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WHAT AFFECTS RETENTION OF CORE CALCULUS CONCEPTS AMONG UNIVERSITY STUDENTS?

A STUDY OF DIFFERENT TEACHING APPROACHES IN CROATIA AND DENMARK

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This paper reports a parallel study of two university calculus courses in Croatia and Denmark using different teaching approaches. Both have lectures to a large group of students but they use different types of exercises. In Denmark, the exercises are student-centred, while the Croatian university uses a teacher-centred approach. The content of the courses are similar regarding the concepts we study in this paper. The students' retention was tested two months after the course exam on these concepts. Our statistical data analysis shows that the Danish students of our sample performed significantly better than the Croatian students of our sample on the conceptual questions, and vice versa for the procedural ones.

INTRODUCTION

The teaching of basic calculus concepts at the undergraduate level is wide and many students who study calculus are not in mathematics study programmes. Calculus at university level is usually taught by professional mathematicians who do not all seem to realize that there may be problems of communication between them and the students who study in non-mathematics study programmes (Maull & Berry, 2000; Guzman *et al*, 1998). When compared to mathematics students, engineering students seem to change their understanding of mathematical concepts as they progress through their studies (Maull & Berry, 2000). In order to gain more insight into the calculus knowledge of non-mathematics students, we investigated the level of retained knowledge in students from technical and natural sciences studies programmes. Our previous survey (Jukić & Dahl, 2010) showed that the students taking part in our experimentation had forgotten a large portion of notions regarding the derivative concept in differential calculus, and furthermore the surveyed students with the lowest course passing grades outperformed the students with high passing grades two months later in our questionnaire. The study reported in the present paper examines the retention of core calculus knowledge at two different non-mathematics student populations.

THEORETICAL BACKGROUND

Conceptual knowledge describes knowledge of the principles and relations between pieces of information in a certain domain and procedural knowledge is knowledge of the ways in which to solve problems quickly and efficiently (Hiebert & Lefevre, 1986). Haapasalo and Kadijevich (2000) redefined conceptual knowledge, highlighting its dynamic nature; it concerns the ability to browse through networks consisting of concepts, rules, algorithms, procedures and even to solve problems in various representation forms. Grundmeier *et al* (2006) showed that students generally choose a procedural over a conceptual way of dealing with problems in integral calculus. Pettersson and Scheja (2008) discovered that students developed their knowledge in integrals in an algorithmic way, not because of misconceptions, but because it was more suitable for them and enabled them to deal functionally and successfully with the presented tasks. Mahir (2009) investigated conceptual and procedural performance in integration in a group of undergraduate students who successfully completed a calculus course. She found that the students did not have satisfactory conceptual knowledge of integration, but those who had some conceptual knowledge, also showed some good procedural performance.

Teaching strategies can roughly be divided into student-centred and teacher-centred teaching (Killen, 2006). In the teacher-centred model, the teacher has direct control over what is taught and how the learners are presented the information they should learn. In the student-centred model, the learner is put at the focus of the teaching/learning process, instead of the teacher. The teacher has less direct control over how and what the students learn. An example of such approach is the use of small group work or cooperative learning. Studies showed that teaching strategies employed in the class can influence the development of one type of knowledge more than another; teacher-centred methods would favour the development of procedural knowledge and student-centred methods would favour the development of conceptual knowledge (e.g. Garner & Garner, 2001; Allen *et al*, 2005).

We examine what calculus knowledge is retained by students from two different mathematical populations two months after the course instruction and examination have taken place. Since these two populations are not completely comparable, we regard this as a parallel study, so caution is needed when making statements comparing the two populations.

THE TWO POPULATIONS: INSTITUTIONAL SETTINGS

In this section we will describe the institutional settings in the two universities where our survey was conducted: the University of Osijek in Croatia and Aarhus University in Denmark. In order to examine the calculus courses and their contexts, lectures and exercises were observed at the universities. Furthermore the teaching materials, exams and curricula were examined and interviews with lecturers, department heads, and teaching assistants were conducted at both universities to gain insight into the similarities and differences of both study programmes.

The Croatian University

The calculus course consists of lecture lessons and exercise lessons where the teaching approach is teacher-oriented. Lectures are given in a traditional form to a large group of students, and exercises are based on direct instructions, used in groups of 30 students where a problem-solving or performance procedure is shown to the students. Conceptual ideas are taught in the context of procedural methods. A first year calculus course is divided in two one-semester courses, entitled Calculus 1 and Calculus 2. Differential calculus is part of Calculus 1 and integral calculus is part of Calculus 2. Part of Calculus 1 is oriented on repetition of high school A-level, using formal mathematical theory, what makes it different from high school mathematics. Also, the majority of the calculus courses are focused on functions in one variable. Every science study programme has its own calculus courses, but these courses have 70% of the content in common. The courses differ not just in course content, but also in the number of teaching hours. They may vary between 60 and 105 hours per semester, altogether for lectures and exercises. The process of examining the students' knowledge begins during the calculus course. Students have several written partial exams with open-ended questions during the semester as a substitution for the final exam at the end of semester. Students have to pass all partial exams and their grade is determined after the last partial exams. Those who fail any of the partial exams during the semester have to take the final exam to pass the course. Students' knowledge in formal mathematical theory in theorem-proof style is also examined. Students get the final grade for both calculus courses separately.

The Danish university

The calculus course is a joint course for all mathematics and science study programmes. The course is organized into traditional lecture lessons and exercise lessons. Lecture lessons are given to a large group of students, but exercise lessons use small group work, based on problem solving where the teaching approach is more student-oriented. A first year calculus course is divided in two courses where functions of one variable and several variables are connected to differential and integral topics. Topics investigated in the questionnaire belong to Calculus 1. Both calculus courses take place during a seven-week half-semester (quarter) period with 63 hours, altogether for lectures and exercises. The process of evaluating students' knowledge starts after Calculus 1, where students take a multiple choice test, which determines whether or not the student can take the final written exam after Calculus 2. The grade obtained in the final exam is a joint grade for Calculus 1 and 2.

About comparing the two universities

The calculus content investigated in this paper belonged to the core of all programmes. One of the major differences between the populations was the teaching methods, but that is not the only difference that might explain how the students answer the questions in our survey. This means that pointing to one single factor causing the difference is not possible, therefore caution is needed and we cannot identify a single cause to the differences in the results of both populations.

METHODOLOGY

We conducted a survey examining a selected number of core concepts in differential and integral calculus through questionnaires given to first year non-mathematics students. The survey took place in the spring of 2009 at University of Osijek and in the autumn of 2009 at Aarhus University.

The Croatian students were given two questionnaires. The first examined their knowledge of derivatives, from Calculus 1, and the second examined their knowledge of integrals, from Calculus 2. The participants were students from the following study programmes: electrical engineering, civil engineering, food technology, physics, and chemistry. 227 students participated in the first questionnaire and were surveyed two months after the exam in differential calculus. 225 students participated in the second questionnaire and were surveyed two months after the exam in integral calculus. More than 94% of the students answered all questions in the first questionnaire and more than 97% of them answered all questions in the second questionnaire.

The Danish students were given one questionnaire combining the questions from the Croatian questionnaires since those concepts are covered in Calculus 1. The students belonged to the following study programmes: biology, chemistry, chemistry & technology, computer science, geology, geo-technology, information technology, molecular biology, medical chemistry, molecular medicine, and nano-science. 147 students participated in the questionnaire. More than 94% of the surveyed students answered all the questions.

The Danish university does not have engineering programmes and the Croatian university does not have all the study programmes surveyed in the Danish university. Since the aim of our parallel study was to examine knowledge retention in non-mathematics students, we do not consider these differences as significant. We wanted to get some insight into the knowledge of non-mathematics students from two different populations, and not in students belonging to a particular study programme. Even though the Danish and Croatian students have met calculus concepts in high school, the university courses provide different approaches to calculus (building calculus conceptions using formal theory) and build relationships between calculus objects (e.g. connecting them with functions of several variables). This diversity in teaching styles between high schools and universities has also been noted by various researchers (e.g. Guzman *et al*, 1998). We wanted to examine the retention of knowledge related to core calculus concepts after university calculus in students coming from different programs, contexts and teaching methods.

Questionnaire design

We designed the questionnaires with multiple choice questions where the wrong options represented typical misunderstandings and errors. Before being given to the students, professional mathematicians and the lecturers of the courses were consulted about the relevance of the questions, formulation, and appropriateness of the options of answers as offering typical misunderstandings. We wanted to examine the students' retention of concepts about derivatives and integrals in a short period of time, since the questionnaire had to be filled out by the students while they attended their class/lecture (the permission to pass the questionnaire during lecture time is easier to get, so multiple choice questions seemed to be a very convenient way to assess the students in a short period of time).

There were four questions about derivatives. The question *Tangent* deals with the geometric interpretation of the derivative of a function at a given point. In *Quotient* our intention was to test the students' knowledge about the quotient differentiation rule. *Composite* examines how the students deal with the derivation of a composite function. *Slope* incorporates several key concepts from differential calculus: slope of tangent line as the derivative of the function f at the given point and the process of differentiation. There were also four questions about integrals. *Area* deals with the geometric interpretation of the integral. It can be argued that among the offered answers "none of the above" would be the correct one, since the answer "the area between the curve $y = f(x)$ and the x-axis for x between a and b" is correct only in the special case where $f(x) \geq 0$ for $x \in [a, b]$ and $f(x)$ is bounded. However, as we had conjectured that the students were likely to overlook or ignore the subtlety of the case of non-positive or non-bounded functions, we did not consider "none of the above" as the right option (and we are aware this can be a potentially contentious choice). *Antiderivative* asked what the anti-derivative of a function is. Depending on the approach that was used in teaching, two of the offered answers could be considered as correct. Therefore, in the data analysis we labelled both possibilities as correct. *Method* asked for the most appropriate/easy method for solving a particular indefinite integral. The integration by parts is considered as the only correct option. The use of substitution for this example is "non-standard", and students would need a table to recall the integral of the logarithm. *Basic integrals* consisted of two indefinite integrals that are usually given in the tables of basic integrals and two possible solutions for each integral. The number of offered options for this question was inspired by the number of possible misunderstandings we considered for each integral. All the questions can be seen in the Appendix. The questions can be grouped as mainly involving either procedural or conceptual knowledge. The conceptual category consists of the questions *Tangent*, *Slope*, *Area* and *Antiderivative*, whereas *Quotient*, *Composite, Method* and *Basic integrals* are classified as procedural. However, this categorisation in conceptual or procedural questions is not absolute. Some questions could be placed in both groups, since they involve both kinds of approaches. For instance several differentiation rules have to be connected in *Composition*, and this, at least in some cases, can be considered as conceptual knowledge. On the other hand, it is possible that some students had experienced tasks like *Slope*, and thus their solution could be based only on recalling the method without any conceptual knowledge. This is the reason why the question is formulated in a little tricky way, so that if the students apply a procedure without carefully thinking, they will fail to answer it correctly. Also, in our case, the students were more exposed to the chain rule of differentiation, unlike the question *Slope*.

RESULTS OF THE QUESTIONNAIRE

Table 1 below shows the distribution of correct answers for all questions in the two populations. No question was answered correctly by all students.

Type	Topic	Question	Croatia		Denmark		Fisher's p-value
			#/total	%	#/total	%	
Conceptual	Differential calculus	Tangent	102/214	46	93/140	66	0.0007
		Slope	34/217	16	51/142	36	<0.0001
	Integral calculus	Area	160/224	71	141/143	99	<0.0001
		Antideriv.	(170+37)/ 223	93	(109+30)/ 142	98	0.0504
Procedural	Differential calculus	Quotient	167/221	76	90/139	65	0.0312
		Composite	144/218	66	94/141	67	1.000
	Integral calculus	Method	142/224	63	64/140	46	0.0011
		Integral a	99/223	46	22/141	16	<0.0001
		Integral b	145/220	65	81/142	57	0.0963

Table 1: Distribution of correct answers. P-values here indicate the size of the differences among the populations.

There was a significant difference in how the Croatian and Danish students answered six of the nine questions, eight if we accept an alpha of 0.10. The Danish students significantly outperformed the Croatian students in almost all the conceptual questions, but in the procedural questions, the Croatian students significantly outperformed the Danish students in four of the five questions. The fifth question (*Composite*) had an almost identical rate of correct answers.

Table 2 below shows how well each of the two populations solved each of the conceptual questions compared to each of the procedural questions.

The results of Table 2, and data from Table 1, show that for the Croatian students there was a significantly different performance in 16 of the 20 comparisons of the two groups of questions. Of the 16 comparisons which showed a significant difference (alpha of 0.10), nine times the procedural question was answered the best, while seven times, the conceptual question had the best answer rate. Hence it appears that there is almost no difference in how the Croatian students answer the conceptual and procedural questions, just a small preference for the procedural ones. Among the procedural questions, the Croatian students achieved better results in the derivative questions than in the integral questions. In the conceptual group, their results were better in the integral questions than in the derivative question.

Croatia		Conceptual				**Denmark**		Conceptual			
		Dif		Int				Dif		Int	
Procedural		Ta	Sl	Ar	An	Procedural		Ta	Sl	Ar	An
Dif	Qu	* p	* p	3356	* c	Dif	Qu	8017	* p	* c	* c
	Co	0001 p	* p	2588	* c		Co	1.000	* p	* c	* c
Int	Me	0011 p	* p	0864 c	* c	Int	Me	0007 c	1151	* c	* c
	Ia	5031	* p	* c	* c		Ia	* c	0001 c	* c	* c
	Ib	0002 p	* p	2207	* c		Ib	1126	0005 p	* c	* c

Table 2: Fisher's p-values comparing answers to the procedural and conceptual questions by population. * denotes p<0.0001. P-values are noted without 0. The letters p (procedural) and c (conceptual) denotes which question had the best answer rate.

For the Danish students there was also a significantly different performance in 16 of the 20 comparisons of the two groups of questions. Of the 16 comparisons which showed a significant difference (alpha of 0.10), three times the procedural question was answered the best, while 13 times, the conceptual question had the best answer rate. Hence, it appears that the Danish students of our sample perform much better at the conceptual questions than at the procedural ones. In the procedural group of questions, the Danish students achieved better results in the integral questions than in the derivative questions. In the conceptual group, their results were better in the integral questions than in the derivative questions.

DISCUSSION AND CONCLUSION

Having in mind that the questionnaires took place only two months after the examination, and that the questions were multiple-choice, we regard the obtained overall results as weak. There was only one question where both populations had a correct answer rate above 80% (*Antiderivative*). The lowest Croatian result is seen in the question *Slope* (16%) and the highest in the question *Antiderivative* (93%). The Danish students achieved the lowest result in the question *Integral a* (16%) and the highest result in the question *Area* (99%).

Both student populations were taught procedural and conceptual knowledge. In terms of long-term retention, procedural knowledge is quite fragile, meaning that procedures are often forgotten quickly or remembered inappropriately (e.g. Allen *et al*, 2005). This is perhaps reflected by the fact that Table 2 shows that 12 times a procedural question did better in comparison with a conceptual question, 20 times the opposite. Also Table 1 shows that no procedural question had a correct answer rate above 76%, while three of eight times, the correct answer rate to a conceptual question was above 90%. Hence, our data lead us to think that the Danish students retained more conceptual knowledge than procedural knowledge, while the Croatian students were almost equally strong/weak in the conceptual and procedural questions. In terms of long-term retention, conceptual knowledge is stable, but possessing conceptual knowledge without procedural fluency is considered to be ineffective (Bosse & Bahr, 2008).

The results of our study can be connected with a long dispute on which type of knowledge is more important and in which order they should be learnt (Rittle-Johnson *et al*, 2001; Haapasalo, 2003). Today, we regard both types of knowledge as important and complementary, thus universities should focus on attaining balance between conceptual and procedural knowledge. Learning new concepts and practicing the skills associated with those concepts are strongly interconnected, therefore, a balance of learning concepts and procedures with explicit connections to those concepts will enhance the long term retention of both (Schoenfeld, 1988).

If we have a look at the results of our two populations, the Croatian students showed significantly better performance in the procedural questions, and the Danish students were significantly better in the conceptual group of questions. The teaching approach at the Croatian university is teacher-centred while it is more student-centred at the Danish university. One may wonder if these results are connected with the teaching approaches. Some studies showed that the teaching strategies employed in class can influence the development of one type of knowledge over the other; teacher-centred on procedural knowledge and student-oriented on conceptual knowledge. Garner and Garner (2001) found similar results in the case of applied calculus examining the retention of students' knowledge after eight months, but Allen *et al* (2005) found significant differences only regarding conceptual knowledge, and no difference in procedural knowledge between students exposed to different teaching strategies in differential equations, examining them after one year. Schumacher and Kennedy (2008), who examined calculus knowledge in students exposed to teacher-centred and student-centred teaching approach, found no statistical significance in success between the two groups of students. The studies that we refer to here had investigated students' retention in courses that only differed in the teaching approach and in the number of course hours. Students in our study also had some further differences in terms of previous training, of course content and of examination styles. Therefore, caution is needed when trying to point to *one* factor explaining the difference. This will be the topic of future research.

APPENDIX

Derivatives questions surveyed with given options for answers

1. Question *Tangent*: What is the geometric interpretation of the derivative of the function

$f: R \to R$ at the point x_0? Offered answers: maximum/minimum of the function f at x_0; slope of tangent line to the curve $y = f(x)$ at x_0; continuity of the function f in the given point; none of the above.

2. Question *Quotient*: Differentiate the function $f(x) = \dfrac{x^2+2}{x^3}$. Offered answers: $\dfrac{x^3(2x)-(x^2+2)(3x^2)}{(x^3)^2}$; $\dfrac{x^3(2x)-(x^2+2)(3x^2)}{x^3}$; $\dfrac{x^3(2x)-x^2(3x^2)}{(x^3)^2}$.
3. Question *Composite*: Differentiate the function $f(x) = \sin^2 6x$. Offered answers: $2\sin(6x)$; $12\sin(6x)$; $12\sin(6x)\cos(6x)$.
4. Question *Slope:* Calculate the slope of the tangent line to the curve $y = (3x)^2$ at the point $x = 1$. Offered answers: 9; 18; 6.

Integral questions surveyed with given options for answers

1. Question *Area*: What is the geometric interpretation of the definite integral $\int_b^a f(x)dx$? Offered answers: The area between the curve $y = f(x)$ and the x-axis for x between a and b; The arc length of the curve $y = f(x)$ on the interval $[a,b]$; continuity of the function f on interval $[a,b]$; none of the above.
2. Question *Antiderivative*: What is an antiderivative of a function f? Offered answers: $\int f(x)\mathrm{d}x$; every function F such that $F'(x) = f(x)$ holds; The set of elementary functions; none of the above.
3. Question *Method*: Which method should be used for computing the integral $\int xe^x dx$? Offered answers: substitution $t = e^x$; integration by parts; trigonometric substitution; none of the above.
4. Question *Basic integrals*:

a. $\int \dfrac{dx}{1+x^2}$ =? Offered answers: (a) $\ln(1+x^2)+C$ and (b) $\arctan x + C$.

b. $\int \dfrac{dx}{x^3}$ =? Offered answers: (a) $-\dfrac{1}{2}x^{-2}+C$ and (b) $\ln(x^3)+C$.

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