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Discussion about the role of teacher authority when making a transition into creative problem-solving in mathematics

Abstract

When introducing a new kind of teaching activity into the classroom, the teacher cannot foresee whether the students will accept it or not. Even when the students do, there is a risk, that they might be neither able nor willing to adopt all aspects of it. The teacher, then, faces a dilemma between, on the one hand, trying to win the students over to this new activity, and on the other hand, exerting an authority that may overrule the students' own initiatives and ideas and which includes the potential danger of "losing" the students. In this article, we present a retrospect re-analysis of an older case where the teacher faces such a dilemma. After analysing video recordings and interviews with the teacher through the lenses of authority, we discuss how the teacher handled this dilemma. Here we take a participationist's perspective and apply theories on authority and positioning. From this perspective, we argue that the teacher's choice of not clearly exerting his authority leaves the students in doubt about whether they fulfilled the requests. We conclude that in the present case a "too nice" atmosphere in the Norwegian classroom had obstructed the teacher's desire to implement a profound change. We come to the general conclusion that teachers need to feel comfortable with being assertive and being willing to exercise authority for the implementation of a new teaching activity, including that of creative problem-solving.

Keywords: classroom discourse, upper secondary education, teacher's authority, participationist perspective, creative problem-solving

Diskussion af lærerens rolle som en autoritet når der foretages en overgang til kreativ problemløsning i matematik

Sammendrag

Når en lærer introducerer en ny undervisningsform i klasseværelset, kan læreren ikke forudse, om eleverne vil acceptere denne. Selv hvis eleverne accepterer undervisningsformen, er der en risiko for, at de ikke vil være villige til, eller i stand til, at antage alle dele af den. Læreren vil i sådanne tilfælde stå i et dilemma – på den ene side, skal han prøve at overtale og vinde eleverne til at acceptere dette nye – eller skal han anvende

sin autoritet der kan tilsidesætte elevernes egne initiativer og ideer, hvorved han risikerer, at "tabe" eleverne. I denne artikel præsenterer vi en retrospektiv analyse af en ældre case, hvor en lærer netop står i et sådant dilemma, og vi diskuterer og analyserer dette ud fra et participationistperspektiv, hvor vi blandt andet anvender teorier om autoritet og positionering. Vi argumenterer for, at lærerens valg om ikke at udøve sin autoritet, efterlader eleverne i tvivl om, hvorvidt de har opfyldt kravene. Vi konkluderer, at det er nødvendigt, at læreren er tryk ved at være insisterende og villig til at udøve autoritet for at kunne implementere en ny form for undervisningsaktivitet, inklusiv kreativ problemløsning. Desuden konkluderer vi, at en for hyggelig atmosfære i det pågældende norske klasserum stod i vejen for lærerens intention om at implementere en gennemgribende forandring.

Nøgleord: klasserumsdiskurs, gymnasieuddannelse, lærerautoritet, participationistperspektiv, kreativ problemløsning

Introduction

This conceptual article applies and discusses the concept of authority as a lens for analysis. We study the classical dilemma in the interplay between the teacher's role as an authority and the students' envisioned creativity unfolding in a problem-solving setting. In the context of problem-solving, creativity would often be perceived as opposed to authority as it would seem, that the students' creativity should unfold better without restrictions imposed by an authoritarian teacher. On the other hand, following the Discursive Approach to mathematics education by Sierpiska (2005), the teacher's role in classroom conversations is characterised by an obligation to lead the discussion in the direction of relevant mathematical ideas, themes, and issues. Here, authority is perceived as a means for deliberately leading the students in a specific direction rather than as a means for setting up restrictions. This article advocates that the teacher's understanding and enactment of authority is particularly crucial for a successful change of well-established rules and routines in the classroom.

Our study's case took place in a Norwegian upper secondary mathematics classroom in 2013. Basically, the dilemma is between a student-centred Western school culture based on democratic values and critical thinking in academia, and (myths about) old-fashioned authoritarian, teacher-centred school teaching. The conflict still exists as an unsolved issue reflected in literature, for example on the teacher's role in problem- and inquiry-based teaching between providing instructions and being a facilitator (e.g., Abou-Hayt et al., 2020). Hmelo-Silver et al. (2007) discuss scaffolding as a pedagogical enterprise, not to be confused with self-learning. Alrø and Skovsmose (2002) proposed a framework for dialogical features, amongst which "the process of identification will provide a resource for further inquiry" (p. 62) through the justification and crystallisation of mathematical ideas.

In the chapter about “Authority and Mathematics Education” in the recent Encyclopedia of Mathematics Education, Fried (2020) refers to Max Weber’s (1947) definition of authority, where it is described as the probability that a group will obey a command. Also, according to Fried (p. 70): “Because of its pervasiveness and dominance, teachers’ authority can conflict with modes of teaching and learning which mathematics education has come to value. Such a conflict arises naturally between teachers’ authority and democratic values. This was studied by Renuka Vithal [...] What was important for Vithal was that the teachers’ authority, although opposed to democracy, could actually live with democracy in a relationship of complementarity”.

Matusov and Marjanovic-Shane discuss authority through the lens of ‘classroom management’ with negative connotations, for example: “some educators criticized classroom management not because it is inefficient, but because it is efficient!” (Matusov & Marjanovic-Shane, 2019, p. 24). Others see classroom management as a means “to create a learning environment that encourages positive social interaction, active engagement in learning, and self-motivation (Burden, 2003, p. 3), without linking it directly to authority. By Burden (2003), classroom management is not seen as opposed to active engagement in learning, these are rather perceived to be joint forces. Recent literature focuses on distribution of authority in the classroom like in Langer-Osuna (2016, 2017), in Langer-Osuna et al. (2020) and in Andresen and Dahl (2018, 2020).

In this article, we wish to nuance and contextualise teacher authority as a lens for analysis including the conflict or not between authority and democracy. Our aim is to renew the discussion about the guiding role of the teacher in a problem-solving and inquiry-based setting. We take a discursive approach and discuss how teacher authority can be linked with the facilitating and support of students’ creativity. Our discussion is based on a retrospective analysis of data from a Norwegian research project carried out in 2013–2015. The project, which focused on change into a problem-solving working style, is further described in this article’s methodology section. The discussion is illustrated by excerpts from classroom observations and interviews with the teacher chosen from the old study. Although data, hence, does not stem from a recent study, our analysis contributes to the current development of understanding and roles of the notion of authority by linking the theoretical perspectives on authority with the teacher’s view on authority as well as with interactions in his classroom.

The research question discussed in this article is:

What role can the teacher’s view on, and enactment of, authority play for the implementation of new teaching activities focusing on support for students’ development of creativity and independency?

Theoretical framework

Discursive approach and collective learning

The study takes a discursive approach to mathematics education and sees learning from a participationist's perspective. The basis is the discursive approach expressed by Forman and McCormick (1995), with collective learning in a participationist's perspective on mathematics teaching and learning. Discourse was here understood as sets of linguistic material that are coherent in organisation and content and enable people to construct meaning in social contexts (Cohen et al., 2000, p. 298). The definitions were adapted from Sfard (2008), with 'discourse' meaning the "different types of communication, and thus of commognition, that draw some individuals together while excluding some others" (p. 91), and 'participationist' denoting "all those theoretical approaches in which learning is conceptualized as participation in classroom discourses" (p. 78). This means that learning is conceptualised as not only being in the heads of people. Lerman (1994) argued for a shift in focus from the individual's 'understanding' to the social nature of thought, knowledge-creation, and learning. In Lerman (2002), the principles of a cultural, discursive psychology were outlined and operationalised as tools of research, including a) Positioning and voice in classroom mathematical practices, and b) Development as a process of thinking/speaking mathematics. Lerman (2002) outlines the principles of a cultural, discursive psychology, where learning is seen as an initiation into the practices of school mathematics, including learning to speak mathematically. The teacher therefore has a vital role in showing what is approved within the discourse, that is, the accountability to the discipline. Furthermore: "In the mathematics classroom, interactions should not be seen as windows on the mind but as discursive contributions that may pull others forward into their increasing participation in mathematical speaking/thinking, in their zones of proximal development" (Lerman, 2002, p. 89).

Thus, in a cultural, discursive psychological view, the students' utterances should not be interpreted in terms of their grasping or understanding certain concepts, explanations, or relations, but rather the answers should be interpreted as acts of participation. This is in line with Sfard's (2008) view of learning as a combination of acquisition and participation, and in line with the Discursive Approach to mathematics education by Sierpinska (2005), who sees the learner of mathematics as an apprentice of mathematical discourse and the teacher as a representative participant of a mathematical culture, who is supposed to create conditions for the initiation of students into this culture. The discursive approach rejects the classical sender–receiver model. It focuses not on transmission of information from one individual to another, but on the participation in an activity of sharing communalities and constructively dealing with the meanings people seem to have in common. Cobb et al. (2010) write about collective learning in terms of the evolution of classroom mathematical practices, with emphasis on intention and meaning. This view contrasts with an approach that treats mathe-

mathematical learning as solely a process of coming to use conventional tools and symbols in socially accepted ways. Further, it goes beyond the barely observed social use, by inferring the taken-as-shared intensions and meanings established by the classroom (social perspective) and the interpretations that individual students make as they participate in communal practices (psychological perspective). These ideas of discursive approach and collective learning in Cobb et al. (2010), Lerman (2002), and Sierpinska (2005) framed our retrospect study of the old case.

The role of the teacher for the students' learning

Gravemeijer argues (2004, p. 126) for “the proactive role of the teacher in establishing an appropriate classroom culture, in choosing and introducing instructional tasks, organising group work, framing topics for discussion, and orchestrating discussion”. Furthermore, Stein et al. (2008, p. 320) emphasise the importance of “using student-developed work as the launching point of whole-class discussions in which the teacher actively shapes the ideas that students produce to lead them to more powerful, efficient, and accurate mathematical thinking”. This is in line with Sfard’s description of the participationist approach as one that “views all the uniquely human capacities as resulting from the fundamental fact that humans are social beings, engaged in collective activities from the day they are born and throughout their lives” (Sfard, 2008, p. 79). Sfard here argues for a commognitive perspective where transformation in discourses is a sign of growth in the complexity of communication, and “the word development refers to discourses, and it encompasses both historical change and individual learning” (Sfard, 2008, p. 276). This means that when learning mathematics, students modify their discourse towards the required properties practised by the academic mathematics community.

Hence, the teacher plays a vital role for the students’ learning including a role for all types of social interactions in the classroom and for the construction of social norms (Cobb et al., 2010) that include the mutual expectations between teacher and students during such interactions. When changing into a new kind of teaching activity, such norms need to be re-established or changed to fit the new activity. Here, the teacher has a key role of not only leading the discussion in a relevant direction, but also in evolving the mathematical practices. Wubbels et al. (2013) argue that teachers’ behaviour can be considered a form of communication and that one cannot NOT communicate whenever one is in company with others. Students will interpret meaning not only from what the teachers say, but from the whole behaviour, including what the teachers do not say.

The teacher’s authority

The teacher–student relationship is structurally an asymmetric relationship due to the differences in power. The patterns of interaction can and will evolve, to for instance make the differences either smaller or greater, hence making the students

begin to behave less or more immaturely, independently, etc. In line with this, based on their research review, Walshaw and Anthony (2008, p. 541) stress that “the nature of discourse in the classroom is not a dialogue between equals, no matter how equitable the goals of classroom community might be and no matter how skilful the teacher is at exploiting and scaffolding the nature of discourse so that knowledge appears to be coconstructed”.

Amit and Fried (2005), referring to Max Weber’s definition (in Amit & Fried, 2005, p. 147), state the following: “A relation of authority exists when one person (or group of people) tends to obey, act on, or accept without question the statements or commands of another person (or group of people...)”. Weberian authority is divided into i) *Legal authority*, based on an established impersonal order and responded to by obedience, ii) *Traditional authority*, based on sanctity or ancient foundations and responded to by loyalty, and iii) *Charismatic authority*, based on an order characterised by its unavailability to ordinary human beings and responded to by devotion (Amit & Fried, 2005, p. 148). As one part of legal authority, Amit and Fried embed expert authority in the mathematics classroom – this is the authority of those who ‘know their subject’. Further, Amit and Fried (2005, pp. 162–163) discuss the question whether intellectual partnership must be opposed to authority, based on their study of students relying on a web of authority relations with friends and family members as well as with the teacher. In the discussion they introduce a revised view of authority in educational settings in the form of a non-localised shared authority. As representative of the revised view of authority they refer to Benne’s idea of ‘anthropogical’ authority. The term anthropogical is coined by Benne as an alternative to pedagogy to stress the need of all human beings at all chronological ages to be re-educated. According to Amit and Fried (2005), anthropogical authority is the third kind of authority discussed by Benne, besides expert authority and rules authority. They quote Benne (Amit & Fried, 2005, p. 164): “The ultimate bearer of educational authority is a community life in which its subjects are seeking fuller and more valid membership. Actual bearers and subjects of this authority must together build a proximate set of mutual relationships in which the aim is the development of skills, knowledge, values, and commitments which will enable the subjects to function more fully and adequately as participants in a wider community life which lies beyond the proximate educational associations”. Furthermore, they state that anthropogical authority addresses the problem that relationships of authority interfere with students’ ability to reflect for themselves and participate in the construction of mathematical ideas. Amit and Fried (2005) conclude that authority interferes this way only when it is imposed from the outside as a case of simple expert authority. Anthropogical authority, in contrast, blurs the division between agent and subject, and shifts emphasis from domination and obedience to negotiation and consent.

Herbel-Eisenmann and Wagner (2010) describe positioning as the ways in which people use action and speech to arrange social structures and stress that

authority and positioning are significant features of all mathematics teaching. As a basis for textual analysis of their data in the form of *stance bundles*, they argue for language patterns linked with four categories of authority: i) *Personal authority* ('I want you to', etc.), ii) *Demands of the discourse as authority* ('we have to', etc.), iii) *More subtle discursive authority* ('you are going to', etc.), and iv) *Personal latitude* ('if you want to', etc.). Herbel-Eisenmann and Wagner (2010) relate the stance bundles to interpersonal positioning as a means for encoding authority of the teacher and discipline, obligations to the teacher and to the discipline, and suggested choices. They stress the importance of being aware of authority structures in mathematics classrooms and reflect about possibilities of promoting students' choice in thoughtful ways. Further, Herbel-Eisenmann and Wagner (2010, p. 61) state: "We are not saying that teachers should release their authority in their classrooms completely. [...] Teachers need to use their authority to exercise their responsibilities for both social and mathematical outcomes." Wagner and Herbel-Eisenmann (2014) elaborate on the four types of authority: The interpersonal positioning suggested by i) *personal authority* language patterns had the sense of the teacher acting as a guide to students. Teachers are placed in a position of responsibility in the classroom and thus direct what happens there. The language patterns in ii) *discourse as authority* suggest that people in general must follow certain rules. The third type was renamed iii) *discursive inevitability* by Wagner and Herbel-Eisenmann (2014). With this structure, there is no explicit reference to obligation, but rather a sense of determination. Like with the previous structure, the authority would seem to rest outside of the context, with no explicit reference to authority. The fourth type, iv) *personal latitude*, is related with human agency as opposed to disciplinary agency. Most of the cases in Herbel-Eisenmann and Wagner (2010) were teacher agency. According to Wagner and Herbel-Eisenmann (2014) the distinction between personal authority and disciplinary authority can be read in the theorisation of positioning theory, particularly in the distinction between *transcendent* and *immanent* factors in social arrangement. The discipline of mathematics is transcendent or outside the experience and choices of people participating in classroom discourse. This transcendence is evident in ii) *discourse as authority* and iii) *discursive inevitability*, whereas i) *personal authority* and iv) *personal latitude* identify authorities within the classroom.

This framework is congruous with our previous work (Andresen & Dahl, 2018, 2020) where we study classroom dialogues from a collective and discursive approach. The present article is based on the same research project but with a different focus and using different data from another teacher with other students. Its framework, though, was expanded and substantiated by the inclusion of ideas about authority (Amit & Fried, 2005; Wagner & Herbel-Eisenmann, 2014; Walshaw & Anthony, 2008).

Methodology

The data came from video recordings from an EU research project (KeyCoMath) that had focused on developing students' strategies for creative problem-solving (Andresen, 2015, 2018). The aim of the EU project was to develop and study teaching that encourages students' activity, inquiry, and intellectual autonomy. The participating teachers all had several years of professional experience and volunteered to develop exploratory and engaging mathematics teaching for the research project. The teachers developed the teaching sequences and tasks themselves. Besides video recordings of Norwegian upper secondary classrooms, data included background materials from the EU project, the researcher's final, personal evaluation interview with the teacher and two group interviews with all the project's eight participating teachers.

This study's recordings (30 hours in total) were done in the autumn of 2013 by the first author. The sequences of the teaching experiment analysed and discussed in this article included four teaching phases: i) The introductory activities, ii) the launching of the main sequence, iii) the running of it, and iv) the conclusion and evaluation in the classroom. Originally, the video recordings were made with the purpose to study the strategies developed by the students under their group work during the novel teaching sequence. In this study, data from the classroom was (re)analysed qualitatively by meaning condensation of oral classroom interactions and interpretation of acts of interaction between the male teacher (pseudonym 'Harald') and the students (Kvale, 1996). During the re-analysis we interpreted the interactions in terms of authority in accordance with the expanded framework described above.

The aim of the re-analysis of the video recordings and students' reports was to identify characteristics (in terms of authority) of the social relations and norms of interaction between Harald and the students and thereby, potentially, to throw light on the apparent discrepancy between the students' willingness to follow the instructions, and Harald's impression of a poor outcome expressed in the interviews. The interviews were analysed qualitatively (Kvale, 1996) with the aim to interpret, in terms of authority, data about how Harald perceived the class, about his prior expectations regarding the novel teaching sequences, and the summary of his evaluation.

It is important to notice that our re-analysis focused on the teacher's use of different kinds of authority. The use of ten years old data is justified, in our opinion, by the fact that the study has the character of a conceptual discussion concerning teacher's types of authority in the context of implementing creative problem-solving in mathematics classrooms: The re-analysis was intended to nuance and contextualise teachers' authority by applying it as a lens for analysis, rather than to create new evidence for the results and outcome of the teaching sessions.

Data and analysis

The teacher Harald in our study introduced creative problem-solving to his class as a novel teaching activity. The aim was to introduce a change in his class which implied (i) more independent and problem-solving ways of working, (ii) a less leading teacher role, (iii) more student autonomy, and (iv) new learning objectives focusing on problem-solving that aligns with Polya (1988), which was part of the goal of the entire project. The sequence included two introductory group work activities on separate problems and subsequently launched the main problem-solving sequence.

During this novel type of teaching, Harald had to weigh and balance two opposing considerations: On the one hand, the students were used to accept and follow the teacher's instructions and to work in a cosy atmosphere having him as the forthcoming and helpful guide. On the other hand, the development towards independency would request students' own initiatives, persistence, and self-confidence. The balance, and dilemma, between these two also had to account for the risk that the class would reject the new type of activities where the students were left on their own. Or, alternatively, that they would accept it, but their independency would obstruct the teacher's leading them into the desired direction. We will discuss this dilemma from the perspective of the teacher's implicit view (in our interpretation) on and enactment of the four types of authority in Wagner and Herbel-Eisenmann (2014).

The Teaching Sequences

The entire sequence was divided into four sub-sections of smaller sequences: i) two introductory group works, ii) the teacher's introduction of the main project, iii) students' group work, and iv) the groups' presentations of their work. Each group submitted a written report to the teacher in advance of their presentation; the reports were close to identical with the PowerPoint presentations. The following excerpts serve to illustrate Harald's mediation of his own expectations and demands to the class, and, how the students responded in each of the four sub-sections.

i) Introductory group work. The complete sub-section lasted 120 minutes. After Harald's brief outline of the essentials of the novel teaching sequence, the students watched a video of two cylinders made by rolling an A4 formatted piece of paper along the short and the long side, respectively. The cylinders in the video got filled up with popcorn (see Figure 1, top left), and Harald asked the students to formulate a relevant problem to solve. Comparing the volume of the two cylinders was the most frequent question among the students. The students spent around 45 minutes on the group work including the time Harald spent on summing up the results on the blackboard. During the group work, the teacher served popcorn to the students.

It appeared to the researchers, based primarily on the concept of *personal authority* by Wagner and Herbel-Eisenmann (2014), that Harald had created an expectant, nice and cosy atmosphere in the classroom where the students immediately engaged in solving the problems. Figure 1 illustrates the informal atmosphere with students feeling free to use the board. During his summary, Harald stressed the variation of results from the measurements and the calculated volumes in a forthcoming tone as if he wanted to praise a diversity of views.

Figure 1. Harald and the students



Next, Harald gave a short lecture (about 8 minutes) on problem-solving, referring explicitly to Polya (1988) and linking to a discussion about the radius a few minutes before. He introduced Polya's four-step process (understand the problem, devise a plan, carry out the plan – solve, look back), but he did not go into details with its *questions to ask* and its *details of (sub-)strategies*. This short lecture resembled normal teaching in the class, and in our interpretation the teacher during the lecture leaned on the *discourse as authority* by Wagner and Herbel-Eisenmann (2014).

After this introduction, the students were introduced to the second group work which again was initiated by a video. This time the problem concerned a box with some water in it. The shape of the box was irregular, but its sides were polygons. The box was turned around and the new problem was to determine the height of the water level after the box had been turned around. To get started, Harald asked the students to make a guess at the new water level, and they voted about the position. In our interpretation, the driving authority in this situation was more like the *discursive inevitability* by Wagner and Herbel-Eisenmann (2014). We based this on the video's presentation of a complex and opaque physical problem which had to be solved by the students.

Like for the first problem, the focus was on the relations between area and volume, but the second problem called for a different strategy. The second task was more advanced and not all students succeeded in solving it. But after the group work, they all had a correct estimation of the water level. The following excerpt illustrates the interaction between Harald and one of the groups. In the excerpt, the students are getting stuck and need help from Harald. Apparently, they asked for help in a way typical for their usual interaction. In the excerpt below, H refers to Harald. It was unfortunately not possible to clearly distinguish the students from each other, hence S does not refer to one particular student:

- H: Just continue with what you are doing
 S: We made a sketch like this
 H: What do you plan to do now? Any ideas? It [the box] is turned around ...
 H: You did some calculations here [points to a prior sketch]
 S: Yes [points]
 H: So, this is the area
 S: Should we count them ... So, there is a line, somehow
 H: Why is there a line here? Or, we could start, ... maybe, it is not here we might start a little too low
 S: [Points to the drawing] here is 1 cm, and then here is 1 cm ...
 H: I mean, one here and one here, what kind of a figure is this?
 S: It must be the triangle
 H: Rectangle
 S: Rectangle here, yes, then we have 231 yes [sitting with the calculator, they point to the drawing, a little pointless]
 H: I think it is a good idea to use this [points to the upper part of the box, on the drawing]

The first part of the excerpt illustrates *personal latitude* (Wagner and Herbel-Eisenmann, 2014), identified during textual analysis by wordings like “what do you plan”, “should we count”, and “we could start”. When Harald takes over, the authority lies in the discipline, that is, in the *discursive inevitability*. Finally, Harald enacts *personal authority* when he gives the hint “I think it is a good idea (...)”.

Apparently, the students in this group neither had an idea about how to start nor a strategy for setting up a problem to solve. When Harald mentioned the area of the figure on their drawing, they took it as a hint and focused on determining the area. In our interpretation, this excerpt illustrates Harald’s kind of ‘paradigmatic enacting’ the *personal latitude* when asking questions to stimulate the students’ generation of ideas: The question “Why is there a line here?” could lead to considerations about the constant volume and the variable height, but Harald changed his mind and talked about the water level. Next, the question “What kind of figure is this?” could lead to recognition of a shape which allowed the students to use a formula for calculation of the area. In our interpretation Harald seeks to trigger the students’ taking over and elaborating on his own questions. This

attempt to trigger students' taking over is what we understand as paradigmatic enacting the personal latitude.

The students then tried to find a way to determine the area of the figure with a sloping side and two right angles and decide to count the squares on the (dimensional) drawing.

- H: You may count the squares, but you must take into account that not all of them are full, then, [points to the sloping side]
 S: [Laughing, pointing to a small piece of a square on the drawing] No ...
 H: You must try this and that
 S: [Looks back in the notebook, looking at the first task with rolling the sheet]
 H: What are you looking at now?
 S: [Mumbles]
 H: Good if you have any ideas, is it not?

The students had succeeded in calculation of the rectangular part of the figure and Harald recommended them to continue with the upper part of the box. They decided to count the squares and Harald's warning that they must take the small ones into account, may be a combined warning and implicit critique of this method. By the comment "You must try this and that", Harald stressed that they were free to decide on their own and, in our interpretation, served to claim inclusiveness. This would resonate with the *personal latitude* since the authority rests on Harald's person and focuses on personal agency. The questions "What are you looking at now?" and "Good if you have any ideas, is it not?" had, in our interpretation, the aim to encourage the students to feel free to get inspired, choose and decide on their own. Thereby, Harald enacted his *personal authority* to support the students. Focus was no longer on agency but rather on the teacher encouraging the students' proceeding with the task.

In one of the other groups, one of the students argued for a strategy which had proved useful in the first task; this group worked independently of Harald's help. The transfer and application of the successful strategy from the first task would illustrate disciplinary authority, that is, the *discursive inevitability*. In parallel, the student might be enacting *personal authority*. The diversity of groups and variations in their capacity was one of the obstacles discussed in advance in the group of teachers in the project.

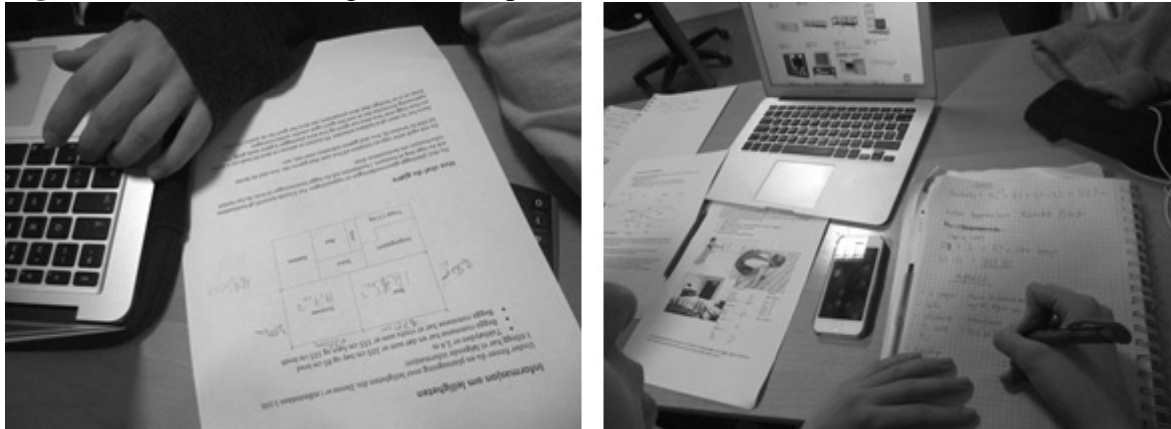
ii) The teacher's introduction of the main project. Harald's introduction to the main sequence was brief. The new problem was very open and consisted of a narrative where students should imagine getting a new flat that needed to be renovated and refurbished, and them having a certain amount of money to spend. Harald handed out a sheet with a dimensional drawing of the flat. The problem did not direct the students towards problem-solving, neither in any advanced nor in a specific mathematical meaning. In our interpretation, Harald enacted *personal authority* in the brief introduction since he gave neither arguments (disciplinary agency) nor paradigmatic examples of strategy (human agency) or solution algorithms (disciplinary agency).

iii) Students' group work. Harald had set up the groups in advance of this session and after naming which students should work together, the students immediately engaged in the group work without any protests against the project or whom they were put in group with. They started calculations of, for instance, area of the flat's walls, and searched on the web to find materials like painting, and the related costs. After a short while, the groups turned their focus of interest to refurbishment, and they searched for furniture, mirrors, posters etc. Apparently, this first part of sequence iii) resembled the normal working habits in the class. Therefore, in our interpretation, authority was not explicitly enacted here.

Harald passed around and small-talked with the groups. There were very few questions, little asking for help, and the atmosphere was vivid but focused. Harald did not interfere with the groups' work but, apparently, wanted to create a good atmosphere and he praised more than he challenged the students. In our interpretation, Harald intended not to enact any kind of authority in this part of sequence iii). This interpretation was supported by Harald's statements in the interviews (see below). In line with Wubbels et al.'s (2013) claim that it is impossible not to communicate, and applying Benne's idea of 'anthropogical' authority to 'classroom life' in which students are seeking fuller and more valid membership, though, we claim that Harald enacts authority by his actions and interactions with the groups. The students, being used to obeying Harald's personal and discursive authority, expect him to interfere and direct them during the group work when necessary for the sake of the discipline/discourse. In our interpretation, hence, Harald implicitly but with *personal authority* gives the students the impression of being 'on track' in the group work.

After a while, Harald noticed that some groups were almost finished and he decided to add a new problem, optional for the faster groups. The new problem was added in the form of a suggestion to groups that wanted to continue and improve their report. Therefore, in our interpretation, it was a case of Harald enacting, and inviting the students to enact, *personal latitude*. The new problem concerned the heating of the flat and encouraged the groups to contemplate various options regarding price, effect, and climate considerations (Figure 2). This new problem might lead to more advanced mathematical treatment like, for example, linear programming, but none of the groups took it that far.

iv) The groups' presentations of their work. The presentations were created in PowerPoint and appeared with little variation. The students were asked to present their final budget to describe how they reached the results, and to reflect about the troubles they had experienced. In general, the budgets were shown in spreadsheets followed by descriptions in the form of narratives of simple calculations with few reflections. Typical reports about troubles concern issues of finding a good offer or suitable materials on the web sites.

Figure 2. Students working on the flat problem

The teacher's perspective

Harald participated in two group interviews as a member of the EU project's teacher group. 1) The first group interview was completed before the novel teaching sequence, and 2) the second group interview was completed after finishing all the novel teaching sequences. Besides, the first author completed 3) a personal interview with Harald immediately after his last session of the novel teaching sequence. Data from these three interviews served to point out Harald's pre-expectations prior to the novel teaching sequence and to summarise his evaluation of it.

1) The first group interview. Harald was asked about his expectations to the teaching experiment. He listed his concerns about the students' willingness to engage with the task in the experimental setting and his worries about what to do if they refused to participate or became apathetic. He had not settled his mind about that at the time. He also worried about his own role in the classroom, and his obligations in and between the lessons. Harald said:

I expect the major difference to be the students' activities in the classroom but also, a huge difference in what we [the teachers] will do, and that my role in the classroom will diminish, in a way. I should, ideally, just be an observer. Now, I will probably have to go in and guide them, to some degree maybe lead them in a fruitful direction and so on, but in general in the usual teaching, your role is very prominent, hence, that will be an important difference, I think.

The other teachers responded to what Harald told. One of them said: "I, actually, experienced that I was rather busy when they [the students] worked. I had to ensure that the groups did not drop out when they got stuck." The same teacher continued talking about a teacher's obligation to help students when they are stuck not by telling what to do but by making them think on their own, ask other questions.

2) The second group interview. Harald explained that the problem was designed to let the students decide on their own what they wanted to find out and calculate. In retrospect, he saw the weakest point in the fact that the students set too simple tasks to solve; like calculating numbers of wooden slats, litres of paint

etc. They spent their creativity not on problem-solving but, rather, on refurbishing and buying secondhand furniture.

In the discussion with the whole group of teachers in the project about making changes, Harald also mentioned experiences from an earlier, similar teaching sequence in another class. At that time, the goals of the session made a crash with the class' expectations, and the students were furious.

3) The personal interview. Immediately after the last session of the experiment, Harald was disappointed and critical of some of the students' efforts with the presentations. He did acknowledge the process descriptions and reflections by a group and others, but in general, most of the students did not really get the point according to his opinion. Several of the groups did not set up and solve problems, only calculations, and they were not critical of their own solutions. For example, one group did not question their own solution with only two lamps in the flat. Harald concluded that next time he should probably poke them a little and challenge their decisions.

Discussion and conclusion

We reinterpreted an old case and studied how the concept of authority could cast new light on this situation. In this way, our article is theoretical. But, as stated above, it does not offer a theoretical discussion about problem-solving. Problem-solving was just the example of a creative student-centred activity. The goal was to become wiser on how the concept of authority could enrich the analysis and discussion. Hence our purpose was not to get new results concerning the teaching and its outcome but as a stepping stone to engage in a general discussion about authority.

Analysis in terms of authority

We have argued that Harald had established and enacted all the four types of authority by Wagner and Herbel-Eisenmann (2014) from the start of the sequences of the teaching experiment analysed and discussed in this article. Consequently, the students immediately accepted his new type of teaching activity, including his decisions about forming the groups. The results of the teaching activity in the form of students' presentations did not, according to our interpretation, reveal the desired change. This impression was supported by Harald's statements in the second and the third interview. The students' presentations revealed few, if any, reflections on the applied strategy, the problem-solving approach and the mathematical content.

There was an apparent discrepancy between, on the one hand, Harald's authority in the well-established student–teacher interactions and norms and practice in the classroom and, on the other hand, the lack of change revealed in the students' outcomes. One could argue that the students might feel, or be, unable

to follow Harald's – to them, controversial – instructions. Taking the well-established classroom relations into account, though, the students would, in this case, be more likely to ask for a clearer guidance, or maybe even to protest. In our interpretation, the students felt comfortable with the project and group work, and they were willing to do their best. In the two introductory tasks, Harald clearly stated a demand of them to be creative and not just follow well-known schemes. These demands were met to a high degree by the class, under Harald's enactment of authority as described in the previous sections. The flat problem, however, did not articulate questions involving neither mathematically demanding calculations nor explicit problem-solving. In our interpretation, the task therefore could not in itself mediate authority, neither in the form of *discourse as authority* nor in the form of *discursive inevitability*. The problems to be solved, strategy discussions and reflections in the group work were, though, understood to follow from the completely open-ended task, after Harald's brief introduction. After launching the work on the main task, Harald neither enacted *personal authority* by, for example, asking thought-provoking and challenging questions, and nor enacted authority in the form of *personal latitude* by, for example, pushing the students in their groups into more demanding situations. When Harald refrained from enacting authority, it was, according to data from the interviews, his deliberate choice. The excerpt from the second introductory problem illustrates an early stage of his choice, when he mainly asks follow-up questions to the students.

Authority as a lens for analysis

It follows from the above that framed in terms of authority, Harald's apparent choice is impossible. As already mentioned, according to Wubbels et al. (2013), students interpret meaning from the teachers' whole behaviour, including what they do not say. By not pushing the students, the teacher would implicitly acknowledge their task-solving working style and its mathematical content during the working sessions. Since Harald was not explicit here, the students, who were unfamiliar with this type of activity, had no chance of knowing whether they had completed the task satisfactorily or not. Nor did Harald explicitly enact authority during the students' presentations, since he insisted neither on reflections nor on considerations about strategy. Harald's actions are in line with the view of mathematics as a process of enculturation into mathematical practices (Erath et al., 2018). Therefore, the teacher needs to be proactive and insist on the new kind of teaching activity. Harald's interpretation of his new role as mainly an observer of the students' activity, and maybe also his worries about "sullen" students, made him reluctant to exercising authority explicitly. In the third interview, Harald reflected on his role and questioned, in our interpretation, his own (impossible) choice when talking about poking the students harder next time.

Framing the re-analysis in terms of authority, hence, leads us to point out the teacher's insistence as an important aspect of the interaction between the teacher and the students.

Further, our re-analysis can support the claim that for a teacher to implement students' learning strategies for creative and independently problem-solving skills, they need to enact authority. This claim follows from our interpretation of data as mentioned above, pointing to lack of authority as an obstacle for implementation of the desired change. Such a request of authority may challenge the myths about authority as a teacher-centred and old-fashioned way of classroom interaction. Instead we would argue, in line with for example Sierpinska (2005), that teachers have a role in actively shaping the ideas that students develop, including learning how to reflect. This is not inconsistent; following Vygotsky, students cannot be expected to invent this purely by themselves, without guidance. "The only good kind of instruction is that which marches ahead of development and leads it" (Vygotsky, 1962, p. 104). It is also seen above in the quote in Fried (2020) relating to Vithal about seeing authority and democracy as complementary. In other words, according to our view, teachers need to be comfortable being assertive and exercise their authority to implement creative problem-solving. The teacher also needs to be proactive when re-negotiating and changing their role as a teacher. This role is an integral part of the social norms of interaction, particularly when re-negotiating or establishing new norms.

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