E. Tutuc, I. Brener, and G. Shvets, "Spectrally  
288 selective chiral silicon metasurfaces based on infrared fano resonances," *Nat. Commun.* **5**, 3892 (2014).
- 289 18. R. Singh, I. A. I. Al-Naib, M. Koch, and W. Zhang, "Sharp fano resonances in thz metamaterials," *Opt. Express* **19**,  
290 6312–6319 (2011).
- 291 19. Y. Wang, S. Jia, and J. Qin, "Tunable fano resonance and enhanced sensing in terahertz metamaterial," *Front. Phys.* **8**  
292 (2021).
- 293 20. X. Romain, R. Degl'Innocenti, F. I. Baida, and P. Boyer, "Tunable polarization-induced fano resonances in stacked  
294 wire-grid metasurfaces," *Commun. Phys.* **4**, 115 (2021).
- 295 21. Y. P. Cao, Y. Y. Wang, Z. X. Geng, J. Liu, Y. P. Yang, and H. D. Chen, "Tuning of fano resonances in terahertz  
296 metamaterials," *J. Appl. Phys.* **117**, 063107 (2015).
- 297 22. Z. Liu, Z. Liu, J. Li, W. Li, J. Li, C. Gu, and Z.-Y. Li, "3d conductive coupling for efficient generation of prominent  
298 fano resonances in metamaterials," *Sci. Reports* **6**, 27817 (2016).
- 299 23. U. Fano, "Effects of configuration interaction on intensities and phase shifts," *Phys. Rev.* **124**, 1866–1878 (1961).
- 300 24. A. A. Darki, A. Parthenopoulos, J. V. Nygaard, and A. Dantan, "Profilometry and stress analysis of suspended  
301 nanostructured thin films," *J. Appl. Phys.* **129**, 065302 (2021).
- 302 25. Y. Yang, B. Cui, Z. Geng, and S. Feng, "Terahertz magnetic and electric mie resonances of an all-dielectric  
303 one-dimensional grating," *Appl. Phys. Lett.* **106**, 111106 (2015).
- 304 26. Y. Shuai, D. Zhao, Z. Tian, J.-H. Seo, D. V. Plant, Z. Ma, S. Fan, and W. Zhou, "Double-layer fano resonance  
305 photonic crystal filters," *Opt. Express* **21**, 24582–24589 (2013).
- 306 27. Y. Wang, D. Stellinga, A. B. Klemm, C. P. Reardon, and T. F. Krauss, "Tunable optical filters based on silicon nitride  
307 high contrast gratings," *IEEE J. Sel. Top. Quantum Electron.* **21**, 108–113 (2015).
- 308 28. F. Brückner, D. Friedrich, T. Clausnitzer, M. Britzger, O. Burmeister, K. Danzmann, E.-B. Kley, A. Tünnermann, and  
309 R. Schnabel, "Realization of a monolithic high-reflectivity cavity mirror from a single silicon crystal," *Phys. Rev.  
310 Lett.* **104**, 163903 (2010).
- 311 29. X. Gao, T. Wu, Y. Xu, X. Li, D. Bai, G. Zhu, H. Zhu, and Y. Wang, "Angular-dependent polarization-insensitive filter  
312 fashioned with zero-contrast grating," *Opt. Express* **23**, 15235–15241 (2015).
- 313 30. M. Niraula, J. W. Yoon, and R. Magnusson, "Concurrent spatial and spectral filtering by resonant nanogratings," *Opt.  
314 Express* **23**, 23428–23435 (2015).
- 315 31. A. Parthenopoulos, A. A. Darki, B. R. Jeppesen, and A. Dantan, "Optical spatial differentiation with suspended  
316 subwavelength gratings," *Opt. Express* **29**, 6481–6494 (2021).

- 317 32. N. Gregersen, "Fourier Modal Method," in *Numerical Methods in Photonics*, (CRC Press, 2017), Optical Sciences  
318 and Applications of Light, chap. 6, pp. 139–194, 1st ed.
- 319 33. B. Bhushan, *Springer Handbook of Nanotechnology*, Springer Handbook of Nanotechnology (Springer Berlin  
320 Heidelberg, 2010).
- 321 34. M. Shearn, X. Sun, M. D. Henry, A. Yariv, and A. Scherer, "Advanced plasma processing: Etching, deposition, and  
322 wafer bonding techniques for semiconductor applications," in *Semiconductor Technologies*, J. Grym, ed. (IntechOpen,  
323 Rijeka, 2010), chap. 5.
- 324 35. M. Tilli, M. Paulasto-Krockel, M. Petzold, H. Theuss, T. Motooka, and V. Lindroos, *Handbook of silicon based*  
325 *MEMS materials and technologies* (Elsevier, 2020).
- 326 36. H. F. Winters and J. Coburn, "Surface science aspects of etching reactions," *Surf. Sci. Reports* **14**, 162–269 (1992).
- 327 37. P. P. Cielecki and E. Skovsen, "A Compact, 3D printable Purge System for Terahertz Spectroscopy," (2021).
- 328 38. P. U. Jepsen, "Phase retrieval in terahertz time-domain measurements: a "how to" tutorial," *J. Infrared, Millimeter,*  
329 *Terahertz Waves* **40**, 395–411 (2019).
- 330 39. J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, *Photonic Crystals: Molding the Flow of Light*  
331 (Princeton University Press, 2008), 2nd ed.
- 332 40. M. Centini, C. Sibilìa, M. Scalora, G. D'Aguzzo, M. Bertolotti, M. J. Bloemer, C. M. Bowden, and I. Nefedov,  
333 "Dispersive properties of finite, one-dimensional photonic band gap structures: Applications to nonlinear quadratic  
334 interactions," *Phys. Rev. E* **60**, 4891–4898 (1999).