**AALBORG UNIVERSITY**
DENMARK

**Structural Quality of Service in Large-Scale Networks**

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*Publication date:*
2005

*Document Version*
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*
Pedersen, J. M. (2005). *Structural Quality of Service in Large-Scale Networks*. Department of Control Engineering, Aalborg University.

# Structural Quality of Service in Large-Scale Networks

Ph.D. Thesis
Submitted to the Faculty of Science,
Aalborg University,
April 2005

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Center for Network Planning
Center for TeleInfrastruktur
Department of Control Engineering

Structural Quality of Service in Large-Scale Networks

Ph.D. Thesis

# Preface

This thesis is a result of my Ph.D. study at Department of Control Engineering, Institute of Electronic Systems, Aalborg University, Denmark. The work has been carried out within the research project "The Structural Impact on Quality of Service Parameters", under supervision of Professor Ole Brun Madsen.

The thesis describes and analyses some network structures, which are useful for designing large-scale networks with well-defined and well-describable global properties. It consists of three parts:

  I  Introduction and Background

 II  Papers

III  Conclusion and Future work

Part II is the most important since it presents most of the background, all methods, and all results. It consists of 8 conference papers, which have been reviewed, accepted, presented and published in conference proceedings, and one manuscript submitted for the journal Computer Communications.

Each chapter of Part II contains one paper, ordered with regard to their contents rather than chronologically. The papers are self-contained and presented as published, with only a few minor corrections of language, typos etc. This means that parts and bits appear in more than one paper. In order to increase readability, and to make the thesis a uniform entity, the following applies to each paper:

- A section labelled "Preface" has been added in the beginning of each paper, describing the context and contribution of the paper as well as crediting the co-authors, and a section labelled "Additional comments" has been added to some of the papers, containing additional comments and results. These additional sections are marked in the section headings by an * to indicate that they were not included in the original papers.

- The papers are presented with a uniform formatting. Information on co-authors is given in the preface of each paper, and the key-words left out. The section

division is not changed, except for the addition of Preface and Additional comments. The numbering of sections, figures, and tables has been unified. Sections are numbered globally for the whole thesis, whereas figures and tables are labelled according to the chapter in which they appear (i.e. the first table in Chapter 4 is labelled Table 4.1).

- All references are assembled in the Bibliography at the end of the thesis. They are listed in alphabetical order, but are the same as in the published papers.

- Where appropriate, figures have been replaced by colour figures. This was not possible in the conference proceedings, but increases readability.

The papers in Chapters 4–5 are the most general papers, describing reliability demands in future access networks and evaluation parameters for such network structures. The papers in Chapters 6–8 deal with different aspects of Topological Routing and 4-regular grid (2D mesh) network structures. The papers in Chapters 9–12 describe different properties of $N2R$ network structures. For a complete overview, please see the thesis roadmap on page xi. A more detailed introduction of each paper is given in the preface added to each paper.

*Aalborg, Denmark, April 2005*        Jens Myrup Pedersen

# Acknowledgements

# Summary

This thesis deals with applying well-ordered and symmetric graph structures in large-scale networks, in order to design network infrastructures possessing well-defined, describable, and predictable properties. The main focus is on wired network infrastructures covering larger geographical areas, but the results also apply to other types of networks, e.g. networks within a single room or single building.

The increasing digitalisation has created the base for two important trends within networking: co-existence and convergence. Before the digitalisation, most networks were dedicated to specific services such as sound, video, and data, but today multi service networks are becoming widely used for a large variety of services and applications. This is clearly seen in the Fiber To The Home implementations, where the same fiber is used for sound, video, data, and combinations hereof. They can also be used for new services, not necessarily known at the time of the infrastructure being designed or implemented. Examples of services under development can be found within areas such as tele-medicine and surveillance. Different applications and services have different requirements to the networks; many applications demand best-effort service only, with few requirements to the network. Others demand high reliability, high bandwidth, short transmission delays, or combinations of such requirements.

Thus, the convergence creates new possibilities, but also leads to increasing demands on the networks, because they must satisfy the very different requirements demanded by various applications and services. Even short outages can be highly critical, and this higher vulnerability to failures and attacks is worsened by an increasing general dependency on networks, both within and between different groups of users, i.e. public institutions, enterprises, and private users. To what extent a given network can deliver a required service/quality depends on a large number of factors. The physical infrastructures play a particularly important role here because they are expensive and difficult to change once established, and because they are expected to have a significantly longer lifetime than the active equipment. This is also true for the Fiber To The Home infrastructures currently being implemented. These are expected to reach most households in Denmark as well as many other countries within the next 15-20 years; the physical topologies/structures of these networks should be carefully chosen, such that they are implemented correctly at the first attempt. For this reason, this thesis primarily relates to these. In order to obtain the best possible support for the demands

posed by different services, the network structures should be designed according to a set of parameters, which have been identified and described in this thesis. This is different from previous approaches, where networks have been planned in an ad-hoc manner, and lines simply added when increasing demands for capacity and QoS required so. This makes it increasingly difficult to maintain an overview, and makes it virtually impossible to provide general global guarantees except for best-effort. Lack of focus on structural properties has resulted in networks, where even a single failure may have unpredictable consequences. Planning based on SQoS (Structural QoS) does, in contrast, focus on structural properties, including support for routing, restoration, and protection schemes.

This thesis analyses different classes of network structures, according to these parameters. The analyses are based on abstractions, where network structures are regarded as graphs. The distances - mainly diameter and average distance - are used to indicate transmission delays and resource usage, while support of algorithms for routing, protection, and restoration as well as connectivity, $k$-diameters, and $k$-average distances are used to determine the levels of reliability supported. Focus is on structures with low nodal degrees, because other structures are hard to physically implement. Most attention is paid to two types of structures, 4-regular grid and $N2R$. For the 4-regular grid, table-free topologically based routing schemes are devised and analysed, as well as methods and extensions facilitating physical implementations, such as hierarchical extensions reducing distances significantly, and pruning, reducing the number of lines while only slightly affecting the distances. The $N2R$ structures are mainly analysed with regard to diameter, average distance, $k$-diameters, and $k$-average distances. The different choices of analysis parameters are partly due to differences in the structures: to fully benefit from the 4-regular grid structures, a high level of overall planning is required, while $N2R$ structures act as a more immediate alternative to the widely used fiber rings. For this reason, $N2R$ structures are also compared to two other 3-regular structures, Double Rings and Degree Three Chordal Rings.

The research conducted is mainly theoretical, but with a clear focus towards applications, and it forms an important and strong fundament for more practically oriented research in using well-ordered graph structures in practical network planning. The work described in the thesis is a first step of research in SQoS-based network planning, and acts as a base for future research, where simulations, test implementations, and development of methods for physical implementation are expected to be the most important areas. The results also form the base for considerations on research in how SQoS-based planning can be used in the planning of wireless networks. Due to factors such as high potential node degrees and high connectivity, these differ notably from wired networks.

# Danish summary – Dansk sammenfatning

Denne afhandling omhandler anvendelse af velordnede og symmetriske grafstrukturer i store netværk, med henblik på at konstruere netværksinfrastrukturer, der besidder veldefinerede, beskrivbare og forudsigelige egenskaber. Der fokuseres på trådede infrastrukturer, der dækker større geografiske områder, men resultaterne dækker også andre typer netværk, f.eks. netværk indenfor ét rum eller én bygning.

Den øgede digitalisering har skabt basis for to vigtige tendenser indenfor netværk, sameksistens og konvergens. Hvor det tidligere var udbredt med dedikerede netværk til enkelte tjenester som f.eks. lyd, billede og data, anvendes multiservicenetværk i stadig højere grad til forskellige typer af tjenester og applikationer. Denne udvikling ses tydeligt i Fiber Til Hjemmet implementationer, hvor den samme fiber anvendes til lyd, billede, data samt kombinationer heraf, og samtidig giver mulighed for nye tjenester, der ikke behøver være kendt på det tidspunkt hvor infrastrukturen designes og implementeres. Eksempler på tjenester under udvikling findes inden for områder som telemedicin og overvågning. Forskellige applikationer og services har forskellige krav til de netværk, de kommunikerer over. Mange applikationer kræver kun best-effort services, der ikke stiller særlige krav til netværkene. Andre kræver høj pålidelighed, mens andre igen kræver høj båndbredde, korte transmissionstider eller flere af delene samtidig.

Konvergensen skaber således nye muligheder, men medfører også øgede krav til netværkene, der skal tilfredsstille de meget forskellige krav, forskellige tjenester og applikationer stiller. Selv korte afbrydelser kan blive meget kritiske, og den øgede sårbarhed over for fejl og angreb forstærkes af en generelt øget afhængighed af netværk, både for og imellem offentlige institutioner, virksomheder og privatpersoner. I hvilken udstrækning et netværk kan levere en ønsket service/kvalitet afhænger af en lang række faktorer. De grundlæggende fysiske infrastrukturer spiller her en særlig vigtig rolle, idet de er dyre og vanskelige at ændre, når først de er etableret. De forventes også at have en betydeligt længere levetid end det aktive udstyr. Dette gælder også for Fiber Til Hjemmet, der er under etablering, og som i løbet af de næste 15-20 år forventes at nå de fleste husstande, i Danmark såvel som i store dele af den øvrige verden; de fysiske topologier/strukturer bør vælges omhyggeligt, således at de etableres korrekt i første forsøg. Denne afhandling vender sig derfor primært mod disse. For at opnå en bedst mulig understøttelse af de krav, forskel-

lige tjenster stiller, bør netværksstrukturerne designes ud fra en række parametre, der er identificeret og beskrevet i denne afhandling. Dette er i modsætning til hidtidige tilgange, hvor netværk typisk har været planlagt ad-hoc. Forbindelseslinier er blevet etableret eller udbygget efterhånden som behovene for kapacitet og QoS øgedes, hvilket har gjort det stadig mere vanskeligt at bevare et overblik, og i praksis umuligt at at give generelle globale garantier ud over best-effort. Manglende fokus på globale strukturelle egenskaber har resulteret i netværk, hvor selv en enkelt fejl kan have uforudsigelige konsekvenser. Planlægning baseret på SQoS (Structural QoS) fokuserer derimod på strukturelle egenskaber, herunder understøttelse af routing, restoration og protection.

Denne afhandling analyserer forskellige klasser af netværksstrukturer i forhold til disse identificerede parametre. Analyserne er baseret på abstraktioner, hvor netværksstrukturer opfattes som grafer. Afstandene - primært diameter og gennemsnitsafstand - bruges til at indikere forsinkelsestider og ressourceforbrug, mens understøttelse af algoritmer til routing, protection og restoration samt sammenhængsgrader, $k$-diameter og $k$-gennemsnitsafstande bruges til at vurdere graderne af pålidelighed der understøttes. Der fokuseres på strukturer med lave gradtal, da andre strukturer vil være vanskelige at implementere rent fysisk. To typer af strukturer, 4-regulær grid og $N2R$, behandles mest dybdegående. For 4-regulær grid foreslås og analyseres tabel-frie topologiske routingprincipper, ligesom der foreslås og analyseres metoder og udvidelser, der er nødvendige for at muliggøre fysiske implementationer. Dette gælder bl.a. hierarkiske udvidelser, der markant reducerer afstandene i store strukturer, og udtynding af disse, der reducerer antallet af linier, men kun marginalt forlænger afstandene. $N2R$-strukturerne er primært analyseret med hensyn til diameter, gennemsnitsafstand, $k$-diametre og $k$-gennemsnitsafstande. Valget af forskellige analyseparametre skyldes til dels de to strukturers forskelligheder: Et optimalt udbytte af de 4-regulære grid-strukturer kræver en høj grad af overordnet planlægning, mens $N2R$-strukturer mere umiddelbart kan anvendes som alternativ til de hyppigt anvendte fiberringe. $N2R$-strukturerne er derfor også direkte sammenlignet med to andre 3-regulære strukturer, dobbeltringe og kordal-ringe af grad 3.

De udførte studier er af teoretisk tilsnit, men med et klart anvendelsesorienteret fokus. De danner et væsentligt og solidt fundament for mere praktisk orienteret forskning i hvordan velordnede grafstrukturer kan anvendes i praktisk netværksplanlæning. Arbejdet præsenteret i denne afhandling er et første skridt i forskning i SQoS-baseret netværksplanlægning, og baner vejen for kommende forskning, hvor simuleringer, test-implementationer og udvikling af metoder til fysisk implementation forventes at blive de vigtigste områder. Resultaterne danner også basis for overvejelser om forskning i, hvordan SQoS-baseret planlægning kan anvendes til planlægning af trådløse netværk. Disse er, bl.a. på grund af høje potentielle gradtal og høj konnektivitet, væsentligt forskellig fra trådede netværk.

# Thesis roadmap

**Part I: Introduction and Background**

**Chapter 1:** Introduction

**Chapter 2:** Background

**Chapter 3:** Problem Formulation

**Part II: Papers**

General

**Chapter 4:** Analysis of reliability demands in FTTH networks, showing a need for physical redundancy.

**Chapter 5:** Determination of SQoS-parameters for evaluating and comparing large-scale networks.

4-regular grid

**Chapter 6:** Basic properties and introduction of hierarchies, Topological Routing and algorithms for fault-tolerance.

**Chapter 7:** Extensions and performance evaluation of hierarchical extensions.

**Chapter 8:** Most important results from Chapters 6-7, more detailed theory and introduction of pruning.

N2R

**Chapter 9:** Basic properties and algorithms.

**Chapter 10:** Selection policy and evaluation of k-average distances and k-diameters, up to 164 nodes. Comparison to Double Ring

**Chapter 11:** k-average distance and k-diameter, up to 400-900 nodes. Comparison to Degree Three Chordal Rings.

**Chapter 12:** Traffic load distribution and schemes for reducing the load on some of the lines at the price of larger efficient distances.

**Part III: Conclusion and Future work**

**Chapter 12:** Conclusion

**Chapter 13:** Future work

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# Part I

# Introduction and Background

# 1 | Introduction

Until recently, most households and businesses were connected by a number of dedicated networks, such as telephony and cable-TV, reflecting their needs for communication, and these dedicated networks were designed to meet the specific requirements for their intended applications. The trend of convergence in networks has led to a change in this view: now, different applications are being designed to run over a common platform. This also creates potentials for bringing new services with strong Quality of Service (QoS) requirements into play.

QoS has become a major research area, and it is expected that many applications with different QoS requirements will be developed within near and far future. Somewhat surprisingly though, almost all research in QoS has focused on the higher layers in the networks, and none or little attention been paid to the physical network structures. This is surprising because these physical structures define the limits of what is possible: no algorithm can perform better than what is allowed by the underlying physical infrastructure.

Until now, most Internet connections have been based on the existing telephony transmission network structures, and attempts have been done to re-use existing methods of network planning from this field. Traditionally, these networks have been designed to provide connectivity rather than to provide connections satisfying QoS requirements, leading to networks lacking global structural properties. As long as connectivity exist, the networks have been upgradeable by increasing the capacity of existing lines, or by adding new lines where needed. Ring structures have been used to guarantee some level of physical redundancy, but they have not been applied in a systematic manner in the access networks.

The lack of global structural qualities causes a number of problems in today's network planning. While restoration can usually be done in case of a single node or line failure, it is not in general possible to do restoration in case of two errors: due to complexity issues, this is so even if physical paths still exist, making it impossible to fully utilize the physical network properties. Mutual dependencies between layers also lead to severe problems in case of network failures. Due to the lack of structural qualities, a set of independent logical paths may not be physically independent, and the existence of a set of physically independent paths does not imply that corresponding logical paths are set up. One of the ideas of Structural Quality of Service (SQoS) is to establish a link between the underlying physical networks and the ability to offer

QoS in the higher layers. Thus, SQoS-based network planning - as opposed to traditional planning methods - aims at planning physical network infrastructures with good, well-defined and well describable global structural properties. Another important benefit of considering SQoS-based planning, is that it brings network planning from an art into a science. In this thesis, this is addressed by proposing the use of symmetric or near-symmetric physical network structures.

In addition to symmetry and connectivity, another main objective when designing network structures is to obtain improved capacity and shorter transmission delays while keeping the costs down. These properties should to the largest possible extent be maintained in case of network failures. The first work on SQoS, which was carried out prior to my work, revealed that choosing the network structures carefully could lead to significant improvements of capacity as well as hop count, the latter showing a potential for shorter transmission delays. These results have been confirmed by the work described in this thesis.

The existing work is extended further by this thesis, which presents fundamental results on SQoS. Two different approaches to designing networks with well-defined SQoS-properties are taken, one for the 4-regular grid structures, and another for the $N2R$ structures. While the results of the two different families of structures are by themselves interesting, the methods are also applicable to other structures and topologies, which may be studied in the future.

The thesis forms a framework for further research in SQoS, opening the road for more practically oriented research, including simulations, test implementations etc. Based on the results presented in this thesis, research on real-world implementations of various well-ordered graph structures including the 4-regular grid structures and the $N2R$ structures has been chosen as a main research area in Center for Network planning (CNP) at Aalborg University. As a part of these activities, a test network is currently being set up in order to test and demonstrate Topological Routing as well as other SQoS-related parameters described in this thesis. This work is carried out in collaboration with Center for TeleInfrastruktur (CTIF), Aalborg University.

# 2 | Background

Since the specific background for each of the papers in Part II of the thesis is contained in the individual papers, the purpose of this chapter is to provide the general background and references. Only little literature exists on Structural Quality of Service (SQoS), and so the chapter is divided into two main sections. The specific background of SQoS is given first, followed by an overview of the most important background knowledge obtained from related research areas.

## 2.1 Specific SQoS background

The concept of SQoS is quite new. It was introduced with the research project "The Structural Impact on Quality of Service Parameters", which started in 2001, and the first publication was presented in a conference in June 2002[MNS02], just after I began working on my Ph.D. My supervisor also introduced me to some of the concepts described in [MKP03], which was not published until October 2003. One objective of this research project was to devise a taxonomy supporting choices of structures and design principles when planning network infrastructures from scratch: this thesis contributes with such a taxonomy, which is also used as a base for many of the other results obtained.

[MNS02] gives an introduction to the problem field of SQoS, and motivates a systematic approach towards the design of network architectures and infrastructures. It puts SQoS into context, provides real-world examples and introduces $N2R$ structures. One of the major contributions is an analysis of convergence of communications. Not only are applications and services converging towards a common communication platform, different networks such as WANs (Wide Area Networks), access networks and MANs (Metropolitan Area Networks), LANs (Local Area Networks), and PANs (Personal Area Networks) are also converging. One of the conclusions of the paper is that further research should focus on the MANs, since in a global perspective, these currently possess the lowest SQoS potentials. This is stressed by the fact that we are confronted with the need for a complete re-implementation of these with fibre optical technology, complemented with new wireless technologies. This forms a historical opportunity to create an improved global infrastructure with an overall high SQoS potential. In order to take advantage of that opportunity, however, systematic approaches for planning networks with high levels of SQoS need to be developed. This

point of view is followed in the thesis, where the major focus is on the MANs and access networks.

[MKP03] introduces two basic approaches for providing SQoS. One is the spanning tree approach, giving an efficient method for planning networks with good structural properties as seen from one node in the network. This method has a drawback since the SQoS parameters are only guaranteed from a single node, and not from all nodes in the network. Therefore, the use of node symmetric graph structures is encouraged, and focus put onto the $N2R$ structures (actually, $N2R$ structures are not in general node symmetric, because two classes of nodes exist, with symmetry within each class). Compared to the definition in [MNS02], the definition of $N2R$ structures is extended by the introduction of an additional parameter: $N2R(p; q; r)$ structures are considered instead of $N2R(p; q)$ structures. For formal definitions of these structures, see Chapter 9. The paper also presents the first calculations of average distance and diameter in $N2R$ structures. The spanning tree is used for reference, indicating that with regard to average distance, $N2R$ structures are close to optimal among the structures where all nodes have degree three. Because of this, and since that due to our knowledge these structures have not previously been considered potential network structures, they have been chosen a main focus area of the work presented in the thesis.

In addition to [MNS02] and [MKP03], the book [Gro04] gives an excellent overview of the design of mesh-based survivable networks, an area closely related to SQoS. It was published in 2004, and unfortunately I did not become aware of it until the final phase of my study. Despite of that, it is probably the best single source introduction to the problem field and methods available: in addition to basic introductions to transport networks and Internetworking, subjects such as failure impacts, survivability principles, graph theory, protection, and restoration are treated in-depth. The book is highly recommended.

SQoS is only one area of research within the whole area of network planning, and should be seen in this context. Before the liberalisation of the telecommunication market, research in network planning was mainly done within the public telecommunication companies, and has accordingly received only little attention from universities and other open research institutions. Again, the best general introduction to the whole field is probably given in [Gro04], which also introduces important trends of research. The formation of parallel network operators has also lead to another main challenge in the construction of infrastructures, namely the conservation of global properties across networks owned and/or operated by different operators. Such global properties are needed in order to avoid unpredictable consequences in case of network failures, e.g. caused by mutual dependencies.

On the physical level today, fiber optical rings are widely used due to their support of

simple routing, restoration, and protection schemes. Tree structures are also widely used, in particular in the access networks, because they are cheap and simple, but they do not offer any means of protection. Both rings and trees are fairly easy to implement in real-world networks, due to their low nodal degrees, even though some optimisation problems arise. These problems are often coupled with the problems of wavelength assignment, as described in e.g. [Sti02]. Many of the physical aspects of designing, planning, and deploying fiber optical networks are treated in [NG02].

## 2.2 Related research areas

Even though the concept of SQoS is quite new, some of the problems within the field have previously been dealt with in other areas. Two areas seem to be the most important, namely graph theory, which is a discipline of Mathematics, and design of computer architectures.

### 2.2.1 Graph theory

Graph theory is of obvious interest because graphs are often used as abstractions of networks. They are particularly well suited for dealing with the structural qualities of networks. Basic notation and concepts can be found in most first-course books on discrete mathematics and graph theory, such as [RW99] or [Tru91]. The most important basics on e.g. routing are also provided in general books on computer networks such as [Tan03].

Background literature more directly related to SQoS has also been found, in particular within two areas, namely $N2R$ structures / Generalized Petersen Graphs and the disjoint-path problems.

$N2R$ structures were presented in [MNS02]. They are a subset of the Generalized Petersen Graphs, discussed in further detail in [FGW71], [Cox50] and [YFMA85]. The latter also discusses the Chordal Rings. The Generalized Petersen Graphs are again a subset of the group of 3-regular graphs, i.e. graphs where all nodes have degree three, also called cubic graphs. An extensive, theoretic study/survey of cubic graphs can be found in [GP95]. [Jør93] presents some interesting results on bounds of diameters in cubic graphs: no cubic graph with diameter 4 (or less) can contain more than 40 nodes. For my work, a more practical approach of calculating the distances rather than bounding them theoretically was applied, but the results of [Jør93] and [GP95] will form a good base for further studies with a more theoretical approach. Results of cubic graphs are interesting, both because they apply to the $N2R$ structures, and because the 3-regular structures are generally interesting from a network planning

perspective.

Independent - or disjoint - paths play an important role in protection schemes. The independent-paths problems are classical problems in graph theory, and a huge number of publications address such problems and variants hereof. [Tar97] gives a good overview of the most relevant fundamentals for my study. It deals with both the $k$-path and independent-$k$-path problems; in the first case, $k$ different paths between two nodes are determined, and in the latter $k$ independent paths are determined. The latter problem is the most relevant when studying protection schemes, but unfortunately it is in general NP-hard to solve. However, an approximation algorithm is proposed. Other papers deal with finding $k$ independent paths in specific types of graphs, such as the Trellis Graphs[NPT97] or Star Graphs[DMS91]. In some cases, these methods can be used in a more general context by mapping arbitrary graphs onto such specific graphs[LPS99].

The independent-path problems also give rise to parameters describing graph structures ability to offer multiple independent paths, and the length of such paths. One such measure is the total$_k$-distance[DEMZ00], another is the $k$-diameter[HLW03]. The latter paper is particularly interesting, because it shows that the Double Ring has the worst possible 3-diameter among all 3-regular, 3-connected graphs. I have just recently become aware of [Bha99], which also provides many interesting results and algorithms for the independent-path problems and variants hereof. It is in place to provide a last reference to [DC04]. This paper applies graph theoretical concepts in order to discuss robustness of various network topologies. It discusses the Moore Bound, providing lower bounds for diameter and average distances, but also a number of other parameters, linking robustness to network topologies. In particular, short distances, node symmetry, and maximum connectivity are considered important properties for a robust topology to satisfy, a statement supporting the choice of making analyses of $N2R$ structures a major part of the thesis.

### 2.2.2   Computer architecture

Multiprocessor systems and parallel processing systems are related to the field of networking by the fact that the processors or machines of such systems are usually connected by well-ordered physical topologies. This is for example the case for the transputer systems, i.e. [inm88]. Many results can be transferred more or less directly to the field of SQoS, but there are also major differences, of which some of the most important are described in Chapter 6. A good overview of different topologies is given in [Kot92], which describes different topologies and provides parameters for comparison and evaluation; more such parameters can be found in [DC04].

One of the most commonly used topologies is the 4-regular grid/ 2D mesh: it is

simple, and routing is straightforward. Much research in the field focuses on fault-tolerant wormhole-routing, e.g. [WS01] and [BC95], but multicast/broadcast routing, e.g. [JCX02] and [JW03], is also an important field of research. As discussed in Chapter 6, these results are not directly transferable to the field of SQoS. There are two major reasons for this. First, the deadlock problems that can occur in wormhole-switched networks do not occur in packet-switched networks. Second, it is generally unacceptable in communication networks to have to disable nodes without failure in order to ensure correct routing in the rest of the network.

The 4-regular grid forms the base for much of the research presented in this thesis. It was originally chosen due to its planarity and routing schemes, but during the studies it turned out to be possible to devise extensions of it in order to better fulfil the requirements of large-scale networks. These extensions have lead to significant results, and accordingly it has become a more important part of the SQoS-based research than expected at first.

Many other topologies have also been studied. The honeycomb [Sto97] is particularly interesting due to the fact that all nodes have degree 3, and because a routing scheme similar to that of the 4-regular grid exists. Related to the honeycomb are also the 2D hexagonal topologies described in [HF82] and [NSZ02], where all nodes have degree 6. [CM00] extends these definitions to a 3D hexagonal topology, where all nodes have degree 12. The honeycomb where all nodes have degree 3 is the most interesting in a SQoS-perspective, because topologies with nodes of high degree are impractical and expensive to deploy over large geographical areas without compromising the line and node independency. The honeycomb has been a part of my study, but mainly due to problems in constructing a hierarchical extension no significant results have been obtained, and thus no paper produced on it. However, the pruning introduced in Chapter 8, which is actually an extension of the 4-regular grid, transforms the 4-regular grid into a topology with structural properties closer to that of the honeycomb, but it still uses the addressing and routing scheme of the 4-regular grid. The schemes developed for handling failures in the 4-regular grid, as described in Chapters 6 and 8, can also be applied to the honeycomb.

A large number of other topologies exist, which in general also suffer from having large node degrees, and/or from being difficult to embed due to non-planarity. These include pyramid networks[CDHhT99], star graphs[MJ94] and hypbercubes[SS88][LKF03] as well as variants hereof such as the extended hypercube[KP92]. These are not considered further.

Multiprocessor systems also use rings and variants hereof. A number of extensions of the ring topologies have been proposed, such as the Generalized Double Rings[MNS02] and the chordal rings. A number of variants of the chordal rings exist, with different node degrees. In addition to the basic properties stated in [Kot92],

an extensive study of chordal rings of degree 3 and 4 was carried out in [Buj03]. This work is only available in Polish, but many of the results have been published in English in [BDZ04b] and [BDZ04a]. I became aware of this research during The 2004 International Conference on Parallel and Distributed Processing Techniques and Applications, June 2004. The degree 3 chordal rings were the most interesting to my research, because they share many properties with the $N2R$ structures. They are discussed and compared to $N2R$ structures in Chapter 11.

## 2.3 Conclusion

In conclusion, the study of state-of-the-art and background in SQoS and related areas, has lead to the following choices:

- Graph theoretic concepts and notation form the base for identifying parameters as well as for describing and evaluating structural properties of networks.

- 4-regular grid structures are considered one main focus area.

- $N2R$ structures are considered another main focus area.

# 3 | Problem Formulation

Until now, most networks have been planned in an ad-hoc manner, leading to networks without global structural properties. The idea of network planning based on Structural Quality of Service (SQoS) principles was introduced prior to the work presented in this thesis, but apart from that no research was done within the field. The aim of the work presented in this thesis has been to:

- Identify and document the needs for SQoS-based planning.

- Identify SQoS-parameters on which the planning should be based.

- Describe and analyse how and to what extent different well-ordered graphs/network structures conform to desired requirements, with particular focus on 4-regular grid and $N2R$ structures.

One challenge in the work has been to identify existing, relevant work, which could form a base for studies on SQoS. While the research field itself contained only few references, particularly relevant literature was found in the area of graph theory. Later in the study, I found that especially the area of parallel processing and computer architecture contained many interesting references as well.

Ideally, SQoS-parameters would have been described and identified prior to the analyses carried out on the different structures. In reality, the set of SQoS-parameters, which have been identified and presented in Chapter 5, have been developed continuously during the three years study, and is therefore not followed completely in the analyses carried out.

Despite of that, the thesis presents a completion of a first step of research in SQoS. The demands for SQoS have been identified, and so have important SQoS-parameters. $N2R$ and 4-regular grid structures have been described and analysed in-depth, clarifying the potentials of SQoS-based planning, as well as describing basic algorithms needed for implementations. Thus, the work described forms the base for moving on to the next step. Simulations and in particular test implementations are needed in order to adjust the various schemes and make them ready for real-world implementations. At the same time, studies can now be made on embedding the structures in such real-world networks.

# Part II

# Papers

# 4 | Reliability Demands in FTTH Access Networks [PKM05]

## 4.1 Preface*

This paper was written in collaboration with Thomas Phillip Knudsen and Ole Brun Madsen, and is the latest and least technical paper included in the thesis. It was presented at the 7th International Conference on Advanced Communication Technology (IEEE/ICACT 2005). The contribution to the thesis can be summarized as follows. The paper:

- Documents some of the most often made implicit and explicit assumptions within the field of network infrastructure planning, namely the needs for reliability in access networks.

The remainder of the thesis focuses on applying the concept of SQoS in order to become able to meet these demands.

## 4.2 Abstract

In this paper, reliability and bandwidth demands of existing, new, and expected future classes of applications running over Fiber To The Home (FTTH) networks to private users and small enterprises are analyzed and discussed. Certain applications such as home security and telemedicine are likely to require high levels of reliability in the sense that the demands for network availability are high; even short times without connectivity are unacceptable. To satisfy these demands, physical redundancy in the networks is needed. It seems to be the case that - at least in the short term - most reliability-critical applications do not require much bandwidth. This implies that redundancy does not need to be by fiber, but can be ensured by e.g. coax, copper or wireless solutions. However, implementing these solutions need careful planning to ensure the physical redundancy. In the long term, it is more likely that physical redundancy by fiber will be requested, either due to reliability-critical bandwidth-critical applications or due to the general dependency on network connections. The paper is ended by a discussion and suggestions for further research. In particular,

further research in combined wired/wireless networks and implementations of fiber redundancy is encouraged.

## 4.3 Introduction

The convergence of communication technologies[MNS02] has led to a large number of applications being capable of operating over a common communications infrastructure. This trend is supported by the implementation of fiber access networks, also known as Fiber To The Home (FTTH), which offer always-on functionality together with huge bandwidths. This can be seen in existing FTTH implementations, where telephony, radio, television, Video on Demand, and broadband Internet are delivered through the same single fiber connection. Since broadband Internet is included, the same fiber is likely to be used for Internet based applications such as surveillance, control of intelligent houses, health monitoring etc.[Lyn00][ND00]. While it is practical and cost-efficient to have one general-purpose network instead of a mixture of dedicated networks, it also affects reliability and dependability: when one single fiber is used for virtually all medias, the increasing dependability on this one connection must be considered. Furthermore, applications such as telemedicine and surveillance, where even a short time without connectivity may be very critical, may by themselves require a level of reliability, which can only be obtained by physical redundancy. Breach of a wired connection is expected to take from hours to days to restore.

Despite the development of reliability-critical applications and the increasing general dependency, most FTTH implementations today do not offer any physical redundancy to the end user. Recent research has dealt with developing and describing robust network topologies[DC04], but since redundancy in the last mile in FTTH networks is assumed to be expensive and often impractical, only little attention has been paid to this. We recently showed that wireless back-up connections could be designed as an integrated part of a FTTH network[PRKM04]. However, the potential bandwidth of a wireless network is much smaller than that of a fiber network due to the limited and shared bandwidth.

The main objective of this paper is to initiate a discussion of reliability and redundancy demands in last-mile FTTH implementations. This is done in two steps.

First, reliability and to some extent bandwidth demands of existing applications as well as applications expected to become widely used within 10-15 years are analyzed. Only applications for private households and small enterprises are taken into account since public institutions and larger enterprises are expected to be connected by individual solutions rather than by general-purpose access networks. In particular,

ordinary access networks are not expected to carry applications such as tele operations, control traffic for nuclear power plants or traffic for a complete emergency central.

Second, a discussion is given on the possibilities of offering physical redundancy in the last mile, and areas for further research are identified.

The work presented complements existing research in application demands, which has mainly focused on bandwidth and Quality of Service. The reliability offered is highly dependent on the physical topologies, and in case of FTTH networks, these are expected to have a long lifetime. Since they are expensive to change once implemented, they should be designed either to meet future reliability demands, or at least in a manner such that they are easily extended to meet these. In order to be able to do so, knowledge of reliability demands is crucial both for research in reliable networks and for practical network planning.

## 4.4   Methods

The analysis of reliability and bandwidth demands is carried out in three steps. Identification of existing applications is done by studying published literature on Internet applications as well as the services offered by major Danish FTTH providers. Identifying applications which will become widely used during the next 10-15 years is harder, because new applications are being developed all the time, and many technical and non-technical factors influence their success and speed of propagation. However, important trends are pointed out.

In the second part of the analysis, applications are classified according to how critical they are with regard to reliability and bandwidth, using rather narrow classification schemes. The applications are first divided into two groups, depending on how critical they are to short-term outages (from a few minutes to a few hours). The group of reliability-critical applications are those, which carry life/death responsibility as well as applications where even short-term outages are likely to lead to significant economic losses. In both cases, it is crucial that no single failure causes the connectivity to break. All other applications are categorized as being non-critical. The reliability-critical applications are also called high-reliability demanding (or simply high-reliability), and the non-critical applications called low-reliability demanding (or simply low-reliability). Since focus is mainly on the last-mile reliability, the importance of applications is seen from a single-user point of view: for example, a small set of private households may be able to live without applications such as WWW for a limited time, whereas disconnecting a whole city or a whole municipality may have other implications. Reducing the risks for such large network failures does not nec-

essarily require redundancy in the last mile, and these problems are not discussed further. For certain groups of applications, it is crucial that at least one application from each group has a high level of reliability, but less important which one. For example, it is crucial for a community that radio and television are not simultaneously unavailable because broadcast communication is important in case of catastrophes, emergencies, wars etc. Since the high-reliability high-bandwidth applications are the most important to consider when designing redundancy, high-reliability applications are treated most in-depth with regard to bandwidth. The applications are divided into two groups - high-bandwidth and low-bandwidth - depending on whether they use high-quality video or not[KPPM04]. This is due to the assumption that applications not using high-quality video are able to run over other access technologies than FTTH, including wireless, copper, and coax. However, Many applications, which can run over slow Internet connections, run better over fast connections, and each of the access technologies offers different bandwidths depending on a number of parameters.

In the last part of the analysis, network reliability and needs for redundancy are discussed in a broader perspective. The classifications given above are loosened up, such that also less reliability-critical applications are discussed.

It is in place to motivate the method used; offering physical redundancy is one way to obtain a high availability, but choices of equipment, wiring, cables, and ducts are also important. In the end, measures such as availability, mean time of failure and mean time of repair may be more appropriate to use - and more interesting for the end users - than redundancy or no redundancy. The reason for using the method described is that the degree of the redundancy is highly dependend on the physical topologies and structures of the network, whereas the other factors are more a matter of equipment. FTTH Networks are expected to have a long lifetime, at least 30 years, and they are designed to be upgradeable by changing end equipment only. Together with the fact that the major costs of FTTH networks are the duct costs, this means that the equipment is both cheaper and easier to upgrade than the physical topologies. Therefore, these topologies should to the largest possible extent be chosen to meet the demands of the future, including demands for redundancy. The applications classified as reliability-critical are applications, for which it is unacceptable that a single cable cut can disconnect the network, independent of the reliability of the equipment used.

## 4.5   Identifying applications

The digitalization of communication has created a base for media convergence, and more and more applications are being designed to communicate over IP. An example is telephony, which has traditionally been sent over circuit switched networks. The

use of Voice over IP is increasing fast, and TV over IP is likely to be the next step. In order to identify the different groups of applications, this section is divided into Applications today, Current applications moving from LAN to WAN environments and New applications. The aim is not to give a complete list of applications, but to cover the most widely used applications today, as well as those expected to become widely used within the next 10-15 years. The list may not cover all groups of applications, but we believe it is representative for the most widely used applications today.

### 4.5.1 Applications today

**Classical Internet applications**

Well known Internet applications today are WWW, Telnet, Email, FTP, and other types of file transfers, e.g. Peer-to-Peer. WWW-traffic can be split into a number of applications including entertainment, information retrieval, home banking, business use and shopping. Many of the applications are client-server applications, where the servers have traditionally been placed at places with dedicated connections. However, private servers are becoming increasingly popular.

**Person-to-person communication**

The Internet is to an increasing extent being used for real-time person-to-person communication such as instant messaging, Voice over IP, chat, and video-chat/video-conferencing. These can be one-to-one, one-to-many or many-to-many.

**Entertainment and broadcast**

Internet-based broadcast services like radio and television, and also other non-interactive services such as Video on Demand, are becoming increasingly popular, and are also implemented in most FTTH solutions. Slightly more interactive services are for example digital television, where the viewers can vote or by other simple means provide feedback while watching television. Today, radio programs where the listeners can participate in the program by phone are popular. This also defines some level of interactiveness, which may be transferred to the television as well, since the bandwidth of FTTH networks allows the viewers to participate through video sessions. Other kinds of interactive entertainment include gaming over networks.

**E-learning**

E-learning is mainly based on technologies already listed, including WWW, file transfers, and real-time person-to-person communication. Therefore, it is not further discussed.

**Home offices**

More and more companies offer the employees the possibility to work fully or partly from home. This brings a number of technologies into play, including various person-to-person communication means and remote access to desk tops and data. A number of new technologies are expected to be used as well, as described in Section 4.5.3.

### 4.5.2   Current applications moving to WAN

**Telemedicine and health care**

Much research today is made in the area of tele health and medicine. This area is not quite new, and many fields exist within it[Fro00]. We shall differ between tele care/monitoring and other applications. Tele care/monitoring includes real-time monitoring of ill or elderly people, for example by sending information on heart rate, cardiograms or lung functions directly to a hospital for real-time analysis. Possibly, information can also be send the other way, and used e.g. for adjusting medication. Other applications such as information search, e-learning, and real-time consultations are covered by previously discussed technologies.

**Remote home control**

Various applications in the field of remote control of homes are being developed, and the perspectives are many: it is possible to control such things as temperature and electric equipment from a remote location, for example by WWW[MAC$^+$01]. This also allows for home security solutions, where the high bandwidth is used for sending audio and video. This can be used to record video/audio clips of alarm events, but also to get real-time video from the home, to watch for example pets and kids.

### 4.5.3   New applications

It is impossible to predict what applications will become the most widely used during the next 10-15 years, but given the trends in communication and networking, the

following factors are expected to be important in application development:

- Allways-on.

- High bandwidth.

- QoS.

The main difference between FTTH and today's broadband solutions is the bandwidth, which allows for real-time video and similar. This will probably lead to further developments in the area of entertainment and video conferencing. While the possible developments in the field of entertainment are virtually unimaginable, the developments in video meetings and video conferencing also hold great potentials. For example, it will become easier to teach people in their private homes, and even the teacher can sit in his private home. Not only the teacher can be seen, but also presentations, videos, and similar, and the communication can be both ways. Similarly, it will become easier to hold meetings over distance, and cooperation over distance will be facilitated.

Machine-to-machine communications is another important field, expected to develop as more intelligent machines are entering the private houses. Until now, transfer of high-quality video has been considered the major bandwidth consumer, but this may change in the future. Today, most companies and institutions make back-ups of data more or less manually. This is often done in short intervals, e.g. each night or each week. In near future, even non-critical private data such as photo albums and home videos are expected to be stored or backed up in storage area networks. It is also expected that working from home will become still more common, all together adding to increasing needs for remote back-ups.

Similarly, it is a common vision that in near future, it will become possible to access private data, desk-top configurations, settings, and applications from any computer connected to the Internet. Making this transparent to the user also requires huge bandwidths.

## 4.6 Analysis of bandwidth and reliability demands

### 4.6.1 Applications today

**Classical Internet applications**

For most of these applications, shorter outages are acceptable, and as such they are not considered reliability-critical. Certain WWW applications such as home banking

may however be critical, because it is important to be able to pay bills and perform other important transactions on time. The increasing use of home banking may also decrease the number of bank branches, thus limiting the alternatives to home banking. Such problems are worsened if telephony uses the same last-mile technology as WWW. Access to WWW may be important in case of wars, disasters or similar. However, it is assumed that information in these cases can as well be carried via TV, radio or some other broadcast media, and therefore it is not considered critical.

The bandwidth demands varies between applications, but in general more bandwidth gives better performance. This is in particular so for transfer of images and video, which are common in file transfers, and also becoming increasingly integrated with WWW-surfing.

A special remark is given to server hosting; the bandwidth demands can vary greatly, but ordinary Internet connections are not assumed to be used for reliability-critical information, and they are therefore categorized as non-critical. If redundancy is offered, they will more likely be used also for reliability-critical applications and data.

Thus, most applications are low-reliability low-bandwidth or low-reliability high-bandwidth. Some high-reliability low-bandwidth applications also exist.

**Person-to-person communication**

As for WWW access, it is assumed that most important information to the public can be delivered through broadcast medias, and therefore most of these person-to-person applications are not considered reliability-critical.

In Section 4.3, it was defined that only reliability demands for ordinary households and small enterprises were considered, but it is not so trivial to define what is an ordinary household. For example, fire fighters, policemen, soldiers, and civil defence people are usually normal people, moving to and from normal houses. Therefore, it is desirable if the normal communication networks offer a reliability satisfactory also for these purposes. Thus, voice communication is categorized as reliability-demanding.

With regard to bandwidth, the video applications require much bandwidth while the rest do not. Therefore, this group of applications contains both low-reliability high-bandwidth, low-reliability low-bandwidth, and high-reliability low-bandwidth applications.

**Entertainment and broadcast**

Entertainment and broadcast contain many different applications. The broadcast applications are considered reliability-critical because they are important in case of catastrophes, disasters, wars, and in general for communication of messages in case of water pollutions, fires etc. It does not matter if radio or television is used: the most important is that there is at least one media available. Other applications in this field are not considered reliability-critical.

With regard to bandwidth, simple radio communication does not require much bandwidth, and even some low-quality television can be sent using only a limited bandwidth. This is especially so if broadcast protocols can be used, eliminating the shared-bandwidth problems of coax and wireless technologies. Some of the other not reliability-critical applications require high bandwidths.

In conclusion, this group of applications contains low-reliability high-bandwidth, low-reliability low-bandwidth, and high-reliability low-bandwidth applications.

**Home offices**

The more specific applications are described and analyzed in other sections, and due to the definition of reliability-critical, they are not classified as such. However, the increasing focus on performance, quality, and reliability of computers and networks in businesses[Gro04] is expected to be reflected in the requirements to home offices, such that the demands will approach those of the business networks. This does, at least, increase the general reliability demands.

### 4.6.2 Current applications moving to WAN

**Telemedicine and health care**

The tele care and monitoring applications are high-reliability, whereas the other health applications are low-reliability.

The bandwidth demands are harder to determine, because the field is developing so fast. Currently, most of the high-reliability applications transmit only small amounts of data, and as long as the applications are machine-to-machine, they are expected to require only low bandwidths. This assumption is supported for a part of the patients, since they are ensured mobility by making it possible to communicate over the mobile phone network, thus not requiring a broadband connection. The demands may increase in the future if the applications are integrated with video monitoring. The bandwidth demands of the low-reliability applications are higher, because they

involve high-quality video presentations and conferencing. However, some of the applications require less bandwidth.

Therefore, this group of applications contains low-reliability high-bandwidth, low-reliability low-bandwidth, and high-reliability low-bandwidth applications. In the future, high-bandwidth high-reliability applications may be developed, but these will probably not become widely used unless the reliability demands can actually be satisfied.

**Remote home control**

The demands for reliability depend on a number of factors, including the application design. For example, applications for controlling windows, doors, and electrical equipment should be designed to handle network failures in a specified manner, and as such they are not reliability-critical. Home security and home surveillance/monitoring are however considered high-reliability demanding because it should not be possible to interrupt their functions by cutting a wire.

The bandwidth demands depend on whether video transmissions are integrated in the applications or not. Today, home surveillance is usually done using sensors, and if a sensor is activated, the alarm is started and a message passed to some control centre and/or a phone number. This does not require much bandwidth. If the alarm is integrated with one or more video cameras, the bandwidth demands are higher. Monitoring of children who are home alone will usually require a number of cameras, which should preferably but not necessarily be able to send data simultaneously. For all the cases listed, human action eliminating the consequences in case of network failures can be taken; monitoring children or pets only make sense if someone can take action if something goes wrong, and for alarm systems, actions can be taken similar to that of the alarm going off. On the other hand, it is also in both cases desirable to have a (possibly low-bandwidth) back-up in order to avoid a collection of houses being disconnected and emptied by thieves. Therefore, such low-bandwidth applications are characterized as high-reliability demanding. In addition, there are other low-reliability low-bandwidth and low-reliability high-bandwidth applications.

### 4.6.3 New applications

During the analysis, a number of low- and high-bandwidth demanding applications were identified, but none of these seem to be reliability-critical. However, it is likely that a wider deployment of FTTH networks will lead to more bandwidth-demanding applications being developed, including applications demanding reliability. This is particularly so if the reliability is offered, such that a base for such applications exist.

## 4.7   The broader perspective

Even for applications not categorized as reliability-critical, there is an increasing dependency on broadband networks. When working from home, critical situations may occur, if an email is to be sent by a certain deadline, or if a lecturer has to give a lecture at a certain time. It may also be necessary to access informations at the WWW, make a voice call or participate in an important video meeting at a given time. Many more cases could be mentioned. It follows that in order to fully benefit from the technological opportunities, the connections must be reliable.

For the FTTH providers, redundancy also plays another important role: it allows for more efficient planning of maintenance and repairs. If physical redundancy is offered, a single error becomes less critical and may not require instant repair.

As stated in Section 4.3 and shortly discussed in Section 4.6.1, only reliability-criticalness for ordinary home end users and small enterprises were considered. However, it is likely to expect an increasing demand for redundancy, either for specific purposes or because of general dependability on networks. Such specific purposes also cover those not considered reliability-critical in this paper.

## 4.8   Conclusion

It turns out that the applications analyzed fall in three main categories: high-bandwidth not reliability-critical applications, low-bandwidth reliability-critical applications, and low-bandwidth not reliability-critical applications.

The main application categories which are reliability-critical are the following.

- Telemedicine including health monitoring.

- Surveillance, home security, home control applications, and other applications communicating machine-to-machine.

- Communication applications such as voice and radio.

Future developments may lead to more integration of high-quality video in these applications, creating demands for both high bandwidth and high reliability.

In addition to these specific reliability-demands, the general dependability on network connections is also expected to increase: the trend of individuals, businesses, and communities relying to an increasing extent on communication networks is expected to continue.

## 4.9   Discussion

The analysis showed that not all applications expected to become widely used within 10-15 years should be vulnerable to single points of failure in the last-mile technologies serving private end users and small enterprises. On the other hand, the most reliability-critical applications seem not to require much bandwidth, indicating that some low-bandwidth alternatives can be used for back-up to the individual private end users and small enterprises.

In an environment of converged communications, these alternatives can be provided using todays broadband solutions such as copper and coax, as far as they are available. However, physical redundancy is only ensured if these networks and the FTTH networks do not share ducts and nodes. Unless specially planned for, they mostly do so. Even if physical redundancy is ensured, it is unsure if there is base for maintaining a complete cabled network only for back-up purposes.

Wireless back-up solutions provide sufficient bandwidth for the most critical applications. This motivates further research to be conducted on design of integrated wired/wireless solutions, where the physical connections are fully independent, and where seamless protection and restoration schemes are supported. In order to obtain a sufficient bandwidth for back-up purposes, the base stations need to be placed not too far from each other, and will most likely be connected by the FTTH fiber infrastructure. It is therefore necessary to connect these base stations in a manner ensuring physical redundancy.

A final possiblity, and probably the most appropriate long-term strategy, is to implement the FTTH networks in a way ensuring complete physical redundancy. Until now, this has been considered impractical and expensive, but apparently this does not always hold. For example, consider a FTTH implementation on a residential street with houses on both sides of the street. Figure 4.1 shows the usual wiring, but if the wiring is done as shown in Figure 4.2 instead, wired redundancy is ensured. While the costs in terms of equipment and fiber may be a bit higher, the duct costs (which are the most significant) are only slightly higher. In order to take advantage of this redundancy, it is however necessary to develop configurations and preferably passive equipment supporting restoration/protection schemes.

It is certain that designing FTTH networks with redundancy is cheaper than designing non-redundant networks and adding the redundancy later on. We therefore believe that the FTTH implementations where possible should be designed either to offer redundancy, or in a way such that redundancy can be added afterwards by changing as little a part of the infrastructure as possible. Doing so also makes it possible to offer redundancy to those requesting it, even when people are moving, without needs for additional investments.

Despite the possibilities of offering wired redundancy, wireless solutions have some major benefits due to the different failure characteristics. In particular, it is more difficult to disconnect a household on purpose, because the wires cannot be cut, making it suitable for surveillance and monitoring. A wireless back-up is also - on a larger scale - less vulnerable to failures caused by e.g. wars or nature catastrophes, but due to the shared-bandwidth problem it would require a larger number of base stations to ensure sufficient capacity in such cases.

As a final remark, it should be mentioned that the most demanding applications are likely to be developed only if they are supported by the infrastructures. Unless sufficient redundancy - even in the last mile - is ensured, the networks will remain unreliable, and it will not be possible to fully benefit from the technological opportunities. Thus, the importance of a carefully planned infrastructure should not be underestimated.
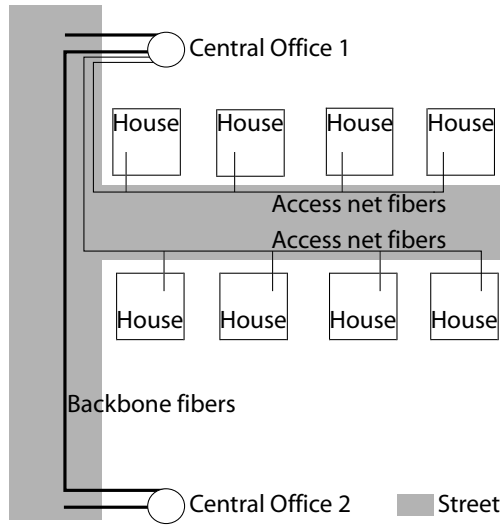
**Figure 4.1:** Typical FTTH-wiring on a residential street with houses on both sides of the street. The solution may be fiber rich (providing a single fiber for each house), or the fibers may be shared.
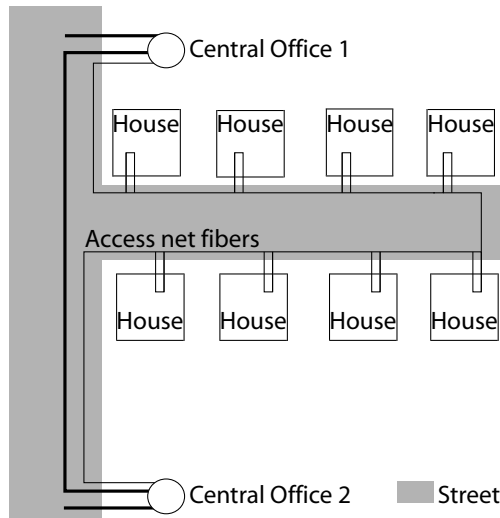


**Figure 4.2:** Proposed FTTH-wiring, ensuring physical redundancy. Compared to Figure 4.1, each house is now provided with a double wiring and connected to two Central offices.

# 5 | An Evaluation Framework for Large-Scale Network Structures [PKM04a]

## 5.1 Preface*

This paper was written in collaboration with Thomas Phillip Knudsen and Ole Brun Madsen, and presented at the Information Technology and Telecommunications Annual Conference 2004 (IT&T 2004). The contribution to the thesis can be summarized as follows. The paper:

- Identifies important SQoS-parameters and their impact on network performance in a broad sense.

It is written after most of the papers on 4-regular grid structures (Chapters 6 – 8), but to a wide extent the parameters have been used in these studies as well as in the studies on $N2R$ structures described in Chapters 9–12.

## 5.2 Abstract

An evaluation framework for large-scale network structures is presented, which facilitates evaluations and comparisons of different physical network structures. A number of quantitative and qualitative parameters are presented, and their importance to networks discussed. Choosing a network structure is a matter of trade-offs between different desired properties, and given a specific case with specific known or expected demands and constraints, the parameters presented will be weighted differently. The decision of such a weighting is supported by a discussion of each parameter. The paper is closed by an example of how the framework can be used. The framework supports network planners in decision-making and researchers in evaluation and development of network structures.

## 5.3    Introduction

Existing copper based access network infrastructures are about to be replaced by new mainly fibre based infrastructures in a large number of countries throughout the world. This is probably the largest investment in IT infrastructure ever, and the duct costs form a considerable part hereof. Once implemented, the physical topologies and structures deployed are expected to have a long lifetime, an assumption supported by the fact that fibre networks are upgradeable by changing end equipment only. The huge investments and expected long lifetimes make it important to choose the physical topologies carefully, and ensure that they fulfil future demands to the largest possible extends while still being cost efficient. While the copper based infrastructures have mainly carried telephony and recently also best-effort data services, the fibre networks introduce a huge increase in bandwidth and support a much larger set of applications. This includes control applications such as tele robotics and tele operations[HKKK01][XT98][XmCjYXd04] as well as audio, video, and multimedia applications. Applications previously confined to LAN environments have already started to migrate to WAN environments, and this trend is expected to continue in near and far future[MNS02] as new applications are developed and supported by the infrastructure. Many of these new applications are not only bandwidth demanding, but also critical with regard to other parameters including QoS parameters and reliability. At the same time, the general dependency on networks is increasing. New protocols such as MPLS and RSVP have been developed in order to satisfy new demands, but given the fact that no protocol can perform better than what is allowed by the underlying physical infrastructure, surprisingly little research has been done in the area of the latter. The ability to offer independent paths for protection, efficient routing schemes, and resistance against attacks or failures are examples of properties closely related to the physical infrastructure. This paper deals with describing such structural qualities of networks and abstracts from equipment specific considerations. While recent research has introduced the concepts of Structural QoS[MNS02] and Sustainable QoS[MK96] as well as a number of network structures[PKM04c][Sto97][MKP03], no commonly agreed upon evaluation parameters have been established, except for very basic parameters such as diameter and average distance, making it difficult to compare and evaluate proposed structures. This paper identifies and presents a set of such evaluation parameters, taking recent studies of attack resistance[DC04] into account as well. The main contribution is the provisioning of an evaluation framework for large-scale network structures that facilitates easy evaluation and comparison of different such structures. A similar framework for parallel processing systems was established in [Kot92], and even though it is not directly applicable here due to the different natures of the systems, some parameters are either the same or closely related. It also contains a number of parameters which complement the work presented

in this paper. The scheme presented is useful both for research and network planning purposes. The number of parameters is kept relatively low, and presented such that relevant parameters are easily identified given knowledge of expected traffic patterns and demands. This also implies that the parameters are of a general character, and that simplifications have been made.

## 5.4 Terminology and definitions

A structure $S$ consists of a set of nodes $N(S)$ and a set of undirected lines $L(S)$ such that each line connects two different nodes. Where unambiguous we just write $N$ and $L$. The numbers of nodes and lines are written $|N(S)|$ and $|L(S)|$. $|N(S)|$ defines the size of $S$. A structure represents a network but is an abstraction from specific physical conditions. All lines are considered identical, whereas the nodes are characterized by their degree: for a node $u$, its degree is the number of lines joined to it, written $deg(u)$. A path $p$ between a node $u$ and another node $v$, is a sequence of nodes and lines $(u = u_0), e_1, u_1, e_2, \ldots, u_{n-1}, e_n, (u_n = v)$, such that each line $e_i$ connects two nodes $u_{i-1}$ and $u_i$. The length of a path corresponds to the number of lines it contains, and so $p$ has length $n$. The distance between two nodes $u$ and $v$ is the length of a shortest path between them, and is written $dist(u, v)$.

Assume that there exists another path $p'$: $(u = u_0), e'_1, u'_1, e'_2, \ldots, u'_{m-1}, e'_m, (u_m = v)$ between $u$ and $v$. $p$ and $p'$ are line independent if $e_i \neq e'_j$ for all $i$ and $j$ such that $1 \leq i \leq n$ and $1 \leq j \leq m$ and node independent if $u_i \neq u'_j$ for all $i$ and $j$ such that $1 \leq i \leq n-1$ and $1 \leq j \leq m-1$. A set of paths are said to be line independent if they are pair wise line independent, and node independent if they are pair wise node independent. Node independence implies line independence for paths longer than one, while the converse does not hold in general. Since in communication networks both nodes and lines can fail, we consider a set of paths independent only if they are all node and line independent. Throughout the paper it is assumed that all structures without failures are connected: for every pair of nodes $u$ and $v$, there exists a path between them. If in case of failures the remaining structure contains no path between $u$ and $v$, we define $dist(u, v) = \infty$ in this remaining structure. Two nodes $u$ and $v$ are said to belong to the same component of a structure if and only if $dist(u, v) \neq \infty$.

## 5.5 Methods

The parameters are divided into quantitative and qualitative parameters. For the quantitative parameters, global and local parameters are used. Global parameters are associated to the structure as a whole, for example $|N|$ and $|L|$, whereas local parameters

are associated to nodes or pairs of nodes. In order to facilitate easy evaluations and comparisons, global parameters are derived from local parameters:

- Let $M$ be a parameter associated to each pair of nodes. For a node $u$, globalizations of $M$ associated to $u$ are determined by taking minimum, average or maximum of $M(u, v)$ over $u$ and all nodes $v$ different from $u$. The obtained parameters are written $M_{min}(u)$, $M_{avg}(u)$, and $M_{max}(u)$ respectively.

- Let $M$ be a parameter associated to each node. Global parameters are obtained by taking the minimum, average or maximum of $M(u)$ over all nodes $u$. The obtained parameters are written $min(M)$, $avg(M)$, and $max(M)$ respectively. These globalization methods are used implicitly in the remainder of the paper.

A parameter is said to be basic if it is not derived from any globalization as described above. In order to keep the number of parameters down, only selected global parameters are included, and the importance of each chosen parameter validated by its importance to performance or implementation. The same applies to the selected qualitative parameters. A classification scheme would be useful for quantifying the qualitative parameters, and has also been developed for some of the parameters with regard to parallel and multiprocessor systems[Kot92]. However, we find it hard to develop a reasonably simple classification scheme, which takes into account all different aspects of the parameters as well as the large variety in demands and constraints that apply to large-scale networks to be deployed under diverse physical conditions. For a concrete network planning case, appropriate aspects and physical conditions can be taken into account and a classification scheme developed.

## 5.6 Identification of parameters

### 5.6.1 Quantitative parameters

**Support for different traffic schemes**

The choice of parameters obviously depends on what kinds of traffic are to be accommodated, and consequently the parameters are divided into traffic types. The structural impact on a networks ability to satisfy application demands is mainly related to distances and connectivity: shorter paths generally imply shorter delays and less overall traffic in the network, and a high connectivity ensures the ability to transmit data even in case of failures. Support for best-effort traffic and traffic, which

is either QoS or reliability demanding, is generally described by the following well known and widely recognized parameters:

- Diameter: $max(dist_{max})$.

- Average distance: $avg(dist_{avg})$.

- Connectivity number: $min(I_{min})$, where the basic parameter $I(u, v)$ is the largest number of independent paths which can be established between $u$ and $v$.

These parameters and the parameters introduced in the following are all build on an assumption of all-to-all traffic. In access networks, a large part of the traffic is to and from main nodes, such as servers or nodes providing access to the Internet. It is not difficult to adjust the parameters to take into account such a different traffic distribution. In order to evaluate the support for applications that are both QoS and reliability demanding, a set of parameters are introduced as a supplement to the three parameters stated above. Three different traffic schemes are considered:

Protected traffic, using $1 + N$ protection: Traffic between two nodes $u$ and $v$ is sent along $k = N + 1$ independent paths. In order to evaluate the structures ability to support this scheme, the measures $k$-diameter and $k$-average distance are used as a generalization of the classical diameter and average distance measures. Since $k$ independent paths between two nodes can generally be chosen in different ways, two approaches are suggested: $k$-$maxdist(u, v)$ is the longest of $k$ independent paths between $u$ and $v$ chosen such that the longest of these paths is shortest possible and $k$-$avgdist(u, v)$ is the average length of $k$ independent paths between $u$ and $v$ chosen such that the average length is smallest possible. It is possible that some sets of paths fulfil the criteria of both measures, but in general the following considerations apply. The choice of paths used for $k$-$maxdist(u, v)$ is optimal in the sense that it reduces the length of the longest path, and can be used for guaranteeing that a connection between $u$ and $v$ can be set up with $k$ independent paths, of which none are longer than $k$-$maxdist(u, v)$. This is useful for applications that are very delay-sensitive yet reliability demanding, and can be used to indicate the structures ability to support such applications. However, this way of choosing the paths is not in general desirable: compared to the choice of paths used for $k$-$avgdist(u, v)$, the average distances can be longer, increasing the likeliness of failures as well as the total amount of traffic generated. Two global parameters are obtained:

- $k$-diameter: $max(k$-$maxdist_{max})$.

- $k$-average distance: $avg(k$-$avgdist_{avg})$.

Assuming that protected paths are set up minimizing the average distances, the $k$-average distance gives a good indication of the traffic generated. If large amounts of traffic are expected to use multiple independent paths set up such that the longest paths are shortest possible, an alternative measure is $avg(k\text{-}maxdist_{avg})$. Another approach, which should be mentioned, is to consider roads instead of paths, where roads are defined as paths with a specified maximum length[BDZ04b]. However, deciding this maximum length is not trivial. Even given specific delay requirements, the delay also depends on protocols, transmission delay etc.

Traffic using restoration: Traffic between two nodes $u$ and $v$ is sent on one path. If a failure occurs, a new path is set up. To describe what guarantees can be provided let $k\text{-}failure\text{-}maxdist(u, v)$ be the largest value of $dist(u, v)$ which can be obtained in any structure constructed by removing from the original structure $k$ nodes or lines different from $u$ and $v$. This leads to the following global parameter:

- $k$-failure-diameter: $max(k\text{-}failure\text{-}maxdist_{max})$.

Given at most $k$ failures, the diameter in the remaining structure does not exceed $k$-failure-diameter. $k\text{-}failure\text{-}avgdist(u, v)$ being the average distance between $u$ and $v$ given $k$ failures of nodes or lines different from $u$ and $v$, $avg(k\text{-}failure\text{-}avgdist_{avg})$ can be used to predict traffic load in case of $k$ failures, but it would be more interesting to analyse the load distribution in case of failures. To calculate $k\text{-}failure\text{-}avgdist(u, v)$ all possible ways of removing $k$ lines or nodes must generally be considered, a huge task.

Protected traffic using $N : M$ protection: Traffic between two nodes is sent on only one path, but backup paths are set up when the connection is established, and traffic can be switched over whenever a failure is detected. The $k$-diameter and $k$-average distance are useful for describing the structures ability to support this kind of traffic. Alternatively, a weighted $k$-average distance, where the primary path is assigned the largest weight, can be used. Schemes combining $N : M$ protection and restoration have been introduced even for QoS-demanding traffic[BBO$^+$03]. Such schemes are able to reuse parts of the primary paths for the backup paths, and it follows that the ability to support such schemes is more similar to the ability to support traffic using restoration. Consequently they can be evaluated by the same parameters.

General remarks: For any pair of nodes $u$ and $v$, $k\text{-}maxdist(u, v)$, $k\text{-}avgdist(u, v)$, and $k\text{-}failure\text{-}maxdist(u, v)$ are all NP-hard to determine for arbitrary structures. Depending on the value of $k$ and the size of the structure, the above suggested methods for finding the paths for determining $k\text{-}maxdist(u, v)$ and $k\text{-}avgdist(u, v)$ may be replaced by simpler algorithms, which may be chosen specifically to suit a given structure. If these algorithms are to be applied in a possible implementation, the

obtained results also reflect the expected performance better than the use of e.g. $k$-$maxdist$ and $k$-$avgdist$. This also relates the parameter to the qualitative parameter algorithmic support.

### Connectivity

It is crucial for both users and network operators that a network does not split into more components in case of failures. The connectivity number introduced is not necessarily a good measure, since even the disconnection of a single node is considered such a split. A more general measure describes the minimum number of nodes to be removed in order to obtain a structure with no component containing more than $|N| - k$ nodes:

- $k$-connectivity number.

### Cost

The cost of a structure mainly deals with the initial cost of establishing it, but the parameters introduced also to a limited extend describe the cost of maintaining it. As the number of nodes defines the size of a structure, the cost is measured by the number of lines. However, in order to take into account the size of the structure, the average node degree can be used instead. This also directly reflects that nodes with large degrees are in general more expensive than nodes with smaller degrees. Noting the one-to-one correspondence for any given structure, the global parameters are:

- Average node degree: $avg(deg)$.

- The number of lines: $|L|$.

Since the dimensioning of nodes and lines depends on the expected amount of traffic, the average and $k$-average measurements yet introduced are also useful. The qualitative parameters embeddability and algorithmic support also influence the cost.

## 5.6.2   Qualitative parameters

### Algorithmic support

Some structures such as rings, double rings, and mesh structures offer simple algorithms for routing and restoration, ensuring that optimal paths can be found and connectivity fully utilized without using large resources on storing and updating tables,

even in case of some limited numbers of failures. The existence of such algorithms is important in order to benefit from structural properties. It is of particular interest that protection schemes are supported, since establishing multiple independent paths with certain constraints or optimisation criteria is in general NP-hard. Algorithms can be evaluated on issues such as complexity and equipment demands, but also on their ability to find good paths, e.g. by evaluating what average distance, diameter, $k$-average distance, and $k$-diameter can be obtained. Depending on the choice of transmission technology, more algorithms can be taken into account. For example, wavelength routing plays an important role in fibre optical networks.

**Embeddability**

Depending on the physical conditions, some structures are easier to embed than other: in general, structures with planar embeddings, i.e. embeddings without crossing lines, are often preferred over structures without planar embeddings, even though a limited number of crossing lines may not be a problem. The embeddability of a given structure is also highly dependent on the specific physical conditions: some areas are better suited for ducts than others (i.e. open land vs. water), and existing ducts, roads or railway lines may be preferable to use or difficult to cross. Embeddability also covers the ability to design the network with variable and inhomogeneous node distributions. This is important when networks are deployed in areas with various node densities, e.g. densely vs. sparsely populated areas. Embeddability also influences the cost of the network.

**Expandability**

Expandability or extensibility describes the ability to insert nodes into a structure after it is implemented without changing its basic properties. It is crucial that it is possible to insert both smaller and larger sets of nodes when needed without having to restructure the network, since such restructurings may require re-establishment of ducts and thus become extremely expensive. Some structures constrain what number of nodes are allowed, such as the double ring and $N2R(50; 9)$ described in Section 5.7. In general, it is preferable if nodes can be inserted one at a time, but in any case the minimum number of nodes required to expand the structure should be smallest possible. On the other hand, it is desirable that structures scale well such that even larger numbers of nodes can be inserted without affecting the quantitative parameters too much. It is also important that nodes can be inserted anywhere in the structure, or least in a way, which causes as few geographical and physical constraints as possible. Expandability also describes the ability to deploy a structure gradually.

**Robustness**

Robustness is related to the quantitative parameters concerning support for different traffic schemes, and deals with the networks ability to resist attacks and failures. [DC04] concluded that especially two properties, node symmetry and optimal connectivity, were important to ensure robustness in network structures: node symmetry ensures that no nodes are more important than others, and thus no parts are particular vulnerable to attacks, and the optimal connectivity ensures the existence of as many independent paths as possible, given the number of nodes and lines provided. Both properties can be satisfied to smaller or larger degree: the $N2R(p;q)$ structures[MNS02] are examples of optimally connected but not necessarily node symmetric structures, which share many of the properties of node symmetric structures. Other quantitative parameters, in particular the connectivity number and $k$-connectivity number, are also useful for evaluating robustness. Traffic load and distribution in case of failures is also important, but usually at least some assumptions on failure probability of nodes and lines must be taken into account in order to determine properties in these regards.

## 5.7   Example

In order to demonstrate how the scheme is used, different structures, each of 100 nodes, are compared. The evaluated structures (see Figure 5.1) are ring, double ring, and $N2R(50;9)$. The choice of evaluation parameters depends on the expected use of the network. For this example, the network is assumed to be an access network implemented by wired (fibre) solutions, and the two most important traffic classes to be best-effort traffic and 1:1 protected traffic. As a result, the selected parameters are average distance, 2-average distance, diameter, connectivity number, line number, and all the qualitative parameters listed in this framework. Since all the structures contain the same number of nodes, the line numbers indicate both cost and average node degree. The comparison is shown in Table 5.1.

The ring is the cheapest structure measured by the number of lines, and also the structure that is most easily embedded and expanded. Furthermore, it is the simplest structure with regard to routing, protection, and restoration, and the symmetries ensure that no part is particularly vulnerable to attacks or failures. On the other hand, it does not perform well with regard to distances, especially not in case of protection or restoration, where it is necessary to route packets all around the ring. These long distances also imply that the network should be dimensioned to handle a large amount of traffic, affecting the cost of lines and node equipment. Connectivity is ensured only in case of one single failure.

Table 5.1: Comparison of selected network structures using selected parameters.

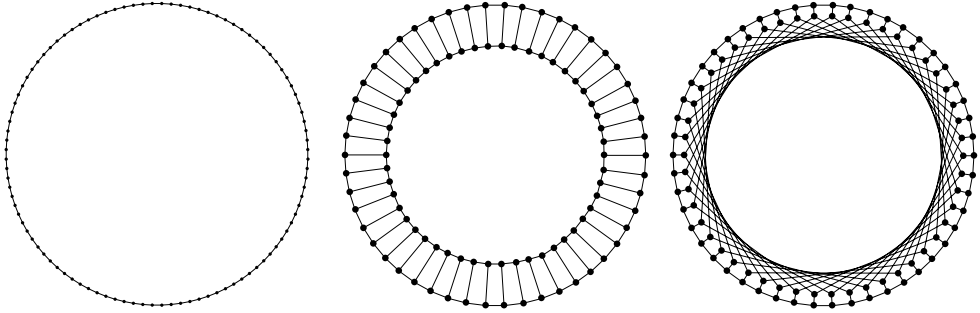| | Ring | Double Ring | N2R$(50; 9)$ |
|---|---|---|---|
| **Avg. distance** | 25.3 | 13.1 | 4.6 |
| **2-avg. dist.** | 50 | 13.6 | 5.5 |
| **Diameter** | 50 | 26 | 7 |
| **Connect. no.** | 2 | 3 | 3 |
| **Line no.** | 100 | 150 | 150 |
| **Algorithmic support** | Simple schemes for routing, protection, and restoration exist. | Simple schemes for routing, protection, and restoration exist. | Routing is more complicated than for the ring and double ring, but still rather simple[PPKM04a]. Restoration/protection more complex, and no known simple schemes fully benefit from the shorter distances. |
| **Embeddability** | The ring is easily embedded in most environments. Planar. | More complicated than the ring, but still planar. [Lóp03] gives an example of how a double ring with 38 nodes can be embedded by considerate network planning. | More complicated to embed, and to our best knowledge no examples exist of it being embedded. It is not planar, and the ducts may be excessively long. Using the same ducts for more lines reduces duct lengths and crossings but also decreases reliability. |
| **Expandability** | Any number of nodes can be added to the structure wherever needed, but distances increase rapidly. | The number of nodes must be even, but nodes can be inserted pair wise wherever needed. Distances increase rapidly. | Nodes must generally be inserted in blocks of $2q = 18$. Distances increase notably slower than for the ring and double ring. |
| **Robustness** | Node symmetric and optimally connected. 2-connected, large distances in case of failures. | Node symmetric and optimally connected. 3-connected, short distances in case of single failures. | Not generally node symmetric; two classes of nodes exist, symmetry within each. Optimally connected, 3-connected. Short distances even in case of multiple failures. |

**Figure 5.1:** Ring, double ring, and $N2R(50; 9)$.

The double ring contains more lines than the ring, and is more difficult to embed and expand. Two independent paths can be established between any pair of nodes using an only slightly more complicated scheme than for the ring. However, fully utilizing the higher connectivity and shorter distances for redundant paths requires a different but still rather simple scheme to be set up. Connectivity is ensured even in case of any two independent failures. Distances are notably lower than for the ring, especially when protection schemes are used: two independent paths can be set up between a pair of nodes without traversing the complete structure, and a second independent path is never more than two hops longer than a shortest path. This reduces the expected traffic load for both protected and unprotected traffic.

The $N2R(50; 9)$ contains the same number of lines as the double ring, but is more difficult to embed. It is not planar, and there are many lines crossing each other. It is also more difficult to expand, even though it can be done without restructuring the whole structure as long as nodes are inserted in blocks of 18. On the other hand, distances increase significantly slower than in the ring and double ring. Finding a shortest path is not difficult, but setting up protection paths is more complex than for the double ring. Distances are notably shorter than for ring and double ring structures. The example illustrates the trade-offs that must be made when choosing a network structure, and that it is hard to decide on a generally preferable structure, partly because crucial parts of such a decision is highly dependent on concrete constraints, demands and expectations. Nevertheless the evaluations give a clear picture of strengths and weaknesses of the structures evaluated.

## 5.8 Discussion

This framework defines a number of quantitative and qualitative parameters for evaluating and comparing network structures. It makes an important tool for developing

alternatives to known network structures like the ring and double ring, and is also useful in order to support decisions in practical network planning. It describes how choosing network structures either generally or in specific cases is a matter of trade-offs between different demands and desired properties, and as such it is rather a tool for clarifying strengths and weaknesses than to point out a best structure given specified demands and conditions. Due to the generality of the framework, the numbers of parameters is kept sufficiently low to maintain an overview of the most important aspects of evaluating and comparing network structures. On the other hand, one should be aware that for more specific purposes or demands, other parameters than those derived in this framework must be taken into account.

# 6 | Topological Routing in Large-Scale Networks [PKM04c]

## 6.1 Preface*

This paper was written in collaboration with Thomas Phillip Knudsen and Ole Brun Madsen, and presented at the 6th International Conference on Advanced Communication Technology (IEEE/ICACT 2004). It is the first of three papers on using the 4-regular grid structures in network planning, and is to a wide extent based on a poster presentation on Topological Routing, which was presented at the Information Technology and Telecommunications Annual Conference 2003 (IT&T 2003) [PKM03]. The contribution to the thesis can be summarized as follows. The paper:

- Identifies the most important differences in using the 4-regular grid structure in multiprocessor systems and large scale networks.

- Devises a number of extensions to meet the demands of large-scale networks, in particular Topological Routing, algorithms for failure handling, and hierarchical extensions. It is shown how Topological Routing can greatly reduce the need for routing tables, which is a major problem in todays networks.

The performances of the hierarchical extensions are evaluated in Chapter 7, which also introduces a few more variants of it. Chapter 8 contains even more variants as well as a more in-depth description of some of the theory presented here.

## 6.2 Abstract

A new routing scheme, Topological Routing, for large-scale networks is proposed. It allows for efficient routing without large routing tables as known from traditional routing schemes. It presupposes a certain level of order in the networks, known from Structural QoS. The main issues in applying Topological Routing to large-scale networks are discussed. Hierarchical extensions are presented along with schemes for shortest-path routing, fault handling, and path restoration. Further research in the area is discussed and perspectives on the prerequisites for practical deployment of Topological Routing in large-scale networks are given.

## 6.3 Introduction

The growth of large-scale networks, particularly the global networks comprising the Internet, has put pressure on traditional routing schemes for such networks; this includes sizes of routing tables and ability to support the increasing demands for QoS. From the field of multiprocessor systems, table-free routing schemes have been known for years. These schemes are not directly applicable to large-scale networks; they rely on the structures having highly regular properties and operating on a limited scale, conditions which are not practical in large-scale networks. Recent work, however, in the field of large-scale networks has proposed the design of networks with global structural properties for the improved support of QoS, termed Structural QoS or SQoS [MNS02][MKP03]. Such network design offers the opportunity for applying concepts from multiprocessor systems to large-scale networks, taking advantage of the global properties to introduce table-free routing. This class of routing schemes, taking advantage of defined structural properties, is labelled Topological Routing. In this paper, characteristics of Topological Routing will be described in relation to large-scale networks.

## 6.4 Methods

The concept of Topological Routing is introduced and related to already known schemes from multiprocessor systems. Two structures known from multiprocessor systems are dealt with in details, and it is analysed how the structural demands of large-scale networks differ from these. This leads to a discussion on how the structures and schemes can be revised in order to satisfy the demands of large-scale networks. A number of problems are identified, and some solutions suggested.

The 4-regular grid structure and the honeycomb structure[Sto97] form the base of the further studies. Both structures support Topological Routing. The structures can be studied as meshes or tori. Throughout this paper, the meshes are considered. However, the algorithms are provided without considering the problems that occur due to nodes on the edge of a structure having smaller degree than the nodes of the structure in general. The algorithms are easily extended to the tori.

## 6.5 Notation

Basic notation is given here. Throughout the paper, more notation is introduced as it is used. A structure $S = N \cup L$ consists of a set of nodes $N$ and a set of undirected lines $L$, such that each line is interconnecting two different nodes. Every node has a

name/address associated to it. A path of length $n$ between a node $u$ and another node $v$, is a sequence of nodes $(u = u_0), u_1, u_2, \ldots, u_{n-1}, (u_n = v)$ such that all pairs of nodes $(u_i, u_{i+1})$ for $0 \leq i \leq n - 1$ are connected by a line $e_{i+1}$. Thus it can also be written $u, e_1, u_1, e_2, \ldots, e_{n-1}, u_{n-1}, e_n, v$. The hop distance between two nodes, $u$ and $v$, corresponds to the length of the shortest path between these two nodes, and is written $d_h(u, v)$. $N(u)$ denotes the set of neighbours of a node $u$ and consists of all nodes $w$ such that $d_h(u, w) = 1$.

In this paper, it is assumed that all structures without failures are finite and connected: for every pair of nodes $u, v \in N$, there exists a path between $u$ and $v$. In case of failures, a structure $S'$ can contain a number of components. A component of $S$ is a set of nodes $N' \subseteq N$ and a set of lines $L' \subseteq L$ so that every such line is incident only with nodes in $N'$. Furthermore, from a node $u \in N'$ a path exists to another node $v$ if and only if $v \in N'$.

## 6.6   Topological Routing

The basic idea of Topological Routing is to use the routing schemes known from multiprocessor systems as described in e.g. [Sto97][CSK90][Wu03]: in all cases an addressing scheme is provided, and from any node any packet can be routed only from knowledge of the address of the current node as well as the destination node. Therefore, the name Topological Routing has been chosen.

The principle is illustrated for the 4-regular grid structure shown in Figure 6.1. Clearly, this scheme will route packets from source to destination in a number of hops corresponding to the sum of the differences of the coordinates in the two directions. This routing principle is generalised in the following.

To every node, an address is associated. These addresses can be formulated by words, coordinates or numbers, forming ordered sequences. Every node knows its own address as well as the addresses of its neighbours. Topological Routing works on a hop-by-hop basis: given a packet $p$ and its destination node $v$, in every node receiving $p$, say $u$, the address of $v$ is compared to the addresses of all neighbours of $u$, and $p$ is sent to an address closest to $v$. The notation $d_a(u, v)$ is used for the address distance between two nodes. How this distance is calculated depends on the chosen addressing scheme. In this paper $d_a((x_1, y_1), (x_2, y_2)) = |x_2 - x_1| + |y_2 - y_1|$ is used as the addressing distance measure for the 4-regular grid. For any node $u$ in the 4-regular grid structure, we write $u = (u_x, u_y)$, where $u_x$ and $u_y$ correspond to x and y coordinates of the address of $u$.

Clearly, the performance of such a scheme depends on both network structure and addressing scheme. In the following, sufficient conditions for an addressing scheme
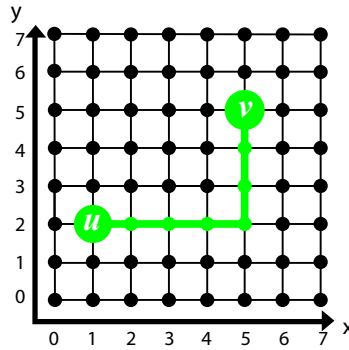
**Figure 6.1:** An example of a 4-regular grid structure. The nodes are addressed according to the coordinate system. Routing a packet from $u$ to $v$ is done hop-by-hop: in every node, the address of $v$ is compared to the current address. Based on the differences in x and y coordinates a next hop is chosen, which reduces the difference in one of the directions. The scheme is not deterministic since in some nodes two next hops can be chosen between. In this example, the packet is always send in the x direction when such a choice is to be made.

to support Topological Routing are stated. Two levels of support are introduced: schemes supporting Topological Routing, and schemes strongly supporting Topological Routing. In schemes supporting Topological Routing, any packet will finally reach its destination, while in schemes strongly supporting Topological Routing, an additional condition is that a chosen path always has to be a shortest path.

Let $S$ be a structure consisting of a set of nodes $N$, and a set of lines $L$. It is assumed that $d_a(u, v) = d_a(v, u) \forall u, v \in N$ where $u \neq v$, that $d_a(u, v)$ is always finite and that $d_a(u, v) = 0 \Leftrightarrow u = v$. If for every pair of distinct nodes, $u, v \in N$, there exists a node $w \in N(u)$ such that $d_a(v, w) < d_a(u, v)$, then $S$ supports Topological Routing: clearly, when routing, the addressing distance is reduced by every hop, and thus a packet routed will always reach its destination, assuming $S$ without errors.

The following further conditions ensure strong support of Topological Routing: assume $u \neq v$. Let $w \in N(u)$ such that $d_a(v, w)$ minimum. Then $z \in N(u)$ must imply that $d_h(z, v) \geq d_h(v, w)$. As a consequence, Topological Routing in such a structure ensures that a shortest path is chosen. In the 4-regular grid $d_h(u, v) = d_a(u, v) = |u_x - v_x| + |u_y - v_y| \forall u, v \in S$. This is sufficient to ensure strong support of Topological Routing. The same is true for the honeycomb structure. The addressing is similar to the scheme above, but extended to three coordinates[Sto97]. This is illustrated in Figure 6.2.
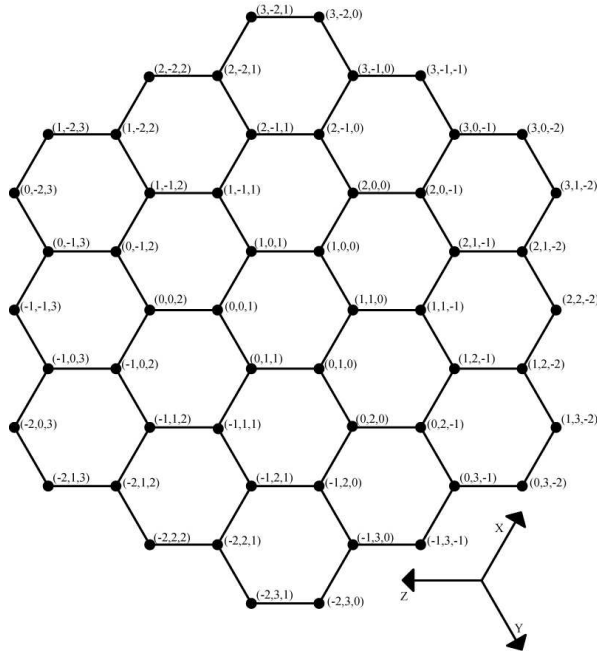
44

**Figure 6.2:** The addressing scheme of the honeycomb structure.

## 6.7 Large-scale network demands

The routing scheme introduced has until now been widely used in multiprocessor systems. These systems differ from large-scale networks in several ways. In order to identify what changes should be made to the scheme in order to make it applicable for large-scale networks, five major differences have been identified.

- Where multiprocessor systems can take advantage of cube and hypercube designs to increase connectivity, and can stack nodes close to each other, large-scale networks must follow an essentially two-dimensional surface, and natural formations as well as human structures form the major constraints on placement of the nodes. Node density varies greatly from deserted landscapes to heavily populated areas.

- Large-scale networks today contain thousands of transport nodes and millions of network termination points. Therefore, the structures must scale very well.

- Traffic in large-scale networks is to an increasing extent Internet traffic; this traffic, opposed to many other traffic types, exhibits a non-localised pattern; hence, most of the traffic passing nodes in networks is transit traffic.

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- Large-scale networks are dynamic entities, which are extended and upgraded continuously, and in which failures occur and must be handled while the network is still in operation. In addition, large-scale net-works have to be established over time. By nature, only add-ons can be performed to change the structure. Consequently, it needs to be extendable so that its coverage area as well as its node density in already covered areas can be expanded.

- Large-Scale networks support many services where continual operation is essential, and therefore there is a need for independent paths in order to support protection and fast restoration.

## 6.8 Revised schemes

The problems are suggested solved by a number of extensions of the scheme. These results all refer to the 4-regular grid structure, but most of them are extendable to the honeycomb structure as well.

### 6.8.1 Hierarchical extension

Hierarchies are introduced in order to deal with scalability issues as well as the need for larger node densities in some areas than in other, because the lower hierarchical levels may exist in selected areas only, as shown in Figure 6.3. This also makes the structure gradually extendable. However, the structure only supports topological routing if all lowest hierarchical layers exist globally, and even in this case it is not supported strongly. In the following, a revised algorithm is presented, which always gives a shortest path, even if parts of the lower hierarchical layers are left out.

Hierarchies are established by adding lines to the basic structure. For constructing just one hierarchical layer, an odd integer $g > 1$ is chosen for granularity. Now every node $u = (u_x, u_y)$ such that $u_x$ and $u_y$ are both divisible by $g$, is connected by new lines to the four nodes $(u_x - g, u_y)$, $(u_x + g, u_y)$, $(u_x, u_y - g)$, and $(u_x, u_y + g)$. More hierarchical layers are constructed in a similar manner. Constructing $n$ layers, these are denoted $H_0, \ldots, H_{n-1}, H_n$, where $H_n$ is said to be the highest layer and $H_0$ the lowest. $H_0 = S$ corresponds to $S$ without hierarchical extensions and is said to be the basic structure. In the rest of the section $k \leq n$. In general, a node $u$ belongs to $H_k$ if and only if $u \in N$, $u_x \equiv 0 \pmod{g^k}$ and $u_y \equiv 0 \pmod{g^k}$. $H_k$ also contain a set of lines, so that every node $u$ is connected to the nodes $(u_x - g^k, u_y)$, $(u_x + g^k, u_y)$, $(u_x, u_y - g^k)$, and $(u_x, u_y + g^k)$, which are all nodes in $H_k$. The hop distance between two nodes, with the restriction that only lines of $\cup H_0, \ldots, H_k$ are used, is written $d_k(u, v)$.

An operator, $round$, is defined, which simply returns a closest integer value of a fraction: for two integers $a$ and $b$, the operator $round(\frac{a}{b})$ determines an integer $I$ such that $|I - \frac{a}{b}|$ is minimum. In all cases where the operator is applied, $b$ is odd. Thus, never more than one value of $I$ exist.

Before stating the algorithm, three basic properties are listed. Let $S$ be a hierarchical 4-regular grid structure, where $g$ is the granularity chosen, and $n$ is the number of hierarchies. Assume for now that all hierarchical layers exist globally.

- Property 1: from a given node $u \in \cup H_0, \ldots, H_k$, the node $u' \in H_{k+1}$ such that $d_a(u, u')$ minimum is unambiguously determined by $u' = (g^{k+1}round(\frac{u_x}{g^{k+1}}), g^{k+1}round(\frac{u_y}{g^{k+1}}))$.

- Property 2: let $u$ be a node in $H_k$, and let $u'$ be the node in $H_{k+1}$ such that $d_a(u, u')$ minimum. Let $v \in N$ and $v'$ be the node in $H_{k+1}$ such that $d_a(v, v')$ minimum. If a shortest path between $u$ and $v$ is passing a node in $H_{k+1}$, a shortest path exists between $u$ and $v$ passing $u'$ and $v'$.

- Property 3: if $d_k(u, v) \leq d_{k+1}(u, v)$ then $d_k(u, v) \leq d_n(u, v)$. Note that this property only holds for $g \geq 5$.

The revised routing algorithm works as follows, assuming $g > 3$. When a packet $p$ with destination node $v$ is received in a node $u$, which is in $H_k$ but not $H_{k+1}$, it is
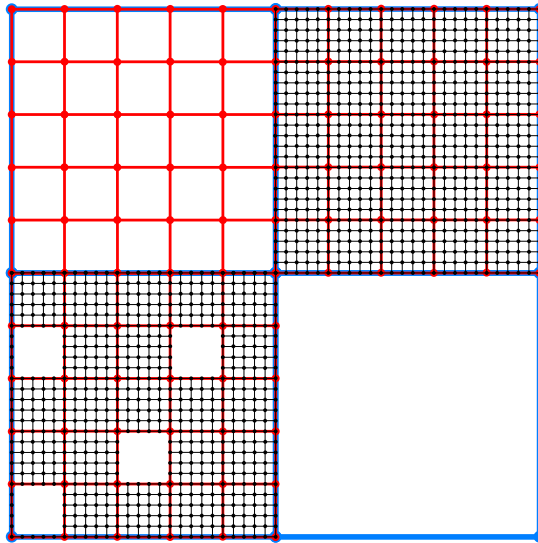


**Figure 6.3:** An example of how hierarchies can be used to create structures, which are more scalable than the basic structure. At the same time, it is gradually extendable and allows for varying node density.
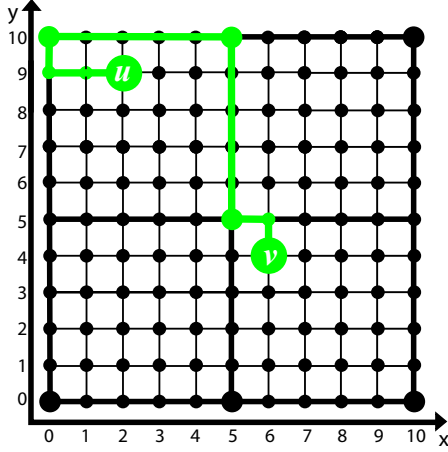
**Figure 6.4:** Hierarchical routing of a packet $p$ from $u = (2,9)$ to $v = (6,4)$. In $u$ it is determined that routing should be done through the upper hierarchical layer, and $p$ forwarded towards $(0,10)$. From $(0,10)$ the packet is forwarded to the neighbour closest to $v$, which is $(5,10)$. The neighbour of $(5,10)$ closest to $v$ is $(5,5)$. From $(5,5)$, the node closest to $v$ is $(6,5)$, and from this node, $p$ is forwarded to $v$.

decided if $p$ should be routed through $H_{k+1}$ (or even higher layers). This is done by deciding if $d_{k+1}(u,v) < d_k(u,v)$, illustrated in Figure 6.4. Due to property 3, higher hierarchies need not be considered. $v$ might not be in $H_k$, but due to property 2, it is sufficient to compare the distances between $u$ and $v''$, $v''$ being the node in $H_k$ such that $d_a(v,v'')$ minimum.

Since $u$ and $v''$ are both in $H_k$, no shortest path uses lines of $H_{k-1}$ or lower layers. Thus, $d_k(u,v'') = \frac{|u_x - v''_x| + |u_y - v''_y|}{g^k}$. If a shortest path exists in $H_{k+1}$ using only lines in $H_k$ and lower layers, this path does also exist in $\cup H_0, \ldots, H_k$, implying that $d_k(u,v'') = d_{k+1}(u,v'')$. If this is not the case, a line in $H_{k+1}$ is contained in a shortest path. Due to property 2, this implies that such a path exists, passing both $u'$ and $v'$, where $u'$ is the node in $H_{k+1}$ such that $d_a(u,u')$ minimum, and $v'$ is the node in $H_{k+1}$ such that $d_a(v,v')$ minimum: $d_{k+1}(u,v'') = d_k(u,u') + d_{k+1}(u',v') + d_k(v',v'')$. Clearly, also this distance is easily calculated given the addresses as well as values of $g$ and $n$: $d_{k+1}(u,v'') = \frac{|u_x - u'_x| + |u_y - u'_y| + |v'_x - v''_x| + |v'_y - v''_y|}{g^k} + \frac{|u'_x - v'_x| + |u'_y - v'_y|}{g^{k+1}}$. The value of $k$ is easily derived from the address of $u$.

If $d_k(u,v'') < d_{k+1}(u,v'')$, routing is done following the normal routing scheme: $w \in N(u)$ so that $d_a(v,w)$ minimum is determined, and $p$ forwarded to $w$. If $d_k(u,v'') > d_{k+1}(u,v'')$, routing along the shortest path happens by forwarding $p$ to a node $w \in N(u)$, such that $d_a(u',w)$ minimum. If $d_k(u,v'') = d_{k+1}(u,v'')$ either scheme may be applied unless $g = 3$, in which case the first scheme must be followed.

If lower layers are omitted in parts of the structure, the above may not hold, since only paths in the highest hierarchical layer can be assumed to exist globally. However, such left out parts of the lower hierarchies are complete squares as shown in Figure 6.3. A problem may arise only if a path should be established between nodes in two different squares, passing one or more left out squares. However, in this case another path of same or shorter length can be established using higher layer nodes. Therefore, the only revision of the scheme needed in case of left out parts, is that routing should be done through $H_{k+1}$ if $d_k(u, v'') = d_{k+1}(u, v'')$.

The algorithm presented ensures that routing upwards and downwards hierarchies are done, so that a shortest path is always chosen. The scheme is easily extended to support different granularity in different layers.

### 6.8.2    Path restoration and Topological Routing

Algorithms for routing in incomplete structures are necessary in order to deal with network failures. Such algorithms have been evolved for the 4-regular grid structure[Wu03][TW00]. However, these algorithms need to be revised to deal efficiently with failures of arbitrary shape in packet switched networks. It is of great importance that the routing is efficient in the sense that short paths are chosen, and the use of tables minimised. At the same time, routing should be possible between any pair of nodes between which a path exists, even in case of failures. Therefore, a new algorithm is proposed that works by constructing small tables in nodes incident to lines, which are unavailable. These tables require update information from the given area of failure only. The scheme presented here deals with a basic 4-regular grid structure.

In a 4-regular grid structure $S = N \cup L$, with a standard $(x, y)$ addressing scheme, suppose that a set of nodes $N'' \subseteq N$ and a set of lines $L'' \subseteq L$ are removed, and let $S'' = N'' \cup L''$. Throughout this paper $S''$ is assumed connected, but this is easily generalised.

If $S - S''$ is connected then $S' = S - S''$. Otherwise, choose a node $u \in S - S''$ . The nodes of $S'$ are then $u$ and all nodes $v$ such that a path between $u$ and $v$ exists in $S - S''$. The lines of $S'$ are the entire set of lines incident with two nodes in $S'$. $S - S'$ is said to be a lake of $S$ and denoted $S_l$.

If $S - S''$ is not connected, the definition clearly depends on the choice of $u$. This reflects the fact that if the network is split into more components, from a node in one such component, all other components seem to be failing.

A node is said to be a border node $(BN)$ of $S_l$ if it is incident with lines in both $S'$ and $S_l$. The set of all nodes of $S_l$ and all lines of $S_l$ not incident with any node in $S'$,

are said to be the interior of $S_l$.

In this paper, only the handling of one such lake is considered. The algorithm is easily extended to deal with any number of lakes. This corresponds to allowing $S''$ not connected.

The presented algorithm, called the lake algorithm, works as follows: when a lake $S_l$ is detected, all $BN$s of $S_l$ maintain a table of the border of $S_l$. Whenever a packet $p$ with destination $v$ could be forwarded by the normal scheme only along lines in $S_l$, a lookup in the table is made. $p$ is then routed around $S_l$, towards a node $w$ on the border so that $d_a(w, v)$ minimum. From here, the standard Topological Routing scheme is again applied. The principle is illustrated in Figure 6.5.
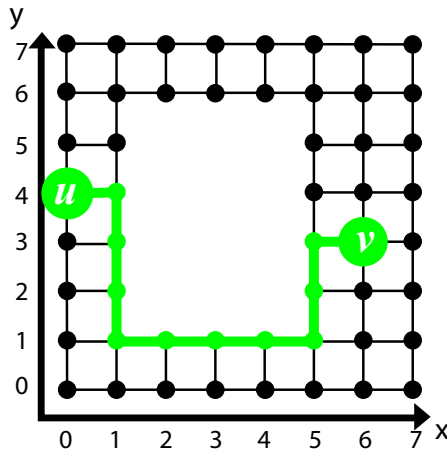


**Figure 6.5:** Routing a packet $p$ from $u$ to $v$ in a structure with a lake. In $(1, 3)$ no line exists to a node closer to $v$, and thus the lake algorithm is applied. $(5, 3)$ is the node on the border of the lake, closest to $v$, and $p$ is forwarded to this node, following a shortest path around the lake.

In the following, consider a structure $S$ containing a lake $S_l$. For every node, $u$, the neighbours of it are ordered as a sequence: $(u_x + 1, u_y), (u_x, u_y - 1), (u_x - 1, u_y),$ $(u_x + 1, u_y), (u_x, u_y + 1), \ldots$. An element is said to be to the right of the preceding element. See Figure 6.6.

When a node $u$ detects that it cannot establish contact to a neighbour node, it becomes aware that a lake has appeared, and $u$ has become a $BN$. The next step is to detect this lake, and collect the information necessary for running the lake algorithm. This is done by sending a left-control-packet $q_u$: from $u$ it is send to the first available node on the left hand side of the detected unavailable node. When a node $v$ receives $q_u$, it is first checked if $u = v$. If this is not the case, $v$ is added to a list carried by $q_u$, storing the addresses of all nodes passed, in that particular order. It is then send to the first available node on the left hand side of the node from which it is was received.
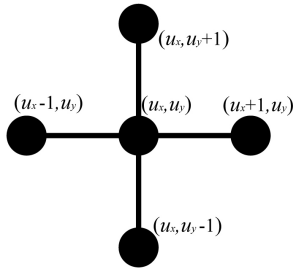
**Figure 6.6:** From $u$ the neighbour nodes are ordered $(u_x + 1, u_y)$, $(u_x, u_y - 1)$, $(u_x - 1, u_y)$, $(u_x, u_y + 1)$. $(u_x, u_y + 1)$ is said to be the first node on the left-hand side of $(u_x + 1, u_y)$, and $(u_x, u_y - 1)$ the first node on the right-hand side of $(u_x + 1, u_y)$.

If $u = v$, the information of nodes passed is stored in a table $T_u$, and $q_u$ terminated. The nodes stored in $T_u$ are the nodes defining the border of $S_l$: the $BN$s are only a subset of these.

Both right-control-packets and left-control-packets can be used. They provide the same information, the only difference being the ordering of the nodes. By sending such control packets in specified intervals, $T_u$ is kept up to date. In the following, $S_{T_u}$ denotes the set of nodes listed in $T_u$ as well as the set of lines connecting all pairs of nodes listed preceding each other in $T_u$. Clearly, $S_{T_u}$ is connected. The length of a shortest path between two nodes $u$ and $v$ both in $S_{T_u}$ using only lines of $S_{T_u}$ is written $d_{S_{T_u}}(u, v)$. When it is experienced that the failure has been recovered from, the table is deleted, and routing is done following the normal Topological Routing scheme.

The same scheme is followed in all other $BN$s. In an implementation of the scheme, some optimisation may be done in order to reduce the number of control packets, by letting control packets carry information for use in all $BN$s, and not only their origin. Now, assume that every $BN$ maintains a table as described. When a packet $p$ with destination $v$ is received in $u$, the following happens:

- It is determined if any node $w \in S' \cap N(u)$ satisfies that $d_a(w, v) < d_a(u, v)$. If this is the case, $p$ is forwarded to such a node.

- If no $w \in S' \cap N(u)$ satisfies $d_a(w, v) < d_a(u, v)$, $p$ would in $S$ be sent on a line which does not exist in $S'$. Now $w \in T_u$ is found such that $d_a(w, v)$ minimum. If $d_a(w, v) = d_a(u, v)$, then $v$ is in the interior of $S_l$, so no appropriate path can be established. In this case, $p$ is discarded. Otherwise $p$ is forwarded to a node $z \in T_u$ such that $d_a(z, v) = d_a(w, v)$. Among these possible nodes, $z$ is chosen as to minimise $d_{S_{T_u}}(u, z)$. A shortest path in $S_{T_u}$ from $u$ to $z$ is determined, and $p$ is routed along this, keeping explicitly specified route information, so that any node passed simply checks this and routes ac-

cordingly. Assuming that the lake does not change during the routing process, $p$ is forwarded to a node $z$ such that $d_a(z, v) < d_a(u, v)$. It is easy to see that a path between $z$ and $v$ of length $d_a(z, v)$ exists in $S'$, and thus $p$ will reach its destination. Some optimisation can be done in this scheme, in order to shorten the path between $u$ and $z$. This can be done by using lines not in $S_{T_u}$. Nodes not in $S_{T_u}$ can also be used, but care must be taken, especially if more lakes exist.

If a packet with an explicitly specified route is received in a node, and the next node listed on the route is unavailable, the explicitly defined route is discarded, and a new route set up using the scheme above.

It might happen that a node $u$ detects a failure, but has to route a packet $p$ with destination $v$ before a table has been set up. One suggestion for handling this case is to forward $p$ as a right- or left packet, being routed as the corresponding control packets. At some point, e.g. when $p$ reaches a node $z$ such that $d_a(z, v) < d_a(u, v)$, $p$ is in $z$ routed according to the general scheme. In $u$ it is possible to copy $p$, and send it as both right- and left packets to provide faster transmission.

The scheme provided could be revised by expanding the set of nodes, which are maintaining tables: larger tables, kept in a larger set of nodes, can reduce the length of paths chosen. This trade-off between table sizes and path lengths comprises an interesting field for further research.

Handling of failures in hierarchical extensions of the structure is not supported by this algorithm. If a packet needs to be routed through higher hierarchies, it is send towards the nearest higher hierarchy node. If this node is unreachable, it has to be determined in which direction the packet should be routed. Further research is needed to clarify how this should be done.

### 6.8.3 Mappings

Large-scale networks must, as mentioned, conform to physical landscapes and can not be built entirely regular; therefore, a mapping from the structure represented by the addressing scheme onto the physical network is necessary. Such mappings, while not arbitrary, can give considerably freedom in placement of nodes and lines in the landscape, as long as two conditions are fulfilled:

First, the two dimensions $x$ and $y$ in the addressing must be preserved as general physical directions in the network; distances measured as a given number of hops, though, need not be of similar length in kilometres. Therefore, what is represented as a grid of squares in the addressing scheme may be mapped such that the four nodes comprising the corners of a square are placed as the corners of an arbitrary trapezoid;

the lines may follow any path through the landscape, as long as they do not overlap.

Second, no two distinct lines or nodes in the addressing scheme may be placed in such proximity in the physical network, that they become a singe point of failure.

It is possible to add nodes of degree one or two to the physical structure, even though they do not exist in the addressing scheme; in this case, such nodes are simply associated with the closest node(s) in the structure.

### 6.8.4 Independent paths for protection

In general, the schemes proposed offer fast line restoration. However, in order to support applications not tolerating any restoration latency, it is necessary to send data through two or more independent paths, an approach recognized as protection. In the following, a scheme allowing for this is introduced. It is first introduced for the 4-regular grid and then extended to the hierarchical extension.

In general, the 4-regular grid offers four independent paths between any pair of nodes. Paths are considered independent only if they share no nodes nor lines.

Assuming the structure to be complete, the routing scheme can be made deterministic and predictable, depending on how lines are chosen when multiple shortest paths are available. If several different such predictable routing schemes are chosen carefully, a corresponding number of independent paths between any pair of nodes can be established. For a rather simple example, assume that a packet $p$ has origin $u$ and destination $v$. Two copies of $p$, namely $p$ and $p'$, are sent from $u$.

In $u$ as well as all other nodes passed until $v$ is reached, the routing follows the general scheme. Whenever a choice of next hop has to be made between two nodes $z$ and $z'$ so that $dist_a(z, v) = dist_a(z', v)$, $p$ and $p'$ are routed differently: in this case, $p$ is routed along the x direction while $p'$ is routed along the y direction.

If $u_x \neq v_x$ and $u_y \neq v_y$, $p$ and $p'$ are send on two independent paths, and if $u_x = v_x$ or $u_y = v_y$, the same path is followed. This scheme always uses as many short paths as are available in the structure. If more paths exist, they cannot be shortest, which makes it necessary to revise the general scheme of always forwarding packets to a node closest to the destination. Two cases must be dealt with:

- If $u_x \neq v_x$ and $u_y \neq v_y$, two paths can be established as stated above. Two more paths are established by routing two additional copies of $p$, $q$, and $q'$, via three intermediate nodes, $v'_q, v''_q, v'''_q$ and $v'_{q'}, v''_{q'}, v'''_{q'}$ respectively. The routing between these is done using the normal Topological Routing scheme. When $v'_q$ has been reached, routing is done towards $v''_q$, from where routing is done towards $v'''_q$. From here $q$ is routed towards $v$. A similar scheme is used for $q'$.

Let $r_x = \frac{v_x - u_x}{|v_x - u_x|}$ and $r_y = \frac{v_y - u_y}{|v_y - u_y|}$. Then $q$ is routed via $v_q' = (u_x - r_x, u_y)$, $v_q'' = (u_x - r_x, v_y + r_y)$, and $v_q''' = (v_x, v_y + r_y)$, while $q'$ is routed via $v_{q'}' = (u_x, u_y - r_y)$, $v_{q'}'' = (v_x + r_x, u_y - r_y)$, and $v_{q'}''' = (v_x + r_x, v_y)$. The lengths of the paths of $q$ and $q'$ are four hops longer than the length of the two shortest paths. No four independent paths from $u$ to $v$ in this structure can have lower maximum, average, or minimum lengths than the set of paths of $p$, $p'$, $q$, and $q'$.

- If $u_x = v_x$ or $u_y = v_y$ (assume the latter) only one shortest path, that of $p$, can be established. Three more packets, $q$, $q'$, and $q''$ are routed in a way similar to the former case. The paths of $q$ and $q'$ are both 2 hops longer than the path of $p$, and constructed by routing through intermediate nodes $v_q'$, $v_q''$ and $v_{q'}'$, $v_{q'}''$ respectively, where $v_q' = (u_x, u_y + 1)$, $v_q'' = (v_x, u_y + 1)$, $v_{q'}' = (u_x, u_y - 1)$, and $v_{q'}'' = (v_x, u_y - 1)$. The path of $q''$ is eight hops longer than the shortest path, with four intermediate nodes $v_{q''}' = (u_x - r_x, u_y)$, $v_{q''}'' = (u_x - r_x, u_y + 2)$, $v_{q''}''' = (v_x + r_x, u_y + 2)$, $v_{q''}'''' = (v_x + r_x, v_y)$. $r_x = \frac{v_x - u_x}{|v_x - u_x|}$.

This approach will clearly yield four independent paths, such that the paths are shortest possible. However, this approach is not directly applicable in case of hierarchical structures. If a packet has to be routed in only one hierarchical layer, the approach will work, but consider that this is not the case. If a packet has to be routed through higher hierarchies, it is encapsulated and routed towards the nearest higher hierarchy node. Therefore, all copies of it are routed towards this node. The approach provided can be used for the routing towards this node, making the paths independent, except for the nodes where a change in hierarchy occurs. The same is true for packets travelling the opposite way in the hierarchies. This results in four line independent paths, of which at least one is shortest possible. Only nodes, which involve hierarchical shifts, are dependent. This might be dealt with by making such nodes more reliable, for example by doubling equipment and power supply.

## 6.9 Conclusion

The fundamentals of a new promising routing scheme for large-scale networks, Topological Routing, have been presented. It has several advantages over the schemes used today: tables are only needed in case of network failures, and even in that case, small tables are sufficient, and only little communication is needed to keep the tables updated. Fast restoration is supported, and protection paths are easily set up. The scheme relies on the network structures satisfying certain global constraints. However, by establishing hierarchies and using appropriate mapping schemes, a high degree of freedom is provided and gradual extension of networks supported. Further

research is needed before Topological Routing can be applied, especially in the fields of hierarchies, restoration, protection, QoS and load balancing. Here the need for a unified standard for construction and classification of network capabilities is crucial.

## 6.10 Additional comments*

Parts of the theory presented in this paper are described in further detail in Chapter 8, but one particular statement deserves an additional comment. In Section 6.8.1, Property 3 states that for $g \geq 5$, that a shortest path from a node in layer $k$ to another node can be found using no higher layers, unless a path through layer $k+1$ is shorter than a shortest path through layer $k$ and lower layers.

First, it is easily seen that this does not hold for $g = 3$. Assume for an example that $g = 3$ and $n_H = 2$, and that a packet is routed from $(7, 1)$ to $(8, 8)$. In the basic structure (layer 0), the distance is 8. Routed through layer 1 (but not 2), the distance is also 8, while through layer 2 the distance is only 6. This is illustrated in Figure 6.7.
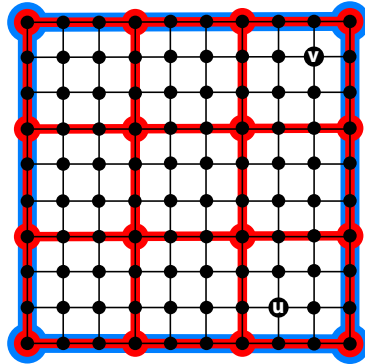


**Figure 6.7:** In layers 0 and 1, the distance between $u = (7, 1)$ and $v = (8, 8)$ is 8, but in layer 2 it is 6.

It is now in place to show that the statement holds for $g \geq 5$. Assume nodes $u$ and $v$, both in layer $k$, and assume that $d_k(u, v) = d_{k+1}(u, v)$. The aim is to show that $d_k(u, v) = d_{k'}(u, v)$ for all $k' > k$.

First, a shortest path between $u$ and $v$ can be found passing either $u' = (v_x, u_y)$ or $v' = (u_x, v_y)$, i.e. including paths $(u_x, u_y), \ldots, u'$ or $(u_x, u_y), \ldots, v'$, where both $u'$ and $v'$ are in layer $k$.

Assume that $d_k(u, u') > 2g + 1$, and let $u_{k+1}$ and $u'_{k+1}$ be the nodes in layer $k + 1$ closest to $u$ and $u'$ respectively. Now, $d_k(u, u_{k+1}) + d_k(u', u'_{k+1}) \leq 2g - 2$. Since $u_{k+1}$ and $u'_{k+1}$ have the same $y$ coordinate, $d_{k+1}(u_{k+1}, u'_{k+1})$ is determined by the distance in the $x$ direction, and does not exceed $\lceil \frac{d_k(u,u')}{g} \rceil$. Thus, $d_{k+1}(u, u') \leq 2g -$

$2+1+\frac{d_k(u,u')}{g}$. It is now easy to see that $d_{k+1}(u,u') \leq 2g-1+\frac{d_k(u,u')}{g} < d_k(u,u')$ when $d_k(u,u') > 2g+1$ and $g \geq 3$. It follows that $d_{k+1}(u,v) < d_k(u,v)$. This holds similarly when $d_k(u,v') > 2g+1$, so it can be assumed that the distance between $u$ and $v$ in layer $k$ does not exceed $2g+1$ in neither the $x$ nor the $y$ direction.

Let $u_{k+1}$ and $v_{k+1}$ be the nodes in layer $k+1$ closest to $u$ and $v$, and let $u_{k+2}$ and $v_{k+2}$ be the nodes in layer $k+2$ closest to $u$ and $v$. These nodes are also the nodes in layer $k+2$ closest to $u_{k+1}$ and $v_{k+1}$ respectively. Since the distance between $u$ and $v$ in layer $k$ does not exceed $2g+1$ in any direction, the distance between $u_{k+1}$ and $v_{k+1}$ in layer $k+1$ cannot exceed $\lceil \frac{2g+1}{g} \rceil = 3$ in any direction. Consider 3 cases:

Case 1: If $u_{k+2}$ and $v_{k+2}$ do not have a common $x$ or $y$ coordinate, then $d_{k+2}(u_{k+2},v_{k+2}) \geq 2$. Furthermore, since $g \geq 5$, and since $u_{k+1}$ and $v_{k+1}$ have no common coordinates, $d_{k+1}(u_{k+1},u_{k+2}) + d_{k+1}(v_{k+1},v_{k+2}) \geq 4$. It follows that $d_{k+2}(u,v) = d_{k+1}(u,v)$. Any shortest path using a line in a layer above $k+2$ can be found passing $u_{k+2}$ and $v_{k+2}$, and since the distance between these two nodes in any layer will be at least 2, it follows that when $d_{k+1}(u,v) = d_k(u,v)$, then $d_k(u,v) = d_{k'}(u,v)$ for all $k' \geq k$.

Case 2: If $u_{k+2} = v_{k+2}$ then no line in layer $k+2$ is used, so $d_{k+1}(u,v) = d_{k+2}(u,v)$, and obviously $d_{k'}(u,v) = d_{k+1}(u,v)$ for any $k' \geq k+1$. It follows that if $d_{k+1}(u,v) = d_k(u,v)$ then $d_k(u,v) = d_{k'}(u,v)$ for all $k' \geq k$.

Case 3: $u_{k+2}$ and $v_{k+2}$ share exactly one coordinate, say $x$. Now, assume a shortest path $p$ between $u_{k+1}$ and $v_{k+1}$ using at least one line of layer $k+2$. $p$ can be constructed like $u_{k+1}, \ldots, u_{k+2}, \ldots, v_{k+2}, \ldots, v_{k+1}$. This path uses no line of layer $k+2$ in the $x$ direction. Now, let $d_{k+1}(x)$ denote the number of lines in the $x$ direction in a shortest path between $u_{k+1}$ and $v_{k+1}$ in layer $k+1$, and let $d_{k+2}(x)$ denote the number of lines in the $x$ direction in $p$ as defined above. Clearly, $d_{k+1}(x) \leq d_{k+2}(x)$.

Similarly to case 1, $d_{k+1}(u_{k+1},u_{k+2}) + d_{k+1}(v_{k+1},v_{k+2}) \geq 2 + d_{k+1}(x)$ because $g \geq 5$. Also similarly $d_{k+2}(u_{k+2},v_{k+2}) \geq 1$. This implies that $p$ is of length at least $3 + d_{k+1}(x)$, and thus $d_{k+2}(u_{k+1},v_{k+1}) = d_{k+1}(u_{k+1},v_{k+1})$. As in Case 1, it follows that $d_{k+1}(u,v) = d_k(u,v)$ and $d_k(u,v) = d_{k'}(u,v)$, for all $k' \geq k$.

# 7 On Hierarchical Extensions of Large-Scale 4-regular Grid Network Structures [PPKM04b]

## 7.1 Preface*

This paper was written in collaboration with Ahmed Patel, Thomas Phillip Knudsen, and Ole Brun Madsen, and presented at The International Conference on Parallel and Distributed Processing Techniques and Applications (PDPTA '04) in 2004. It is the second of three papers on 4-regular grid structures. The contribution to the thesis can be summarized as follows. The paper:

- Evaluates the performance of the hierarchical extensions of the 4-regular grid structures, as proposed in Chapter 6. It turns out that these reduce distances in the structures significantly.

- Variants of these extensions are also proposed and evaluated: extended and skew meshes. The purpose of these extensions is better support of real-world implementations.

The work is continued in Chapter 8, in particular by the introduction of pruning.

## 7.2 abstract

It is studied how the introduction of ordered hierarchies in 4-regular grid network structures decreases distances remarkably, while at the same time allowing for simple topological routing schemes. Both meshes and tori are considered; in both cases, non-hierarchical structures have power-law dependencies between the number of nodes and the distances in the structures. The perfect square mesh is introduced for hierarchies, and it is shown that applying ordered hierarchies in this way results in logarithmic dependencies between the number of nodes and the distances, resulting in better scaling structures. For example, in a mesh of 391876 nodes, the average distance is reduced from 417.33 to 17.32 by adding hierarchical lines. This is gained

by increasing the number of lines by 4.20% compared to the non-hierarchical structure. A similar hierarchical extension of the torus structure also results in logarithmic dependencies, the relative difference between performance of mesh and torus structures being less significant than for non-hierarchical structures, especially for large structures. The skew and extended meshes are introduced as variants of the perfect square mesh and their performances studied, and it is shown that while they allow for more flexibility in design and construction of structures supporting topological routing, their performances are comparable to the performance of the perfect square mesh. Finally, suggestions for further research within the field are given.

## 7.3    Introduction

Routing schemes applied in large-scale networks such as the Internet have until recently operated on a best-effort basis only. This has been sufficient for many applications such as email, news, webbrowsing, and file transfers, but the needs for reliable connections are increasing as the dependency on communication networks grows: as a consequence of the convergence of communications, several different medias are becoming able to communicate over the same physical lines such as telephony, television, video, and data traffic. At the same time, an increasing amount of control and distributed applications are being developed to communicate over the Internet, such as the many teleoperation and telerobotics projects, e.g. [HKKK01][XT98][XmCjYXd04]. This convergence of communications is expected to continue in near as well as far future[MNS02].

Routing schemes based on large tables are under severe pressure in todays Internet. In 2001, experiments showed that it took on average three minutes to recover from interdomain path failovers. For some path failovers it took up to 15 minutes before the routing tables were stabilished[LABJ01]. The dependency on exact and updated tables is a major problem for the routing schemes, since the size of the Internet makes it impossible to maintain updated tables of the complete Internet topology; not only would this require huge tables, it would also be extremely bandwidth and ressource demanding to keep them updated. Furthermore, the provisioning of two or more physical independent end-to-end paths is necessary in order to provide connections that are reliable even in case of equipment failures, broken cables etc., and even if routing tables are exact and updated, this is an extremely difficult task in todays complex Internet topology.

Reducing the needs for large tables while at the same time offering easy ways to determine several independent paths, can be done by designing networks with global structural and topological properties such as the 4-regular grid network structure[PKM04c] (which is actually not 4-regular since edge and corner nodes have

degrees less than four). It is suggested a base for a future access network infrastructure, making the study of it highly actual: many countries are expected to let fiber networks replace the existing copper-based infrastructures in near future, and for these infrastructures to benefit from global structural and topological properties, it must be well planned. Changing the physical structure after implementation is costly due to the high duct costs, and the physical structures are expected to have long lifetimes as the networks are upgradeable by changing end equipment only. This paper shows how hierarchical extensions of the 4-regular grid network structure can be used for reducing distances, especially in large-scale structures.

## 7.4   Terminology

Abstractions of networks, called structures, are studied. A structure consists of a set of nodes and a set of lines, such that each line connects two nodes. Lines are considered undirected: if a pair of nodes $(u, v)$ is connected by a line, so is $(v, u)$. A path between a source node $u$ and a destination node $v$ is a set of nodes and lines $(u = u_0), e_1, u_1, e_2, u_2, e_{n-1}, u_{n-1}, e_n, (u_n = v)$, where each line $e_i$ connects the nodes $u_{i-1}$ and $u_i$. The path length is determined by the number of lines between the source and destination node; in the case above, the path is of length $n$. The distance between two nodes $u$ and $v$ is written $d(u, v)$ and is equal to the length of the shortest path between $u$ and $v$. Note that $d(u, v) = 0$ if and only if $u = v$. The size of a structure is equal to the number of nodes it contains. The degree of a node is the number of lines joined to it.

## 7.5   Background

Structural QoS[MNS02] and Sustainable QoS[MK96] were introduced recently and deal with parameters related to architecture and structural properties of networks. This motivates an approach of designing physical network infrastructures that are able to support reliability and QoS demanding applications. [PKM04c] investigates an approach of designing network structures, which by their high degree of regularity support a simple and essentially table-free routing scheme known as topological routing. It is shown how topological routing can solve a large number of the problems faced in todays routing schemes, but the networks must be well structured and organized for such a scheme to work. It suggests that the 4-regular grid structure is used for topological routing: the nodes are addressed from a cartesian coordinate system, such that every node has a coordinate address $(x, y)$ associated to it: let $dim_x$ and $dim_y$ be positive integers. Then every integer coordinate set $(x, y)$, such that

$0 \leq x \leq dim_x$ and $0 \leq y \leq dim_y$, is the address of a node. Every node has associated to it such an address, and no two nodes have the same address. Two variants are used, the mesh and torus. In the mesh, a line between two nodes $(x_1, y_1)$ and $(x_2, y_2)$ exists if and only if $|x_2 - x_1| + |y_2 - y_1| = 1$, in the torus additional lines exist connecting nodes where either $|x_2 - x_1| = dim_x$ and $y_1 = y_2$ or $|y_2 - y_1| = dim_y$ and $x_1 = x_2$.

A packet is routed from source to destination in such a structure on a hop-by-hop basis: in every node, it is forwarded to a node with the smallest possible distance to the destination, which is easily determined. The packet only needs to carry its destination address, and every node only needs to know the adresses of its neighbours. In the torus, every node must also know $dim_x$ and $dim_y$.

While such structures are used for multiprocessor systems, and much research in this area has dealt with them (e.g. [Wu03][TW00]), they suffer from severe scalability problems in order to be used for large-scale networks: there is a power-law dependency between the number of nodes and distances in the structures. For example, the average distance in a square mesh structure of 10000 nodes is 66.67. This should be compared to the following: in 1998 average path lengths of the Internet were measured, and it was shown that the Internet at that time had an average path length of around 11-24, depending on location[FPLZ98]. In 1999, appr. 88000 different nodes (routers) were found in the Internet[BC99].

Hierarchical extensions of the 4-regular grid structure were introduced in [PKM04c]: a set of hierarchical lines are added to a structure, such that a revised topological routing scheme is supported, while at the same time the distances are shortened. The revision of the routing scheme is based on the fact that deciding whether routing should be done in higher hierarchies is done in each node, provided knowledge of a few global parameters. If routing is to be done through a higher hierarchical layer, a shortest path can always be found using the closest higher hierarchy node, which is easily determined. When the highest layer to be used is reached by the packet, the basic routing scheme is used. As a result, a shortest path between any two nodes is easily determined, taking the hierarchical lines into account.

In this paper such a way of constructing hierarchical lines is evaluated by studying how the addition of such lines decreases distances.

## 7.6 Methods

Structures are evaluated by calculating and comparing distances. Let $u_1, \ldots, u_n$ be the nodes of a structure. Then for every node $u_j$, the average distance from $u_j$ to all

other nodes is calculated as $d_{avg}(u_j) = \dfrac{\sum\limits_{i:\, i \neq j,\, 1 \leq i \leq n} d(u_i, u_j)}{n-1}$. Three measures are used for evaluation: average distance, worst-case average distance, and diameter. The average distance is given by $\dfrac{\sum\limits_{j:\, 1 \leq j \leq n} d_{avg}(u_j)}{n}$, and the worst-case average distance by the maximum value of $d_{avg}(u_j)$ over all $j$ such that $1 \leq j \leq n$. The diameter is the maximum value of $d(u_i, u_j)$ over all distinct values of $i$ and $j$, $1 \leq i, j \leq n$. All calculations were performed with computer aid. However, only a subset of the calculations were necessary to perform due to symmetries.

The hierarchical extension of a mesh structure is given as follows: granularities $g_x$ and $g_y$ are positive integers, chosen for each direction. The number of hierarchical layers $n_H$ must also be chosen among the positive integers. For $0 \leq x \leq dim_x$ and $0 \leq y \leq dim_y$ every node $u = (x, y)$ such that $x \equiv 0 \pmod{g_x^i}$ and $y \equiv 0 \pmod{g_y^i}$, $1 \leq i \leq n_H$, is said to belong to the $i^{th}$ hierarchical layer. The lines of this layer connect $u$ to the nodes $(x + g_x^i, y)$, $(x - g_x^i, y)$, $(x, y + g_y^i)$, and $(x, y - g_y^i)$ that exists in the structure. $g_x$ and $g_y$ must be chosen odd in order to support the revised topological routing scheme, and so this is also assumed. Note that a node belonging to the $i^{th}$ hierarchical layer also belongs to every $j^{th}$ hierarchical layer, $j < i$.

The main model used is the perfect square mesh. In addition to the conditions above, $g_x = g_y$ (simply written $g$) and $dim_x = dim_y = g^{n_H}$. This model is highly regular and symmetric but has a drawback concerning flexibility: given a specified granularity, only a few distinct values of $dim_x$ and $dim_y$ within a specified range are supported. As a result, only structures of certain sizes can be constructed. For example in case of $g = 5$, only structures of size 36, 676, 15876, 391876, 9771876, etc. exist. An example of the perfect square mesh is shown in Figure 7.1. A similar model is used for the torus structures, except that $dim_x = dim_y = g^{n_H} - 1$. The hierarchical lines are defined slightly different, such that each node $u = (x, y)$ of the $i^{th}$ layer is connected by hierarchical lines to the nodes $(x + g_x^i \pmod{(dim_x + 1)}, y)$, $(x - g_x^i \pmod{(dim_x + 1)}, y)$, $(x, y + g_y^i \pmod{(dim_y + 1)})$, and $(x, y - g_y^i \pmod{(dim_y + 1)})$ that are different from $u$, where $a \pmod{b} = kb + a$, $k$ being the smallest integer such that $kb + a \geq 0$. This definition implies that the $n_H^{th}$ hierarchical layer contains no lines.

Perfect square mesh and hierarchical torus structures with $g = 3, 5, 7, 9, 11$ were evaluated, and for each value of $g$, a number of different values of $n_H$ were used. Two additional sets of calculations were performed, dealing with variants of the perfect square mesh. The skew mesh is the first variant, and the restrictions of the perfect square mesh relaxed such that $g_x \neq g_y$, and consequently $dim_x \neq dim_y$. This allows for more flexibility in the design, and might be used for designing structures
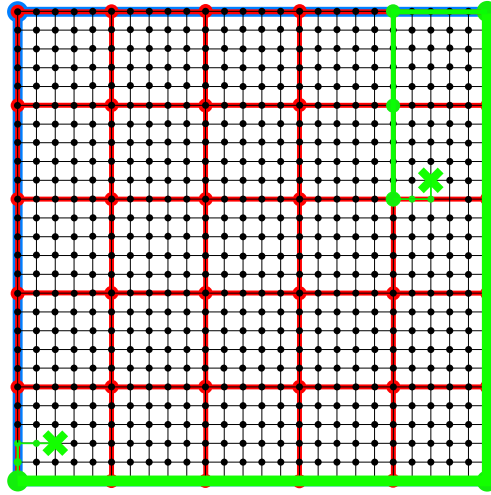
**Figure 7.1:** The perfect square mesh with $g = 5$ and $n_H = 2$. It is indicated how the distance between the two marked nodes has been reduced from 34 to 12 by hierarchical lines.

of different sizes and shapes. The structures were evaluated for $g_x = 3$ and $g_y = 11$, and the performance compared to the perfect square mesh. The other variant is the extended mesh, which can be obtained from a perfect square mesh by changing the interval for node coordinates to $-\lfloor \frac{dim_x}{2} \rfloor \leq x \leq \lfloor \frac{3dim_x}{2} \rfloor$ and $-\lfloor \frac{dim_y}{2} \rfloor \leq y \leq \lfloor \frac{3dim_y}{2} \rfloor$. This structure was evaluated for $g = 5$, and compared to the perfect square mesh.

## 7.7 Results

The results are illustrated in Figures 7.2–7.10. For every choice of granularity, a limited number of calculations were performed: for the selected values of $g$, all structures of size appr. 100000 and smaller were considered, with a few calculations for larger structures as well.

Despite the small number of calculations, we believe that the indications of logarithmic dependencies are reliable due to the well ordered structures. This is supported by two facts. First, the structures with $g = 3$ and $g = 5$ allow for the largest number of calculations for each structure, and they all show virtually perfect logarithmic dependencies between structure sizes and the various distance measures. Second, the diameter does clearly increase linearly with $n_H$, and therefore close to logarithmically with the number of nodes. This is true for both the hierarchical torus, the perfect square mesh, and the variants for all values of $g_x$ and $g_y$. Some measurements of very small structures seem to differ slightly from the rest, but all structures
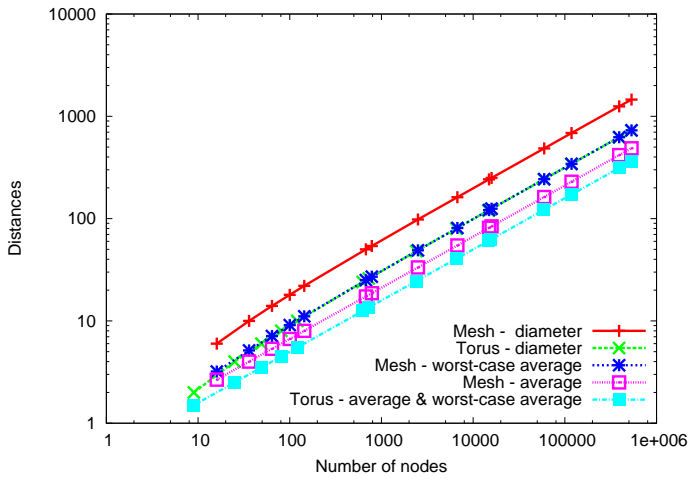
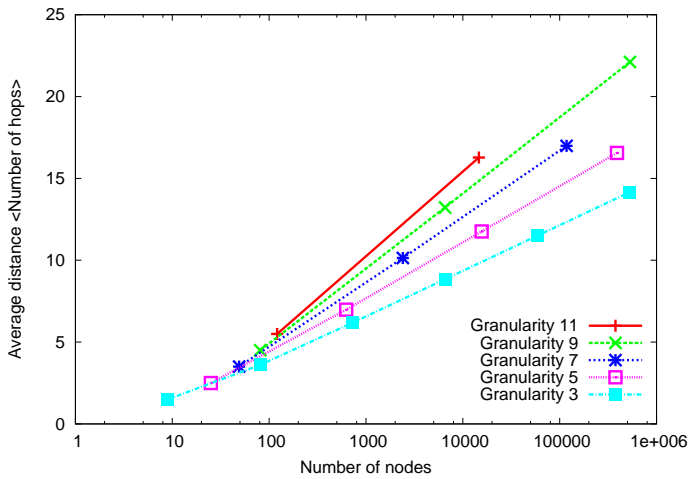**Figure 7.2:** Distances in non-hierarchical mesh and torus structures.



**Figure 7.3:** Avg. distances in hierarchical torus.

soon approximate a logarithmic dependency.

Figure 7.2 shows how the non-hierarchical approach results in power-law dependencies between the number of nodes and distance measures. Even though distances are shorter in the torus than in the mesh, none of the structures scale well, and in large structures distances are huge.

These results should be compared to the distances in the hierarchical torus shown in Figures 7.3–7.5 and perfect square mesh shown in Figures 7.6–7.8. The logarithmic
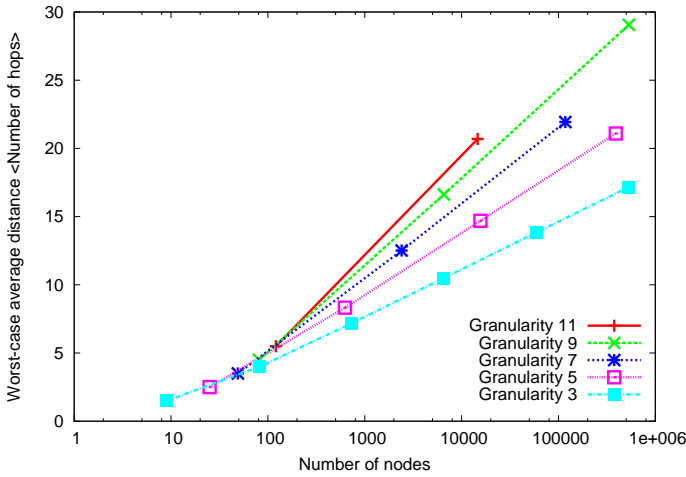
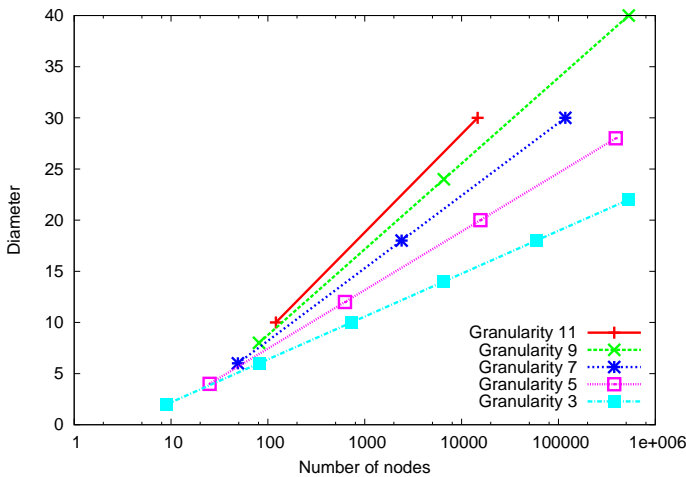**Figure 7.4:** Worst-case average distances in hierarchical torus.



**Figure 7.5:** Diameters in hierarchical torus.

dependencies between the number of nodes and distances are interesting, but it is also worth noting that the distances are kept reasonable low, even for very large structures. For example, the hierarchical torus with $g = 5$ and $n_H = 3$ is a structure of size 15625 with average distance 11.76, worst-case average distance 14.70, and diameter 20. The non-hierarchical torus of same size has average distance and worst-case average distance 62.50, and diameter 124. This gain is obtained by adding 1300 lines to the original 31250, an increase of 4.16%.
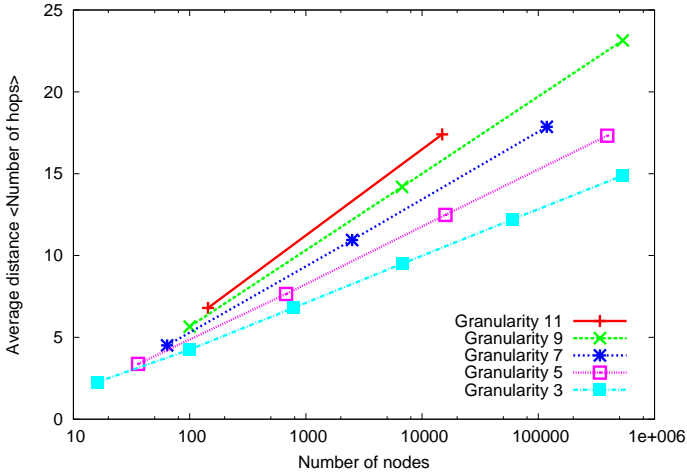
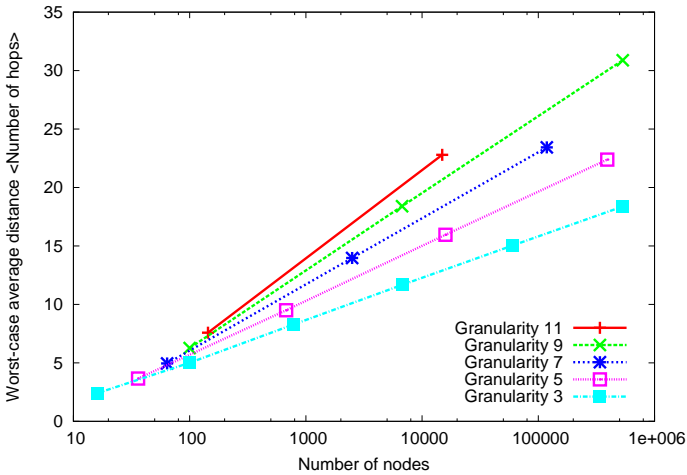**Figure 7.6:** Average distances in perfect square mesh.



**Figure 7.7:** Worst-case average distances in perfect square mesh.

The perfect square mesh shows the same pattern as the hierarchical torus. Among the non-hierarchical structures, the torus performs considerably better than the mesh, but for the hierarchical extensions the relative differences are smaller, especially for large structures.

The perfect square mesh with $g = 5$ and $n_H = 3$ contains 15876 nodes with average distance 12.48, worst-case average distance 15.96, and diameter 22. A square mesh structure of this size, without the hierarchical extension, has corresponding dis-
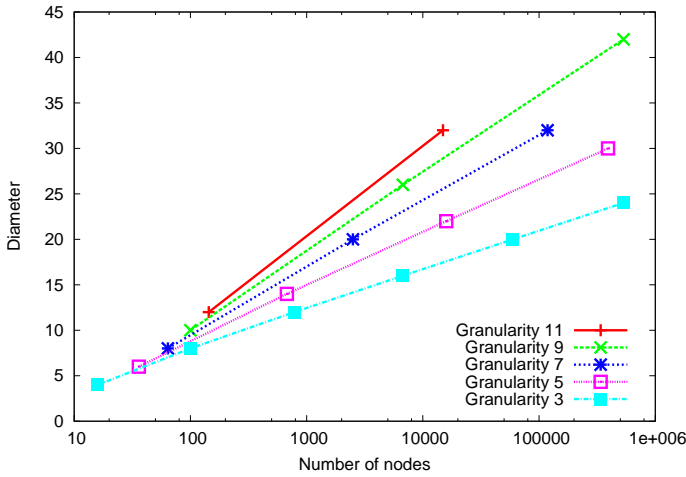
**Figure 7.8:** Diameters in perfect square mesh.

tances 84.00, 125.01, and 250. The number of lines is 32864, while it is 31500 in the non-hierarchical structure, an increase of 4.33% or 0.086 lines per node. $g = 5$ and $n_H = 4$ gives a perfect square mesh of size 391876. In this case, average distance, worst-case average distance, and diameter are 17.32, 22.39, and 30 respectively. They should be compared to the non-hierarchical structure of the same size, with corresponding distances 417.33, 625.00, and 1250 respectively. The number of lines in the non-hierarchical structure is 782500, while it is 815364 in the hierarchical extension, an increase of 4.20%, or 0.084 lines per node.

As expected, the smaller $g$ is chosen, the shorter the distances are. On the other hand, for small choices of $g$ more lines are added. $g = 3$, the smallest possible, and $n_H = 5$, gives a structure of size 59356, with average distance, worst-case average distance and diameter 12.20, 15.03 and 20 respectively. 15004 lines are added to the non-hierarchical structure of 118584 lines, an increase of 12.65%, or 0.25 lines per node. Without hierarchies, the corresponding distances are 162.67, 243.00, and 486.

Figures 7.9–7.10 show the performances of the variants of the perfect square mesh, compared to the perfect square mesh with performance closest to those of the variants.

The skew mesh with $g_x = 3$ and $g_y = 11$ performs similarly to the perfect square mesh with $g = 9$ in terms of both average distance, worst-case average distance, and diameter. The skew mesh does, however, require a larger number of hierarchical lines. Since no structures of comparable size exist, this can be seen by comparing the number of hierarchical lines relative to the number of nodes. The perfect square meshes of sizes 6724 and 532900 have 0.027 respectively 0.025 hierarchical lines per
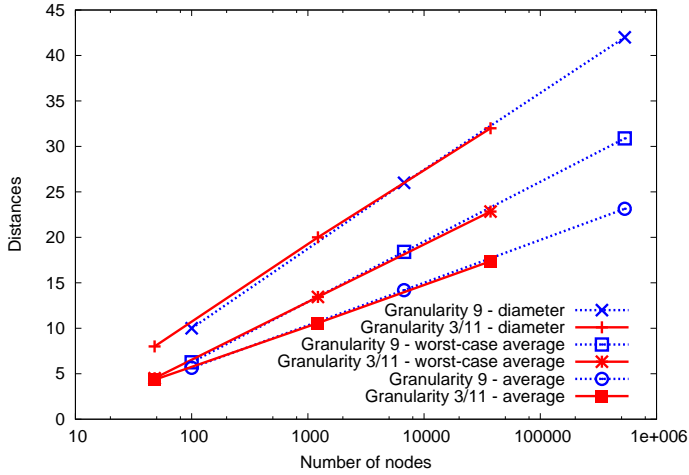
**Figure 7.9:** Skew mesh with granularity 3/11 and perfect square mesh with granularity 9.
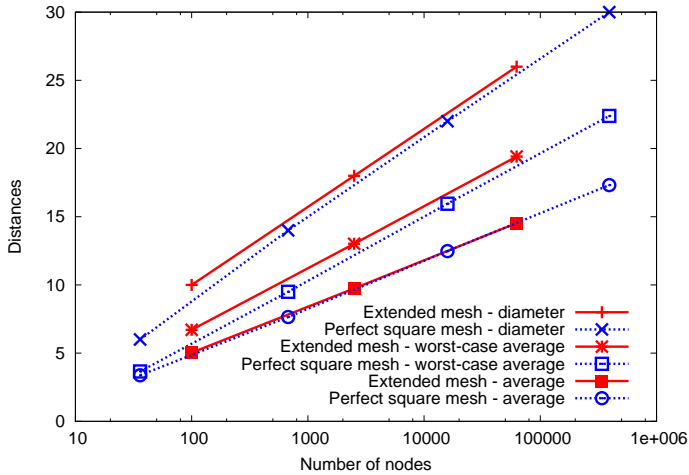


**Figure 7.10:** Extended mesh compared to perfect square mesh, both with granularity 5.

node, while the skew mesh of size 37296 has 0.064 hierarchical lines per node.

The extended mesh with $g = 5$ performs comparable to the perfect square mesh with $g = 5$, with only slightly larger diameters and worst-case average distances. All distance measures are approximating those of the perfect square mesh for large numbers of nodes. As with the skew mesh, it is not possible to compare two structures of the same size, because only structures of different sizes are supported. Since the granularity is the same for both variants, the number of hierarchical lines are close

to each other, being only a bit smaller for the extended mesh. The extended mesh of size 62500 has 0.081 hierarchical lines per node.

## 7.8 Discussion and conclusion

It was shown that ordered hierarchies as introduced are useful for reducing distances in 4-regular grid network structures significantly, especially for large-scale structures. The main model used was the perfect square mesh and its torus counterpart. While there are power-law dependencies between the number of nodes and average as well as worst-case average distances in the non-hierarchical structures, the corresponding dependencies for the hierarchical structures are logarithmic and leads to noticeable smaller distances. For example, the perfect square mesh with $g = 5$ and $n_H = 4$ is a structure of size 391876 with average distance only 17.32, compared to an average distance of 417.33 for a similar sized mesh structure without hierarchies. The performance difference between mesh and torus, which is noticeable for non-hierarchical structures, is considerably smaller for the hierarchical structures, especially for large structures. Even for small values of $g$, the number of hierarchical lines added to the structure is quite small compared to the total number of lines. Even though these hierarchical lines are longer than the non-hierarchical lines in the sense that they connect nodes with larger distances in the basic structure, they can follow the same ducts, minimizing the costs. The small number of hierarchical lines implies a similar limited increase in the node degrees, which is another important cost factor.

Two variants were introduced as alternatives to the perfect square mesh, the skew mesh and the extended mesh. As both show logarithmic dependencies between size and distances, they can be used in concrete network planning when no suitable perfect square mesh exists.

However, the skew mesh has a drawback since more hierarchical lines are used for this structure than for a perfect square mesh of the same size and performance. An alternative to this structure, which may prove to be better, is obtained by placing a number of perfect square mesh structures adjacent to each other, connected at the edges. This allows for more different shapes than the skew mesh, but further research is required for evaluation of such an approach.

On the other hand, the extended mesh has performance almost as good as the perfect square mesh, using even slightly fewer hierarchical lines per node. It may be useful for constructing structures of sizes different from what is supported by the perfect square mesh. It may also be possible to use it as a base for constructing cheaper and better performing hierarchical structures than the perfect square mesh, where more nodes are placed closer to higher hierarchical nodes, reducing especially diameters

and worst-case distances.

With this contribution, the 4-regular grid structures have been shown to form a suitable base for fiber-based access networks, offering high connectivity and reasonably small distances. However, efficient protection and restoration schemes still have to be developed for hierarchical extensions of the structures.

In order to create a cost-efficient alternative to the ring structures widely used today, it should also be considered if the number of lines, and consequently the average node degree, could be reduced. As the number of hierarchical lines is small compared to the total number of lines even for small values of $g$, focus should be on the lines of the basic structure. A scheme reducing this number, and thus the connectivity, while maintaining the support of topological routing and relatively short distances, would be a major contribution here.

## 7.9 Acknowledgements

## 7.10 Additional comments*

If the 4-regular grid structures are to be implemented in the lowest levels of a network, it is likely that most traffic will be directed to and from a single node, or at least a limited set of the nodes, providing connectivity to the higher layers. If such a node can be chosen with some degree of freedom, the most suitable candidates are those with the lowest distances to other nodes. This gives rise to two other parameters, namely minimum average distance and minimum diameter, where the latter corresponds to what is in graph theory known as the radius. The measures are defined as follows:

- Minimum average distance: The average distance from a node $u$ to all other nodes, where $u$ is chosen among all nodes such that this value is smallest possible.

- Minimum diameter: The maximum distance from a node $u$ to any other node, where $u$ is chosen among all nodes such that this value is smallest possible.

They were calculated for perfect square meshes with $g = 3$-11. Figures 7.11–7.14 show the results compared to the average distances and diameters. As for the average

distance and diameter, the new measures still grow logarithmically with the number of nodes, but the minimum diameter grows only half as fast as the diameter. The minimum average distance also remain significantly lower than the average distance. Similar results were obtained for the skew mesh with $g_x = 3$ and $g_y = 11$, and for the extended mesh with $g = 5$. These are shown in Figures 7.16–7.17. It should be noted that in all of these structures, there exist nodes from which both minimum average distance and minimum diameter can be obtained.



**Figure 7.11:** Minimum average distance and diameter in perfect square mesh with $g = 3$.



**Figure 7.12:** Minimum average distance and diameter in perfect square mesh with $g = 5$.
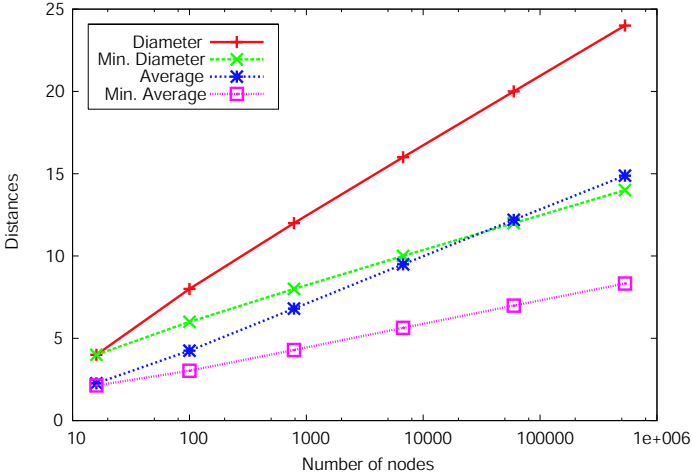
**Figure 7.13:** Minimum average distance and diameter in perfect square mesh with $g = 7$.
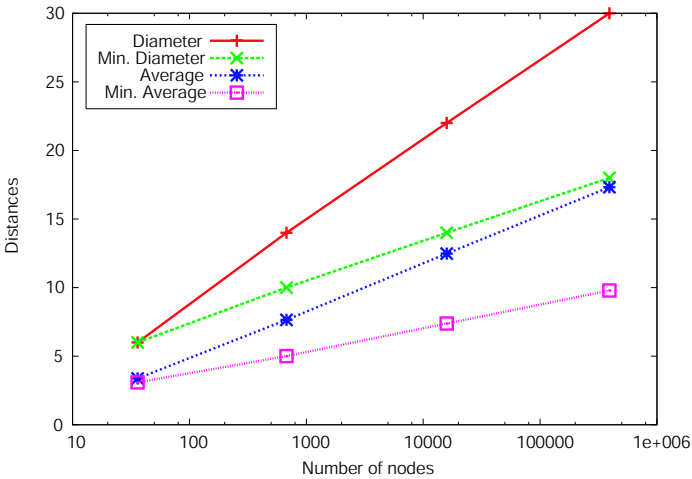


**Figure 7.14:** Minimum average distance and diameter in perfect square mesh with $g = 9$.

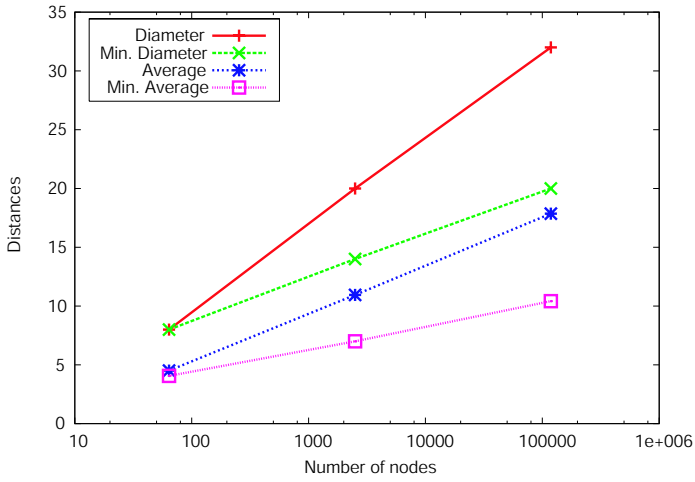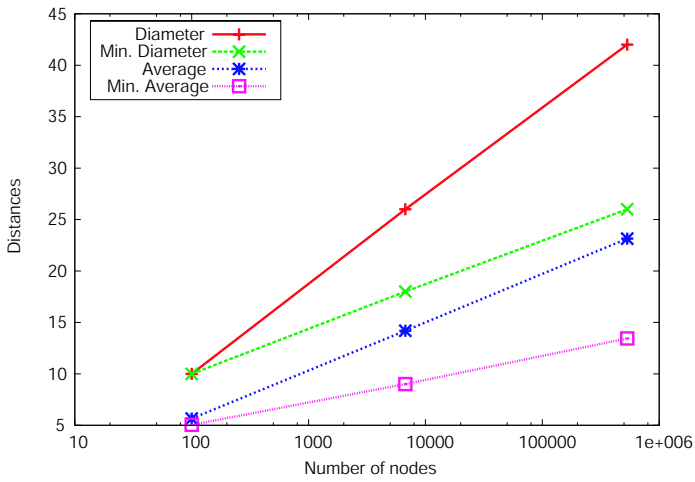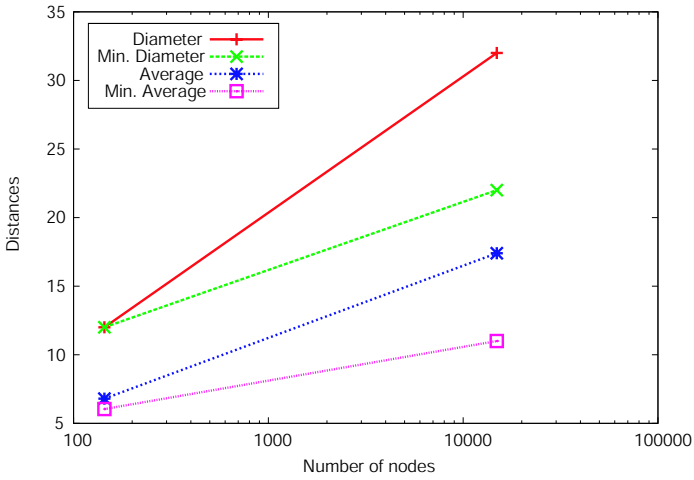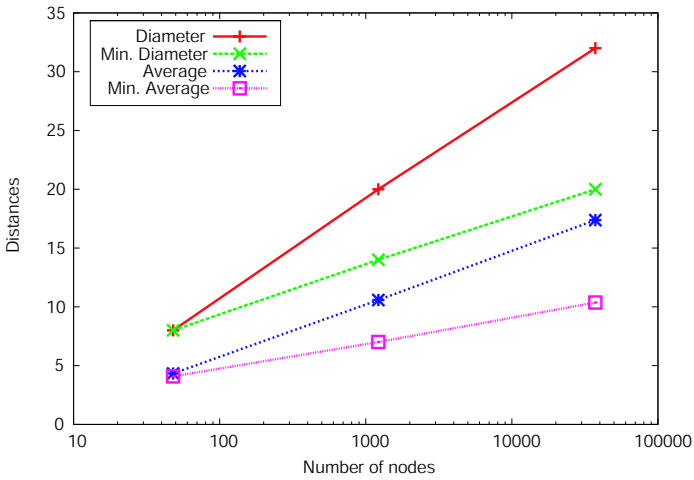**Figure 7.15:** Minimum average distance and diameter in perfect square mesh with $g = 11$.



**Figure 7.16:** Minimum average distance and diameter in skew mesh with $g_x = 3$ and $g_y = 11$.
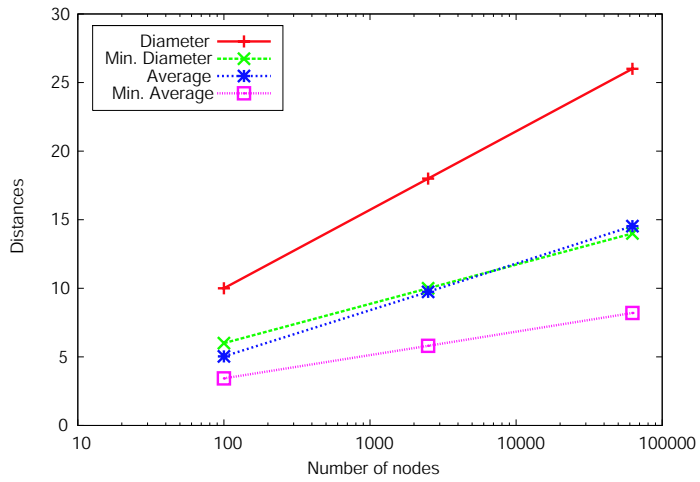
**Figure 7.17:** Minimum average distance and diameter in extended mesh with $g = 5$.

# 8 | Applying 4-Regular Grid Structures in Large-Scale Access Networks [PPKM]

## 8.1 Preface*

This paper was written in collaboration with Ahmed Patel, Thomas Phillip Knudsen, and Ole Brun Madsen. It is submitted to the journal Computer Communications, but has not yet been reviewed. It is the last of three papers on the 4-regular grid structures. It is mainly an extension of the two papers presented in Chapters 6–7, and the most important results from these papers have been included here as well, but new results are also presented. The contribution to the thesis can be summarized as follows. The paper:

- Contains more details on the most important theory introduced in Chapter 6.

- Introduces and evaluates pruning as an addition to the hierarchical extension proposed in Chapter 6 and evaluated in Chapter 7. Pruning facilitates implementation and decreases the number of lines remarkably while only slightly affecting the distances.

## 8.2 Abstract

4-regular grid structures have been used in multiprocessor systems for decades due to a number of nice properties with regard to routing, protection, and restoration, together with a straight-forward planar layout. These qualities are to an increasing extent demanded also in large-scale access networks, but concerning protection and restoration these demands have been met only to a limited extent by the commonly used ring and tree structures. This paper discusses essential aspects of applying 4-regular grid structures in such networks, and suggests extensions of the basic structures in order to deal with restoration, protection, scalability, embeddability, flexibility, and cost.

## 8.3   Introduction

The Internet is to an ever increasing extent becoming a part of every day life for people all over the world. While it was traditionally used for best-effort services such as email, news, FTP, and to some extent WWW, a large variety of applications have now been developed that demand a higher level of QoS and reliability. In order to support this, new protocols such as Intserv and Diffserv have been developed together with different protection and restoration schemes.

While most households today receive telephony, television, and radio by dedicated technologies for each medium, this is expected to change as virtually all media are becoming able to communicate via Internet protocols and consequently make use of the same physical connections[MNS02]. Together with an increasing use of telerobotics[HKKK01][XmCjYXd04], tele operations[XT98], and other critical applications, the access networks are becoming a critical part of the whole communication infrastructure. Even though protocols are being developed to ensure reliability, the physical network structures limit what level of reliability can be offered: communication between two nodes is only possible if there is a physical connection between them. In a tree-based network structure there exists only one path between any pair of nodes, making it vulnerable to attacks and failures. Obvious and commonly used alternatives to tree structures are ring structures, which offer connectivity in case of any single failure. However, given the expected demands of reliability, this is likely to become insufficient in near future.

The study of structural and topological properties of networks is highly relevant because Fiber To The Home is about to replace the old copper-based telephony infrastructure in many countries worldwide. This is a unique opportunity to implement a structure almost from scratch. At the same time, it is a huge task, especially because it requires a huge amount of duct digging. This together with the fact that fibre infrastructures are expected to have a long lifetime because they are upgradeable by changing end equipment only, is a major argument for choosing network structures with good and predictable properties.

It has been shown that node symmetry, maximal connectivity, and regularity are important properties to satisfy for robust network structures[DC04]. A regular structure is a structure where all nodes are connected by the same number of lines $n$, and given $n$, such a structure is said to be $n$-regular. 2-regular structures are equivalent to rings, and inherently node symmetric and maximally connected. The next step, discussed in this paper, is to consider 3-regular or 4-regular structures which are preferably node symmetric and maximally connected, but at least sufficiently regular and symmetric to benefit from these properties.

Among the 3-regular structures, the group of $N2R(p;q)$ structures has been intro-

duced as a generalization of double rings. Another set of 3-regular structures, the honeycomb structures, have been introduced for multiprocessor systems[Sto97], but most of these results are directly transferable to large-scale networks. However, some hierarchical extensions must be developed for these structures to perform reasonably well with regard to average distances and diameters, and this challenge has not yet been met[PKM04c].

4-regular grid structures have been used for multiprocessor systems in decades due to their nice properties with respect to routing and restoration, and they were recently suggested used as a base for access network structures as well[PKM04c][PPKM04b].

The aim of this paper is to present basic properties of 4-regular grid structures in a large-scale network perspective, together with a number of extensions that can be made in order to deal with some major differences between multiprocessor systems and large-scale networks. The work done in the field so far is purely theoretical, but this paper forms a base for starting practical tests and experiments.

The remainder of the paper is organized as follows: Section 8.4 introduces preliminaries, background, and notation. Section 8.5 introduces the basic 4-regular grid structure, which forms a base for the extensions provided in Sections 8.6 – 8.8. Section 8.6 introduces restoration and protections schemes. The two other important extensions introduced are the hierarchical extension (Section 8.7) and the pruning (Section 8.8). Section 8.9 provides some tools for embedding the structures in real-world networks, and Section 8.10 ends the paper with a conclusion and suggestions for further work.

## 8.4   Preliminaries

Throughout this paper, network structures are studied. The definition of a structure is similar to the definition of a graph, and can be used for modelling a network, abstracting away from specific physical conditions. Node equipment, transmission technologies, wiring, and bandwidth are not taken into consideration.

A structure consists of a set of nodes and a set of lines, such that each line interconnects two nodes. Lines are bidirectional: if a pair of nodes $(u, v)$ is connected by a line, so is $(v, u)$. A path from a source node $u$ to a destination node $v$ is a sequence of nodes and lines $(u = u_0), e_1, u_1, e_2, u_2, \ldots, e_{n-1}, u_{n-1}, e_n, (u_n = v)$, where every line $e_i$ connects the nodes $u_{i-1}$ and $u_i$. It is assumed that $u_i \neq u_j$ whenever $i \neq j$. Only connected structures are dealt with, i.e. for each pair of nodes $(u, v)$ in the structure, there exists at least one path between $u$ and $v$. The length of a path is determined by the number of lines it contains; in the previous case, the path is of length $n$. The distance between two nodes $u$ and $v$ is written $d(u, v)$ and is determined by the length

of the shortest path between them.

Two different paths between a pair of nodes $(u, v)$ are said to be line independent if they share no lines, and a set of paths between $u$ and $v$ are said to be line independent if they are pairwise line independent. Similarly, two different paths between a pair of nodes $(u, v)$ are said to be node independent if they share no nodes except for $u$ and $v$, and a set of paths between $u$ and $v$ are said to be node independent if they are pairwise node independent. It is easy to see that two node independent paths must be line independent, but that the converse is not in general true.

For a node $u$, the set of nodes $v$ such that $d(u, v) = 1$ are said to be the neighbours of $u$, and two nodes are said to be connected if and only if they are neighbours. The degree of a node corresponds to the number of neighbours it has. If all nodes of a structure have the same degree $n$, the structure is said to be $n$-regular. The size of a structure is given by the number of nodes it contains.

A number of parameters for evaluation of structures[PKM04a] are referred to throughout the paper, and defined in the following. Let $S$ be a network structure consisting of a set of nodes $N$ and a set of lines $L$:

- Average average distance: the average average distance is obtained by taking the average of $d(u, v)$ over all pairs of nodes $u \neq v$, where $u, v \in N$.

- Worst-case average distance: The worst-case average distance is obtained by taking the maximum over all nodes $u$ of the average of $d(u, v)$ for all nodes $v$, $u \neq v$, where $u, v \in N$.

- Diameter: The diameter is obtained by taking the maximum of $d(u, v)$ over all pairs of nodes $u \neq v$, where $u, v \in N$.

- Cost: Since the representations of structures abstract from specific physical conditions such as node equipment, transmission technologies, bandwidth, and line ducts and lengths, it is hard to estimate the cost of a structure as such. We use either the number of lines or the average node degree to indicate the cost.

## 8.5   The basic structure

Let $dim_x$ and $dim_y$ be positive integers. They define a 4-regular grid structure $S$ with node set $N$ and line set $L$ as follows. Every node in $N$ is associated to a pair of integer coordinates $(x, y)$ such that $0 \leq x \leq dim_x$ and $0 \leq y \leq dim_y$, and every such coordinate pair is associated to a node. Furthermore, no two nodes are associated to the same pair of coordinates. Consequently, there are exactly $(dim_x + 1)(dim_y + 1)$ nodes in $S$. If a node $u$ is associated to a coordinate pair $(x_u, y_u)$, we write $u =$

$(x_u, y_u)$ to ease notation. The definition of lines depends on whether the mesh or torus is dealt with.

The lines of the 4-regular mesh is given as follows: two nodes $(x_u, y_u)$ and $(x_v, y_v)$ are connected by a line if and only if $|x_u - x_v| + |y_u - y_v| = 1$. Despite not being regular, we still refer to it as the 4-regular mesh. A torus is obtained by adding a set of lines, such that two nodes $(x_u, y_u)$ and $(x_v, y_v)$ are connected also if either $|x_u - x_v| = dim_x$ and $y_u = y_v$ or $|y_u - y_v| = dim_y$ and $x_u = x_v$. The torus is regular, node symmetric, and maximally connected, which is not the case for the mesh. The mesh nevertheless possesses most of the qualities related to these properties, except for the unevenly distributed traffic and the less robust nodes on the edges. Since the mesh is planar and thus easily embedded on a surface, this paper focuses on the mesh, and when refering to the 4-regular grid structure, the mesh is implicitly assumed. However, most of the results are easily extended to the torus, even though certain parts, such as routing, become more complicated.

A node $(x, y)$ such that either $x = 0$, $x = dim_x$, $y = 0$ or $y = dim_y$ is said to be an edge node, and the four nodes $(0, 0)$, $(dim_x, 0)$, $(0, dim_y)$, and $(dim_x, dim_y)$ are said to be corner nodes. While in general there exist four node independent paths between any pair of nodes, there exist no more than three or two node independent paths between a pair of nodes if one of the nodes is an edge or corner node, respectively. It is also easy to see that the distances to and from edge and corner nodes are generally larger than those to and from nodes in the middle of $S$.

### 8.5.1 Routing

The purpose of the routing scheme presented is to make it possible to send packets from one node to another using a shortest path. This is done using hop-by-hop routing, such that when a node receives a packet of which it is not the intended destination, it is forwarded to a neighbour. Doing this without tables, relying only on node addresses, is known as Topological Routing[PKM04c]. Topological Routing is especially beneficial in large-scale networks, because maintaining tables of the complete network topology is a resource-consuming task.

Let $p$ be a packet with destination $(x_v, y_v)$. Whenever $p$ is received by a node $(x_u, y_u)$ it is determined if it has reached its destination. If this is not the case, it is forwarded using the following algorithm, which also applies if $(x_u, y_u)$ is the source node:

- Let $\Delta x = x_v - x_u$ and $\Delta y = y_v - y_u$.

- If $\Delta y < 0$, $p$ can be forwarded to $(x_u, y_u - 1)$, and if $\Delta y > 0$ to $(x_u, y_u + 1)$.

- If $\Delta x < 0$, $p$ can be forwarded to $(x_u - 1, y_u)$, and if $\Delta x > 0$ to $(x_u + 1, y_u)$.

If $\Delta y = 0$ or $\Delta x = 0$, the path is uniquely determined. Otherwise two possiblities exist and a choice must be made. A random choice can be made, one direction can be given highest priority such that it is followed whenever possible, or the packet can be send in the direction with the highest value of $\Delta$. The advantage of the latter approach is that the number of potential paths is the highest possible in every intermediate node, i.e. in case of an arbitrary failure the risk of having to route along a longer path is minimized. This choice can also be used by protection schemes.

## 8.6   Restoration and protection

The routing scheme introduced always results in a shortest path, given that the structure is as defined, without failing nodes or lines. However, both lines and nodes do fail from time to time, and therefore it must be possible to route even in case of one or more failures: in any case where a path exists between two nodes, the routing scheme should be able to find it. Some applications tolerate a certain delay or jitter, which allows time for establishing a new path, while others are more critical with respect to delays and must be able to communicate smoothly, even in case of failures. This is handled by restoration and protection schemes.

The restoration scheme allows for restoration and for choosing paths in networks with failures. This is handled by lake algorithms of which a more in-depth discussion, with a slightly different definition of lakes, is found in[PKM04c]. Let $S$ be a 4-regular grid structure and assume that a set of nodes $N'$ and a set of lines $L'$ are missing or out of order. Furthermore, any line connected to a node in $N'$ is considered to belong to $L'$. Let $S'$ denote the structure without failing nodes and lines (that is $S' = S - L' - N'$).

A set of nodes $N'' \subseteq N'$ and lines $L'' \subseteq L'$ are said to form a lake $A$ if in a standard $(x, y)$ planar representation of $S$ as shown in Figure 8.1, it is possible to draw a (not necessarily straight) line from any element (node or line) in $A$ to any other element in $A$ without crossing any nodes or lines not in $A$. Furthermore, in this planar representation, it must not be possible to draw a line from an element in $A$ to an element in $N'$ or $L'$, which is not in $A$, without crossing an element in $S'$.

A node in $S'$ is said to be a border node of $A$, if it is in $S$ connected to a line in $A$. In the following it is assumed that only one lake $A$ exists in $S'$, but this is easily generalized, keeping in mind the fact that no node in $S'$ is a border node to more than two lakes.

When a node $u$ detects that a line or node connected to it experiences a failure, it becomes aware that a lake has appeared, and that consequently it has become a border node. The next step is to collect the information necessary to be able to route packets around the lake. This is done by using either left control packets, right control packets

or both. Consider $S$ mapped onto a standard $(x, y)$ coordinate system as shown in Figure 8.1. From $u$ a left control packet $q_{\text{left}}$ is sent along the first available line on the left hand side of the detected unavailable line/node. Initially it contains only information stating that it is a left control packet with origin $u$. When a node $v$ receives $q_{\text{left}}$, it first checks if $v$ is the origin of $q_{\text{left}}$. If this is not the case, $v$ is added to a list carried by $q_{\text{left}}$, keeping record of all nodes passed, as well as their order. $q_{\text{left}}$ is then forwarded along the first available line on the left hand side of the line from which is was received. When $q_{\text{left}}$ is received by its origin, the list of nodes passed is stored in a table $T_u$, called a lake table, and $q_{\text{left}}$ is killed. The nodes stored in this table define the border of $A$ seen from $u$ (there may exist several borders of $A$ such that no path exists between any pair of nodes from different borders). Note that not all nodes on the border are actually border nodes. Right control packets are defined in a similar manner, replacing the occurences of "left" in the above definition by "right". For every border node $u$ of $A$, such a table $T_u$ is kept updated by sending right and/or left control packets within specified intervals. When it is determined that the failing link has recovered from the failure, $T_u$ is deleted and routing again done as usual. If $u$ is a border node of two lakes, a table is maintained for each lake.
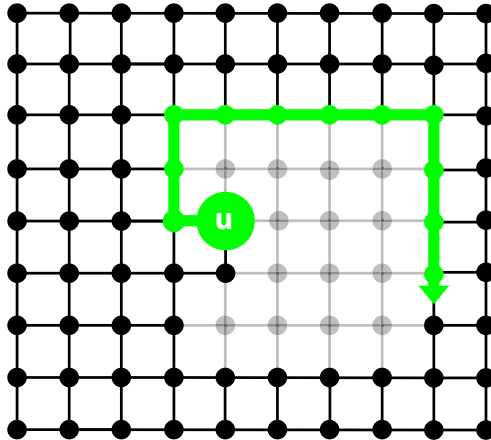


**Figure 8.1:** The routing of a left packet (green line) from $u$ ensures it traverses around the lake (shaded). For any lake $A$, a line can be drawn between any two elements of it, without crossing any element not in $A$.

When a packet $p$ with destination $w$ is received in $u$, the following happens: if in $S'$ there exists a neighbour $u'$ of $u$ such that $d(u', w) < d(u, w)$, $p$ is forwarded from $u$ to $u'$ as usual. If, however, there is no such node, a lookup is made in $T_u$, and a node $v$ in $T_u$ is chosen such that $d(v, w) < d(u, w)$, and such that $d(v, w)$ is smallest possible. A shortest path from $u$ to $v$ using the nodes of $T_u$ is now determined, and $v$ is sent to the first node on this path along with the path specification. In any node of this explicitly defined path, it is forwarded simply to the next node of the path. In case a line of this path is failing, the path is discarded and $p$ treated like any other

packet. Given that the lake is or becomes stable during the routing process and that a path to the destination, it is ensured that the packet reaches its destination in a finite number of hops. If $T_u$ contains no node $v$ such that $d(v, w) < d(u, w)$, the packet is either treated as if no table exists (described below), or it is discarded. If the table is updated and contains no such node $v$, $w$ is either in $A$, or it is unreachable from $u$, possibly because the removal of $A$ has disconnected $S$.

Different schemes can be used to optimize the setup of paths given a table $T_u$. The simplest solution is to define the path as the list of nodes traversed by a right or left control packet, but in some cases gains can be obtained by discarding loops, or even by using nodes not listed in $T_u$. Maintaining tables of a larger part of the structure than just the border of $A$ may be useful in order to determine shorter paths, and it may be possible to improve performance by storing tables in a larger set of nodes around a lake, such that alternative routing can be done before a packet reaches the border. However, there is a trade-off between different factors including path lengths, setup times in case of failures, restoration time, and resource usage in terms of storage capacity and control traffic.

In case only one line or node fails, and even in case a few lines/nodes are failing, lake tables can be generated fast since the left and right control packets only need to traverse a few nodes. However, it can happen that $u$ receives a packet $p$ with destination $w$, which could in $S$ only be sent along a line in $A$, before a table $T_u$ has been created. In this case, a chance is taken to send the packet right or left (or both) around the lake (using a scheme similar to that of left and right control packets) for example until it reaches a node $v$ such that $d(v, w) < d(u, w)$, from where it is routed either normally or by using lake algorithms. It is also possible to simply discard the packet.

The protection scheme allows for choosing paths with protection. In general up to four node independent paths can be set up between any pair of nodes, but if one of the nodes is an edge or corner node, fewer paths exist. Assume that $(x_u, y_u)$ and $(x_v, y_v)$ are nodes in $S$, that none of them are edge nodes, and that a packet $p$ is to be sent from $(x_u, y_u)$ to $(x_v, y_v)$.

Clearly, either $x_u \neq x_v$ or $y_u \neq y_v$ (or both). Without loss of generality, it is necessary only to consider two cases: in the first case $x_u \neq x_v$ and $y_u \neq y_v$, and in the second case $x_u \neq x_v$ and $y_u = y_v$. In both cases it can be assumed without loss of generality that $x_u < x_v$ and $y_u \leq y_v$.

In the first case, four node independent paths are established by duplicating the packet and routing each copy as follows:

1. $(x_u, y_u)$, $(x_u, y_u + 1)$, ..., $(x_u, y_v)$, $(x_u + 1, y_v)$, ..., $(x_v, y_v)$. This path has length $y_v - y_u + x_v - x_u$ and is a shortest path.
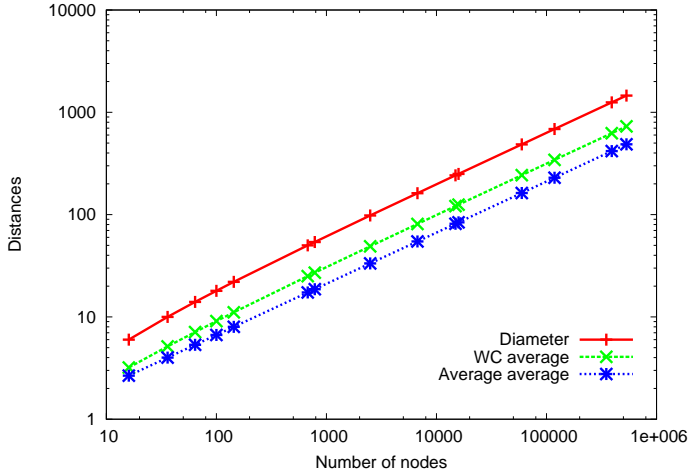
**Figure 8.2:** Distances in 4-regular grid mesh with no hierarchical extension.

2. $(x_u, y_u)$, $(x_u + 1, y_u)$, ..., $(x_v, y_u)$, $(x_v, y_u + 1)$, ..., $(x_v, y_v)$. This path has length $y_v - y_u + x_v - x_u$ and is a shortest path.

3. $(x_u, y_u)$, $(x_u, y_u - 1)$, $(x_u + 1, y_u - 1)$, ..., $(x_v + 1, y_u - 1)$, $(x_v + 1, y_u)$, ..., $(x_v + 1, y_v)$, $(x_v, y_v)$. This path has length $y_v - y_u + x_v - x_u + 4$.

4. $(x_u, y_u)$, $(x_u - 1, y_u)$, $(x_u - 1, y_u + 1)$, ..., $(x_u - 1, y_v + 1)$, $(x_u, y_v + 1)$, ..., $(x_v, y_v + 1)$, $(x_v, y_v)$. This path has length $y_v - y_u + x_v - x_u + 4$.

In the second case, the first three node independent paths are established by sending copies of the packet as follows:

1. $(x_u, y_u)$, $(x_u + 1, y_u)$, ..., $(x_v, y_u = y_v)$. This path has length $x_v - x_u$ and is a shortest path.

2. $(x_u, y_u)$, $(x_u, y_u + 1)$, $(x_u + 1, y_u + 1)$, ..., $(x_v, y_u + 1)$, $(x_v, y_u = y_v)$. This path has length $x_v - x_u + 2$.

3. $(x_u, y_u)$, $(x_u, y_u - 1)$, $(x_u + 1, y_u - 1)$, ..., $(x_v, y_u - 1)$, $(x_v, y_u = y_v)$. This path has length $x_v - x_u + 2$.

The fourth node independent path can be established in two ways. In some cases where either $(x_u, y_u)$, $(x_v, y_v)$ or both are neighbours to an edge node, it is possible that only one of them exists:

- $(x_u, y_u)$, $(x_u - 1, y_u)$, $(x_u - 1, y_u + 1)$, $(x_u - 1, y_u + 2)$, $(x_u, y_u + 2)$, ..., $(x_v + 1, y_u + 2)$, $(x_v + 1, y_u + 1)$, $(x_v + 1, y_u)$, $(x_v, y_u = y_v)$.

- $(x_u, y_u)$, $(x_u - 1, y_u)$, $(x_u - 1, y_u - 1)$, $(x_u - 1, y_u - 2)$, $(x_u, y_u - 2)$, ..., $(x_v + 1, y_u - 2)$, $(x_v + 1, y_u - 1)$, $(x_v + 1, y_u)$, $(x_v, y_u = y_v)$.

In both cases the path length is $x_v - x_u + 8$.

## 8.7 Hierarchical extension

A hierarchical extension is introduced in order to deal with the fact that distances in 4-regular grid structures increase as shown in Figure 8.2, which results in considerably larger distances than in todays Internet. For example, the average distance in a square mesh structure of 10000 nodes is 66.67, while in 1998 average path lengths of the Internet were measured, and it was shown that the Internet at that time had an average path length of around 11-24, depending on location[FPLZ98]. In 1999 appr. 88000 different nodes (routers) were found in the Internet[BC99].

The extension is introduced in two steps: first an additional set of lines, hierarchical lines, are defined, and next a revised routing scheme is presented in order to make use of these new lines. At the end of the section, the performance of the hierarchical extension is evaluated.

### 8.7.1 Physical extension

Let $S$ be a 4-regular grid structure, and let $g_x$ and $g_y$ be positive integers defining the granularity in $x$ respectively $y$ directions. Furthermore, $n_H$ must be a non-negative integer defining the number of hierarchies. If $n_H = 0$, there are no hierarchical lines added to the structure. While the definitions in the following are valid for all $g_x > 0$ and $g_y > 0$, we assume that both $g_x$ and $g_y$ are chosen odd such that $g_x \geq 3$ and $g_y \geq 3$ in order to support the revised routing schemes.

Let $(x, y)$ be a node of $S$. As given by the definition of the 4-regular grid structure, it is connected by basic lines to the nodes $(x + 1, y), (x - 1, y), (x, y + 1), (x, y - 1)$ that exist in $S$. For any $i$, $0 \leq i \leq n_H$ such that $x \equiv 0 \pmod{g_x^i}$ and $y \equiv 0 \pmod{g_y^i}$, a node $(x, y)$ is said to belong to layer $i$. For $i > 0$ this means that it is connected by hierarchical lines to the nodes $(x + g_x^i, y), (x - g_x^i, y), (x, y + g_y^i)$, and $(x, y - g_y^i)$ that exist in the structure. These lines are also said to be lines of layer $i$. As such, the introduction of hierarchies is the introduction of an additional set of lines. The main model used in this paper is the Perfect Square Mesh. In addition to the conditions above, $S$ must satisfy that $dim_x = g_x^{n_H}$, $dim_y = g_y^{n_H}$, and $g_x = g_y$.

For short, we write $g = g_x = g_y$. An example of such a structure, illustrating how performance gains are obtained, is shown in Figure 8.3.
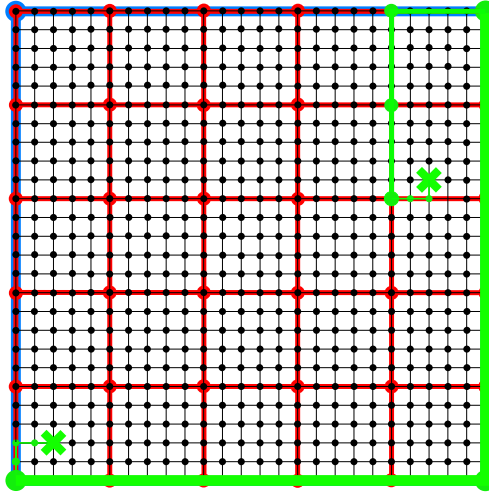


**Figure 8.3:** A Perfect Square Mesh with $g = 5$ and $n_H = 2$. The distance between the marked nodes is reduced from 34 to 12 by the addition of hierarchical lines.

## 8.7.2 Revised routing scheme

Before introducing the revised routing scheme, some notation is necessary. For any pair of nodes $u = (x_u, y_u)$ and $v = (x_v, y_v)$, both belonging to layer $i$ ($0 \leq i \leq n_H$) of a structure with hierarchical extension, let $d_i(u, v) = \frac{|x_u - x_v|}{g_x^i} + \frac{|y_u - y_v|}{g_y^i}$. Furthermore, let $u_j = (x_{u_j}, y_{u_j})$ and $v_j = (x_{v_j}, y_{v_j})$ be the nodes of layer $j$ ($0 \leq j \leq n_H$) such that $d_0(u, u_j)$ and $d_0(v, v_j)$ are smallest possible.

The revised routing scheme makes use of a number of easily established properties. Assume that $S$ is a 4-regular grid structure with $g_x$, $g_y$, and $n_H$ chosen as above. The scheme is designed for the Perfect Square Mesh with $g > 3$, but even if $g_x \neq g_y$ and/or $g = 3$, it can be easily modified to always result in a shortest path. In these cases it may be necessary to perform more calculations because path lengths in higher layers must be calculated.

1. Let $u = (x, y)$ be a node of $S$. Then the node $u_i$ is easily determined by $(g_x^i \operatorname{round}(\frac{x}{g_x^i}), g_y^i \operatorname{round}(\frac{y}{g_y^i}))$, where $\operatorname{round}(a)$ determines the integer $I$ such that $|a - I|$ is smallest possible. Since $g_x$ and $g_y$ are odd, such rounding is always unique.

2. Let $u$ and $v$ be nodes of $S$, and assume for some value of $i < n_H$ that no path $p_{i+1}$ between $u$ and $v$ using at least one line of layer $i + 1$ exists, which is shorter than the shortest path using only lines of layer $i$ or lower, $p_i$. Then there exists no path using lines of layer $i + 2$ or higher, which is shorter than $p_i$.

3. Let $u$ and $v$ be nodes of $S$, and assume for some $i$ that a shortest path between $u$ and $v$ uses some line of layer $i$. This implies that there exists a shortest path between $u$ and $v$ that contains all nodes $u_j$ and $v_j$, for all $j \leq i$.

The routing scheme works as follows, assuming that a packet $p$ is to be sent from $u$ to $v$, $u$ belonging to layer $i$ but not layer $i + 1$. $p$ can be sent either through a node of layer $i + 1$ or through nodes of layer $i$ and below only. If it is sent through layer $i+1$, it is forwarded to a node $u'$ among $(x_u + g_x^i, y_u), (x_u - g_x^i, y_u), (x_u, y_u + g_y^i), and (x_u, y_u - g_y^i)$ that minimizes $d_0(u_{i+1}, u')$. If it is not sent through layer $i+1$, two schemes can be used. In case $g_x \neq g_y$, only the first scheme guarantees that a shortest path is always chosen:

1. Let $u'$ be a neighbour of $u$ that belongs to the highest possible layer while still satisfying $d_0(u', v) < d_0(u, v)$. Then $p$ is sent to $u'$.

2. $p$ is forwarded to any neighbour $u'$ of $u$ that minimizes $d_o(u', v)$.

If $p$ is sent through layer $i$, a shortest path from $u$ to $v$ that passes at least one node of layer $i$ can be constructed passing both $u_i$ and $v_i$ due to property (3), and the same holds for any shortest path from $u$ to $v$ that passes at least one node of layer $i + 1$: such a shortest path exists passing both $u_i$, $u_{i+1}$, $v_{i+1}$, and $v_i$.

Therefore, when deciding if routing should be done through layer $i+1$, it is sufficient to compare the distance from $u = u_i$ ($u$ belongs to layer $i$) to $v_i$ using layer $i$ and layer $i + 1$. Note that routing through layer $i$ does not necessarily imply that any line of this layer is actually used. The distance between $u_i$ and $v_i$ using layer $i$ is given by $d_i(u, v_i)$ while the distance using layer $i + 1$ is given by $d_i(u, u_{i+1}) + d_{i+1}(u_{i+1}, v_{i+1}) + d_i(v_i, v_{i+1})$. All these nodes are easily determined due to property (1).

Cases can occur where the distances using layer $i$ and layer $i + 1$ are identical, and similarly if routing is done using layer $i$, it may happen that more than one neighbour of $u$ can be selected for forwarding. In this case, routing can be done using any such layer and line. Usually the lowest hierarchical layer will be chosen.

Routing can of course also be done using layer $i + 2$ or higher. This decision is taken whenever a packet reaches a node of layer $i + 1$. This is sufficient due to properties (2) and (3).

The presented scheme always determines a shortest path, and selects the appropriate hierarchical layer, ensuring maximum benefit from the hierarchical extension.

The restoration and protection schemes introduced in Section 8.6 have not yet been extended to deal with the problems, which occur in hierarchical structures. The protection scheme is easily extended to provide four node independent paths between any pair of nodes within each layer, making it possible to establish line independent paths from source to destination even in a hierarchical structure. However, the same nodes may be used when shifting from one layer to another. This may be managed by improving the reliability of nodes belonging to first or higher layers, but increasing distances are still a problem: the additional hops as specified in the scheme are added at each layer. The restoration scheme must also be extended, especially in order to handle situations where a packet is to be routed from a layer $i$ to layer $i+1$, but where the nearest layer $i+1$ node is out of order or not reachable.

### 8.7.3   Performance

The performance of the hierarchical extension in terms of average average distances, worst-case average distances, and diameters were measured by calculating those values for a number of hierarchical structures. The results are shown for the basic structures (Figure 8.2) and for the Perfect Square Mesh (Figures 8.4–8.6). While the basic structures have power-law dependencies between the number of nodes and the distances, the dependencies are logarithmic for the Perfect Square Mesh. Further details of this study can be found in[PPKM04b].
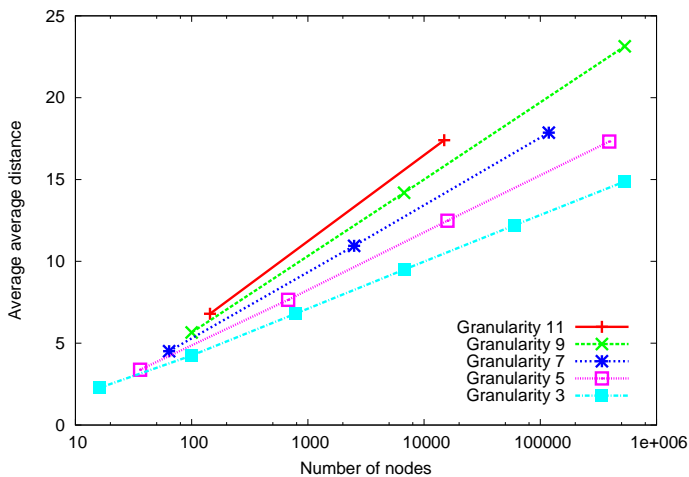


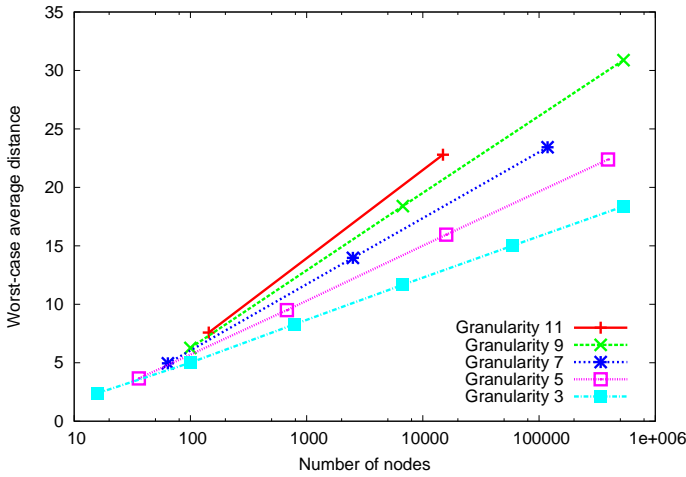**Figure 8.4:**  Average average distances in Perfect Square Mesh.

**Figure 8.5:** Worst-case average distances in Perfect Square Mesh.
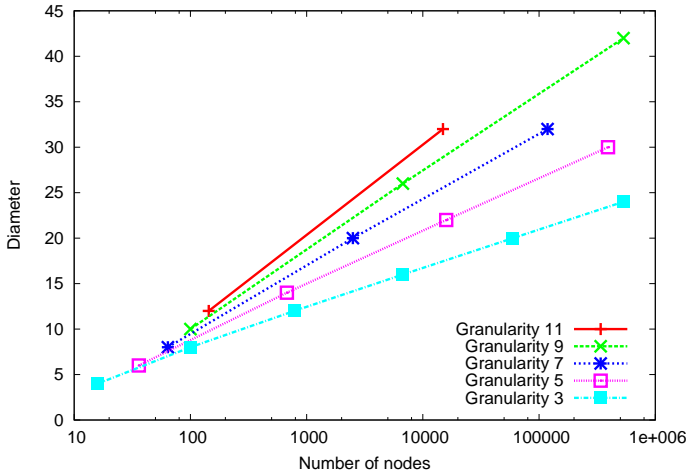


**Figure 8.6:** Diameters in Perfect Square Mesh.

## 8.8 Cost reduction

Since ring structures have average degree 2, and double ring structures as well as Generalized Petersen Graphs[MKP03] have average degree 3, the 4-regular grid structure is more expensive: the torus is 4-regular and consequently the average degree is 4, while the mesh has a slightly lower average degree due to edge and corner nodes. For a structure of for example $26 \cdot 26$ nodes, the average degree is 3.85, approaching 4 for larger structures. The hierarchical extensions generally add to the number of

**Table 8.1:** Average degree and the contribution to this from basic lines in Perfect Square Mesh with $g = 5$.

| Size | Avg. degree | % of lines basic |
|---|---|---|
| $n_H = 1$, 36 nodes | 3.56 | 93.8 |
| $n_H = 2$, 676 nodes | 4.04 | 95.3 |
| $n_H = 3$, 15 876 nodes | 4.14 | 95.8 |
| $n_H = 4$, 391 876 nodes | 4.16 | 96.0 |

lines and node degrees, but their contribution to the total number of lines and average degree is limited, as can be seen from Table 8.1.

The high costs of the 4-regular grid structures make it difficult to apply them in real-world networks since rings offer a level of protection which has been satisfactory so far. However, cheaper variants of the 4-regular grid structure may form better solutions than the rings even in the short term, while at the same time allowing for gradual extensions. In the following, two ways of pruning are discussed.

The first approach is to remove every second line in one of the directions as shown in Figure 8.7, leading to a honeycomb structure. This approach does seem feasible: it is cheaper than the 4-regular grid, it can be extended/upgraded to a 4-regular grid structure, and addressing and routing have been described, with properties similar to those of the 4-regular grid structure[Sto97]. However, no hierarchical extension has been developed or proposed so far. The rest of the section deals with another approach, making use of the fact that the number of hierarchical lines is very small compared to the number of basic lines. This motivates an approach of reducing only the number of lines in the basic structure, see Figure 8.8. This is done by removing all basic lines in the $y$ direction except for those co-located with lines of a higher hierarchy. In other words, two nodes $(x_u, y_u)$ and $(x_v, y_v)$ are connected by a basic line if and only if one of the following is true:

- $y_u = y_v$ and $|x_u - x_v| = 1$.

- $x_u = x_v$, $|y_u - y_u| = 1$ and $x_u \equiv 0 \pmod{g_x}$.

The performances of such a pruned Perfect Square Mesh with $g = 3$ and $g = 9$ are shown in Figures 8.9–8.10. It can be seen that while the number of lines, and thus the average degree as well as the cost, is significantly reduced, the average average and worst-case average distances are only slightly larger. For the structures with 6724 nodes, the number of lines is reduced by 29.2% for the Perfect Square Mesh with $g = 3$, and by 43.3% for the Perfect Square Mesh with $g = 9$. The average distances are increased by 0.024% respectively 0.35%. The corresponding values for the structures with 532 900 nodes are reductions in the number of lines of 29.6% resp. 43.8%,
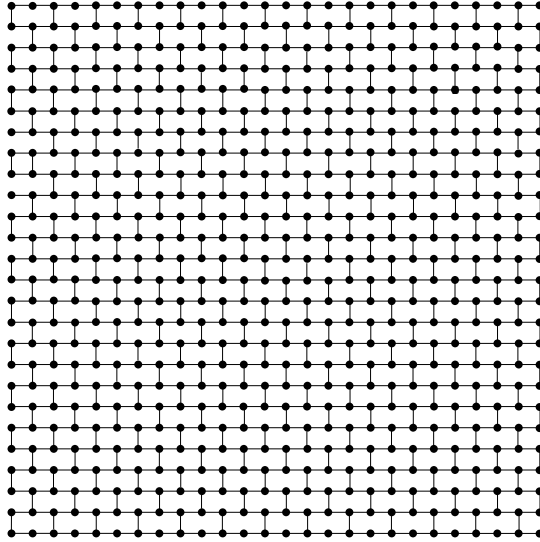
**Figure 8.7:** Pruning a basic structure to obtain a honeycomb.
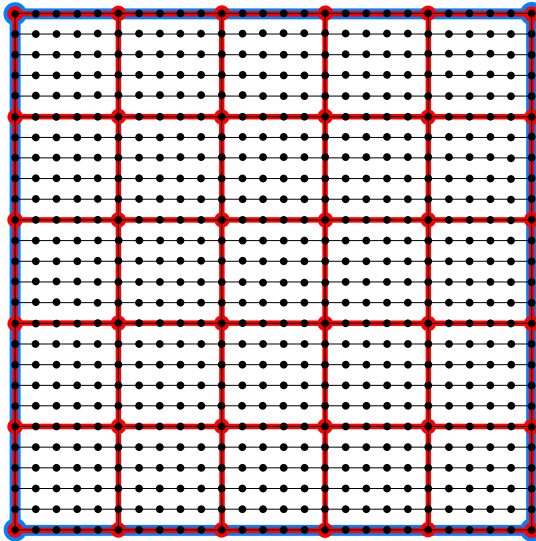


**Figure 8.8:** Pruning of a Perfect Square Mesh with $g = 5$ and $n_H = 2$ by removing all basic lines in the $y$ direction, except those co-located with lines of higher hierarchies.

and an increase in average distance of $<0.001\%$ resp. $0.003\%$. The calculations were done with a precision of 3 decimals, but for $g = 3$ with 532 900 nodes, the calculations returned identical values. Due to the fact that only distances between pairs of nodes for which all shortest paths use no hierarchical lines are affected, the
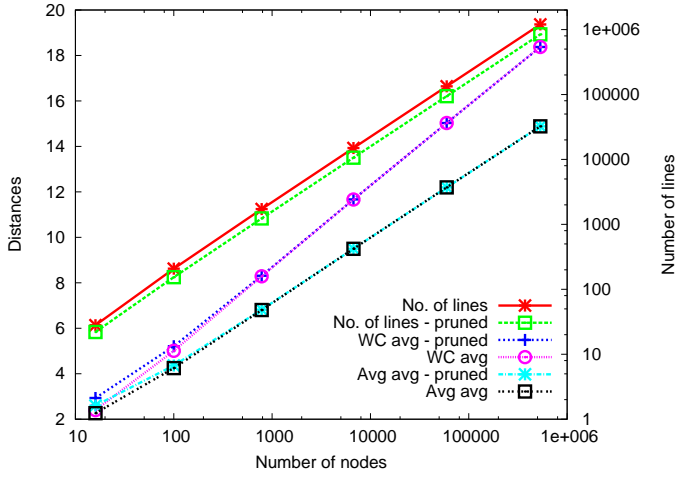
**Figure 8.9:** Comparison of Perfect Square Mesh to pruned Perfect Square Mesh for $g = 3$.
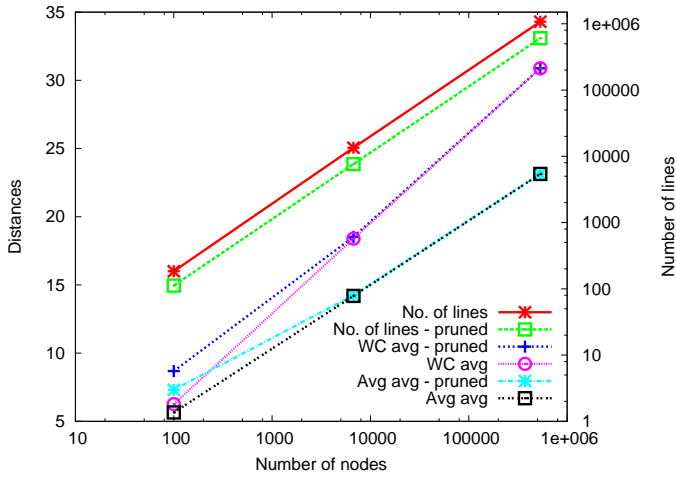


**Figure 8.10:** Comparison of Perfect Square Mesh to pruned Perfect Square Mesh for $g = 9$.

differences are smallest for large structures and structures with small granularity. The diameters are the same for pruned and non-pruned structures. While the distances are only slightly affected by this pruning, the reliability is better in the non-pruned structures due to the higher connectivity.

This approach to pruning requires a revised routing scheme. Assume that a packet is send from a node $u = (x_u, y_u)$ to another node $v = (x_v, y_v)$. A revised routing scheme must be used in case all of the following conditions are true:

- $x_u \neq 0 \pmod{g_x}$.

- $x_v \neq 0 \pmod{g_x}$.

- $y_v \neq y_u$.

- $\lfloor \frac{x_u}{g_x} \rfloor = \lfloor \frac{x_v}{g_x} \rfloor$.

In this case, $d_0(u, v)$ is different from the corresponding value in the non-pruned structure. Two alternative path lengths are calculated, corresponding to using either $(g_x \lfloor \frac{x_u}{g_x} \rfloor, y_u)$ or $(g_x \lceil \frac{x_u}{g_x} \rceil, y_u)$ as intermediate node. The corresponding values of $d_0(u, v)$ are $|y_u - y_v| + (x_u \bmod g_x) + (x_v \bmod g_x)$ and $|y_u - y_v| + 2g_x - (x_u \bmod g_x) - (x_v \bmod g_x)$ respectively. Depending on which one results in the shortest path, the packet is sent towards one of these intermediate nodes in case routing is done using only the basic layer.

In all other cases, a shortest path is ensured by forwarding packets along the $x$ direction in the basic layer only if routing along the $y$ direction of the basic layer does not reduce the distance to the destination. It is necessary though to adjust the protection scheme in all cases.

## 8.9   Embedding of structures

The studies conducted so far have mainly concentrated on topological properties of network structures, and abstractions from real networks have been made to large extents. However, in order for the structures to be applied in real-world networks, there must exist ways of mapping them onto the physical level.

In order to decide if it is possible to map some structure into a real-world network, it is necessary to consider a number of physical conditions: existing structures and ducts may be preferable to use, location and density of nodes may depend on the density of homes and offices, some places are more suitable for ducts than others etc. Due to the dependability on such specific conditions, it is not possible to present one single method for embedding the structures. Instead we provide a number of useful tools, which can be combined in order to develop structures that are possible to use in specific cases.

### 8.9.1   Varying hierarchical depth

From a Perfect Square Mesh, it is possible to omit either one or more of the lowest hierarchical layers in parts of the structure, without affecting the routing scheme. This allows for making a structure more dense in some areas than in others. It also
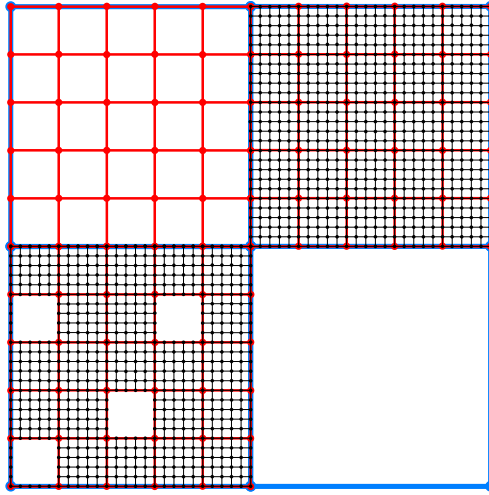
**Figure 8.11:** Flexibility obtained by removing areas of lowest hierarchical layers.

makes it possible to extend a structure after it is initially deployed, because omitted layers are easily added later on, and it allows for constructing structures of different sizes. An example of a structure with such omitted areas is shown in Figure 8.11.

The areas to be left out must be selected carefully. Removing a single node or an arbitrary set of nodes may result in the routing scheme no longer guaranteeing that a shortest path is always chosen. In the following, areas of the lowest layer that can be left out are described, and a generalization to more layers presented.

Assume a Perfect Square Mesh, with $dim_x$, $dim_y$, and $g = g_x = g_y$, and let $a$ and $b$ be positive integers such that $ag \leq dim_x$ and $bg \leq dim_y$. $a$ and $b$ then define an area $A$. A node $(x, y)$ belongs to the inner of $A$ if and only if $(a - 1)g < x < ag$ and $(b - 1)g < y < bg$. $A$ also has a border. It consists of the four corner nodes $((a - 1)g, (b - 1)g)$, $((a - 1)g, bg)$, $(ag, (b - 1)g)$, and $(ag, bg)$ as well as four sections, each defined by the nodes with coordinates fulfilling the conditions below, respectively:

- $x = (a - 1)g$ and $(b - 1)g < y < bg$.

- $x = ag$ and $(b - 1)g < y < bg$.

- $y = (b - 1)g$ and $(a - 1)g < x < ag$.

- $y = bg$ and $(a - 1)g < x < ag$.

When $A$ is left out, this is done by removing all nodes in the inner of $A$ as well as all lines connected to at least one node in the inner of $A$. Nodes belonging to the border

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of $A$ can be removed as well, but the conditions are slightly more complicated: no node belonging to the inner of $A$ belongs to the inner or border of any other area, but a node belonging to the border of $A$ may belong to the border of up to three other areas.

The four corner nodes of $A$ cannot be removed at this step because they also belong to layer 1, but other border nodes can be removed section-wise. Within each section either none or all are removed, and if a section is removed so are all basic lines connected to at least one node in the section. A section can only be removed if no node in it is connected to a node outside it, except for the corner nodes of $A$.

It is interesting but also rather trivial to realize that for two nodes $u$ and $v$ in a structure where an area of the lowest layer is left out, the distance is the same as in the full structure. Let $S$ be a Perfect Square Mesh, and let $S'$ be identical to $S$, except that one area $A$ (say, the inner of it) is left out. This implies that there exist integers $a$ and $b$ such that the nodes with $x$ coordinates $(a - 1)g < x < ag$ and $y$ coordinates $(b - 1)g < y < bg$ are left out.

Assume for contradiction that there are two nodes, $u$ and $v$, in $S'$ between which a shortest path is longer than a path between $u$ and $v$ in $S$. This implies that any shortest path between $u$ and $v$ in $S$ uses at least one node from the inner of $A$, which again implies that there exist two nodes $u'$ and $v'$ on the border of $A$ such that any shortest path between them passes only nodes of the inner of $A$ and thus is shorter in $S$ than in $S'$. Assume that $u'$ has coordinates $(x_1, y_1)$, and that $v'$ has coordinates $(x_2, y_2)$, and note that neither $u'$ nor $v'$ are corner nodes of $A$. Without loss of generality it is assumed that $x_1 \leq x_2$ and $y_1 \leq y_2$.

Three cases are considered:

- $x_2 - x_1 < g$ and $y_2 - y_1 < g$. If $x_1 = x_2$ or $y_1 = y_2$, assume without loss of generality that $x_2 = x_1$. $y_2 - y_1 < g$ and thus $x_1 = (a - 1)g$ or $x_1 = ag$, and in both cases a path exists: $(x_1, y_1), (x_1, y_1 + 1), \ldots, (x_1 = x_2, y_2)$. This path has length $y_2 - y_1$, equal to $d_0(u', v')$. So, assume $x_1 \neq x_2$ and $y_1 \neq y_2$. Then $x_1 = (a - 1)g$ and $y_2 = bg$ or $x_2 = bg$ and $y_1 = (b - 1)g$. In the first case there exists a path $(x_1, y_1), (x_1, y_1 + 1), \ldots, (x_1, y_2), (x_1 + 1, y_2),$ $\ldots, (x_2, y_2)$, and in the second case a path $(x_1, y_1), (x_1 + 1, y_1), \ldots, (x_2, y_1),$ $(x_2, y_1 + 1), \ldots, (x_2, y_2)$. Both paths are of length $x_2 - x_1 + y_2 - y_1$, and thus equal to $d_o(u', v')$, and a contradiction is obtained.

- $x_2 - x_1 = g$ and $y_2 - y_1 < g$. This implies $x_1 = (a - 1)g$ and $x_2 = ag$. Two paths $p_1$ and $p_2$ are constructed, where $|p_1|$ and $|p_2|$ denote their respective lengths. $p_1$ contains the layer 1 line connecting $(x_1, (b-1)g)$ and $(x_2, (b-1)g)$ while $p_2$ contains the layer 1 line connecting $(x_1, bg)$ and $(x_2, bg)$. Thus, $p_1 = (x_1, y_1), (x_1, y_1 - 1), \ldots, (x_1, (b-1)g), (x_2, (b-1)g), (x_2, (b-1)g + 1), \ldots,$

$(x_2, y_2)$ and $p_2 = (x_1, y_1), (x_1, y_1 + 1), \ldots, (x_1, bg), (x_2, bg), (x_2, bg - 1),$ $\ldots, (x_2, y_2)$. $|p_1| = y_1 + y_2 - 2(b-1)g + 1$ and $|p_2| = 2bg - y_1 - y_2 + 1$. This implies that $|p_1| + |p_2| = 2 + 2g$. Since $x_2 - x_1 = g$, $d_0(u', v') \geq g$. It must then be true that $|p_2| > g$. If $|p_2| \geq g + 2$ then $|p_1| \leq g$, and thus $|p_2| = g + 1$. Since $|p_2| = 2bg - y_1 - y_2 + 1$ this implies that $2bg - y_1 - y_2 = g$. Since $g$ is odd, $y_1 - y_2$ is odd, but then $y_1 \neq y_2$, implying that $d_0(u', v') = g + y_2 - y_1 \geq g + 1$, but then the distance is not shorter in $S$ than in $S'$, a contradiction.

- $x_2 - x_1 < g$ and $y_2 - y_1 = g$. This case is similar to the case above, and a final contradiction obtained.

The property also holds if one or more sections of border nodes of $A$ are removed: these nodes are only connected to each other and to nodes of the first layer, and their removal does therefore not increase any distances.

Since the distance between no pair of nodes is increased in $S'$ compared to $S$, the existing routing scheme only needs to be changed in nodes of borders of left-out areas. While initially algorithms similar to the lake algorithms seem necessary, it can actually be solved by simpler means.

According to the proof stated, two approaches will both result in a shortest path being chosen in all cases:

- Routing always using the highest hierarchical. layer in which a shortest path exists.

- Whenever a packet is forwarded from a node of the border of a left-out area, and it should normally be forwarded to a node of the inner of this area, it is instead forwarded towards the nearest higher-hierarchy node.

The principle described can be used for other layers than the basic layer. However, no node of layer $i$ can be removed if it is connected by any lines of a layer $< i$.

For a layer $i$, and positive integers $a$ and $b$ such that $ag^i \leq dim_x$ and $bg^i \leq dim_y$, an area $A$ of layer $i$ is defined by the nodes such that $(a-1)g^i \leq x \leq ag^i$ and $(b-1)g^i \leq y \leq bg^i$. A node $(x, y)$ belongs to the inner of $A$ if and only if $(a-1)g^i < x < ag^i$ and $(b-1)g^i < y < bg^i$. The inner of $A$ can be removed only if all nodes not belonging to layer $i$ within the inner of $A$ have been removed. Sections of border nodes of $A$ are defined as in the basic layer, and can be removed under the same conditions with the addition that a node cannot be removed if it is connected by lines of a layer $< i$. Nodes on the corner of $A$ cannot be removed at this step because they are also nodes of layer $i + 1$. The nodes of layer $n_H$ are all corner nodes of $S$, and it makes no sense to remove them, since this would imply a removal of the whole structure.

The distances of a structure with various left-out layers, may differ from those of a Perfect Square Mesh, depending on which layers a left out, and further analysis is needed in each case to clarify this. However, for each such structure the distance between any pair of nodes equals the distance between the same pair of nodes in a full Perfect Square Mesh. Since the second approach to pruning described in Section 8.8 only applies to the lowest hierarchical layer, the gains of pruning may be less significant if combined with leaving out layers, depending on the extent to which layers are left out.

It was previously[PKM04c] proposed to merge Perfect Square Meshes side by side in order to obtain structures with different sizes and shapes than the Perfect Square Mesh while still maintaining $g_x = g_y$. Actually, this corresponds to leaving out a number of lowest layers as well as removing the highest layer. Leaving out the highest layer does affect performance, and it should also be noted that it is possible to do so only if the structure remains connected. Furthermore, if the highest hierarchical layer is removed, care must be taken as to which areas are left out, unless the routing scheme is revised.

## 8.9.2 Varying granularity

It was previously shown[PPKM04b] that hierarchical structures different from the Perfect Square Mesh (i.e. $g_x \neq g_y$) also result in logarithmic dependencies between the number of nodes and distances in the structure. An example of such a skew structure with $g_x = 11$ and $g_y = 3$ is shown in Figure 8.12. Applying this in combination with leaving out layers makes it necessary to refine the routing scheme in order to ensure that a shortest path is always chosen. As shown in Figure 8.13, the structures with $g_x = 11$ and $g_y = 3$ have performances close to those of the Perfect Square Mesh with $g = 9$. However, as can be seen in Tables 8.2 – 8.3, the skew mesh requires relatively more hierarchical lines and contains more edge nodes than the Perfect Square Mesh, resulting in more expensive and less robust structures.
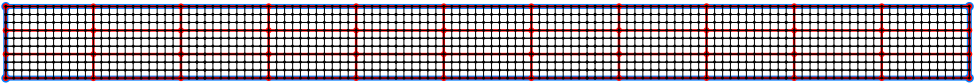


**Figure 8.12:** A structure with $g_x = 11$, $g_y = 3$ and $n_H = 2$.

It is also possible to vary the granularity from layer to layer: instead of choosing one value of $g_x$ and one value of $g_y$, $g_{x_i}$ and $g_{y_i}$ are chosen for every $i, 0 < i \leq n_H$. Now $dim_x = g_{x_1} g_{x_2} \ldots g_{x_{n_H}}$ and $dim_y = g_{y_1} g_{y_2} \ldots g_{y_{n_H}}$. The definition of hierarchical nodes and lines as well as the routing scheme must be adjusted accordingly.
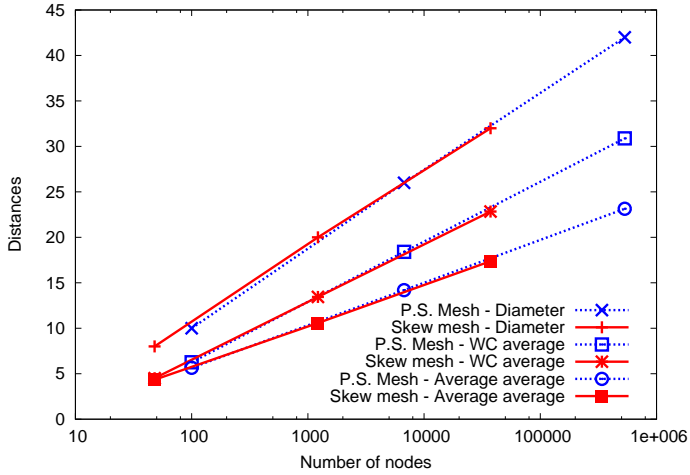
**Figure 8.13:** Performances of skew structures with $g_x = 11$ and $g_y = 3$ compared to performances of Perfect Square Mesh with $g = 9$.

**Table 8.2:** Contribution from hierarchical lines to the average node degree and the relative number of edge nodes for structures with $g_x = 11$ and $g_y = 3$.

| Size | Avg. degree from h-lines | % of nodes on edge |
|------|--------------------------|--------------------|
| $n_H = 1$, 48 nodes | 0.17 | 58.3 |
| $n_H = 2$, 1220 nodes | 0.14 | 21.3 |
| $n_H = 3$, 37 296 nodes | 0.13 | 7.28 |

**Table 8.3:** Contribution from hierarchical lines to the average node degree and the relative number of edge nodes for Perfect Square Mesh with $g = 9$.

| Size | Avg. degree from h-lines | % of nodes on edge |
|------|--------------------------|--------------------|
| $n_H = 1$, 100 nodes | 0.080 | 36.0 |
| $n_H = 2$, 6724 nodes | 0.055 | 4.82 |
| $n_H = 3$, 532 900 nodes | 0.051 | 0.55 |

Another promising extension of the Perfect Square Mesh, the extended mesh, maintains the square shape, but the size is redefined, such that $-\lfloor \frac{g^{n_H}}{2} \rfloor \leq x, y \leq \lfloor \frac{3g^{n_H}}{2} \rfloor$, as seen on Figure 8.14. It was evaluated for $g = 5$, and performance was close to the Perfect Square Mesh with the same granularity[PKM04c], as can be seen in Figure 8.15. Using this approach to various extents makes it possible to construct structures of different sizes than the Perfect Square Mesh definitions allow for.
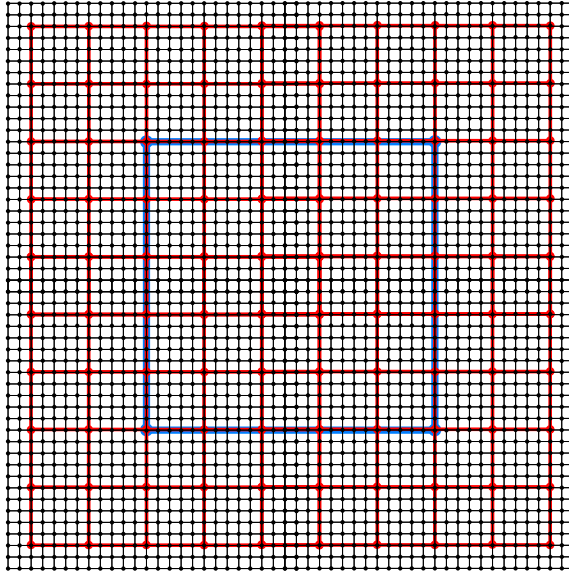


**Figure 8.14:** Extended structure with $g = 5$ and $n_H = 2$.
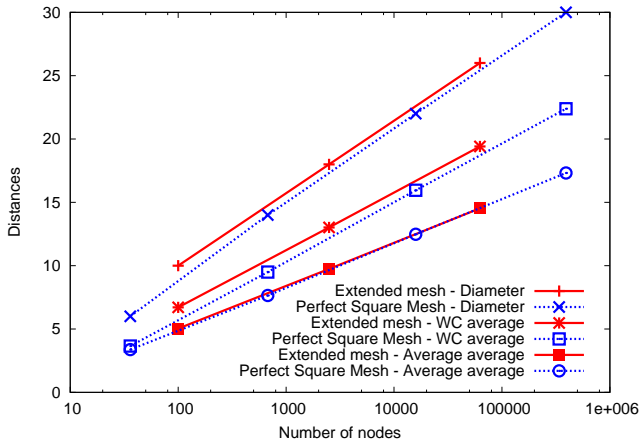


**Figure 8.15:** Performances of extended structures with $g = 5$ compared to performances of Perfect Square Mesh with $g = 5$.

# 8.10   Conclusion and further work

This paper has discussed a number of aspects on how to apply the 4-regular grid structures in large-scale communication access networks. Due to the nice properties of these structures, including simple routing and restoration schemes, they have been deployed in multiprocessor systems for decades. However, they are not directly applicable for use in large-scale networks. Four major barriers have been dealt with in this paper, namely restoration/protection, scalability, cost, and embeddability.

With regard to restoration, a simple scheme was introduced, facilitating fast and easy restoration by the concept of lake algorithms: if only a small part of the network is failing, only small tables are needed, and consequently only little communication is necessary in order to create and maintain these tables. It is, however, necessary to revise the lake algorithms if they are to be applied to hierarchically extended structures as described below. How this is done is not yet fully clarified, and should be subject to further investigation. We suggest that simulations are used in order to try out different approaches.

A simple way of establishing up to four independent paths was introduced for protection. In the basic structure (and generally within each hierarchical layer) the paths are node independent, but in the hierarchical extension, nodes are shared wherever a packet moves from one hierarchical layer to another. In some cases only two or three independent paths can be set up. This is the case if the source or destination nodes are edge/corner nodes, or if the routing is done in a pruned structure and either source or destination node belongs only to the basic layer.

A hierarchical extension was proposed in order to deal with the scalability problem. This approach results in logarithmic dependencies between the number of nodes and the distances in the structure. Depending on the chosen granularity, the number of hierarchical lines was small compared to the number of lines in the basic structure.

To deal with the cost problem, an approach was suggested where in a hierarchical structure some lines of the basic layer are left out. As with any other way of reducing the number of lines, it reduces the connectivity, but the distances in the structure were hardly affected.

Due to the many physical and demographic conditions and constraints, it is not possible to provide a general method for embedding these structures in real-world networks. This paper presented a tool-case with different approaches which can be used and combined to different extents.

So far, the structures have only been dealt with theoretically, and we suggest that future research should have a more practical focus. Restoration and protection schemes should be tested by simulations and small-scale tests, particularly for hierarchical

structures, where such results are necessary to clarify how certain situations should be handled.

Further studies should also be conducted on how the structures can be embedded in real-world networks. We suggest that case studies are performed under different geographical and demographic conditions in order to clarify how and to what extent the structures can be embedded, and in order to indicate what other tools may be needed in order to facilitate embeddings.

## 8.11    Acknowledgements

We would like to thank University College Dublin for hosting Jens Myrup Pedersen and Thomas Phillip Knudsen as visiting researchers during the work of this paper, as well as Nikita Schmidt for his helpful comments.

## 8.12    Additional comments*

In Section 7.10, minimum average distance and minimum diameter were introduced and determined for the Perfect Square Mesh, skew mesh and extended mesh. It should be noted that pruning a hierarchical structure as described in Section 8.8 does not change neither minimum average distance nor minimum diameter.

# 9 | Generalized Double Ring Network Structures [PPKM04a]

## 9.1 Preface*

This paper was written in collaboration with Ahmed Patel, Thomas Phillip Knudsen, and Ole Brun Madsen, and presented at The 8th World Multi-Conference On Systemics, Cybernetics and Informatics (SCI 2004). It is the first of four papers on the $N2R$ structures, and the contribution to the thesis can be summarized as follows. The paper:

- Gives the basic properties with regard to symmetry and isomorphisms.

- Devises a basic routing algorithm.

- Determines average distance and diameter, and compare these to the values of the Double Ring. $N2R$ is shown to be superior with regard to these parameters.

The work is continued in Chapters 10–11, with the evaluation of $k$-average distances and $k$-diameters. Chapter 12 also complements the work, by evaluating traffic load and exploring potentials of implementing the structures by combining wired and wireless technologies.

## 9.2 Abstract

This paper describes and studies generalizations of the well known double ring network structures. Two classes of structures are studied, the $N2R(p;q)$ and $N2R(p;q;r)$ structures, of which the former is a special case of the well known Generalized Petersen Graphs. Basic properties of these structures are shown, indicating that they form a suitable base for future access network infrastructures. The first result is that every $N2R(p;q;r)$ structure is isomorphic to a $N2R(p;q)$ structure $N2R(p;q')$, and it is shown how $q'$ is determined. Consequently, the rest of the paper focuses on the $N2R(p;q)$ structures. Results on the Generalized Petersen Graphs provide necessary and sufficient conditions for a $N2R(p;q)$ structure to be

node or line symmetric, and a table-free routing scheme always determining a shortest path between any pair of nodes is presented. Next, the performance in terms of average distances and diameters is evaluated and compared to the performance of double rings. This comparison shows that the $N2R(p;q)$ structures are superior to the double rings with regard to distances. For example, a $N2R(p;q)$ structure with 1000 nodes has average distance 12.0 and diameter 18, while a similar sized double ring has average distance 125.6 and diameter 251. Finally suggestions for further research are given.

## 9.3   Introduction

One of the largest network planning tasks ever is the migration from copper based to fibre based access networks, which is expected to take place in large parts of the world within near future. These new access networks form a major investment in IT infrastructure, and due to the high duct costs of deploying such networks, the basic physical structures of the networks are assumed to have a long lifetime. This assumption is further supported by the fact that fibre networks are upgradeable by changing only end equipment. As a consequence, the physical network structures should be chosen carefully from the beginning. So far, fibre rings have been widely used due to a number of good properties: two node independent paths exist between any pair of nodes, ensuring operation even in case of failures, restoration is fast and simple, and routing is easy. It is also easy to add nodes to the structure.

However, the ring structures also have drawbacks: distances increase fast and linearly with the number of nodes, and the protection and restoration schemes are limited since only one spare path is available. Furthermore, using this spare path may cause delay or jitter in transmissions since its hop length can be as large as the number of nodes in the ring. This has been sufficient for the best-effort traffic of the Internet accommodated so far, but applications moving from LANs into WANs (e.g. [HKKK01][XT98][XmCjYXd04]), the convergence of communications[MNS02], and the general dependency on networks are likely to increase demands to more than what is supported by rings. A future-proof infrastructure needs to take this into account, and thus alternatives to the rings must be developed.

3-regular structures are interesting because they have the potential of offering three independent paths between any pair of nodes, while at the same time being cost efficient. However, such structures are generally more complicated with regard to routing, restoration, protection, and setup: at least they must be organised in a planned and well organised manner in order to benefit from the higher connectivity. An obvious 3-regular generalization of the ring structure is the double ring. This paper discusses two generalizations of the double ring structure, the $N2R(p;q)$ and

$N2R(p; q; r)$ structures.

The rest of the paper is organized as follows: Section 9.4 describes the notation used in the paper, Section 9.5 provides an overview of preliminaries, background, and previous research, Section 9.6 explains the methods used, Section 9.7 contains the results obtained, and Section 9.8 concludes the paper with a discussion of the results and suggestions for further research.

## 9.4   Notation

Throughout this paper, network structures are studied. A structure consists of a set of nodes and a set of lines, such that each line interconnects two nodes. Lines are considered undirected: if a pair of nodes $(u, v)$ is connected by a line, so is $(v, u)$. A path from a source node $u$ to a destination node $v$ is a set of nodes and lines $(u = u_0), e_1, u_1, e_2, u_2, e_{n-1}, u_{n-1}, e_n, (u_n = v)$, where every line $e_i$ connects the nodes $u_{i-1}$ and $u_i$. The path length is determined by the number of lines between the source and destination node; in the previous case, the path is of length $n$. The distance between two nodes $u$ and $v$ is written $d(u, v)$ and is determined by the length of the shortest path between them. For a node $u$, the set of nodes $v$ such that $d(u, v) = 1$ are said to be the neighbours of $u$. Two nodes are said to be connected if and only if they are neighbours. The degree of a node corresponds to the number of neighbours it has. If all nodes of a structure have the same degree $n$, the structure is said to be $n$-regular. The size of a structure is given by the number of nodes the structure contains. All indexes are calculated modulo $p$, where the value of $p$ will be clear from the context. For any integer $n$ the value of $n \pmod p$ is determined as $n + kp$, where $k$ is the smallest integer such that $n + kp \geq 0$. This definition implies that e.g. $-1 \pmod 5 = 4$.

## 9.5   Background

The $N2R(p; q)$ structures were proposed used for networks in 2002[MNS02], and the $N2R(p; q; r)$ structures were introduced as an extension in 2003[MKP03]. To define the structures let $p$ and $q$ be positive integers such that $q < \frac{p}{2}$ and $gcd(p, q) = 1$. A $N2R(p; q)$ structure then consists of two interconnected rings, both with $p$ nodes. The outer ring consists of the nodes $o_0, o_1, \ldots, o_{p-1}$, such that every node $o_n$ is connected to $o_{n-1}$ and $o_{n+1}$, while the inner ring consists of the nodes $i_0, i_1, \ldots, i_{p-1}$ such that every node $i_n$ is connected to $i_{n-q}$ and $i_{n+q}$. The rings are interconnected such that every node $o_n$ is connected to $i_n$. For a $N2R(p; q; r)$ structure it is additionally assumed that $r$ is a positive integer, that $r < \frac{p}{2}$ and that $gcd(p, r) = gcd(q, r) =$

1. A $N2R(p;q;r)$ structure is defined similar to a $N2R(p;q)$ structure, except for the outer ring: any node $o_n$ is connected to $o_{n-r}$ and $o_{n+r}$ instead of $o_{n-1}$ and $o_{n+1}$. Examples are shown in Figures 9.1–9.2. It is easily seen that $N2R(p;q;1)$ is equivalent to $N2R(p;q)$ for all values of $p$ and $q$.
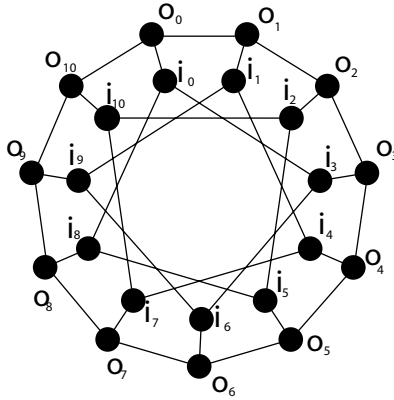


**Figure 9.1:** $N2R(11;3)$.
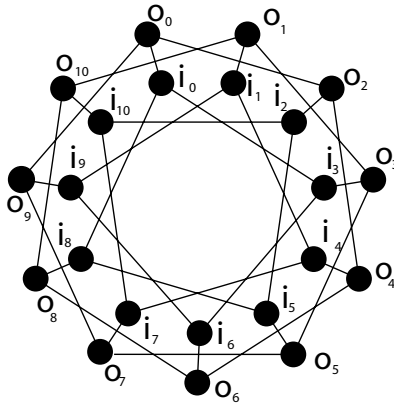


**Figure 9.2:** $N2R(11;3;2)$, isormorphic to $N2R(11;2;3)$ and to the $N2R(11;3)$ structure shown in Figure 9.1.

The class of $N2R(p;q)$ structures is a special case of the Generalized Petersen Graphs, which have long been known in the area of graph theory. For the Generalized Petersen Graphs it is not generally assumed that $gcd(p,q) = 1$, but according to [FGW71] this special case, which is exactly the class of $N2R(p;q)$ structures, was described for the first time in 1950[Cox50]. Only few results have been established regarding its properties as a network structure. This paper aims at establishing some basic results in this regard.

## 9.6 Methods

Graph theoretic methods are used to show that every $N2R(p;q;r)$ structure is isomorphic to a $N2R(p;q)$ structure. As a consequence of this, the remainder of the paper considers only $N2R(p;q)$ structures. This also implies that existing results of the Generalized Petersen Graphs are directly transferable to the complete class of $N2R(p;q;r)$ structures. Two such result are used in this paper, providing necessary and sufficient conditions for node and line symmetry.

Next, a simple table-free routing scheme always determining shortests paths is introduced. This routing scheme takes into account only node addresses and the global parameters $p$ and $q$. The scheme may not be optimal with regard to algorithmic complexity, but due to its generality and correctness it forms a base for further research and optimization.

To evaluate the performance of the structures, average distances and diameters for $N2R(p;q)$ structures are calculated for all structures with up to 2000 nodes, as well as for a few larger structures. The average distance is the average over the distances between all pairs of distinct nodes, and the diameter the largest distance between any pair of distinct nodes. The results were compared to those of the double ring (corresponding to $N2R(p;1)$). For every value of $p$, a number of $N2R(p;q)$ structures exist, which in this paper is handled as follows: given $p$, $q$ is chosen as to minimize the average distance. If more values of $q$ satisfies this, $q$ is chosen among these to minimize the diameter. In far most cases, this choice of $q$ also minimizes the diameter.

## 9.7 Results

### 9.7.1 Isomorphisms

It is first shown that every $N2R(p;q;r)$ structure is isomorphic to a $N2R(p;q)$ structure $N2R(p;q')$, and how $q'$ is determined from $p$, $q$, and $r$. Consider a $N2R(p;q;r)$ structure such that the outer ring connects the nodes in the following order: $o_0, o_r, o_{2r}, o_{3r}, \ldots, o_{-2r}, o_{-r}, o_0$. The nodes of this ring are renamed such that every node $o_n$ is assigned the new name $o'_{\frac{kp+n}{r}}$, where $k$ is the smallest integer such that $\frac{kp+n}{r}$ is a nonnegative integer. Since $n < p$ it is easily seen that for each $o_n, 0 \leq n \leq p - 1$, distinct new names are chosen. The nodes of the outer ring are connected in the following order: $o'_0, o'_1, o'_2, \ldots, o'_{p-2}, o'_{p-1}, o'_0$, so that every node $o'_n$ is connected to $o'_{n-1}$ and $o'_{n+1}$. The nodes of the inner ring are renamed in the same manner: every node $i_n$ is renamed $i'_{\frac{kp+n}{r}}$. Clearly, the lines connecting

the outer and inner rings are maintained: a line exists between two nodes $o'_n$ and $i'_m$ if and only if $n = m$. It should also be noted that for every node $o'_n$ or $i'_n$, $n$ satisfies $0 \leq n \leq p - 1$, and that no two nodes have been assigned the same new name. Thus, it remains to show that every node $i'_n$ is connected to exactly two other nodes of the inner ring, $i'_{n-q'}$ and $i'_{n+q'}$. Consider the original name of $i'_n$, $i_{rn}$. This node is connected to two nodes of the inner ring, with the original names $i_{rn+q}$ and $i_{rn-q}$. These nodes are renamed $i'_{\frac{kp+rn+q}{r}} = i'_{n+\frac{kp+q}{r}}$ and $i'_{\frac{k'p+rn-q}{r}} = i'_{n-\frac{k'p+q}{r}}$ respectively, where $k$ is the smallest integer such that $\frac{kp+rn+q}{r}$ is a nonnegative integer, and $k'$ the smallest integer such that $\frac{k'p+rn-q}{r}$ is a nonnegative integer. Since $\frac{kp+q}{r} = \frac{-k'p+q}{r}$ the obvious candidate for $q'$ is $\frac{kp+q}{r}$. If $\frac{kp+q}{r} \geq \frac{p}{2}$ $q'$ is instead chosen as $p - \frac{kp+q}{r}$. It only remains to realize that $gcd(p, q') = 1$: this follows from the fact that $gcd(p, q) = gcd(p, r) = gcd(q, r) = 1$. The fact that every $N2R(p; q; r)$ structure is isomorphic to a $N2R(p; q)$ structure has now been established.

## 9.7.2 Symmetric structures

A structure is said to be node symmetric (or node similar) if the structure "looks the same" from every node and line symmetric if it "looks the same" from every line. [FGW71] shows that a $N2R(p; q)$ structure is node symmetric if and only if $q^2 \equiv 1 \pmod{p}$ or $q^2 \equiv p - 1 \pmod{p}$, and that the only line symmetric $N2R(p; q)$ structures are $N2R(4; 1)$, $N2R(5; 2)$, $N2R(8; 3)$, $N2R(10; 3)$, $N2R(12; 5)$, and $N2R(24; 5)$. $N2R(10; 2)$ would be line symmetric as well, but since $gcd(10, 2) \neq 1$ it is not considered a $N2R(p; q)$ structure.

Even though not all $N2R(p; q)$ structures are node symmetric, only two classes of nodes exist, one for each ring, and there is symmetry within each such class. Furthermore, all nodes are equally important with concern to connectivity, which adds to the robustness. However, only node symmetry can ensure that the impact on link load in case of failures is the same no matter which node is removed. It should be noted that there exist three node independent paths between any pair of nodes; a 3-regular structure satisfying this is said to be optimally connected. According to [DC04] node symmetry and optimal connectivity are important properties to satisfy for robust network structures.

## 9.7.3 Routing schemes

The outset for the proposed routing scheme is hop-by-hop routing: in each node, it is determined to which neighbour a given packet should be forwarded in order to obtain a shortest path. Since a number of calculations must be performed in each

node, further research could deal with optimizing this approach by letting each packet carry additional routing information, or even complete path information. However, the scope for now is to set up a scheme which makes it possible to decide which of the three neighbours a given packet should be forwarded to. The scheme proposed is inspired by the routing scheme known as Topological Routing[PKM04c].

While such a scheme is obvious for ring and double ring structures, it is less obvious for the $N2R(p;q)$ structures. This paper presents a general approach, which works for all choices of $p$ and $q$. It might be possible to find simpler schemes for specific choices or subsets of choices of $p$ and $q$. Assume that the nodes are addressed such that the indexes increase while following the outer ring in clockwise direction (as in Figures 9.1–9.2).

First, some basic functions are introduced: $d_{o+}(o_s, o_d)$ denotes the distance from a node $o_s$ on the outer ring to another node $o_d$ on the outer ring, with the restriction that only the outer ring is used, and that it is forwarded in clockwise direction. $d_{o+}(o_s, o_d) = d - s \pmod{p}$. Similarly, $d_{o-}(o_s, o_d)$ denotes the similar distance, but sending the packet counter-clockwise instead of clockwise, so $d_{o-}(o_s, o_d) = p - d_{o+}(o_s, o_d)$. Furthermore $d_o(o_s, o_d) = min\{d_{o+}(o_s, o_d), d_{o-}(o_s, o_d)\}$. In a similar manner, $d_{i+}(i_s, i_d)$ denotes the distance from $i_s$ to $i_d$, both on the inner ring, with the restriction that only the inner ring is used, and it is forwarded clockwise. $d_{i+}(i_s, i_d) = k$, where $k$ is the smallest nonnegative integer such that $s + kq \pmod{p} = d$. As was the case for the outer ring, $d_{i-}(i_s, i_d) = p - d_{i+}(i_s, i_d)$ and $d_i(i_s, i_d) = min\{d_{i+}(i_s, i_d), d_{i-}(i_s, i_d)\}$. Four different cases are considered; in each case $o$ or $i$ indicates if a node is on the outer or inner ring, while the index $s$ or $d$ indicates if the node is source or destination. The source node is the actual node, where the routing decision is taken. The index $n$ is also being used for an intermediate node. In addition to $d_i$ and $d_o$, also $d_{ioi}$ and $d_{oio}$ are used. The former is the distance between two nodes of the inner ring, using a path including at least one line of the outer ring, and the latter is the distance between two nodes of the outer ring, using a path including at least one line of the inner ring. Finally, for integers $a$ and $b$ such that $a \leq b$, $int(a, b)$ defines the set of integers $\{a \pmod{p}, a + 1 \pmod{p}, \ldots, b \pmod{p}\}$, where $p$ is an integer given from the context.

Routing from $o_s$ to $o_d$: Using only the outer ring results in path length $d_o(o_s, o_d)$. The alternative is to use the inner ring. A shortest path using a line of the inner ring can always be found that uses the line from $o_s$ to $i_s$. From $i_s$ the packet is forwarded along the inner ring to a node $i_n$. From $i_n$ the packet is then forwarded to $o_n$, and along the outer ring to $o_d$. Note that $o_n$ may equal $o_d$. $n$ is chosen within $int(d - \lfloor \frac{q}{2} \rfloor, d + \lfloor \frac{q}{2} \rfloor)$. For each value of $n$, the distance from $o_s$ to $o_d$ is given as follows: $d_{oio}(o_s, o_d) = 2 + d_i(i_s, i_n) + d_o(o_n, o_d)$. $n$ is chosen to minimize this

distance. If $d_o(o_s, o_d) > d_{oio}(o_s, o_d)$, the packet is forwarded to $i_s$. Otherwise the packet is routed as follows: if $d_{o-}(o_s, o_d) \geq d_{o+}(o_s, o_d)$ it is forwarded to $o_{s+1}$, and otherwise to $o_{s-1}$.

Routing from $i_s$ to $i_d$: Using only the inner ring results in path length $d_i(i_s, i_d)$. A shortest path using a line of the outer ring can always be found that passes $o_d$, from which it is forwarded to $i_d$. A node $i_n$ must be determined from which the packet should be forwarded to $o_n$. $n$ must be chosen within $int(d - \lfloor \frac{q}{2} \rfloor, d + \lfloor \frac{q}{2} \rfloor)$, and such that $d_{ioi}(i_s, i_d) = 2 + d_i(i_s, i_n) + d_o(o_n, o_d)$ is minimized. If $d_i(i_s, i_d) > d_{ioi}(i_s, i_d)$ and $s \neq n$, the packet is forwarded to $i_{s+q}$ if $d_{i+}(i_s, i_n) \leq d_{i-}(i_s, i_n)$, and to $i_{s-q}$ otherwise. If $d_i(i_s, i_d) > d_{ioi}(i_s, i_d)$ and $s = n$, the packet is forwarded to $o_s = o_n$. If $d_i(i_s, i_d) \leq d_{ioi}(i_s, i_d)$ the packet is forwarded to $i_{s-q}$ if $d_{i-}(i_s, i_d) < d_{i+}(i_s, i_d)$ and to $i_{s+q}$ otherwise.

Routing from $i_s$ to $o_d$: A shortest path from a node of the inner ring to a node of the outer ring will always use exactly one of the lines interconnecting the rings. Therefore the path from $i_s$ to $o_d$ can be constructed as $i_s, \ldots, i_n, o_n, \ldots, o_d$, where $n$ may equal $d$. The main task is to find the appropriate value of $n$, and once again it must be chosen within $int(d - \lfloor \frac{q}{2} \rfloor, d + \lfloor \frac{q}{2} \rfloor)$. $n$ is chosen to minimize $d(i_s, o_d) = 1 + d_i(i_s, i_n) + d_o(o_n, o_d)$. If $n = s$, the packet is forwarded to $o_n = o_s$. Otherwise it is routed as follows: If $d_{i+}(i_s, i_n) \leq d_{i-}(i_s, i_n)$, the packet is forwarded to $i_{s+q}$, otherwise it is forwarded to $i_{s-q}$.

Routing from $o_s$ to $i_d$ is done similarly: A shortest path from $o_s$ to $i_d$ can be written $o_s, \ldots, o_n, i_n, \ldots, i_d$. $n$ must be chosen within $(s - \lfloor \frac{q}{2} \rfloor, s + \lfloor \frac{q}{2} \rfloor)$, minimizing $d(o_s, i_d) = 1 + d_o(o_s, o_n) + d_i(i_n, i_d)$. If $n = s$, the packet is forwarded to $i_n = i_s$. Otherwise it is routed as follows: if $d_{o+}(o_s, o_n) \leq d_{o-}(o_s, o_n)$, the packet is forwarded to $o_{s+q}$, and otherwise it is forwarded to $o_{s-q}$.

### 9.7.4 Performance

The performance of $N2R(p; q)$ structures, $q$ selected as described in Section 9.6, has been evaluated and compared to the performance of the double rings. The results for structures of up to 2000 nodes are shown in Figures 9.3–9.4. Earlier studies showed that in terms of average distance, the $N2R(p; q)$ structures are close to optimal among the 3-regular structures at least up to a few hundred nodes[MKP03]. Our calculations show that even for large structures, average distances are kept low. For example, the $N2R(p; q)$ structure of 1000 nodes has average distance 12.0, while the $N2R(p; q)$ structure of 2000 nodes has average distance 16.4. For the double rings, the corresponding values are 125.6 and 250.6. Additionally, we found that for 5000 nodes the average distance of the $N2R(p; q)$ structure is 25.1, and for 10000 nodes it is 34.8. The pattern is the same for the diameters. For example, the diameter of the
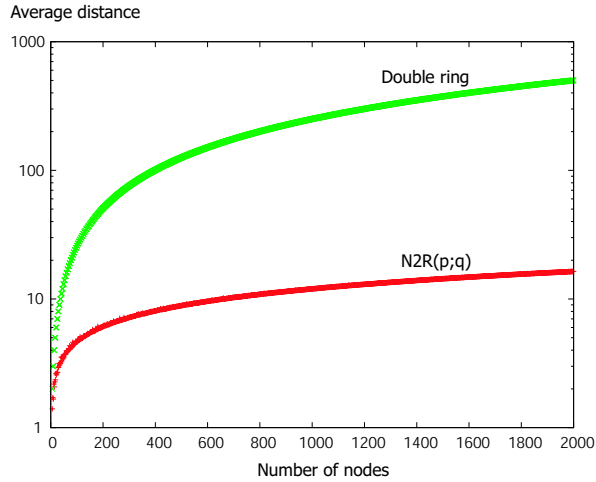
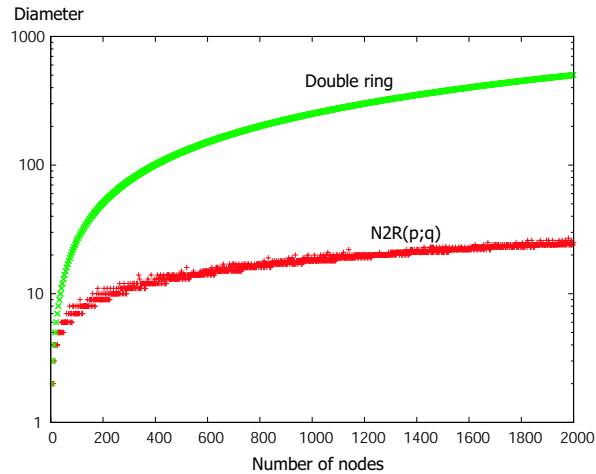**Figure 9.3:** Avg.distances of $N2R(p;q)$ and double ring structures.



**Figure 9.4:** Diameters of $N2R(p;q)$ and double ring structures.

$N2R(p;q)$ structure of 1000 nodes is 18, and the diameter of the $N2R(p;q)$ structure of 2000 nodes is 25. The corresponding values for the double rings are 251 and 501. In addition to the results shown, the diameter of the $N2R(p;q)$ structure of 5000 nodes is 38, and it is 52 for the structure of 10000 nodes. The plots of performance of $N2R(p;q)$ structures shown in Figures 9.3–9.4 are not smooth: even though both average distance and diameter generally increase with the number of nodes, some numbers of nodes allow for better performance than others. This is most noticeable for the diameter. For example, the $N2R(p;q)$ structure of 1000 nodes has diameter

18, while the structure of 936 nodes has diameter 20.

$N2R(p; q)$ structures are superior to double rings, both in terms of average distances and diameters, making it possible to build larger structures while keeping the distances low. If, for example, the diameter is to be kept under 15, the largest double ring can accommodate no more than 58 nodes, while the largest $N2R(p; q)$ structure can accommodate as much as 730 nodes. Even though diameters and average distances of $N2R(p; q)$ structures increase much slower with the number of nodes than those of the double rings, it is limited how large $N2R(p; q)$ structures can be constructed while keeping average distances and diameters at a reasonable level.

## 9.8   Discussion and conclusion

Generalizations of double ring network structures, $N2R(p; q)$ and $N2R(p; q; r)$ structures, have been presented in a network-structure perspective. The first major result was that every $N2R(p; q; r)$ structure is isomorphic to a $N2R(p; q)$ structure $N2R(p; q')$, where $q' = \frac{kp+q}{r}$ or $q' = p - \frac{kp+q}{r}$. As a consequence of this, results from graph theory where a generalization of the class of $N2R(p; q)$ structures is known as the Generalized Petersen Graphs, can be applied to all $N2R(p; q; r)$ structures. Two such results are that a $N2R(p; q)$ structure is node symmetric if and only if $q^2 \equiv 1 \pmod{p}$ or $q^2 \equiv p - 1 \pmod{p}$ and line symmetric if and only if it is one of the following: $N2R(4; 1)$, $N2R(5; 2)$, $N2R(8; 3)$, $N2R(10; 3)$, $N2R(12; 5)$ or $N2R(24; 5)$. A table-free routing scheme was introduced that uses hop-by-hop routing and always determines a shortest path. Finally, the performance of $N2R(p; q)$ structures in terms of average distance and diameter was evaluated and compared to double rings. These distances are remarkably smaller than for double rings, and even for a structure of 1000 nodes, the average distance is only 12.0, and the diameter 18. For comparison, the corresponding values for a double ring of the same size are 125.6 and 251. Together with the fact that the structures are optimally connected and node symmetric, or close to node symmetric, this indicates that they are superior alternatives to rings and double rings in access networks, given that appropriate embeddings can be found.

Even though $N2R(p; q)$ structures form a suitable base for future access networks, we believe that some extension is necessary for very large networks in order to keep average distances and diameters low. It is not clear if such an extension can be based on 3-regularity: [RN02] showed that nodes with very large degrees are important in order to ensure connectivity and reduce distances in random structures, but it may be possible to avoid such very large nodes by considerate design. Further research could deal with clarifying this question, with and developing such extensions. These extensions should ensure that easy routing, restoration, and protection schemes are

supported together with global 3-connectivity, i.e. the existence of three node independent paths between any pair of nodes.

Further research in $N2R(p;q)$ structures should deal with a number of other problems as well: a more efficient routing scheme for finding shortest paths is needed, and routing schemes should be developed which can determine two or three independent paths in an optimal or near-optimal manner; this is crucial in order to support protection schemes, and thus to take full advantage of the 3-connectivity offered. Determining these paths should take into account that the choices of first and second paths affect the possibilities of choosing more paths. Actually, finding optimal solutions to these disjoint-path problems is known to be NP-hard for arbitrary structures. If the problems are easier for $N2R(p;q)$ structures is not known. Restoration is also crucial: even for applications which do not require path protection, fast restoration in case of line or node failures should be supported by the routing scheme. Such schemes for $N2R(p;q)$ structures have yet to be developed. Another open and interesting question deals with the choice of $q$ given $p$: in the evaluation scheme presented in this paper, $q$ was in every case chosen as to minimize the average distance. For further research, as well as for practical network planning purposes, it is necessary to clarify to what extent $q$ can be chosen such that more parameters are optimized upon simultaneously, and to develop methods for finding appropriate values of $q$ given a set of optimization parameters.

## 9.9   Acknowledgements

The authors would like to thank University College Dublin for hosting Jens Myrup Pedersen and Thomas Phillip Knudsen as visiting researchers during the work of this paper.

## 9.10   Additional comments*

The minimum average distance and minimum diameter as introduced in Section 7.10 are also relevant for the $N2R$ structures, but due to the higher degree of symmetry, they only differ slightly from the average distances and diameters. The difference between diameter and minimum diameter cannot exceed one, and the difference between average distance and minimum average distance will always be less than one. Two factors must be taken into account.

First, there is a chance that $q$ could be chosen differently given $p$ if minimum diameter and average distance were to be minimized rather than diameter and average distance. Calculations for $p \leq 2000$ show that it is in all cases possible to find at least one

structure (i.e. one value of $q$) fulfilling both of these two optimisation criteria:

- Given $p$, $q$ is first chosen to minimize diameter. If this results in more candidates, $q$ is chosen to minimize average distance.

- Given $p$, $q$ is first chosen to minimize minimum diameter. If this results in more candidates, $q$ is chosen to minimize minimum average distance.

It should be noted that the order of optimization criteria (diameter, average distance) used here is different from that used in the paper. This is due to the results of [PKM04b], presented in Chapter 10, but as stated in this paper, it is in most cases possible to find structures minimizing both average distance and diameter.

Over all these 1998 values of $p$, 2594 structures fulfil the first criteria, and only 7 of these do not fulfil the second: for $p = 165$ two structures fulfil the first criteria, but only one fulfills the second, and for $p = 1100$ 7 structures fulfil the first criteria, while only one fulfills the second.

It is not ensured though, that in a given structure a node exists from which both minimum average distance and diameter can be obtained. However, this was the case for all 2587 structures fulfilling both criteria stated above.

Among the 1998 values of $p$, it was only possible to find two values of $p$, 9 and 35, for which a minimum diameter was smaller than the smallest possible diameter.

It was only possible to obtain minimum average distances sligthly smaller than the average distances. When for each value of $p$, $q$ was chosen according to the criteria above, the average distances were on average over the 1998 values of $p$ 0.020% higher than the minimum average distances. The largest difference was 2.99%, and in 708 cases there were no differences at all.

# 10 | Comparing and Selecting Generalized Double Ring Network Structures [PKM04b]

## 10.1 Preface*

This paper was written in collaboration with Thomas Phillip Knudsen and Ole Brun Madsen, and presented at The Second IASTED International Conference on Communication and Computer Networks (CCN 2004). It is the second of four papers on $N2R$ structures and presents a continuation of the work presented in Chapter 9. The contribution to the thesis can be summarized as follows. The paper:

- Devises and evaluates a policy for selecting the best $N2R$ structures given the number of nodes.

- Determines $k$-average distance and $k$-diameter for $N2R$ structures with up to 164 nodes. $N2R$ is shown to be superior to the Double Ring with regard to these parameters.

The work is continued in Chapter 11, where $k$-average distance and $k$-diameter are evaluated for even larger structures and compared to the Degree Three Chordal Rings.

## 10.2 Abstract

$N2R(p;q)$ network structures were introduced recently as a generalization of double rings, and they were shown to be superior compared to double rings in terms of average distance and diameter. For a given number of nodes, there is only one double ring, but often more different $N2R(p;q)$ structures. These different structures have different properties, and choosing a best structure depends on what properties are considered important. For both research and planning purposes it is beneficial to have a simple scheme for optimally choosing $q$ given $p$. We show that for structures of up to 4000 nodes, it is in 1919 of 1998 cases possible to choose a structure minimizing both average distance and diameter. For structures of up to 164 nodes, mini-

mizing average distance and diameter results in structures where 2-average distance, 3-average distance, 2-diameter, and 3-diameter are minimized or nearly minimized.

## 10.3   Introduction

The convergence of communications[MNS02] together with an increasing amount of bandwidth-demanding applications has led to the beginning deployment of Fiber To The Home (FTTH) infrastructures. In many countries worldwide, FTTH has been recognized as the only way of offering sufficient bandwidth, and plans for deploying these infrastructures are being made.

Not only the need for bandwidth, but also the needs for reliability and protection are expected to increase. First, existing reliability-demanding applications previously confined to LAN environments are moving to WAN environments, and new applications are being developed, including telerobotics[HKKK01][XmCjYXd04] and teleoperations[XT98]. Second, various medias such as TV, radio, and telephony are to an increasing extent being transmitted via the Internet, a trend expected to continue as FTTH solutions are deployed and efficient broadcast protocols developed. This adds to the dependency on stable Internet connections and motivates that they should not be too vulnerable to attacks and failures.

Since fiber networks are upgradeable by changing end equipment such as senders and transmitters without changing the fibers, and since careful planning makes it possible to increase the number of fibers without digging new ducts, the basic physical structures chosen for FTTH infrastructures are expected to have a long lifetime. As changing the physical structures in access networks is likely to imply digging of new ducts, it is often very expensive, and therefore the physical structures should be chosen carefully from the beginning: they should be cost-efficient while still providing small distances and support appropriate levels of protection, such that services are not disconnected in case of failures. The demands for reliability and protection exclude the otherwise attractive trees, and to ensure connectivity even in case of two independent failures, ring structures are also insufficient. 3-regular, maximally connected structures are interesting because they for any number of nodes have the smallest number of lines, while still ensuring connectivity between any pair of nodes even in case of any two independent failures. Together with node symmetry, maximal connectivity is an important property to satisfy for robust networks[DC04]. Double rings are 3-regular and maximally connected, and they are simple extensions of rings. However, recent research has shown that generalized double rings known as $N2R(p;q)$ structures, which are again special cases of the Generalized Petersen Graphs[FGW71], perform considerably better with respect to average distance as well as diameter while still being 3-regular and maximally connected[MKP03]. While double rings

are node symmetric, this is not necessarily the case for $N2R(p;q)$ structures, but they are nearly node symmetric as only two classes of nodes exist, with symmetry within each class.

For a given number of nodes, only one ring and one double ring exist, and average distance and diameter increase proportional to the number of nodes. Thus, if a network of a given size is needed, it is not difficult to determine what ring or double ring would be the appropriate choice. This is more difficult for the $N2R(p;q)$ structures. Given an (even) number of nodes, it is in many cases possible to construct different structures with different properties. The ability to select an appropriate structure given the number of nodes is crucial, both for practical network planning and for further research in network structures. If $N2R(p;q)$ structures are to be compared with other network structures, it complicates these comparisons that different numbers of $N2R(p;q)$ structures exist for different numbers of nodes, especially if multiple parameters are used for comparison, and different $N2R(p;q)$ structures are optimal with respect to different parameters.

## 10.4   Preliminaries

The idea of studying the physical structures impact on large-scale networks ability to support QoS and reliability demanding applications was presented recently with the introduction of the concepts Sustainable QoS[MK96] and Structural QoS[MNS02]. [MNS02] also introduced the $N2R(p;q)$ structures for use in a network-planning context. A more thorough description of these structures can be found in [PPKM04a], which also raises the questions discussed in this paper.

A network structure $S$ is a set of nodes and a set of lines, where each line interconnects two nodes. Lines are bidirectional, so if a pair of nodes $(u, v)$ is connected, so is $(v, u)$. A structure can be considered a model of a network, abstracting from specific physical conditions such as node equipment, medias, and wiring. The definition of a structure is similar to that of a simple graph in graph theory. A path between two distinct nodes $u$ and $v$ is a sequence of nodes and lines: $(u = u_0), e_1, u_1, e_2, u_2, \ldots, u_{n-1}, e_n, (u_n = v)$, such that every line $e_i$ connects the nodes $u_{i-1}$ and $u_i$. The length of a path corresponds to the number of lines it contains, so in the case above, the path is of length $n$. The distance between a pair of distinct nodes $(u, v)$ corresponds to the length of the shortest path between them and is written $d(u, v)$. This paper considers only connected structures, i.e. between every pair of distinct nodes there exists a path. Two paths between a pair of nodes $(u, v)$ are said to be independent if they share no lines and nodes except for $u$ and $v$. Similarly, a set of paths are said to be independent if the paths are pairwise independent. The size of a structure equals the number of nodes it contains.

Let $p$ and $q$ be positive integers, such that $p \geq 3$, $q < \frac{p}{2}$, and $gcd(p, q) = 1$. $p$ and $q$ then define a $N2R(p; q)$ structure $S$ as follows: $S$ consists of two rings, an outer ring and an inner ring, each containing $p$ nodes. The nodes of the outer ring are labeled $o_0, o_1, \ldots, o_{p-1}$, and the nodes of the inner ring labeled $i_0, i_1, \ldots, i_{p-1}$. Thus, $S$ contains $2p$ nodes. For each $i$ such that $0 \leq i \leq p - 1$ there exists a line between each of the following pairs of nodes:

- $(o_i, o_{i+1(mod\ p)})$ (lines of the outer ring).

- $(i_i, i_{i+q(mod\ p)})$ (lines of the inner ring).

- $(o_i, i_i)$ (lines connecting the two rings).

The classical double ring with $2p$ nodes obviously corresponds to $N2R(p; 1)$. An example of another $N2R(p; q)$ structure is shown in Figure 10.1.

This paper compares different $N2R(p; q)$ structures such that structures with the same number of nodes are compared to each other. This is done by comparing all different $N2R(p; q)$ structures for each value of $p$: these are obtained by considering in principle all values of $q$ allowed by the definition of $N2R(p; q)$. However a number of values of $q$ are discarded due to the following easily verified fact: for a fixed value of $p$, let $q_1 < q_2$ fulfil for $i = 1, 2$ that $q_i < \frac{p}{2}$ and $gcd(q_i, p) = 1$. Then $N2R(p; q_1)$ is isomorphic to $N2R(p; q_2)$ if $q_1 q_2 = 1(mod\ p)$ or $q_1 q_2 = p - 1(mod\ p)$. For such two isomorphic structures, $q_2$ is discarded and only $q_1$ considered.
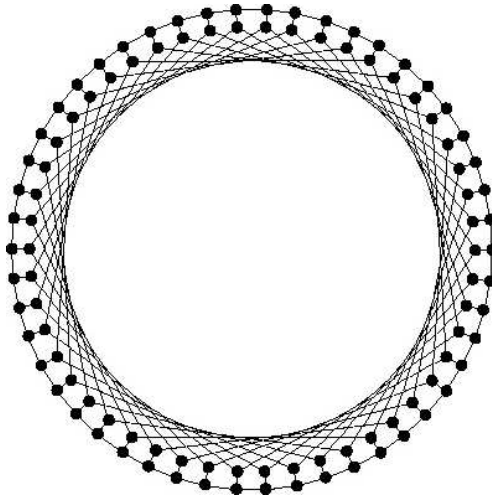


**Figure 10.1:** $N2R(50; 9)$.

## 10.5   Parameters for comparison

The most commonly used parameters for evaluation and comparison of network structures are diameter and average distance, defined as follows.

- Average distance: The average of $d(u, v)$ taken over all pairs of distinct nodes.

- Diameter: The maximum of $d(u, v)$ taken over all pairs of distinct nodes.

These parameters are primarily useful for standard Internet applications using shortest paths. With the increasing amount of reliability-demanding applications, it is expected that a number of future applications will use various protection schemes, and as a consequence hereof, several independent paths between given pairs of nodes must exist simultaneously. This motivates the introduction of the following parameters[PKM04a]:

- $k$-average distance: For every pair of distinct nodes $(u, v)$, $k$ independent paths between $u$ and $v$ are constructed such that the sum of the lengths of these paths is smallest possible. The $k$-average distance is then obtained by taking the average of these sums over all pairs of distinct nodes.

- $k$-diameter: For every pair of distinct nodes $(u, v)$, $k$ independent paths between $u$ and $v$ are constructed such that the longest of these paths is shortest possible. The $k$-diameter is the maximum over the lengths of these longest paths.

Due to the 3-connectivity of all $N2R(p; q)$ structures, these parameters are considered for $k = 2, 3$. 1-diameter and 1-average distance equal diameter and average distance. Certain other parameters also used for evaluation of network structures or graphs, such as the connectivity number and the cost in terms of number and nodes/lines, are not relevant for this study as they attain the same values for all $N2R(p; q)$ structures of the same size.

## 10.6   Choosing $q$ given $p$

Both average distance and diameter are easily determined using well-known algorithms such as Dijkstras algorithm or similar. Thus, for every value of $p$ it is easy to determine the optimal structure in the sense of either of these parameters. No efficient method is known for determining $k$-diameter and $k$-average distance for $k > 1$, and

the calculation time using brute-force methods increases exponentially with the number of nodes. However, due to the high level of symmetry together with the relative small number of lines, we were able to calculate those parameters for all structures of up to 164 nodes.

Obviously the choice of $q$ given $p$ can be made in many ways, considering one or more parameters defined above, either by selecting primary and secondary parameters or by defining some weighted average of the parameters. In each case, the choice and/or weight of parameters must be subject to expected network requirements, providing no general method for selecting a "best" structure. It further complicates the selection if the structures are large because $k$-average distance and $k$-diameter may be impossible to determine.

Probably the simplest method is to minimize average distance, diameter or both. This approach is evaluated in two steps. First, the correlation between minimizing average distance and minimizing diameter is evaluated. Second, it is studied to what extent minimizing average distance and diameter minimizes the other parameters.

Since both average distances and diameters are easily calculated even for large structures, this was done for all structures of up to 4000 nodes. First, for every value of $p$ the structures with minimum diameter were selected, and among these the structures with smallest average distance were selected. Second, for every value of $p$ the structures with minimum average distance were selected, and among these the structures with smallest diameter were selected. In the following, values obtained these ways are compared to minimum values. For each value of $p$, and for each parameter, the minimum value is that of the given parameter in the $N2R(p; q)$ structure where this parameter is smallest possible. Thus, a minimum value is assigned to every parameter for every value of $p$.

1998 cases were considered ($3 \leq p \leq 2000$), and it turned out that in 1919 (96.0%) of these, the two approaches led to the same set of structures. In 1758 of these 1919 cases, only one value of $q$ minimized both average distance and diameter.

If a network is to be chosen with specific requirements, it is possible that either diameter or average distance is considered much more important than the other. This makes it easy to choose between the two approaches in the remaining 79 cases, where only one of the parameters can attain its minimum value. However, it is useful to evaluate each of these approaches in order to obtain a general rule of thumb on how to choose the structures.

Figures 10.2–10.3 show how average distances and diameters vary from the minimum values using each of the two approaches. Only values of $p$ for which no structure minimizes both average distance and diameter are shown. If the diameter is minimized first, the average distances over the 79 cases are on average 0.12% and in the worst
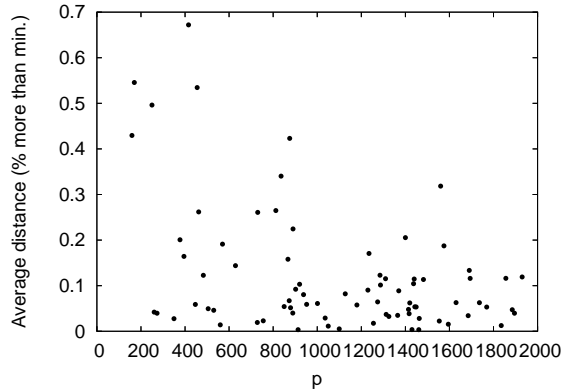
**Figure 10.2:** In 79 cases, average distance and diameter cannot both be minimized. For each value of $p$, a structure with the smallest average distance is chosen among those with minimum diameter.
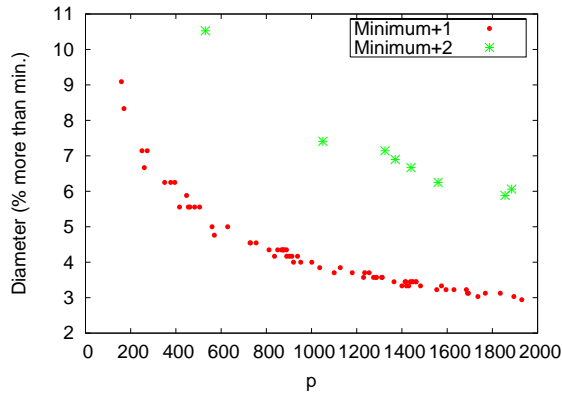


**Figure 10.3:** In 79 cases, average distance and diameter cannot both be minimized. For each value of $p$, a structure with the smallest diameter is chosen among those with minimum average distance.

case 0.67% higher than the minimum values. Minimizing the average distance first gives diameters, which over the 79 cases are on average 4.6% and in the worst case 10.5% higher than the minimum values. It is not surprising that these values are relatively higher than those of the average distances since the diameter is always an integer. In 71 cases, the diameter is one higher than the minimum, and in 8 cases it is two higher, as shown in Figure 10.3. Taking all 1998 cases into account, minimizing the diameter first leads to average distances on average 0.0048% higher than minimum, and minimizing the average distance first, on average leads to a diameters 0.18% higher than minimum.

The second step was to evaluate not only average distance and diameter, but also the other parameters introduced previously. It was evaluated to what extent these
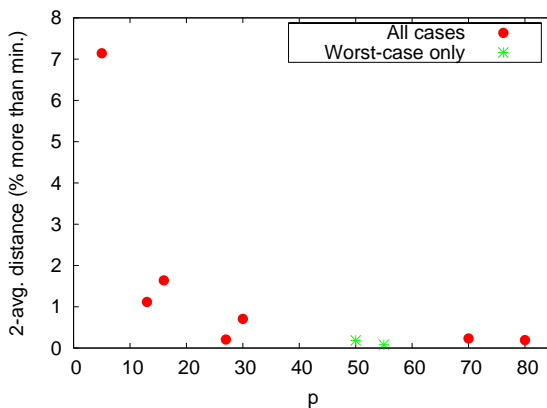
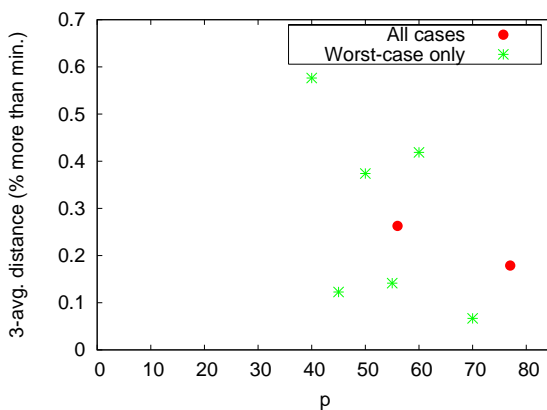**Figure 10.4:** 2-avg. distance compared to minimum values.



**Figure 10.5:** 3-avg. distance compared to minimum values.

parameters are minimized when minimizing average distance and diameter. In all cases considered ($3 \leq p \leq 82$) average distance and diameter can be minimized simultaneously, but in 6 cases ($p = 40, 45, 50, 55, 60, 70$) two choices of $q$ minimize both. In each of these cases, there exists a single value of $q$, which is superior because it performs better than or equal to the other with respect to all parameters. In these cases both superior (best-case) and worst-case values are used for comparison. Since in every of these cases 2-diameter and 3-diameter are independent of this choice of $q$, the worst-case results only differ from the best-case results for 2-average and 3-average distances.

The relative differences between minimum values and values obtained by the methods presented, are shown for each of the parameters in Figures 10.4–10.7. In each
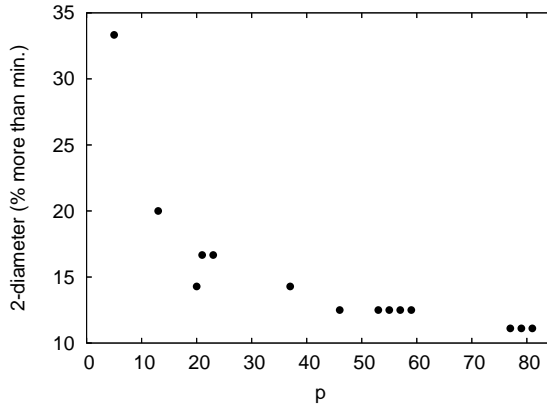
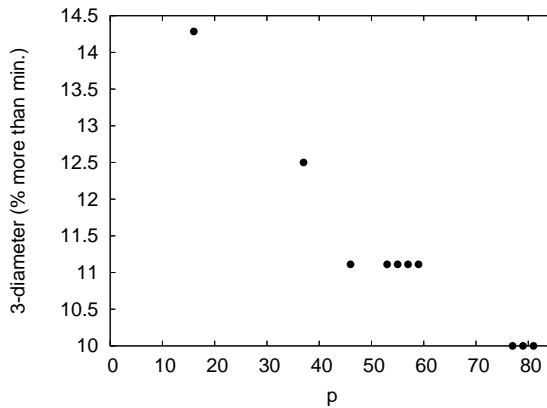**Figure 10.6:** 2-diameter compared to minimum values.



**Figure 10.7:** 3-diameter compared to minimum values.

figure, only the cases for which the methods sketched do not necessarily minimize the respective parameter are shown. In 56 of the 80 cases (70.0%), all parameters attain the minimum values, and in additional 4 cases (5.0%), there exist two values of $q$ minimizing both average distance and diameter, where in each case exactly one of these minimizes all parameters.

Figure 10.4 shows how in 7 of 80 cases (8.8%), the 2-average is not minimum, and in 2 cases (2.5%) the 2-average can be minimum, depending on the choice between two values of $q$. Over the 7 cases, the 2-average distance is on average 1.6%, and over the 2 cases either 0 or 0.13% higher than the minimum values. Over all 80 cases, the 2-average is on average 0.14% (best-case and worst-case) higher than the minimum values. In Figure 10.5 it is shown how in 2 of 80 cases (2.5%) the 3-averages are

not minimum, and in 6 cases (7.5%) the 3-average can be minimum, depending on the choice between two values of $q$. Over the 2 cases, the 3-average distances are on average 0.22%, and over the 6 cases either 0 or 0.28% higher than the minimum values. Over all 80 cases, the 3-averages are on average 0.0055% (best-case) or 0.027% (worst-case) higher than the minimum values. Figure 10.6 shows how in 14 of 80 cases (17.5%) the 2-diameters are not minimum. Over these cases, the 2-diameters are on average 15.1%, and over all 80 cases on average 2.6% higher than the minimum values. In Figure 10.7 it is shown how in 10 of 80 cases (12.5%) the 3-diameters are not minimum. Over these cases, the 3-diameters are on average 11.2%, and over all 80 cases on average 1.4% higher than the minimum values. In no case the 2-diameter or 3-diameter exceeds the minimum value by more than one.

## 10.7   Comparison to Double Rings

As a by-product of the calculations made, it was possible to compare the performance of $N2R(p; q)$ structures to the performance of double rings with respect to $k$-average distance and $k$-diameter for $k = 2, 3$, as an addition to existing comparisons of diameter and average distance. The comparisons are shown in Figures 10.8–10.9, and for each value of $p$, $q$ has been selected to minimize average distance and diameter.

For the six values of $p$ where this gave more possible values of $q$, the superior was selected. According to the previous results shown in Figures 10.4–10.5, this choice only influenced the values of 2-average in 2 cases, and the value of 3-average in 6 cases. In none of these cases, the difference exceeded 0.6%, an insignificant difference compared to the differences between the parameters for $N2R(p; q)$ and double ring structures.

It is seen that $N2R(p; q)$ structures are superior to the double rings with respect to all parameters: for the largest structures considered, $p = 82$, $N2R(82; 11)$ was selected as the best structure, and no other $N2R(82; q)$ structure performs better in terms of any of the parameters. This structure has 2-average distance 12.9, 3-average distance 22.4, 2-diameter 10, and 3-diameter 11. The corresponding values for the double ring are 43.3, 104.7, 42, and 82. Thus, the values for the $N2R(82; 11)$ structure are only 29.7%, 21.4%, 23.8%, and 13.4% of those of the double ring. The differences clearly increase with the size of the structures. For $N2R(p; q)$ structures, the differences between 2-diameter and 3-diameter are considerably smaller than for the double rings, and the same applies to 2-average distance and 3-average distance.

$k$-average distance and $k$-diameter generally increase with the structure size. However, there are values of $p$ for which a larger structure exists, performing better in terms of one or more parameters. For the 73 values of $p \leq 75$ it is in 33 cases
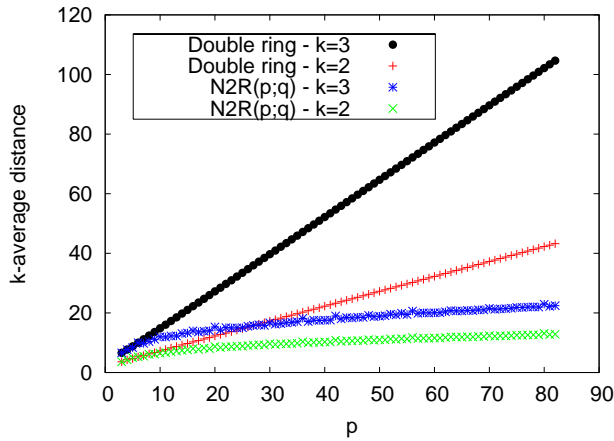
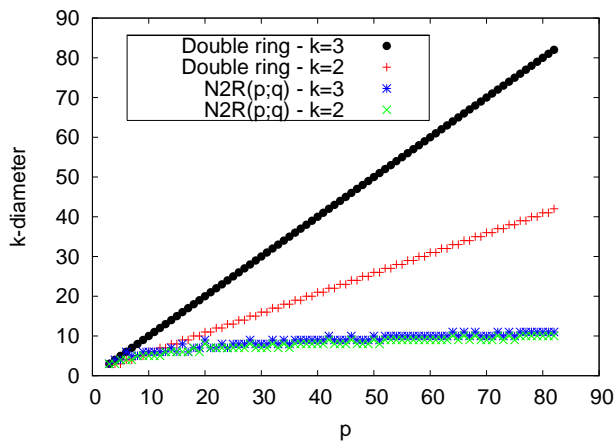**Figure 10.8:** $k$-avg. distance for $N2R(p;q)$ and double rings.



**Figure 10.9:** $k$-diameter for $N2R(p;q)$ and double rings.

(45.2%) possible to find a larger value of $p \leq 82$ that performs better in terms of at least one of the four parameters introduced in this paper, and in 20 cases (27.4%) it is possible to find a larger value of $p$ with all four parameters being smaller or equal. In each of these cases, at least one parameter is improved. Taking also average distance and diameter into account, the corresponding numbers of cases are 38 (52.1%) and 20 (27.4%). In general the gains of choosing a larger structure are small, but in some cases they are significant. For instance, choosing $N2R(21;8)$ instead of $N2R(20;3)$ reduces average distance by 0.25 (6.9%), 2-average distance by 0.33 (3.8%), 3-average distance by 1.04 (6.8%), diameter by 1 (16.7%), 2-diameter by 1 (12.5%), and 3-diameter by 2 (22.2%).

## 10.8   Conclusion and discussion

This paper has discussed important aspects of choosing an optimal generalized double ring structure, known as $N2R(p; q)$, given the number of nodes. While $p$ is given, there are different ways of choosing $q$; we showed that minimizing average distance and diameter in most cases also minimizes other parameters. The basic parameters considered were diameter and average distance, and these parameters were calculated for all structures with up to 4.000 nodes. In 96.0% of these cases it is possible to select a structure minimizing both average distance and diameter. In the remaining cases two approaches were considered, given $p$. The first approach was to select all values of $q$ minimizing the average distance, and among these candidates select $q$ to minimize the diameter. The second approach was to select all values of $q$ minimizing the diameter, and among these candidates select $q$ to minimize the average distance. Over all values of $p$, minimizing the average distance first leads to diameters on average 0.18% higher than the minimum values, and minimizing the diameter first leads to average distances on average 0.0048% higher than the minimum values. The corresponding worst-case values are 10.5 % and 0.67%. Especially the latter approach leads to virtually optimal results in all cases, and we believe that it will be hard to find a better approach for choosing optimal structures with respect to these two parameters.

These two approaches were evaluated also with respect to four other parameters, namely 2-average distance, 3-average distance, 2-diameter, and 3-diameter. Due to calculation complexity, these parameters were evaluated only for structures of up to 164 nodes, and in all cases it was possible to find values of $q$ minimizing both average distance and diameter. In general $q$ is uniquely determined, but in 6 cases two candidates for $q$ exist, resulting in different values of either one or both of 2-average and 3-average distance. In all of the 6 cases, the $k$-diameter is independent of this choice. Choosing the best value of $q$ in these cases leads to the four parameters being on average 0.14%, 0.0055%, 2.6%, and 1.4% higher than the minimum values. We conclude that this approach gives surprisingly good results, and therefore provides a good method for selecting overall optimal structures. However, it is not in all cases possible to minimize all parameters, and if one or more parameters are considered particularly important, $q$ may be chosen better using some other method.

As a by-product of the calculations made, it was also shown that the $N2R(p; q)$ structures perform and scale notably better than the double rings with respect to the parameters introduced. For instance, for $N2R(82; 11)$, the values of the four parameters were only 13-30% of those of the double ring. It is especially interesting that the differences between 2-diameter and 3-diameter and between 2-average distance and 3-average distance are considerably smaller than for the double ring, indicating that carefully selected $N2R(p; q)$ structures do not suffer from the ring/double ring

phenomenon, where backup-paths must take the "long" way around the ring[DC04].

While it is useful to be able to choose an optimal structure in terms of the various parameters discussed in this paper, the embeddability must also be taken into account, and doing so is probably the largest and most important challenge for further research on this problem: an otherwise optimal structure may prove impossible or very expensive to deploy in the real world. $N2R(50; 9)$ shown in Figure 10.1 is an example of a structure which may be hard to deploy by wired networks if distinct lines may not share ducts. If distinct lines share ducts, the robustness is affected, so it might be a more promising strategy to combine wired and wireless networks. Research and practical experience is needed in order to obtain more knowledge on this.

It is also still needed to develop efficient routing and protection schemes such that two or three independent paths can be easily established. Likewise, efficient restoration schemes should be developed. No optimal solutions are known for establishing several independent paths between a given pair of nodes and doing so would also require some definition of optimality. The definitions of $k$-average and $k$-diameter illustrate different approaches to this. The results of this paper provide a reference-model for such routing schemes because the length of two or three independent paths obtained by proposed routing schemes can be compared to $k$-average distance and $k$-diameter.

Since a brute-force approach was used to determine $k$-average distance and $k$-diameter, we were not able to obtain results for structures larger than 164 nodes. It may be possible to develop faster optimal or near-optimal algorithms, which can be used to test whether the minization criteria used in this paper are suitable also for larger structures. The brute-force calculations of this paper can be used for reference when evaluating the performance of such an algorithm, at least for $p \leq 82$.

# 11 | Distances in Generalized Double Rings and Degree Three Chordal Rings [PRM05a]

## 11.1 Preface*

This paper was written in collaboration with Tahir M. Riaz and Ole Brun Madsen, and presented at The IASTED International Conference on Parallel and Distributed Computing and Networks, IASTED PDCN 2005. It is the third of four papers on $N2R$ structures, and presents a continuation of the work presented in Chapter 10. The contribution to the thesis can be summarized as follows. The paper:

- Uses the selection policy devised in Chapter 10 to evaluate $k$-average distance and $k$-diameter for structures with up to 400-900 nodes.

- Shows that this selection policy can also be applied to Degree Three Chordal Rings.

- Compares $N2R$ to Degree Three Chordal Rings with respect to average distance, diameter, $k$-average distance, and $k$-diameter, and shows that $N2R$ is superior with respect to all these parameters.

The paper is the last paper on distances in $N2R$ structures.

## 11.2 Abstract

Generalized Double Rings ($N2R$) are compared to Degree Three Chordal Rings ($CR$) in terms of average distance, diameter, $k$-average distance, and $k$-diameter. For each number of nodes, structures of each class are chosen to minimize diameter and average distance, an approach which is shown to result in all other parameters being either minimized or nearly minimized. Average distance and diameter are compared for all structures with up to 1000 nodes, and $k$-average distances and $k$-diameters for all structures with up to 400-900 nodes. $N2R$ are shown to be superior with regard to these parameters, especially for large structures.

## 11.3 Introduction

When designing a network or parallel computing system, interconnection topologies are important. Much research has been conducted in order to compare different topologies, in particular for multiprocessor systems[Kot92], but comparison parameters have been determined also for large-scale communication networks[PKM04a]. Most of the compared topologies contain nodes of degree four or more.

In general, it is important to keep the costs and thus the node degrees as low as possible, and for communication infrastructures this in particular so; nodes are often placed in different physical locations, spread over large geographical areas. Not only node equipment but also digging is costly and should be kept to a minimum, and a limited set of potential ducts such as roads often limit the number of possible physical paths. Therefore it is not surprising that most communication infrastructures have until now been based on trees and rings. Trees offer no redundancy, while rings offer connectivity in case of any one arbitrary failure. Unfortunately, two failures will split the network, and even a single failure leads to notably larger distances and thus higher transmission delays and traffic loads. Currently, the convergence of communications is leading to an increasing dependency on the Internet[MNS02], a trend supported by the fact that a large number of applications are being developed requiring both Quality of Service and reliability. These include home automation[ND00], tele operations[XT98], and tele robotics[HKKK01]. These needs for reliability can to some extent be satisfied by using wireless back-up as a supplement to Fiber To The Home solutions for the last-mile access networks[PRKM04], but there is an urgent need for developing more robust topologies for the higher layers, in particular local and regional backbones. In order to increase reliability while keeping the costs down, 3-regular 3-connected topologies are interesting. They have the smallest possible node degree, while still providing connectivity in case of any two independent failures. Double rings[MKP03] are simple 3-regular 3-connected topologies, which offer easy routing, restoration, and protection schemes, but suffer from large distances. Two alternative classes of 3-regular 3-connected topologies are the Generalized Double Rings also known as $N2R$[MKP03], which is a subset of the Generalized Petersen Graphs[FGW71], and the Degree Three Chordal Rings, e.g. [Kot92][BDZ04b] (for simplicity we write $N2R$ and $CR$ throughout the paper). $CR$ and $N2R$ share many properties: in addition to being 3-regular and 3-connected, they are not in general planar, they can be expanded in similar ways and they have fairly short diameters[YFMA85]. However, there are also a few important differences: $CR$ are node symmetric, while $N2R$ contain either one or two classes of nodes with symmetry within each class. Another difference is that $CR$ are based on one main ring while $N2R$ are based on two main rings. Since they perform comparable with regard to those quantitative and qualitative parameters, the distances are important

when selecting which topology should be used for some network or parallel computing system. In this paper, we compare the two classes of structures with regard to a number of different distance parameters. The results apply to physical as well as logical level networks, making them interesting in a broad context.

## 11.4 Preliminaries

A structure is a set of nodes and a set of lines, where each line interconnects two nodes. Lines are bidirectional, so if a pair of nodes $(u, v)$ is connected, so is $(v, u)$. A structure can be considered a model of a network, abstracting from specific physical conditions such as node equipment, media, and wiring, and the definition is similar to that of a simple graph: a path between two distinct nodes $u$ and $v$ is a sequence of nodes and lines: $(u = u_0), e_1, u_1, e_2, u_2, \ldots, u_{n-1}, e_n, (u_n = v)$, such that every line $e_i$ connects the nodes $u_{i-1}$ and $u_i$. The length of a path equals the number of lines it contains, so in the case above, the path is of length $n$. The distance between a pair of distinct nodes $(u, v)$ equals the length of the shortest path between them, and is written $d(u, v)$. This paper considers only connected structures, i.e. between every pair of distinct nodes there exists a path. Two paths between a pair of nodes $(u, v)$ are said to be independent if they share no lines or nodes except for $u$ and $v$, and a set of paths are said to be independent if the paths are pair wise independent. The size of a structure equals the number of nodes it contains.

$N2R$ structures are defined as follows. Let $p$ and $q$ be positive integers, such that $p \geq 3$, $q < \frac{p}{2}$, and $gcd(p, q) = 1$. $p$ and $q$ then define a structure $N2R(p; q)$, which consists of two rings, an outer ring and an inner ring, each containing $p$ nodes. The nodes of the outer ring are labeled $o_0, o_1, \ldots, o_{p-1}$, and the nodes of the inner ring labeled $i_0, i_1, \ldots, i_{p-1}$. Thus, it contains $2p$ nodes. For each $i$ such that $0 \leq i \leq p-1$ there exists a line between each of the following pairs of nodes:

- $(o_i, o_{i+1(mod\ p)})$ (lines of the outer ring).

- $(i_i, i_{i+q(mod\ p)})$ (lines of the inner ring).

- $(o_i, i_i)$ (lines connecting the two rings).

The classical double ring with $2p$ nodes obviously corresponds to $N2R(p; 1)$. An example of a $N2R$ is shown in Figure 11.1. One more restriction to $q$ given $p$ applies throughout the paper: given $p$, let $q_1 < q_2$ fulfil for $i = 1, 2$ that $q_i < \frac{p}{2}$ and $gcd(q_i, p) = 1$. Then $N2R(p; q_1)$ is isomorphic to $N2R(p; q_2)$ if $q_1 q_2 = 1(mod\ p)$ or $q_1 q_2 = p - 1(mod\ p)$. For such two isomorphic structures, $q_2$ is discarded and only $q_1$ considered a permitted value.
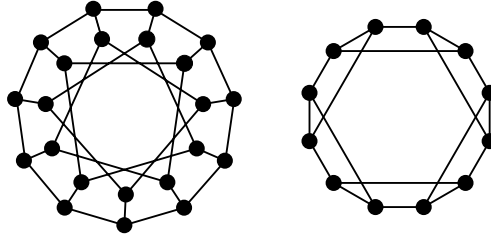
**Figure 11.1:** $N2R(11;3)$ and $CR(12;3)$.

$CR$ structures are defined as follows. Let $w$ be an even integer such that $w \geq 6$, and let $s$ be an odd integer, such that $3 \leq s \leq \frac{w}{2}$. $w$ and $s$ then define $CR(w,s)$ with $w$ nodes labeled $u_0, \ldots, u_{w-1}$. For $0 \leq i \leq w - 1$ there exists a line between each of the following pairs of nodes:

- $(u_i, u_{i+1(mod\ w)})$.

- $(u_i, u_{i+s(mod\ w)})$, for $i$ even.

An example of a $CR$ is shown in Figure 11.1.

### 11.4.1   Evaluation parameters

Widely used distance measures for network topologies are average distance and diameter, indicating transmission delays as well as traffic load.[PKM04a].

- Average distance: The average of $d(u,v)$ taken over all pairs of distinct nodes.

- Diameter: The maximum of $d(u,v)$ taken over all pairs of distinct nodes.

For real-time applications where even short transmission outages are not acceptable, protection schemes are used. For this, $k$ paths are established when the connection is set up. Traffic can be sent simultaneously along all these $k$ paths, or along only one path, keeping the last $k - 1$ path(s) ready for immediate use whenever a failure is detected. In both cases, long restoration times are avoided. The $k$-measures $k$-average distance and $k$-diameter reflect the considerations of average distance and diameter:

- $k$-average distance: For every pair of distinct nodes $(u,v)$, $k$ independent paths between $u$ and $v$ are constructed such that the sum of the lengths of these paths is smallest possible. The $k$-average distance is the average of these sums over all pairs of distinct nodes.

- $k$-diameter: For every pair of distinct nodes $(u, v)$ $k$ independent paths between $u$ and $v$ are constructed such that the longest of these paths is shortest possible. The $k$-diameter is the maximum over the lengths of these longest paths, over all pairs of distinct nodes.

Since both $N2R$ and $CR$ are 3-regular, these parameters are considered for $k = 2, 3$. 1-average distance and 1-diameter equal average distance and diameter. Where not confusing, we will simply write $k$-average instead of $k$-average distance.

## 11.5 Methods

The first step is to determine which structures to compare. In order to facilitate a comparison, it is desirable to have only one $N2R$ and one $CR$ of a given size, or a limited number with parameters close to each other. This is especially so for general purpose networks where more parameters are used for selection; assume that a network structure is to be chosen, which should have short average distance and diameter. It is little interesting if one structure performs well with regard to average distance and another structure belonging to the same class performs well with regard to diameter, if no structure of that class perform satisfactory with regard to both. For both $CR$ and $N2R$, there can exist several structures of the same size. For $N2R$ a policy was introduced for selecting $q$ given $p$[PKM04b]:

- Select the values of $q$ such that the diameter is minimum.

- Among those values of $q$, select those such that the average distance is smallest possible.

It was shown for $p \leq 2000$ that this leads to structures, which are close to optimal with regard to average distance, and for $p \leq 82$ they were also shown to be optimal or close to optimal for $k$-averages and $k$-diameters. $k$-averages and $k$-diameters were not evaluated for larger structures. In this paper, we evaluate the same selection policy for $CR$ and compare it to the related policy where average distance is minimized first, and $s$ then chosen among these possible values to minimize the diameter. For average distance and diameter, this comparison is performed for all structures with $w \leq 1000$. Average distances and diameters are determined by simply calculating these values for all structures, using standard shortest-path algorithms and making use of the symmetries. Since no efficient algorithms are known for determining $k$-average and $k$-diameter, they are calculated using brute-force algorithms. Therefore, the policies were only evaluated for $w \leq 100$ with regard to those parameters. Since

the two policies result in the same structures for $w \leq 100$, no policy comparison were made here.

Based on the results obtained, the policy of selecting first diameter and then average distance is used in the rest of the paper, providing a base for comparison of all parameters. First, diameter and average distance are compared. Due to the selection policy, for each number of nodes all selected structures within each class have the same average distance and diameter. The results are derived from the calculations carried out in order to compare the selection policies.

For the $k$-measures, the parameters can be determined for significantly larger structures if they are calculated only for good values of $q$ and $s$ rather than for all permitted values. This also reduces the number of structures for which the $k$-measures are evaluated. Therefore, they are for each value of $p$ or $w$ determined only for the values of $q$ or $s$ determined by the selection policy. While it is not guaranteed for any $k$-measure that the minimum value is obtained, any other choice would imply a trade-off between average distance/diameter and one or more $k$-measures. For general purpose networks, average distance and diameter would in many cases be considered most important, and the presented selection policy therefore used anyway. Given this selection, 2-average, 3-average, 2-diameter, and 3-diameter were evaluated for structures with up to 900, 500, 800, and 400 nodes respectively. In some cases multiple $N2R$ and $CR$ exist for each number of nodes, but as the $k$-measures for these structures turn out to be close to each other, no further selection is done.

## 11.6 Results

### 11.6.1 Selection policies for $CR$

**Average distance and diameter**

Two approaches were evaluated for choosing $s$ given $w$. In the first approach, for every value of $w$, all values of $s$ minimizing the diameter are selected, and among these, $s$ is chosen to minimize average distance. In the second approach all values of $s$ minimizing the average distance are selected, and among these, the values resulting in the smallest diameter are selected.

In 434 of the 498 cases, there exist structures minimizing both average distance and diameter. Figures 11.2–11.3 illustrate the resulting average distances and diameters compared to the optimal values in the remaining 64 cases. With minimized diameters, the average distances are on average over these 64 cases 0.27% higher than minimum, and with minimized average distances, the diameters are on average 4.90% higher. Over all 498 cases, the differences are on average 0.035% and 0.63%. We choose to
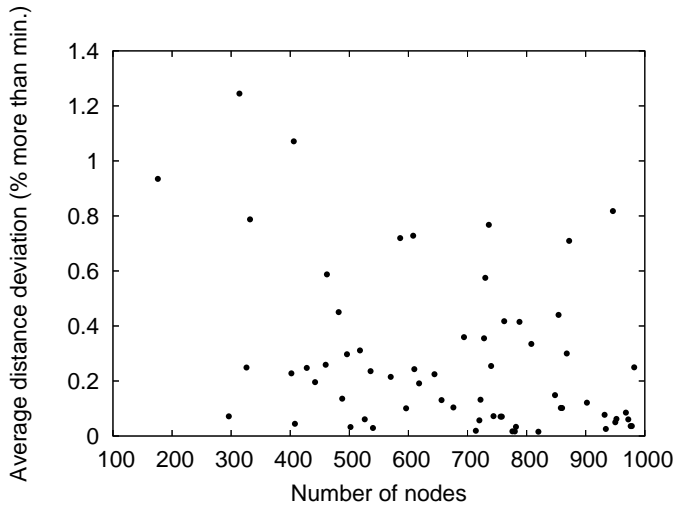
**Figure 11.2:** The difference between minimum average distances and minimum average distances when diameters are minimized.
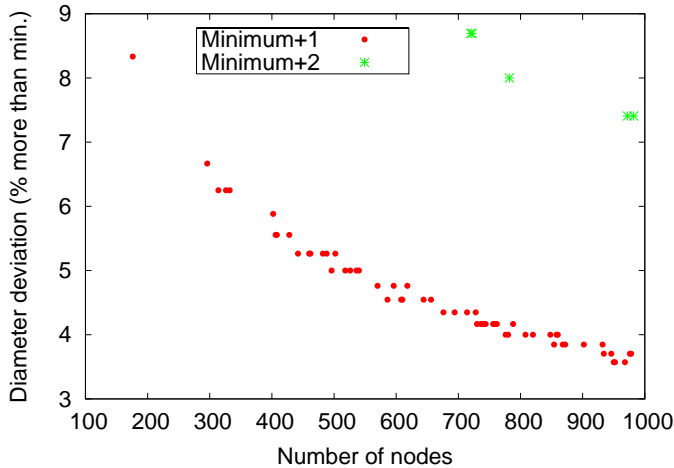


**Figure 11.3:** The difference between minimum diameters and minimum diameters when average distances are minimized. In these 64 cases, the diameter is either 1 or 2 higher than minimum.

use the first approach for our studies, but both nearly minimize the parameters.

## $k$-average and $k$-diameter

We evaluate to what extent the optimal $k$-average and $k$-diameter is obtained when for each value of $w$, the values of $s$ are selected using the chosen selection policy. This is done for $6 \leq w \leq 100$, a total of 48 values. For 31 of these, all chosen values

of $s$ minimized all parameters, and for additionally 5 values of $w$, at least one of the chosen values of $s$ minimizes all parameters. In the remaining cases, the choices of $s$ result in structures, which are close to minimal as can be seen in Figures 11.4–11.5. They show how much larger the $k$-measures are for the selected values of $s$ compared to minimum values. Only values of $w$ and $s$ not minimizing the respective parameters are shown; if for $w$ some but not all selected values of $s$ minimize a parameter, the values not minimizing $s$ are marked wc (for worst-case).
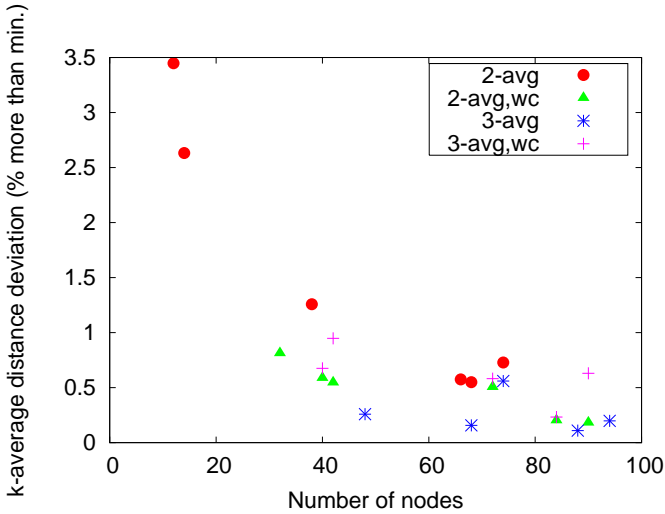


**Figure 11.4:** 2-average and 3-average compared to minimum values in the 12 respectively 10 cases when they are not always minimized. wc indicates that some but not all choices of $s$ minimize the parameter.
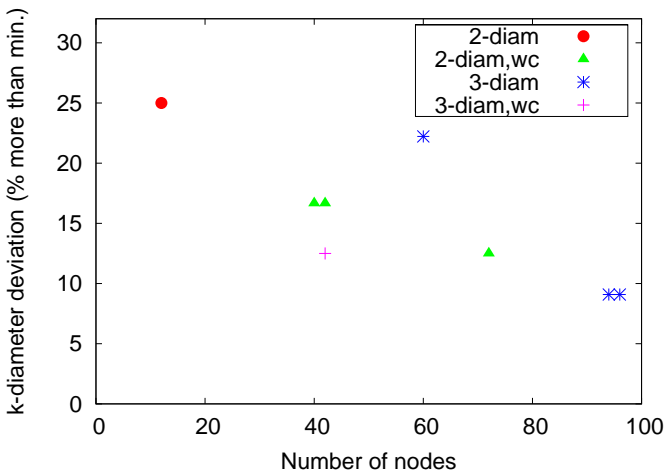


**Figure 11.5:** 2-diameter and 3-diameter compared to minimum values in the 4 respectively 4 cases when they are not always minimized. wc indicates that some but not all choices of $s$ minimize the parameter.

Over all 48 values of $w$, the 2-averages are on average 0.19% higher than the minimum valuess in the best-cases and 0.25% higher in the worst-cases. For the 3-averages, the corresponding values are 0.027% and 0.091%. For 2-diameters, the values are 0.52% and 1.48%, and for 3-diameters 0.84% and 1.10%. For $w = 60$, the 3-diameter is 2 higher than the minimum value, but in all other cases the difference in $k$-diameter does not exceed one. We conclude that the proposed selection policy nearly minimizes the $k$-measures, and so this approach can be used. Through the rest of the paper, when referring to $CR$ and $N2R$ structures, they are implicitly assumed to be selected in this manner.

### 11.6.2   Comparison of $N2R$ and $CR$

**Average distance and diameter**

In general, $N2R$ have lower average distances and diameters than $CR$, as can be seen in Figures 11.6–11.7. For small structures the differences are limited, and in 2 cases (12 and 20 nodes), $CR$ are better than $N2R$ with regard to both average distance and diameter. In all other cases, $N2R$ are better than or equal to $CR$ with regard to both parameters. The differences generally increase with the size of the structures, but are significant even for some rather small structures. For example, average distances and diameters of $N2R(25; 7)$ and $CR(50; 9)$, both minimizing both parameters within each class of structures, are 3.49 respectively 5 for the former and 3.94 respectively 7 for the latter. For $N2R(400; 91)$ and $CR(800; 109)$ the average distances are 10.9 respectively 15.5, and the diameters 17 respectively 25. Thus, using $N2R$ instead of $CR$ in these cases reduces the average distance by 29.4% and the diameter by 32.0%.

**$k$-average and $k$-diameter**

Figures 11.8–11.11 show the $k$-averages and $k$-diameters of all selected $CR$ compared to all selected $N2R$. The chosen selection policy can result in several different structures of the same size, in which case they are all shown in the plots. For $w \leq 900$ there are on average 1.91 $CR$ and 1.23 $N2R$ for each number of nodes, but the $k$-measures only differ slightly. Over all considered values of $w$ and $p$, the maximum $k$-measures are on average 0.0078% - 0.17% larger than the minimum for $CR$ and 0.0024%-0.078% larger than the minimum for $N2R$. The largest differences are found for the $k$-diameters.

$N2R$ structures are in general superior with respect to all four parameters. For all structures with 44 or more nodes, $N2R$ are better than $CR$ in terms of at least one parameter, and equal to or better than $CR$ with respect to all parameters. For structures with fewer than 44 nodes, the picture is more mixed, and the differences generally
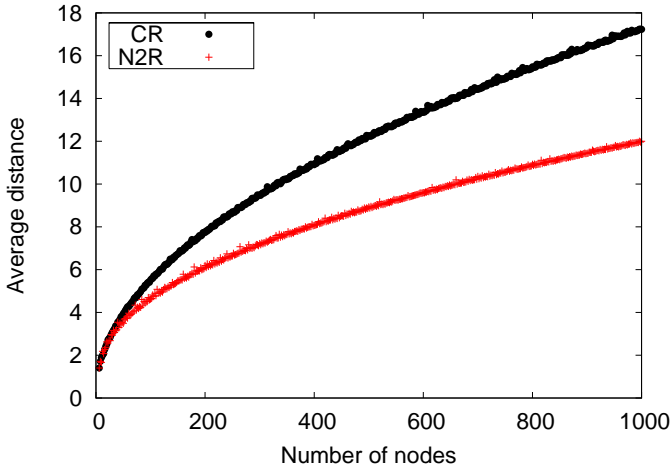
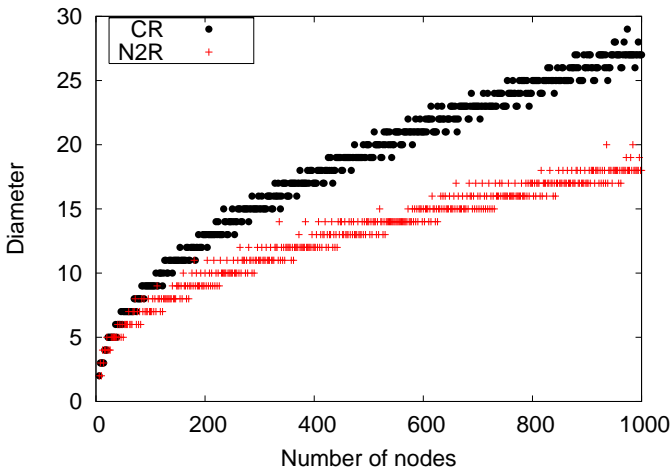**Figure 11.6:** Average distances of $N2R$ and $CR$.



**Figure 11.7:** Diameters of $N2R$ and $CR$.

small: in 8 cases $N2R$ are better with regard to at least one parameter and better than or equal with regard to all other parameters. In 2 cases $N2R$ and $CR$ are equal in terms of all parameters, and in 7 cases $N2R$ are best with regard to some parameters and $CR$ best with regard to others. 2 cases remain. In each of these there exist one $N2R$ but multiple $CR$. For $w = 32$, $s = 13$ implies that all parameters are equal, while for $s = 7$ and $s = 9$ $CR$ have slightly lower 2-average than $N2R$. This choice does not affect the other parameters. For $w = 40$, $CR(40; 9)$ has the same 2-average as $N2R$, but is better in terms of all other parameters. $CR(40; 11)$ has 2-average
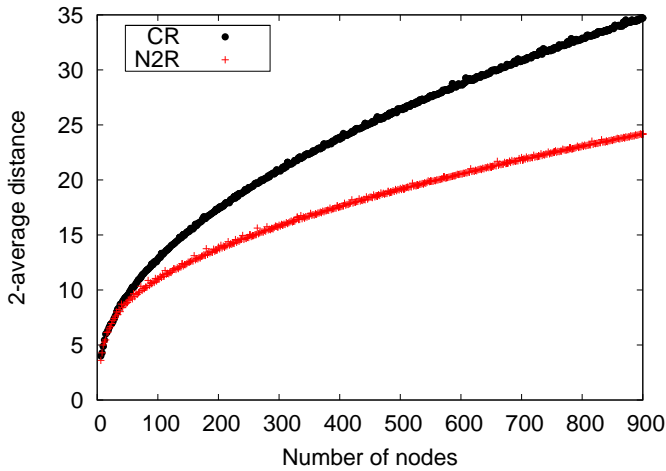
**Figure 11.8:** 2-averages of $N2R$ and $CR$.
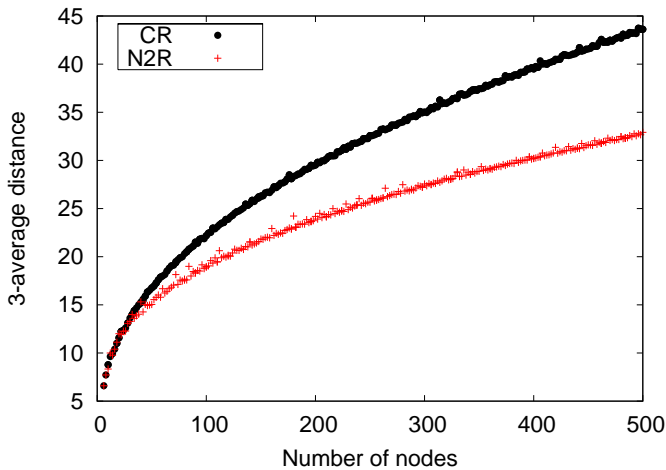


**Figure 11.9:** 3-averages of $N2R$ and $CR$.

slightly higher, but is also superior to the $N2R$ in terms of all other parameters.

As for the average distance and diameter, the differences increase with the size of the structures. For 400 nodes, $CR(400; 47)$ and $N2R(200; 19)$ minimize both average distance and diameter within each class of structures. $CR$ has 2-average 23.72, 3-average 39.5, 2-diameter 18, and 3-diameter 20. The corresponding values for $N2R$ are 17.56, 30.2, 14, and 15. Thus, choosing $N2R$ instead of $CR$ in this case reduces the parameters by 26.0%, 23.6%, 22.2%, and 25.0% respectively.

**Figure 11.10:** 2-diameters of $N2R$ and $CR$.



**Figure 11.11:** 3-diameters of $N2R$ and $CR$.

## 11.7   Conclusion and discussion

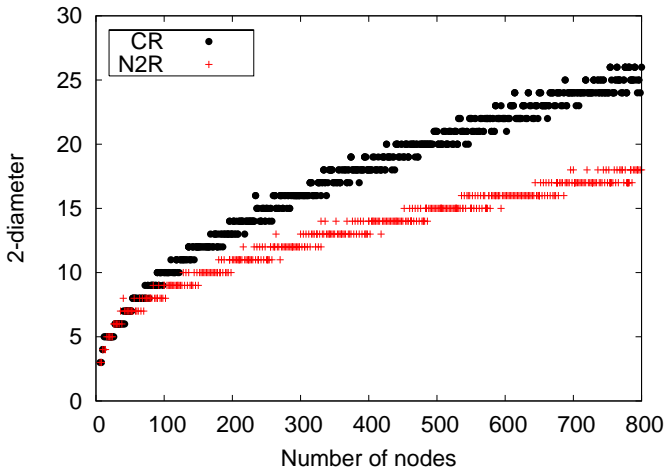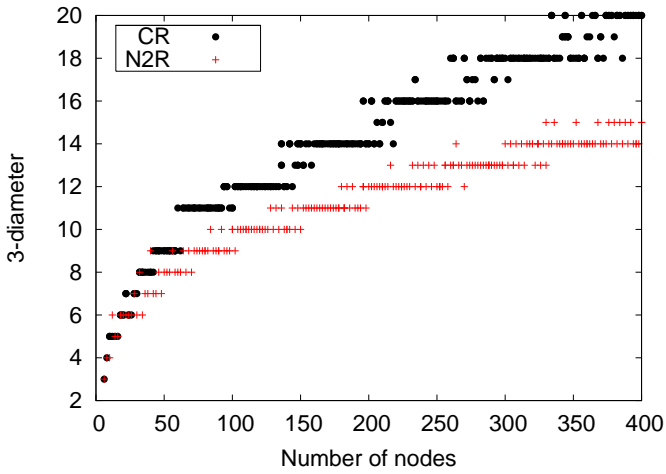Two important results were obtained. First, a policy for selecting the best Degree Three Chordal Rings ($CR$) given the number of nodes was devised and evaluated. It was shown that selecting $s$ to minimize diameter and to the largest possible extent also average distance leads to structures, which are close to optimal with regard to average distance, $k$-average distance and $k$-diameter, $k = 1, 2$. This selection policy facilitates the comparison of $CR$ to other structures, and this was applied to obtain the most interesting result, namely the comparison of $CR$ to Generalized Double Rings ($N2R$), which share many properties with $CR$. As the selection policy was previously shown to be good also for $N2R$, they are selected in the same way.

Average distance and diameter were compared for structures with up to 1000 nodes. For 2 small structures, $CR$ performed better than $N2R$ with regard to both of these parameters, but for large structures $N2R$ performed considerably better than $CR$. For structures with 800 nodes, the average distance of $N2R$ is 29.4% and the diameter 32.0% lower than for $CR$. 2-average distance, 3-average distance, 2-diameter, and 3-diameter were calculated for all structures with up to 900, 500, 800 and 400 nodes respectively. For structures with less than 44 nodes, the differences between $N2R$ and $CR$ were small, even though $N2R$ generally performed better than $CR$. For structures with 44 or more nodes, $N2R$ performed better in all cases, with the differences between each of the four parameters generally increasing with the size of the structures. For instance, for structures with 400 nodes the four parameters are 22.2-26.0% lower for $N2R$ than for $CR$.

Using $N2R$ instead of $CR$ in logical or physical networks may require more careful planning because simple rings are so easily extended to $CR$, but if this careful planning is done, the perspectives are promising and facilitate the design of networks with shorter distances and thus shorter transmission delays as well as lower traffic loads. While our results indicate that $N2R$ are superior to $CR$, it must also be taken into consideration that $CR$ are more extensively studied with regard to routing and transmission abilities. Therefore, we would like to encourage further research to deal with these aspects of $N2R$.

# 12 | Traffic Load on Interconnection Lines of Generalized Double Ring Network Structures [PRM05b]

## 12.1 Preface*

This last paper was written in collaboration with Tahir M. Riaz and Ole Brun Madsen, and presented at The 7th International Conference on Advanced Communication Technologiy (IEEE/ICACT 2005). It is the last of four papers on $N2R$ structures, but the results are not directly related to those presented in Chapters 10–11. The contribution to the thesis can be summarized as follows. The paper:

- Shows that the subset of the lines connecting the two rings in $N2R$ structures carry significantly lower loads than the other lines given shortest-path routing and all-to-all traffic, in particular for large structures. This can facilitate implementation in real-world networks by combining wired and wireless networks.

- Devises two routing schemes, reducing the load on these Interconnection Lines further, at the price of higher efficient average distances and diameters.

## 12.2 Abstract

Generalized Double Ring ($N2R$) network structures possess a number of good properties, but being not planar they are hard to physically embed in communication networks. However, if some of the lines, the interconnection lines, are implemented by wireless technologies, the remaining structure consists of two planar rings, which are easily embedded by fiber or other wired solutions. It is shown that for large $N2R$ structures, the interconnection lines carry notably lower loads than the other lines if shortest-path routing is used, and the effects of two other routing schemes are explored, leading to lower loads on interconnection lines at the price of larger efficient average distance and diameter.

## 12.3   Introduction

Many applications such as tele robotics[HKKK01][XmCjYXd04] and tele operations[XT98] are currently migrating from LAN to WAN environments. This trend is expected to continue and will put a huge pressure on Internet infrastructures at all levels, in terms of not only bandwidth but also reliability[MNS02]. While fiber networks offer almost unlimited bandwidths, it is still necessary to develop physical network topologies, which offer sufficient levels of reliability. Most networks are today based on ring topologies, which offer two independent paths between any pair of nodes. While being easy to implement and embed, they suffer from large hop counts, and even though easy protection and restoration schemes are supported, they do not handle failures well: any single failure results in notably larger hop counts, implying a huge increase in traffic load as well as transmission delay, and in case of two failures, the network is disconnected. The Generalized Double Rings ($N2R$) structures introduced recently[MKP03] offer 3 independent paths between any pair of nodes and high levels of symmetry. However, like other 3-regular structures with fairly short distances such as chordal rings[BDZ04b], they are not planar and thus hard to physically implement by fiber without compromising the line independency.

While no other wired or wireless technology offer a bandwidth comparable to that of fiber networks, wireless technologies are developing fast, and the idea of combining wired and wireless networks to obtain network structures with good structural properties seems interesting. Despite expected technological developments, it is likely to be suitable only for structures where the wireless parts carry significantly lower traffic than the wired parts. It was indicated that using shortest-path routing in $N2R$ structures, some lines would carry a limited amount of traffic[PPKM04a]. This is investigated further in this paper, forming a base for designing networks, which are fairly easy to implement and possess good structural properties. To our knowledge, load distribution has not been studied in this perspective before.

## 12.4   Preliminaries

A network structure $S$ is a set of nodes and a set of bidirectional lines, where each line connects two nodes. A structure can be considered a model of a network, abstracting from specific physical conditions such as node equipment, medias, and wiring. The definition of a structure is similar to that of a simple graph in graph theory. A path between two distinct nodes $u$ and $v$ is a sequence of nodes and lines: $(u = u_0), e_1, u_1, e_2, u_2, \ldots, u_{n-1}, e_n, (u_n = v)$, such that every line $e_i$ connects the nodes $u_{i-1}$ and $u_i$. The length of a path corresponds to the number of lines it contains, so in the case above the path is of length $n$. The distance between a pair

of distinct nodes $(u, v)$ corresponds to the length of the shortest path between them, and is written $d(u, v)$. This paper considers only 3-connected structures, i.e. between every pair of distinct nodes there exist three different paths, which share no nodes or lines. The size of a structure equals the number of nodes it contains. A structure has a planar representation if it can be drawn with no lines or nodes crossing or overlapping each other. A structure with a planar representation is said to be planar. Average distance and diameter of a structure are defined as follows. The average of $d(u, v)$ over all pairs of distinct nodes $u$ and $v$ is said to be the average distance, and the maximum of $d(u, v)$ over all pairs of distinct nodes is said to be the diameter.

$N2R$ structures are defined as follows. Let $p$ and $q$ be positive integers, such that $p \geq 3$, $q < \frac{p}{2}$, and $gcd(p, q) = 1$. $p$ and $q$ then define a $N2R(p; q)$ structure $S$, which consists of two rings, an outer ring and an inner ring, each containing $p$ nodes. The nodes of the outer ring are labeled $o_0, o_1, \ldots, o_{p-1}$, and the nodes of the inner ring labeled $i_0, i_1, \ldots, i_{p-1}$. Thus, $S$ contains $2p$ nodes. For each $i$ such that $0 \leq i \leq p-1$ there exists a line between each of the following pairs of nodes:

- $(o_i, o_{i+1(mod\ p)})$ (lines of the outer ring: outer lines).

- $(i_i, i_{i+q(mod\ p)})$ (lines of the inner ring: inner lines).

- $(o_i, i_i)$ (interconnection lines).

$N2R(p; 1)$ is called the Double Ring ($DR$), and the diameter given by $\lfloor \frac{p}{2} \rfloor + 1$. Since the diameter increases linearly with the structure size, it is useful for reference purposes.

The set of lines of the inner ring is denoted $L_i$, the set of lines of the outer ring is denoted $L_o$, and the set of interconnection lines is the denoted $L_{io}$. Even though $N2R$ structures are not in general planar, any $N2R$ structure from which one of the sets of lines $L_i$, $L_o$ or $L_{io}$ is removed has a planar representation. Furthermore, any $N2R$ structure can be physically implemented in a way where only the lines of either $L_i$, $L_o$ or $L_{io}$ are crossing each other. Figure 12.1 shows $N2R(11; 3)$ drawn according to the definition and as two (planar) rings, where only interconnection lines need to cross each other, making them candidates for the wireless part of the network.

For a given $N2R$ structure, the average-path load is defined for each of the set of lines $L_i$, $L_o$, and $L_{io}$ as follows. Assume that paths are set up between any pair of distinct nodes, giving a total of $p(2p - 1)$ paths, $p_1, \cdots, p_{p(2p-1)}$. Any such path $p_j$ of length $|p_j|$ consists of $|p_j|_{L_i}$ lines of $L_i$, $|p_j|_{L_o}$ lines of $L_o$, and $|p_j|_{L_{io}}$ lines of $L_{io}$. Note that these values depend on how the shortest paths are chosen. $\frac{\sum_{j=1}^{p(2p-1)} |p_j|_{L_i}}{p(2p-1)}$ is the average-path load on inner lines, $\frac{\sum_{j=1}^{p(2p-1)} |p_j|_{L_o}}{p(2p-1)}$ the average-path load on outer lines,
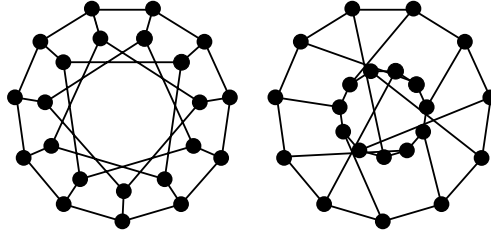
**Figure 12.1:** $N2R(11;3)$ drawn in two different ways.

and $\frac{\sum_{j=1}^{p(2p-1)} |p_j|_{L_{io}}}{p(2p-1)}$ the average-path load on interconnection lines. Adding these three values, the total average-path load is obtained, equaling the average distance if all paths are chosen to be shortest paths. Any shortest path between nodes of the same ring will use 0 or 2 interconnection lines, and any shortest path between nodes of different rings will use exactly one interconnection line[PPKM04a]. This implies that the average-path load on interconnection lines is between 0.5 and 1.5, implying a limited traffic load on these. Where it does not lead to confusion, we may simply write load instead of average-path load.

Routing policies are introduced, which constrain the use of interconnection lines. In this way, one path is chosen between each pair of distinct nodes, but it does not need to be a shortest path. Taking the average over these path lengths, the efficient average distance is obtained, given that routing policy. Similarly, the efficient diameter is obtained by taking the maximum over these path lengths.

## 12.5 Methods

The study is carried out in three steps. In each step, different policies for structure selection and routing apply. Structure selection policies are used for choosing $q$ given $p$, reflecting that for each value of $p$ several different structures can exist with different characteristics.

In the first step, the load on interconnection lines is compared to the load on other lines. $q$ is for each value of $p$ initially chosen to minimize diameter and to the largest possible extent also average distance. It was shown[PKM04b] that this leads to structures with average distance minimized or nearly minimized. This selection policy may result in several values of $q$ being chosen. Routing, or path selection, is done using shortest paths in three variants; first, the shortest paths are chosen to minimize the load on interconnection lines, second they are chosen to minimize the load on inner lines, and finally they are chosen to minimize the load on outer lines. When several values of $q$ exist, further selection is done for each of the three routing schemes by

choosing $q$ to minimize the load on the lines of which the load is minimized. Thus, for each value of $p$, the lowest possible load for each set of lines is obtained.

In the second step, two approaches to further reduce the load on interconnection lines, at the price of higher efficient average distances and diameters, are studied. $q$ is chosen as before, but where this results in several values of $q$, only those resulting in the lowest possible load on interconnection lines are chosen. During this step, two routing schemes are evaluated. Both use shortest paths between nodes in different rings, but for pairs of nodes in the same ring, restrictions on the use of interconnection lines apply. This is done for each of the schemes as follows, where $x_{diam}$ and $x_{avg}$ must be chosen in each case. In Routing Scheme 1 ($RS1$), a path containing interconnection lines is chosen if and only if the lengths of all paths not containing interconnection lines exceed either the diameter of the structure or $x_{diam}\%$ of the diameter of the $DR$ with the same number of nodes, whichever value is largest. In Routing Scheme 2 ($RS2$), a path containing interconnection lines is chosen if and only if the lengths of all paths not containing interconnection lines exceed the length of a shortest path by at least $x_{avg}\%$.

The two schemes are evaluated separately. First, $x_{diam}$ is varied in steps of 10, and evaluated for $x_{diam} = 0, 10, 20, \ldots, 100$. Next, $x_{avg}$ is also varied in steps of 10, i.e. $x_{avg} = 10, 20, \ldots, 100$. $x_{avg} = 0$ is not used. At the end of this step, $RS1$ and $RS2$ are compared. For each considered set of values of $p$, $q$, and $x_{diam}$, a value of $x_{avg}$ is determined, which results in a structure with the same load on interconnection lines. If no value of $x_{avg}$ satisfies this, $x_{avg}$ is first determined by the lowest value of $x_{avg}$ resulting in the load on interconnection lines being lower than that of $RS1$. An adjustment is then made by allowing an additional number of paths to use the interconnection lines, such that the load on interconnection lines equals that of $RS1$. These paths are chosen to minimize efficient average distance and to the largest possible extent also efficient diameter. Now, for each considered value of $p$, the two ways of obtaining a certain load on interconnection lines are compared by efficient average distance and efficient diameter.

$q$ was in the previous steps chosen to minimize diameter, average distance, and load on interconnection lines given shortest-path routing. If the revised routing schemes are used, this may not be optimal. In the last step, it is studied if other values of $q$ perform better when $RS1$ is used, varying $x_{diam}$ from 0 to 100 in steps of 10. Given $p$ and $x_{diam}$, it is determined which value of $q$ results in the best performance. Using efficient average distance, efficient diameter, and average-path load on interconnection lines as performance parameters, this is done as follows. For the considered values of $p$ and $x_{diam}$, all permitted values of $q$ with diameter and average distance less than or equal to the efficient diameter and efficient average distance respectively, are evaluated. The resulting efficient average distance, efficient diameter, and average-path

load on interconnection lines are then compared to the values obtained in the second step.

All calculations are perfomed for all $p \leq 1000$ on a standard PC, using C programs. All paths constructed are either shortest paths or paths running along the inner or outer ring, and together with the symmetries, this makes it possible to calculate all the desired values within acceptable calculation times.

## 12.6  Results

Figure 12.2 shows that for large values of $p$, interconnection lines carry significantly lower loads than other lines, using shortest-path routing and avoiding interconnection lines if possible. For $p$ small, the interconnection lines carry appr. 33% of the total load, a number decreasing as $p$ increases. The distribution of the remaining load depends on the chosen routing strategy. Figures 12.3–12.4 show the potentials when reducing the load on the different sets of lines. For $p \geq 45$ the interconnection lines allow for the lowest loads, but for $p < 45$ the picture is more mixed.

By revising the routing scheme, it is possible to reduce the load on interconnection lines significantly, but it has its costs in terms of average distance and diameter. $RS1$ leads to distances and loads as shown in Figures 12.5–12.10. In Figures 12.5, 12.7, 12.9, $x_{diam}$ is varied in steps of 10, but only a selection of these results are shown in Figures 12.6, 12.8, 12.10 to increase readability. Among the 998 considered values of $p$, there are 73 cases where more than one value of $q$ exist, and in some of these cases,
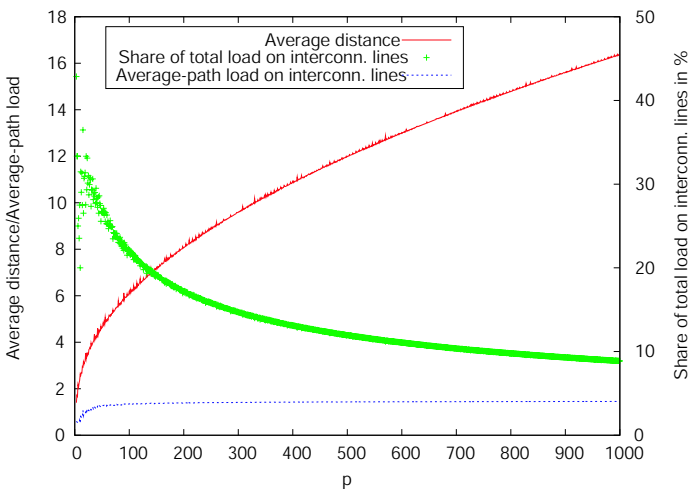


**Figure 12.2:**   Contribution from interconnection lines to average distance, assuming shortest-path routing and avoiding interconnection lines where possible.
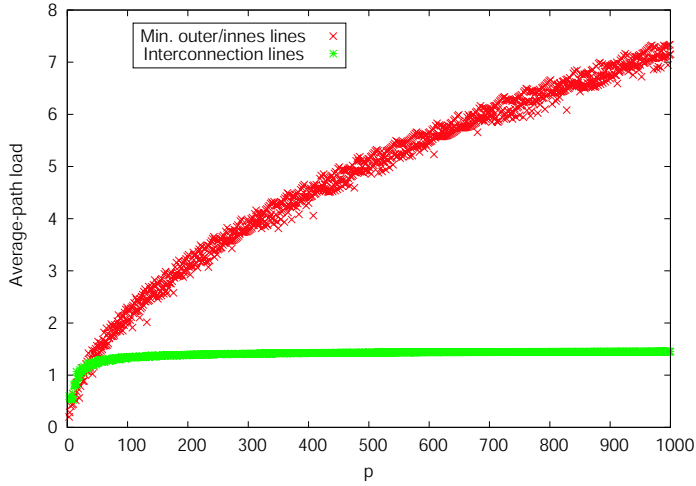
**Figure 12.3:** Average-path loads with shortest-path routing, minimizing the load on interconnection, outer or inner lines. Of the two latter, only the minimum value is shown for each value of $p$.
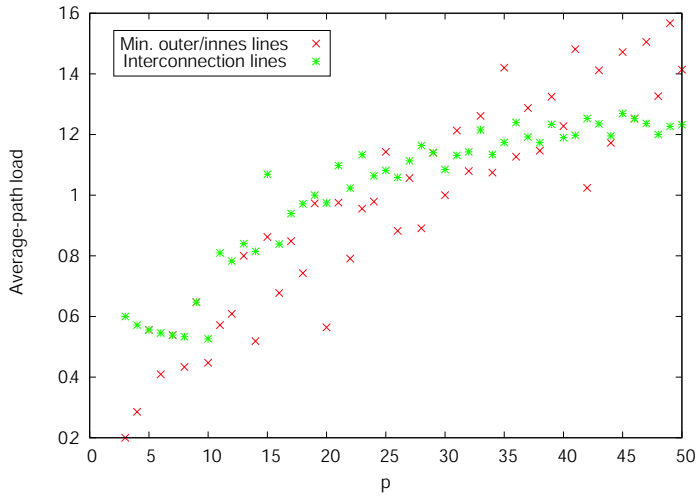


**Figure 12.4:** Average-path loads with shortest-path routing, minimizing the load on either interconnection, outer or inner lines. Of the two latter, only the minimum value is shown for each value of $p$.

the efficient average distances depend on further selection of $q$. Over these 73 cases and the 11 values of $x_{diam}$ from $0 - 100$, the average difference between maximum and minimum efficient average distance is 0.42% of the minimum. In no case the difference exceeds 2.35%. $q$ is chosen to minimize the efficient average distance. This choice affects no other parameters.

$RS2$ leads to distances and loads as shown in Figures 12.11–12.13. The further selec-
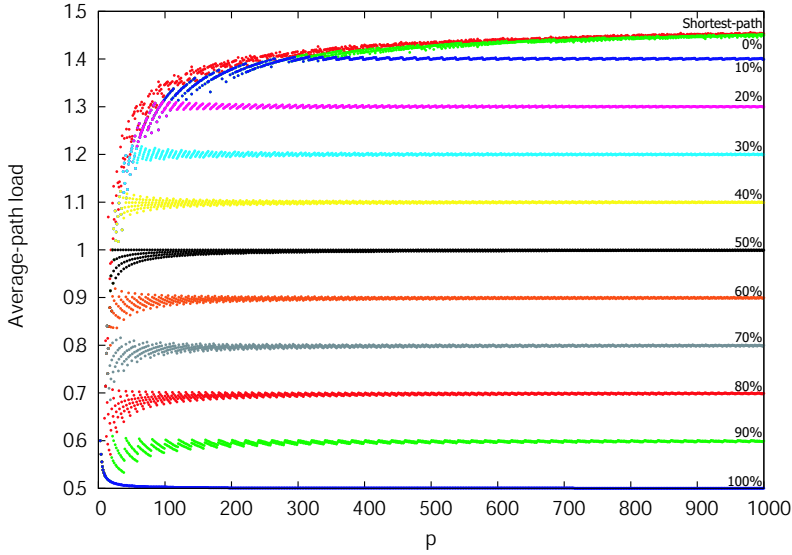
**Figure 12.5:** Interconnection line loads using $RS1$, varying $x_{diam}$.
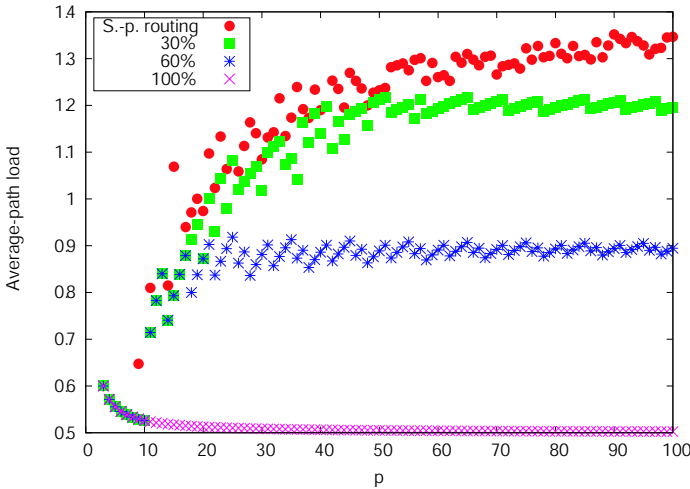


**Figure 12.6:** Interconnection line loads using $RS1$, varying $x_{diam}$.

tion of $q$ is slightly more difficult here, because the choice of $q$ affects the line load on interconnection lines as well as efficient average distance and diameter. Over the 73 cases with multiple values of $q$ and the 10 values of $x_{avg}$ $(10 - 100)$, the differences between maximum and minimum values are on average 1.48% (load on interconnection lines), 2.30% (efficient average distance), and 4.35% (efficient diameter) of the minimum. First, $q$ is chosen to minimize the load on interconnection lines, which re-
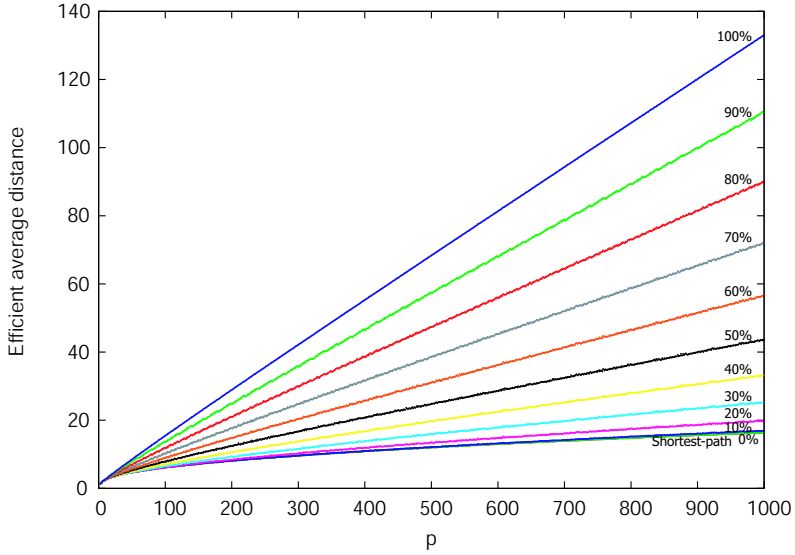
**Figure 12.7:** Efficient average distances using $RS1$, varying $x_{diam}$.
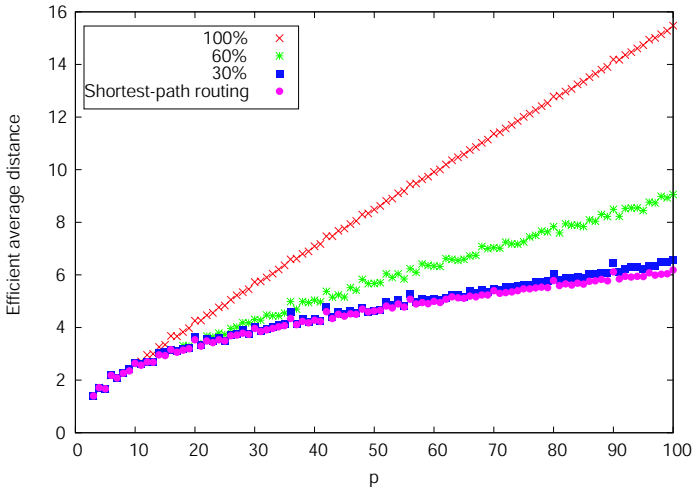


**Figure 12.8:** Efficient average distances using $RS1$, varying $x_{diam}$.

duces the number of values of $p$ with multiple values of $q$ to on average (over the 10 values of $x_{avg}$) 33.9. From this point, $q$ is chosen to minimize efficient diameter, and where this leads to multiple candidates finally to minimize efficient average distance. Over the on average 33.9 values of $p$ with multiple values of $q$, this leads to efficient average distances 0.043% over the minimum obtained when minimizing the load on interconnection lines. In no case the chosen efficient average distance exceeds the
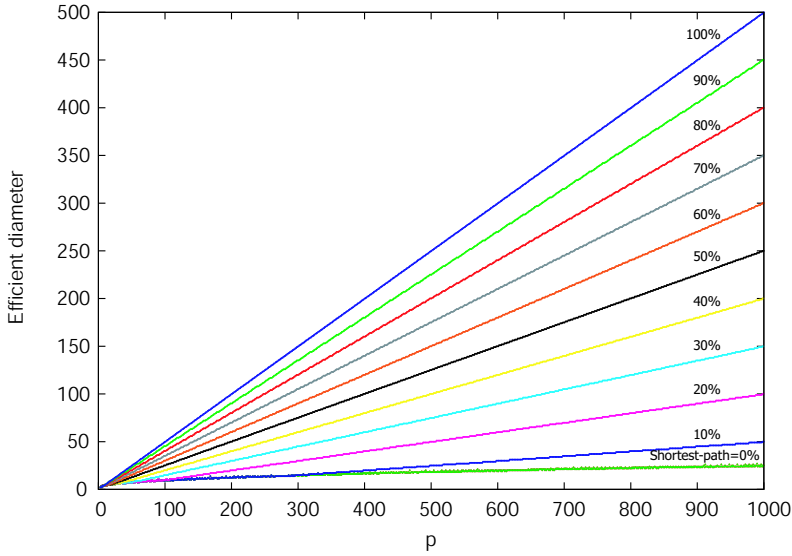
**Figure 12.9:** Efficient diameters using $RS1$, varying $x_{diam}$.
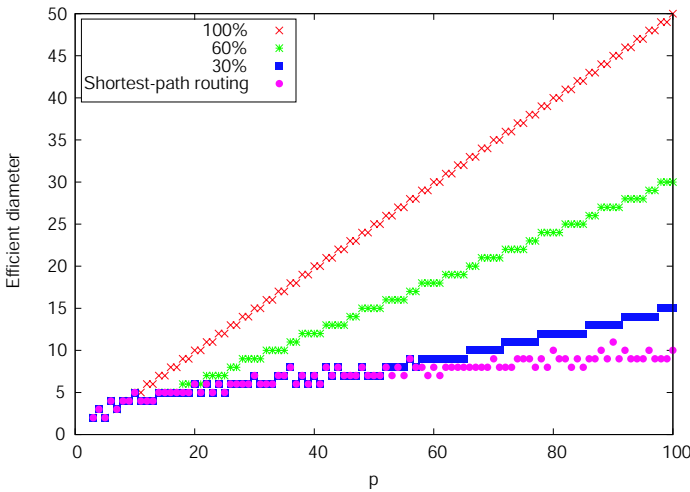


**Figure 12.10:** Efficient diameters using $RS1$, varying $x_{diam}$.

minimum value by more than 1.15%.

A direct comparison of the two approaches shows that in order to obtain the same load on interconnection lines, $RS2$ resulted in larger or equal efficient diameters and smaller or equal efficient average distances than $RS1$. For each value of $p$, the differences in some cases depend on the value of $q$, in which case $q$ is chosen first to
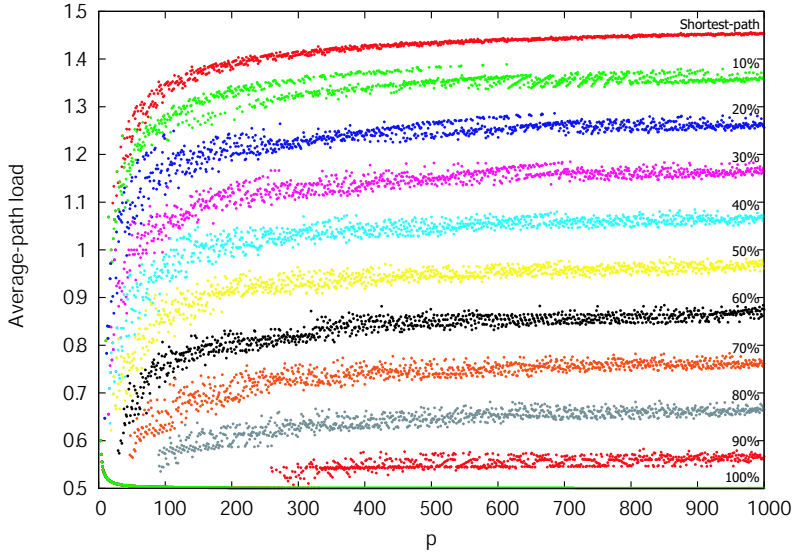
**Figure 12.11:** Interconnection line loads using $RS2$, varying $x_{avg}$.
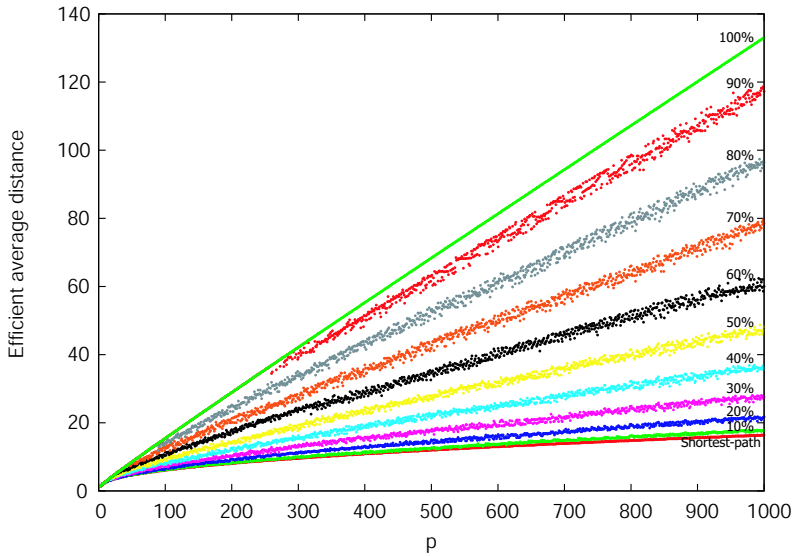


**Figure 12.12:** Efficient average distances using $RS2$, varying $x_{avg}$.

maximize the relative difference in efficient average distance, and second to minimize the relative difference in efficient diameter, giving an impression of the trade-offs. In general, the relative differences become smaller when $p$ becomes large, which is illustrated for $x_{diam} = 60$ in Figure 12.14. Table 12.1 shows for each value of $x_{diam}$
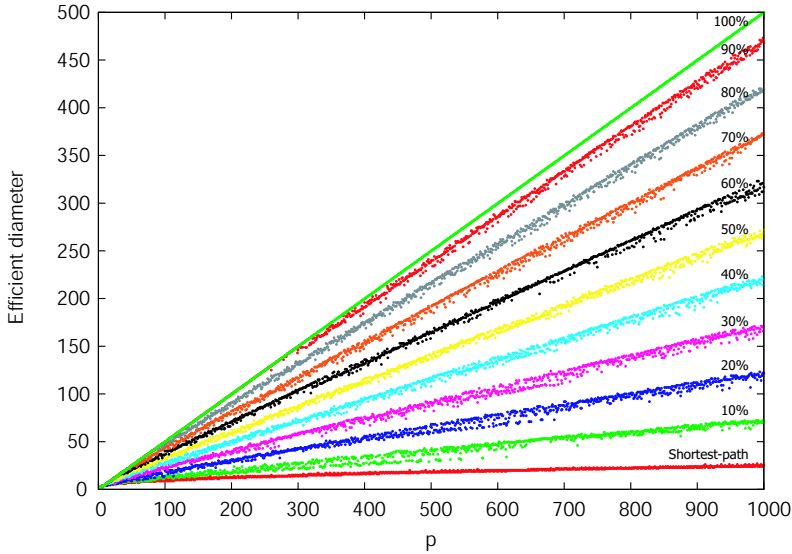
**Figure 12.13:** Efficient diameters using $RS2$, varying $x_{avg}$.
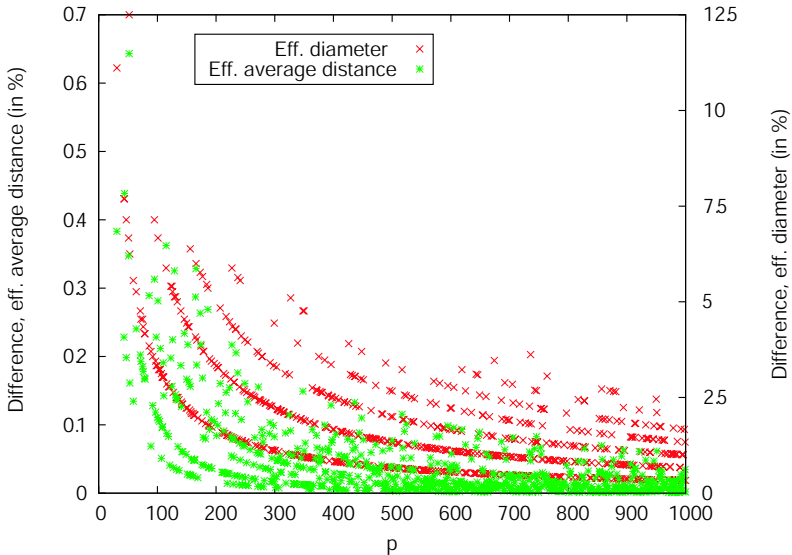


**Figure 12.14:** Reduction in efficient average distance/increase in efficient diameter with $RS2$ instead of $RS1$, $x_{diam} = 60$ and same interconnection load.

the relative differences in efficient average distance and diameter.

For all considered values of $p$ and $x_{diam}$, it was determined if another value of $q$ would result in a better performance than the values of $q$ determined during the sec-

**Table 12.1:** Efficient average distances and diameters of $RS2$ compared to $RS1$ for values of $p$ where they are not equal. Differences in % of $RS1$-values.

| $x_{diam}$ | No. $p$'s | Avg. diff, eff. avg. | Max. diff, eff. avg. | Avg. diff, eff. diam. | Max. diff, eff. diam. |
|---|---|---|---|---|---|
| 0 | 334 | 0.0531% | 0.784% | 6.84% | 25.0% |
| 10 | 578 | 0.0631% | 0.784% | 7.10% | 25.0% |
| 20 | 658 | 0.0791% | 0.784% | 5.24% | 25.0% |
| 30 | 635 | 0.0755% | 0.784% | 3.61% | 25.0% |
| 40 | 600 | 0.0614% | 0.637% | 2.79% | 20.0% |
| 50 | 554 | 0.0446% | 1.02% | 2.12% | 16.7% |
| 60 | 692 | 0.0436% | 0.643% | 1.79% | 12.5% |
| 70 | 639 | 0.0377% | 1.35% | 1.48% | 14.3% |
| 80 | 635 | 0.0323% | 0.877% | 1.40% | 14.3% |
| 90 | 643 | 0.0236% | 0.340% | 1.15% | 7.69% |
| 100 | 0 | 0% | 0% | 0% | 0% |

**Table 12.2:** Reductions in eff. avg. distance, choosing $q$ differently.

| $x_{diam}$ | Number of $p$'s | Avg. red. | Max red. |
|---|---|---|---|
| 0 | 43 | 0.317% | 2.52% |
| 10 | 349 | 0.270% | 2.52% |
| 20 | 511 | 0.629% | 2.52% |
| 30 | 554 | 0.654% | 2.52% |
| 40 | 587 | 0.547% | 2.52% |
| 50 | 635 | 0.474% | 2.45% |
| 60 | 604 | 0.364% | 2.35% |
| 70 | 676 | 0.295% | 2.92% |
| 80 | 719 | 0.240% | 3.47% |
| 90 | 643 | 0.134% | 1.79% |
| 100 | 49 | 0.0864% | 1.79% |

ond step. It turned out that in every case, the load on interconnection lines and efficient diameter remained the same, but for some values of $p$ it was possible to reduce the efficient average distance. The results are listed in Table 12.2; for each value of $x_{diam}$, the number of values of $p$ for which at least one better $q$-value exist is shown together with the potential maximum and average reductions, taken over these values of $p$.

## 12.7   Conclusion and discussion

It was shown that using shortest-path routing, the load on interconnection lines is limited for any $N2R$ structure, and for $p \geq 45$ the average-path load on interconnection lines is smaller than the loads on inner and outer rings. The differences increase with the size of the structures. On average, the shortest paths use 0.5-1.5 interconnection lines, and even though this number grows fast towards 1.5 with the size of the structures, the average path length increases significantly faster. It should however be kept in mind that the absolute traffic load on interconnection lines grow faster with the size of the structures, because more nodes create more traffic. This is not reflected by the measures used in the paper.

It was also shown that the load on interconnection lines can be further reduced by changing the routing scheme, but this also implies significantly larger efficient average distance and efficient diameter; to reduce the load on interconnection lines to approximately 0.5, the efficient diameter approaches that of the $DR$. Two revised routing schemes were proposed. Given the decreased interconnection line load, one minimized the efficient diameter, and the other the efficient average distance, but it turned out that the differences between them were in general insignificant. If networks are implemented combining fiber/wireless solutions, it may be appropriate to use such a revised routing scheme to prefer the use of fiber lines, also reflecting the fact that fiber transmissions are faster and with fewer errors than wireless transmissions; a longer path using only fiber and allowing for optical switching may be better than a shorter combined fiber/wireless path.

Structures were chosen to minimize diameter, average distance, and load on interconnection lines, and even with the revised routing schemes, this seems to be a fairly good choice. The results indicate that networks with $N2R$ topologies can be implemented physically by using wireless solutions for some or all of the interconnection lines. However, this requires more research in combining wired and wireless networks into one common network.

In access networks, a large part of the traffic is usually one-to-all traffic, going to and from a gateway to the Internet. In this case, the traffic will most likely not be distributed evenly on the interconnection lines, and it might be advantageous to implement some of these lines by fiber, and some by wireless technologies.

# Part III

# Conclusion and Future work

# 13 | Conclusion

The results of the work have been described in the papers in Part II, where each paper is self-contained with its own conclusions. This chapter therefore gives an overview of the results.

One part of the study was to identify directions for further research. This has been done in the individual papers, but for the directions to take also the overall conclusion presented in this chapter into account, suggestions for further research are presented in Chapter 14.

Prior to my study, SQoS was presented, but only little knowledge established except for motivations and ideas. During my study, motivations for SQoS have been further documented, and methods given for describing and determining structural properties of networks. Properties of 4-regular grid structures have been determined, and extensions developed in order to obtain better SQoS-properties. One of the initial ideas of SQoS was to use $N2R$-structures to obtain networks with better structural properties. By now, these have also been extensively analysed.

In Chapter 4, the demands for reliability in the access networks were determined. Together with the work presented in [MNS02] and [MKP03], this documents the demands for SQoS at all levels in the networks. The increasing demands for reliability even in FTTH access networks are expected to make single points of failure unacceptable, especially if the networks are to be used for critical applications such as surveillance and tele medicine. To avoid such single points of failure, physical redundancy is needed, either provided by fiber or by some other technology.

Chapter 5 identifies the most important SQoS parameters. These are useful for analyzing and comparing network structures, but the parameters presented can also be taken into account when actually planning network infrastructures. Both quantitative and qualitative parameters are identified, and it is devised how the parameters reflect different network usages. There is a particular focus on global structural properties, which is in line with the concept of SQoS.

Chapters 6–8 deal with the 4-regular grid structures. These structures are mainly interesting due to their planarity, simplicity, symmetry, and support for Topological Routing. They have been used in multiprocessor systems for many years, so they are well known both theoretically and practically, but the significant differences between large-scale networks and multiprocessor systems pose some interesting challenges.

The structures are difficult to implement or embed because of the regularity and relatively high nodal degrees, and large structures suffer from large distances. Extensions have been developed in order to cope with these problems, resulting in scalable and more flexible structures. The most important extensions are the following:

- Lake algorithms, making communication possible even in networks with failures.

- Hierarchical extensions, where hierarchical lines are added. These reduce the distances significantly, and simple routing and protection schemes are still supported.

- Schemes for varying the hierarchical depths, allowing for more flexible deployment and gradual implementation.

- Pruning, reducing the number of lines while only slightly affecting distances and routing algorithms.

Chapters 9–12 analyse the $N2R$ structures and compare them to two other 3-regular structures, Double Rings and Degree Three Chordal Rings. Basic properties are shown, and a shortest-path routing scheme devised. The structures are mainly analysed with regard to a number of distance parameters: average distance, diameter, $k$-average distance, and $k$-diameter. A main problem of evaluating $N2R$ structures, and comparing them to other, is the choice of selection policy. Given the number of nodes, different $N2R$ structures exist with different properties, and when comparing on multiple parameters, it is necessary to select one structure for each number of nodes. A selection policy is devised, which minimizes or nearly minimizes all of the distance parameters. The $N2R$ structures are shown to be superior with regard to the distance parameters used. An extension of the $N2R$ structures was proposed in [MKP03]: originally, $N2R$ structures were constructed with two parameters, $p$ and $q$ (and written $N2R(p;q)$), but [MKP03] introduced a third parameter $r$ (written $N2R(p;q;r)$). It is shown that every such $N2R(p;q;r)$ structure is isomorphic to a $N2R(p;q)$ structure. The $N2R$ structures are not in general planar, and the crossing lines make them difficult to physically implement. It was shown that under assumption of all-to-all traffic distribution, some of the lines carry significantly lower loads than the rest, and if these could be implemented by wireless technologies, the remaining structure would be planar. Ongoing research deals with other ways of implementing them, see Chapter 14.

The papers presented in this thesis form a theoretical base for SQoS-based network planning and for further research within the field. Directions for further research are given in Chapter 14.

# 14 │ Future work

This section is based partly on the conclusions presented in Part II. The papers presented in Part II themselves explore different approaches to the field, and each paper also devises suggestions for further research. Throughout the papers, many different directions for further research have been suggested, and it is not difficult to suggest even more. In the following, I have chosen to sketch what I believe are the most important. These are chosen in order to obtain the following:

- A stronger theoretical base for SQoS.

- An extended framework for evaluating and comparing network structures.

- A scheme for handling SQoS across network layers.

- Testing, optimization, and demonstration of algorithms for routing, restoration, and protection.

- Schemes for SQoS-based planning of physical network infrastructures.

- Knowledge on SQoS in wireless and combined wired/wireless networks.

## 14.1 A stronger theoretical base for SQoS

In this thesis, two potential classes of structures have been described and analysed: the 4-regular grid structures and the $N2R$ structures. Similar analyses should be conducted on more classes of structures, in particular if 3-regular or 4-regular structures with properties better than those of the $N2R$ respectively 4-regular grid structures are found. With regard to methods, most of the qualitative results obtained for 4-regular grid and $N2R$ structures have been obtained by calculations rather than mathematical analysis. For future studies, it would be interesting to obtain such more analytical results, possibly by establishing more precise bounds for the specific structures. Analytical results could also be obtained on how to choose $q$ given $p$ for $N2R$ structures, or on how to choose $s$ given $w$ for Degree Three Chordal Rings.

Development of reference models and bounds for regular structures is another interesting field: the Maximum Spanning Tree approach[MKP03] provides lower, achievable bounds for diameter and average distance, measured from one node to all other

nodes. The Moore Bound[DC04] gives a lower bound for the diameter given the number of nodes, from which also a bound for the average distance can be determined. However, there is still a need for better bounds for average distance, and for bounds for $k$-average distances and $k$-diameters. These are particularly interesting for 3-regular and 4-regular structures.

For the 4-regular grid structure, a hierarchical scheme was proposed, ensuring the scalability. While $N2R$ structures (and Degree Three Chordal Rings) scale much better than rings and double rings, they cannot grow infinitely large, and some hierarchical extension is therefore needed. This does not appear to be simple: ideally, such a hierarchical extension should connect a number of $N2R$ structures, and preferably preserve the 3-connectivity. This implies that (at least) three nodes from each "basic" $N2R$ structure should be chosen, but no obvious way of choosing these exists. When this selection has been performed, all such selected nodes could again be connected by a $N2R$ structure, facilitating multiple hierarchies.

## 14.2 An extended framework for evaluating and comparing network structures

The framework presented in Chapter 5 assumes to some extent either all-to-all or one-to-all traffic. With the structures abilities to offer three independent paths between any pair of nodes, other traffic generation schemes are likely to occur, and parameters reflecting these schemes better should be determined. In order to obtain realistic results, it may be appropriate to use simulations rather than calculations for this, because so many parameters come into play, including the distribution of the nodes generating the most traffic. For example, three nodes of a $N2R$ structure may provide the connectivity to higher layers, and thus create the most traffic. Thus, the selection of these nodes has an impact on the traffic distribution and vice versa.

If coupled with the research on SQoS-based planning of physical network infrastructures (see Section 14.5), the framework could be extended to include measures related to possible implementations such as fiber and duct lengths. In this way, it could be used for designing automatic tools for SQoS-based network planning.

## 14.3 A scheme for handling SQoS across network layers

So far, SQoS has mainly been established on the physical level, by devising parameters and principles, which should be taken into account when designing physical network structures. Defining and devising such structures, with good and predictable

properties, is indeed an important part of the SQoS research field, but it cannot stand alone, and questions of how the properties of the physical layer can translate into good properties in higher layers are still to be answered. These questions arise in particular when routing and physical structures become related, such as for the 4-regular grid structure.

One possibility is to let the mappings from physical to logical layers preserve the good properties, and another to assign properties related to SQoS to lines in the higher layers of the network. The schemes developed should on one hand ensure maximum benefit from the SQoS-properties, and on the other hand facilitate a smooth integration with the unstructured networks widely used today.

## 14.4 Testing, optimization, and demonstration of algorithms for routing, restoration, and protection

For the 4-regular grid structures, a number of algorithms for Topological Routing, restoration, and protection were proposed in Chapters 6–8. For the $N2R$ structures, a routing scheme was proposed in chapter 9. In order to test, optimize, and demonstrate these algorithms, it is necessary to perform either simulations or make test implementations.

Initiatives have already been taken concerning the latter, as a part of the C3/I3 project run by Center for Network Planning and Center for TeleInfrastruktur, Aalborg University. A test network is currently being set up with 36 PCs as well as a central monitoring and configuration unit. It is expected that this project will result in better optimized algorithms as well as specifications of hardware requirements and standards, bringing SQoS and Topological Routing from theory to practice.

The algorithms presented in this thesis are based on the structures fulfilling rather strict requirements, making it hard to implement networks gradually and take full advantage of the structural qualities in the intermediate steps, where they are not fully completed. The lake algorithms of Topological Routing allow routing even in incomplete structures, but do not ensure that shortest paths are always chosen. Algorithms taking full or almost full advantage of the structural properties even in structures fulfilling less strict requirements would be a major contribution. A slightly different approach could be to develop algorithms supporting stepwise implementations of network structures, benefiting from the structural qualities at certain defined intermediate steps. The latter approach appears to be simpler, but also weaker, in particular in case of network failures.

## 14.5   Schemes for SQoS-based planning of physical network infrastructures

One of the main problems in using well-ordered structures in the design of physical large-scale networks is the embedding: how to implement the structures in the real world in a cost-efficient manner without compromising independency or violating structural qualities?

Research has already been initiated within this field at Center for Network Planning, Aalborg University, and until now one publication has been produced, dealing with the 4-regular grid structure[RPM05]. The aim of this research is to create semi-automatic and in the long term full-automatic tools for embedding these structures onto potential physical ducts such as the road network. It is the plan to test the embedding strategies and tools by using case studies, and to test the results on dedicated test networks such at that described in Section 14.4.

The ongoing research at Center for Network Planning also deals with $N2R$ structures, and development of tools for embedding these. In Chapter 12 it was proposed to implement the $N2R$ structures by combining wired and wireless technologies. Another approach is also being studied, namely that of implementing the inner ring in one duct; it is cheaper and more practical, but also reduces the robustness[JPP05]. The main focus of this research so far, is to study how the embeddings affect the reliability. This also brings other questions into play: in particular, $q$ may be chosen differently in order to increase the robustness even if this increases the distances, or the $N2R(p; q; r)$ structures may be used for the implementations.

When I began working on this field in 2002, the assumption was that due to the implementation of Fiber To The Home, there is a unique opportunity to design new communication infrastructures from scratch, without having to take existing ones into account. Since the implementation of Fiber To The Home has now taken off, and since many of these networks are designed without global structural qualities in mind, it will sooner or later be beneficial to have methods for transforming existing networks into well-ordered structures.

A different aspect of the latter deals with gradual or stepwise implementations. As mentioned in Section 14.4, a complete network structure cannot be implemented in a single step. It is therefore important to be able to make a stepwise implementation, with good, predictable, and describable SQoS-properties even in the intermediate steps. In order to support this, schemes for gradual implementations and definitions of intermediate steps should be devised for each class of structures. Preferably, this should be done with sufficient degrees of freedom, such that the implementations can adapt to the specific physical conditions and constraints that apply in concrete network planning tasks.

## 14.6 SQoS in wireless and combined wired/wireless networks

At this stage, SQoS has been discussed with a major focus on wired MANs and access networks, in particular Fiber To The Home. With the increasing use of multi service networks, and with the increasing focus on ubiquitous computing and mobility, the demands for reliability, predictability, and classical QoS properties are likely to increase significantly also in the wireless area. For these reasons, SQoS should also be considered in wireless networks, and in combined wired and wireless networks.

In order to do so, a number of differences between wired and wireless networks must be taken into account. One of the main constraints in wired networks is the limited connectivity, with nodes of relatively low degree. In wireless networks the situation is the opposite, with potentially very high levels of connectivity. Problems arising from mobility, such as hand-over problems, form another group of problems, which occur only in wireless networks.

Wired and wireless networks can be combined in many ways. Designing combined wired and wireless network infrastructures and topologies, as proposed and discussed in Chapter 12, is one example. Another well-known combination is found in the cell phone systems, where the cell phone connects wirelessly to base stations, which are then connected by some wired networks. The future is expected to bring many new combinations of wired and wireless networks, where, in addition to hand-over problems, there is a need for ensuring SQoS through both wired and wireless parts of the network.

A first important step in bringing SQoS into wired and combined networks, is to extend the definitions and abstractions in order to handle the different characteristics of wired and wireless lines. It is also crucial to develop methods to support improved schemes for hand-over and other mobility related problems.

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