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Statistics of Rain Attenuation Revisited

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Abstract

Millimeter frequency satellite links are prone to variations in attenuation over a large range due to rain, more than 30 dB. Traditionally, it has been considered to be part of meteorological conditions, unavoidable variations. Statistical distributions have been fit-to-data type with little or no physical insight. It is the purpose of this paper to test the hypothesis that the dynamic variations are Ricean or Rayleigh type fading, caused by multipath from the rain. This opens the possibility of exact distributions of Doppler variations and of the recently studied rain fade slope, which must follow a Student's t-distribution with the Doppler spread as parameter. The theory is tested on experimental data published in [1].

1. Introduction

The growth of satellite communications in the 1970'ies spurred a number of investigations on the propagation channel, mainly on the attenuation due to hydrometeors. Overviews may be found in [2-5] with many details on the effect of rain drop size and distributions, correlation with rain rates and other phenomena, but with a few exceptions the variability of the channel was not considered in detail. When extreme attenuation occurred in practice it was supposed to be 'the effect of deep fades caused by intense rain cells of limited horizontal extent' [4]. One exception is the work of [6] who analyzed a slab of rain, where both amplitude and phase fluctuations were derived, depending on antenna properties. Their channel model of a coherent field superimposed on a collection of random contributions is similar to the one in this paper, but no statistical distributions were derived. The main message of the present paper is that the deep fades may not be due to any meteorological phenomenon, but a natural consequence of a sum of random complex contributions. The analogy with mobile communications is stressed.

More recently, there has been considerable focus on details of the channel through rain, where the speed of change of attenuation is considered, expressed as a rain fade slope measured in dB/s [7, 1]. The distribution of slopes were compared with an ad hoc distribution, while it is shown here that during the heavy fading periods the unconditional distribution is the well known Student's-t.

2. Statistics of Attenuation

It is generally accepted that a collection of absorbing and scattering drops will give rise to exponential decay

$$P(L) = e^{-\gamma L} \tag{1}$$

where γ is the extinction coefficient, the sum of γ_a and γ_s for absorption and scattering.

Expressing the attenuation in dB we find that

$$A(dB) = 4.34\ \gamma L = N\ D(f,a)\ L\ \ dB \tag{2}$$

where N is the number of drops per volume, L is the effective length of the path, and D is a drop function of frequency, size and shape of drop. We are not concerned with the actual attenuation, more the statistical properties.

Lin [8] finds by analyzing many different rain events that the percentage of raining time that A(dB) exceeds a certain value is lognormally distributed to a good accuracy, i.e. the log of A in dB follows a normal distribution, so in a sense the received power in Watts is log-lognormal distributed. Lin argues that a product of a number of random γ will explain this distribution. This is correct, but there is no justification that the total γ is the product of individual γ, it is difficult to imagine a product of dBs. On the contrary, if the rain consists of a number of layers with different γ_i the total attenuation will be the sum of the individual γ_i, or

$$\gamma = \gamma_1 + \gamma_2 + \gamma_3 + \ldots + \gamma_n \tag{3}$$

leading to a normal distribution of A(dB), i.e. a lognormal distribution of the linear power. This, on the other hand, does not agree with the experimental results, so we must look elsewhere. The lognormal distribution is given by

$$P(A \geq x) = \frac{1}{2} erfc\left[\frac{\log_{10} x - \mu}{\sqrt{2}\sigma}\right] \tag{4}$$

Lin finds that the standard deviation σ lies between 0.4 and 0.7 for satellite paths. We accept that the lognormal distribution for A(dB) describes the attenuation well from experiments, but it lacks a theoretical foundation. The lognormal is a versatile distribution with two independent parameters, which can fit many cases with positive variables.

Figure 1 shows a comparison between a Rayleigh distribution and a lognormal according to eq. 4 for μ=0.85 and σ=0.43 with quite a close fit over a 20 dB range, so it is natural to connect the Rayleigh distribution with the dynamics of rain attenuation. The Rayleigh distribution occurs physically from the summation of many scattered fields with arbitrary phase, with normal distribution for the real and imaginary part.

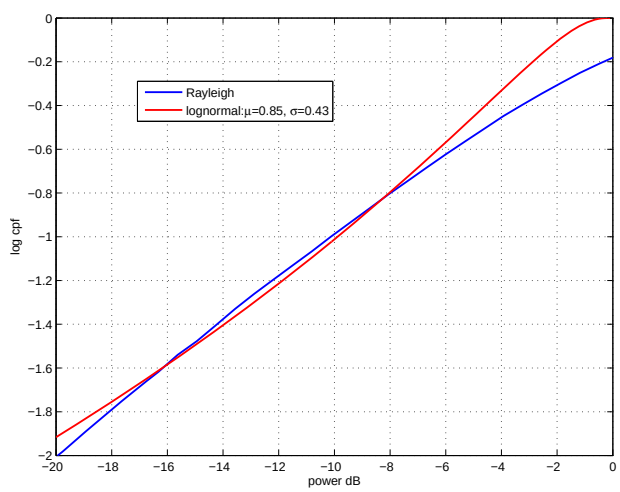

Figure 1 Comparison between theoretical Rayleigh distribution relative to the mean power and the lognormal distribution for the power in dB, known to be a good model for experimental channels. For the lognormal the mean value is 0.85 and standard deviation =0.43.

We conclude that the Rayleigh distribution is a possible candidate with the implication that a major part of the dynamic variation may be due to scattering towards the antenna and not due to absorption.

3. Statistics of rate of change of attenuation

Recently, there has been considerable interest in how fast the channel changes with implications for fade - mitigation techniques. For a Rayleigh channel the Doppler frequency is Student's-t distributed [9] with a Doppler spread equal to ω_D. According to [9] the power gradient in dB/s has the same distribution with a spread proportional to the Doppler spread

$$s_{\gamma'} = 20\omega_D / \log(10) \quad \text{dB/s} \quad \text{and the pdf} \tag{5}$$

$$p(\gamma') = 0.5 \frac{s_{\gamma'}^2}{(s_{\gamma'}^2 + \gamma'^{\,2})^{\frac{3}{2}}} \quad \text{with corresponding CDF} \tag{6}$$

$$P(\gamma' < \Gamma') = 0.5 + 0.5 \frac{\Gamma'}{\sqrt{s_{\gamma'}^2 + \Gamma'^2}} \quad . \tag{7}$$

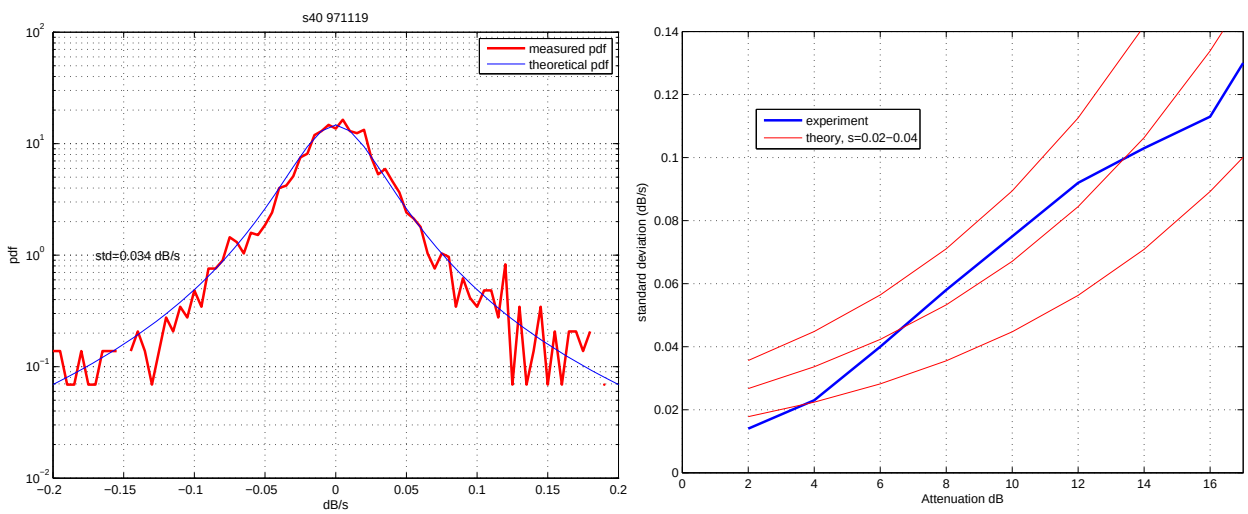

Figure 2a. An example of fade slope distribution where experimental results [1] are compared with the theoretical Student's-t distribution. f=40GHz.The standard deviation is 0.034dB/s. b) standard deviation of fade slopes conditioned on attenuation in dB for Gaussian statistics for three values of unconditional standard deviation and experimental results[1] for 20, 40, and 50 GHz combined

Based on the experimental results described in [1] Figure 2a shows a comparison with the Student's-t distribution. Note the similarity with the ad-hoc distribution in [7]

$$p(\gamma') = \frac{2s_{\gamma'}^3}{\pi(s_{\gamma'}^2 + \gamma'^{\,2})^2} \tag{8}$$

which however was valid for the slope conditioned on the attenuation. Figure 2b shows the fade slope versus attenuation for three different standard deviations in theory. For the theoretical complex Gaussian case (Rayleigh fading) the conditional distribution is normal with zero mean and a variance inversely proportional to the power [9]

$$\sigma^2 = \frac{s_{\gamma'}^2}{2P} \tag{9}$$

or expressed differently, the variance is proportional to the attenuation in linear terms, not in dB. The comparison with experiment again indicates good agreement, suggesting that the dynamics of attenuation is governed by multipath propagation.

4. Conclusion

The main contribution of the paper is to show that a major part of the attenuation of a satellite signal through rain may not be due to absorption, but the result of multipath propagation from rain scatterers to the antenna. The history of a rain event will be shown to follow the scenario of a Ricean distribution in the beginning with a strong direct signal on top of the scattered signals and a subsequent period where the scattering dominates and the envelope is Rayleigh distributed. During the latter period analytical results for the fade slopes are well known from mobile communication systems, expressed by the Student's-t distribution.

The importance of the results lie not so much in the actual distributions, but the physical insight they give. If the deep fades are not due to bursts of high absorption, but rather a cancellation effect, then there might be interesting implications for fading mitigation techniques. The fact that the channel varies very slowly, considering the velocity of rain and the high carrier frequencies, can be understood from an analysis of the Doppler spreads in the antenna beam.

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