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# Parametric Modeling and Pilot-Aided Estimation of the Wireless Multipath Channel in OFDM Systems

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**Abstract**—In this paper we present a refined model of the wireless multipath channel along with a thorough analysis on the impact of spatial smoothing techniques when used for improved channel estimation. The state-of-the-art channel estimation algorithm for pilot-aided OFDM systems is robustly designed and operates without knowledge of the time-varying multipath propagation delays in the wireless channel. However, algorithms exploiting knowledge of these time-varying delay parameters can outperform the state-of-the-art solution. We demonstrate from simulations how the Unitary ESPRIT algorithm together with spatial smoothing techniques exhibit a promising potential for multipath propagation delay estimation. Furthermore, we show that the optimum smoothing parameters depend notably on the channel model assumed, specifically in terms of the dynamical behavior of the multipath delays.

## I. INTRODUCTION

During the last decade, the technique of *orthogonal frequency-division multiplexing* (OFDM) has entered and settled within several wireless standards, e.g. European digital audio broadcasting, IEEE 802.11a wireless local area networking and 3GPP *long term evolution* (LTE). The reasons for OFDM being widely selected are manifold. A few motivations include the flexibility in spectrum occupancy, robustness against inter-symbol-interference and easy integration with multiple antenna techniques.

Today, even higher data rates are demanded - calling for larger digital constellation sizes and coherent detection. Channel estimation is therefore required and commonly achieved using pilot symbol transmissions. In principle, the channel estimation may be conducted in a completely non-parametric manner. However, this approach conflicts with the requirement of high data rates due to the dimensionality of the estimation problem and also due to the time-varying behavior of the wireless channel (expensive time-frequency overhead of pilot symbols). With the aim of lowering the dimension of the estimation task and the amount of pilot symbols needed, a parametric structure of the wireless multipath channel is typically imposed [1]–[4]. Yet, the parametric channel model assumed in scientific literature and wireless standards [5] does not adequately reflect dynamic environments, e.g. with a mobile receiver. For instance, the multipath propagation delays, the inter-delay gaps and the overall number of delays are often

modeled as persistently fixed - even though the receiver is assumed to be moving. Furthermore, it is common to include modeling of the Doppler frequency shifts experienced by the receiver [2], [4] - despite the fact that Doppler shifts and delay fluctuations are indisputably related. Hence, the default and widely used modeling of the wireless channel is counterintuitive and inadequate.

When employing the state-of-the-art channel estimator [1] (robust design), the fluctuating behavior of the multipath delays are of no importance since a continuum of equally powered channel components is assumed. However, this robust design yields an irreducible performance degradation which is avoidable if instead a channel estimator presupposing knowledge of the time-varying delays is used. Hence, if sufficiently accurate delay estimates can be obtained, the robust state-of-the-art channel estimator [1] can be outperformed. Yet, for this opposing solution to earn practical attention it requires a sufficiently accurate/realistic model of the wireless multipath channel.

In recent literature [2] the ESPRIT algorithm [6] has been proposed to serve as initial multipath delay acquisition tool for pilot-aided OFDM systems. The ESPRIT algorithm is an eigenvalue decomposition based method which exhibits satisfactory estimation performance when the multipath propagation delays in the channel model stay persistently fixed. However, in more realistic scenarios the propagation delays will fluctuate over time, the overall number of delays will change and also the inter-delay gaps will vary. Thus, depending on the individual realizations of the channel the delays will sometimes tend to cluster while other times tend to be more dispersed. Such effects are typically not captured by the channel models in use. Accordingly, promising simulation-based algorithm performance may implicitly give rise to erroneous comprehension - directly inherited from the inappropriate channel modeling.

In this paper we present an advanced multipath channel model which manages to mimic an increased amount of real-world channel effects. Compared to the default state-of-the-art channel model, this advanced model is of supplementary dynamic nature and therefore allows for interesting simulation-based comparisons. In terms of channel estimation perfor-

mance we compare the state-of-the-art algorithm [1] with the *linear minimum mean squared error* (LMMSE) estimator [2] using Unitary ESPRIT [7] as multipath delay estimation tool. Additionally, a key contribution of this paper is a thorough analysis of the performance gain obtained when applying a spatial smoothing scheme for improved delay estimation accuracy. The smoothing scheme is also employed in [2], yet no analysis of its impact is provided and no justification for the smoothing parameters are given. We investigate how to optimize the smoothing parameters depending on the dynamical behavior of the wireless multipath channel model assumed.

The remaining parts of this paper are organized as follows. In Section II a scenario involving an OFDM system is described and the signal model is presented. The channel models considered are introduced and discussed in Section III. In Section IV we briefly describe the main principles of the ESPRIT algorithm. Performance evaluations are conducted and compared in terms of Monte-Carlo simulations in Section V. Concluding remarks are provided in Section VI.

## II. OFDM SIGNAL MODEL

We consider a single-input single-output OFDM system designed with a total of  $N$  subcarriers. The effective spectrum occupied by the system is often adjusted by forcing certain subcarriers inactive, for instance at each edge of the overall bandwidth. Accordingly, only  $N_u \leq N$  subcarriers are used for actual transmissions.

The OFDM signal is generated as follows. Initially, a stream of raw information bits are modulated onto a set of PSK/QAM symbols which are then multiplexed with a sequence of  $M$  pilot symbols. After multiplexing the sequence consists of exactly  $N_u$  symbols  $x_1, x_2, \dots, x_{N_u}$ , and these are intended for transmission. Finally, OFDM modulation by means of an IFFT is performed and a cyclic prefix is inserted.

The received signal is OFDM demodulated by discarding the samples corresponding to the cyclic prefix and the  $N$  time-domain samples left are exposed to a FFT. We assume that the channel remains static during transmission of each OFDM symbol and furthermore that the duration of the cyclic prefix exceeds the maximum excess delay of the channel. The OFDM demodulated signal at the receiver is then given as

$$\mathbf{r} = [r_1, r_2, \dots, r_{N_u}]^\top = \mathbf{X}\mathbf{h} + \mathbf{w}, \quad (1)$$

where  $\mathbf{X} = \text{diag}\{x_1, x_2, \dots, x_{N_u}\}$  is a diagonal matrix built from the transmitted symbols and  $\mathbf{h} = [h_1, h_2, \dots, h_{N_u}]^\top$  contains as components the channel frequency responses at the  $N_u$  active subcarriers. Circular symmetric additive white Gaussian noise contributions with variance  $\sigma^2$  are contained in the vector  $\mathbf{w} = [w_1, w_2, \dots, w_{N_u}]^\top$ .

### A. Pilot Symbol Observations

The received pilot symbol observations are used to estimate the channel frequency response at all subchannels carrying non-redundant data symbols. Conveniently, we define the following subset of indices

$$\mathcal{P} := \{p(1), p(2), \dots, p(M)\} \subset \{1, 2, \dots, N_u\},$$

which identifies the  $M$  subcarriers used for pilot symbol transmissions. We extract the  $M$  equations from (1) corresponding to the indices contained in  $\mathcal{P}$  and define

$$y_m := \frac{r_{p(m)}}{x_{p(m)}}, \quad m = 1, 2, \dots, M,$$

which we can appropriately and compactly formulate as

$$\mathbf{y} := (\mathbf{X}_{\mathcal{P}})^{-1} \mathbf{r}_{\mathcal{P}} = \mathbf{h}_{\mathcal{P}} + (\mathbf{X}_{\mathcal{P}})^{-1} \mathbf{w}_{\mathcal{P}}, \quad (2)$$

meanwhile the subscript notation should be obvious to interpret. We assume that all pilot symbols hold unit power, whereby the statistics of the noise term  $(\mathbf{X}_{\mathcal{P}})^{-1} \mathbf{w}_{\mathcal{P}}$  remains unchanged. Hence, the observations available in (2) are known to the receiver due to the pilot symbol data and  $\mathbf{y}$  yields the true channel frequency responses (at the pilot subcarriers) embedded in zero-mean complex Gaussian noise. To properly estimate the channel frequency responses at *all* active subcarriers, i.e. the vector  $\mathbf{h}$  in (1), a parametric model of the wireless channel is invoked. In this way the dimension is notably reduced since the task is now altered to estimate only a relatively small number of channel model parameters.

## III. MULTIPATH CHANNEL MODELS

Two different multipath channels are presented in this section. The overall model for these two channels is the same and the first configuration described is simpler but unrealistic with respect to certain physical interpretations. The second configuration described is more dynamic and sophisticated while easier to accept from a physical point of view. In the entire paper we assume a non-line-of-sight, far-field scenario where *only* the receiver is moving.

The model commonly used to describe a time-varying multipath channel impulse response is given by

$$g(t, \tau) = \sum_{\ell=1}^{L(t)} \alpha_{\ell}(t) \delta(\tau - \tau_{\ell}(t)), \quad (3)$$

where  $\delta$  is the Dirac delta. Each complex-valued amplitude  $\alpha_{\ell}$ ,  $\ell = 1, 2, \dots, L(t)$ , is typically modeled as a wide-sense stationary, zero-mean complex Gaussian process [1]–[4]. The processes  $\{\alpha_{\ell}\}$  are furthermore assumed to be mutually uncorrelated, i.e. the channel described by (3) is a so-called *wide-sense stationary* and *uncorrelated scattering* [8] (WSSUS) Rayleigh fading channel.

### A. Static Reference Channel

The simpler and static channel model is described according to a relaxed version of (3) reading

$$g(t, \tau) = \sum_{\ell=1}^L \alpha_{\ell}(t) \delta(\tau - \tau_{\ell}). \quad (4)$$

The overall number  $L$  of echoes in the channel is fixed and also the delay parameters  $\{\tau_{\ell}\}$  are persistently static. All amplitude processes  $\{\alpha_{\ell}\}$  are assumed to share the same normalized autocorrelation function, given in terms of the zeroth-order Bessel function of the first kind. Accordingly, the normalized

Doppler power spectrum associated with each echo is bathtub-shaped and usually referred to in terms of Clarke or Jakes, see [9, Sec. 3.2] and the references therein. Such modeling is based on the assumption of a *uniform scattering environment*, a scenario which is difficult to accept by physical means. Specifically, it is hard to imagine a propagation environment such that the transmitted signal is scattered into plenty reflections arriving uniformly from every direction, all equally delayed, and thereby combining into one of the  $L$  dominant echoes in the channel. Nonetheless, such a channel model is usually assumed, e.g. by 3GPP in [5].

### B. Dynamic Channel

A more realistic model would allow for the delay parameters to fluctuate over time as a result of receiver mobility. Also, the overall number of echoes in the channel may change from time to time due to blocking obstacles in the environment. Hence, a channel impulse response as described by (3) is appropriate and notably more realistic than the model in (4). Initially, for  $\ell = 1, 2, \dots, L(t)$ , the channel echoes are modeled as

$$\alpha_\ell(t) = \sqrt{\frac{Q_\ell}{R}} \sum_{r=1}^R \exp(j2\pi f_D \cos(\theta_{\ell,r})t + j\varphi_{\ell,r}), \quad (5)$$

where  $Q_\ell$  is the average power of the  $\ell$ 'th echo,  $f_D$  denotes the maximum Doppler frequency and  $\{\varphi_{\ell,r}\}$  are i.i.d. uniform initial phases. In contrast to the uniform scattering environment, each channel echo  $\alpha_\ell$  in (5), is (heuristically) modeled from  $R$  azimuth excited subcomponents centered around a nominal angle of arrival  $\bar{\theta}_\ell$ . Specifically, the modeling reads

$$\bar{\theta}_\ell \stackrel{\text{i.i.d.}}{\sim} \mathcal{U}(-\pi, \pi) \quad \text{and} \quad \theta_{\ell,r} | \bar{\theta}_\ell \stackrel{\text{i.i.d.}}{\sim} \text{vM}(\bar{\theta}_\ell, \kappa),$$

where the notation  $\text{vM}(\bar{\theta}_\ell, \kappa)$  refers to the *von Mises* distribution with location parameter  $\bar{\theta}_\ell$  and concentration parameter  $\kappa \geq 0$ , see [10] for details. In this setup the channel echoes do *not* share the same normalized autocorrelation function and the Doppler power spectra are therefore individual too.

Following the modeling suggestion in [11], it is convenient to let transitions of arising channel echoes occur according to a homogeneous Poisson process with rate  $\lambda_A$ . Assigning i.i.d. exponential lifetimes with mean  $1/\lambda_B$  to the echoes then results in  $L(t)$  being a Poisson distributed random variable with  $\mathbb{E}[L(t)] = \lambda_A/\lambda_B$ . For simplicity and due to our receiver mobility assumption, it is furthermore convenient to model the delay fluctuations from straight line advancements, i.e.

$$\tau_\ell(t) = \tau_{\ell,0} + \frac{f_D \cos(\bar{\theta}_\ell)}{f_c} (t - t_{\ell,0}), \quad t \geq t_{\ell,0},$$

where  $f_c$  denotes the carrier frequency of the communication system and  $t_{\ell,0}$  is the birth time of the  $\ell$ 'th echo. The distribution of the initial delays  $\{\tau_{\ell,0}\}$  can be specified as desired - a simple choice is to select the uniform distribution on an appropriate interval. The average power terms  $\{Q_\ell\}$  may then be assigned according to an exponentially decaying function (i.e. the power delay profile is specified). The straight line advancements of the multipath delays are illustrated in

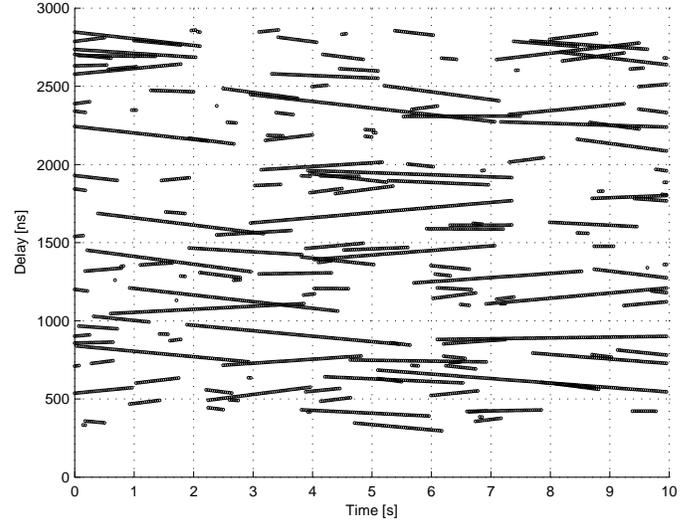


Figure 1. Contiguous realization of the dynamic channel with maximum Doppler frequency  $f_D = 100\text{Hz}$  and carrier frequency  $f_c = 2\text{GHz}$ .

Fig. 1 which reports a ten seconds realization of the dynamic channel with  $\mathbb{E}[L(t)] = 15$  delays on average. As can be seen from the figure the channel exhibits a reasonable amount of dynamical behavior, e.g. the overall number of delays is changing over time and also the straight line patterns of the delays are quite apparent.

The simpler and more static channel model described comprises the state-of-the-art reference. The intention with the more realistic and dynamic channel model described is to mimic a time-varying and fluctuating behavior of  $L(t)$ ,  $\{\tau_\ell(t)\}$  and  $\{|\tau_\ell(t) - \tau_k(t)|\}$ . Our goal is to investigate how incorporation of such dynamics affects the pilot-aided channel estimation performance.

### IV. PROPAGATION DELAY ESTIMATION

Assuming the reference channel model (4) as described in Section III-A, we reformulate the observation model (2) as

$$\mathbf{y} = \mathbf{T}(\boldsymbol{\tau})\boldsymbol{\alpha} + \mathbf{n}, \quad (6)$$

where we have introduced a  $M \times L$  matrix  $\mathbf{T}(\boldsymbol{\tau})$ , the vector  $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_L]^\top$  and the additive noise vector  $\mathbf{n}$ . The matrix  $\mathbf{T}(\boldsymbol{\tau})$  depends on the delay parameters and the pilot symbol positions in such a way that its  $(m, \ell)$ 'th entry reads

$$\mathbf{T}_{m,\ell} = \exp\left(-j2\pi \frac{p(m)}{N} \frac{\tau_\ell}{T_s}\right), \quad \begin{array}{l} m = 1, 2, \dots, M, \\ \ell = 1, 2, \dots, L, \end{array}$$

where  $T_s$  denotes the sampling time of the communication system. Notice that the  $L$  columns building up the matrix  $\mathbf{T}(\boldsymbol{\tau})$  are of identical structure and by system design the parameters  $N$ ,  $T_s$  and  $\mathcal{P}$  are known - only the delays  $\{\tau_\ell\}$  are unknown. The theoretical covariance matrix associated with  $\mathbf{y}$  reads

$$\mathbf{R} := \mathbb{E}[\mathbf{y}\mathbf{y}^H] = \mathbf{T}(\boldsymbol{\tau})\mathbf{A}\mathbf{T}^H(\boldsymbol{\tau}) + \sigma^2\mathbf{I}_M, \quad (7)$$

where we have implicitly assumed that any component of  $\boldsymbol{\alpha}$  is statistically independent of any component of  $\mathbf{n}$ . Furthermore,

$\mathbf{A} := \mathbb{E}[\boldsymbol{\alpha}\boldsymbol{\alpha}^H]$  is a  $L \times L$  diagonal matrix due to the uncorrelated scattering assumption. Notice in (7), that since the delay parameters are assumed static the covariance matrix  $\mathbf{R}$  does not change over time.

Now, any vector in the null space of  $\mathbf{T}^H(\boldsymbol{\tau})$  is an eigenvector of  $\mathbf{R}$  with associated eigenvalue  $\sigma^2$ . Therefore, the particular eigenvectors of  $\mathbf{R}$  not belonging to the null space of  $\mathbf{T}^H(\boldsymbol{\tau})$  are all associated with eigenvalues strictly greater than  $\sigma^2$ . This key fact provides insight on how the signal subspace and the noise subspace can be separated according to the individual magnitudes of the eigenvalues. From a proper design of the set  $\mathcal{P}$ , the structure inherited by the matrix  $\mathbf{T}(\boldsymbol{\tau})$  allows for two specific submatrices to be related by a simple rotational (i.e. unitary) transform. Estimation of this unitary transform is essentially how the ESPRIT algorithm is used to estimate the unknown delay parameters, see [2].

Obviously, the theoretical covariance matrix  $\mathbf{R}$  is not available. Instead the ESPRIT algorithm is applied to some ‘prudent’ estimate of the matrix. Observations which we denote by  $\{\mathbf{y}_k\}$  are collected temporally, and in a generic manner we arrange  $K$  of such vectors in the  $M \times K$  matrix

$$\mathbf{Y} := \begin{bmatrix} | & | & & | \\ \mathbf{y}_1 & \mathbf{y}_2 & \cdots & \mathbf{y}_K \\ | & | & & | \end{bmatrix}. \quad (8)$$

The estimate used could then be the sample covariance matrix

$$\hat{\mathbf{R}} := \frac{1}{K} \mathbf{Y} \mathbf{Y}^H \quad \text{or} \quad \tilde{\mathbf{R}} := \frac{1}{2} (\hat{\mathbf{R}} + \mathbf{J} \hat{\mathbf{R}}^T \mathbf{J}),$$

where  $\tilde{\mathbf{R}}$  is the centrosymmetric equivalent<sup>1</sup> of  $\hat{\mathbf{R}}$ . Here  $\mathbf{J}$  denotes the  $M \times M$  reversal matrix with 1’s in its entire anti-diagonal and 0’s elsewhere, see [12, Sec. 4.8, 6.5.8].

If instead we assume the more realistic and dynamic channel model (3) as described in Section III-B, the entire situation is crucially altered. In (6), the delay parameter  $\boldsymbol{\tau} = \boldsymbol{\tau}(t)$  is now time-variant and the basis of the underlying signal subspace is therefore changing over time (potentially, the dimension changes too, e.g. while gathering data for the matrix  $\mathbf{Y}$ ). Essentially, the rotational transform to be estimated is time-variant since the delay parameters no longer stay fixed and hence, the basic assumptions for ESPRIT are not satisfied. Yet, by considering only time frames of sufficiently short duration, the delay fluctuations can be considered negligible. Finally, to achieve improved estimation accuracy and reduced complexity we employ Unitary ESPRIT [7], not standard ESPRIT.

#### A. Spatial Smoothing

To decrease any disturbing impact from the time-varying delay parameters it seem obvious to use an observation matrix  $\mathbf{Y}$  where  $K$  is as small as possible. With  $K$  small, only a few observations are collected in the time direction and this fact complies well with the rigorous latency requirements of today’s communication systems. If the number of pilot

symbols  $M$  is relatively large and if the set  $\mathcal{P}$  is designed appropriately, we can apply a so-called spatial smoothing technique. By doing so we artificially build up more time-direction observations by suffering on overall dimension (aperture) in the frequency direction. By applying a vertical sliding window of size  $M_1 \leq M$  to the  $M \times K$  matrix in (8) we obtain a new observation array of size

$$M_1 \times K(M - M_1 + 1).$$

Notice how the attribute of wide-sense stationarity in the frequency domain (inherited from the uncorrelated scattering assumption in the delay domain) is paramount when applying the smoothing window. Obviously, the number  $M_1$  should be chosen according to a trade-off between aperture and estimation accuracy. Choosing  $M_1$  smaller generates more snapshots while is (simultaneously) penalized by poorer ability to resolve closely displaced delay parameters. Notice that with  $K = 1$  the data matrix  $\mathbf{Y}$  in (8) has unit rank and consequently  $\hat{\mathbf{R}}$  only holds a single nonzero eigenvalue. In this case we should indeed make sure that  $M - M_1 + 1$  exceeds the total number of delays in the channel - otherwise there are not enough nonzero eigenvalues for ESPRIT to process. Spatial smoothing techniques are commonly employed to decorrelate coherent signal sources, see e.g. [13] and the references therein.

#### V. PERFORMANCE EVALUATION

In this section we evaluate the pilot-assisted channel estimation performance of the LMMSE estimator from [2] using Unitary ESPRIT as delay estimation tool. For all configurations considered we evaluate uncoded *bit-error-rate* (BER) performance of the OFDM system. We investigate the impact of spatial smoothing as a function of the window size  $M_1$  and the two different channel models are treated separately. We consider a 3GPP LTE alike scenario with system parameters:

$$N = 2048, N_u = 1200, T_s = 32.55\text{ns}, M = 200.$$

The duration of the cyclic prefix is  $4.69\mu\text{s}$ , corresponding to  $144 T_s$ -samples. A total of 14 OFDM symbols are transmitted every millisecond and four of these carry  $M = 200$  pilots each. We assume the pilot symbols to be evenly positioned along the  $N_u = 1200$  active subchannels with a fixed spacing of six subcarriers, i.e.

$$\mathcal{P} = \{3, 9, 15, \dots, 597, 603, \dots, 1185, 1191, 1197\}. \quad (9)$$

The set of pilot symbol positions  $\mathcal{P}$  in (9) represents a uniform linear array of sensors with maximum overlap. The carrier frequency of the system is assumed to be  $f_c = 2\text{GHz}$  and we consider a receiver traveling at walking speed, i.e. the maximum Doppler frequency is assumed to be  $f_D = 10\text{Hz}$ . The digital modulation scheme used is QPSK (gray coded), both for data symbols and pilot symbols.

#### A. Performance in Static Reference Channel

As the static reference channel we employ the 3GPP EVA-profile from [5, Annex B.2] which constantly holds  $L = 9$  multipath echoes with fixed delays and its maximum excess

<sup>1</sup>The theoretical covariance matrix in (7) is Toeplitz when the subcarrier spacings between adjacent pilots are all identical.

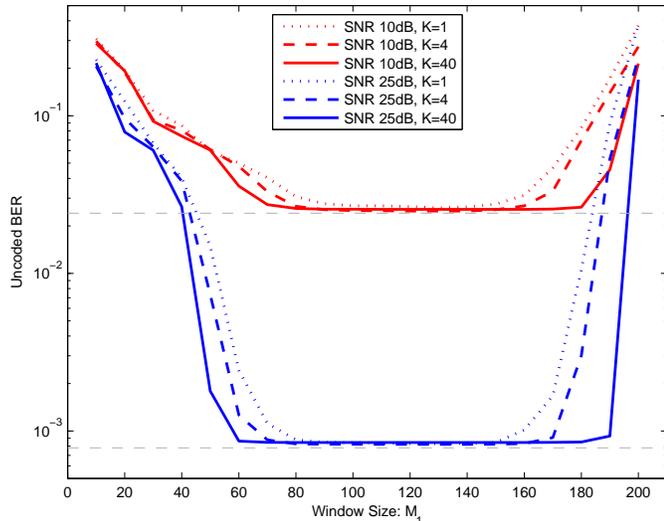


Figure 2. BER performance as a function of  $M_1$ . The two grey-dashed lines indicate the BER performance at 10dB and 25dB of *signal-to-noise ratio* (SNR) using true/known channel frequency responses.

delay is approximately half the duration of the cyclic prefix. To visualize how the window size  $M_1$  impacts the overall system performance, we consider a span from  $M_1 = 200$  towards  $M_1 = 10$ , corresponding to no smoothing and full-scale smoothing, respectively. Figure 2 reports the uncoded BER-performance of the OFDM system as a function of the window size  $M_1$ . We always feed the true number of delays (i.e.  $L = 9$ ) directly to Unitary ESPRIT, since estimation of the number of channel echoes is not an objective in this paper. In Fig. 2, it is interesting to note that a rather wide range of window sizes are leading to the same degree of performance (near to that of using known channel coefficients). Even with  $K = 1$  we realize that near-optimal performance is achievable. However, additional smoothing is required and the range of window sizes inheriting splendid performance is more tight when  $K$  is smaller. Notice also the immediate and steep performance gains obtained when  $M_1$  decreases from its maximum value  $M = 200$ . This behavior partly reflects the fact that rank is building up in the covariance matrix estimate, cf. the discussion at the end of Section IV. Finally, recall that the inter-delay gaps are persistently fixed in this scenario and hence, the resolvability issues for Unitary ESPRIT to deal with are identical/constant for *all* individual channel realizations.

### B. Performance in Dynamic Multipath Channel

With a channel inheriting additional dynamical behavior we now repeat the same simulation study as just described in the previous section. Hence, we wish to visualize the impact of the window size  $M_1$  in a scenario where the delay resolvability issue is non-constant across the individual realizations of the channel. For simulation technical reasons the dynamic channel holds fifteen echoes on average<sup>2</sup>, i.e.  $L(t)$  is Poisson

<sup>2</sup>Basically, we require  $P(L(t) = 0)$  to be negligible.

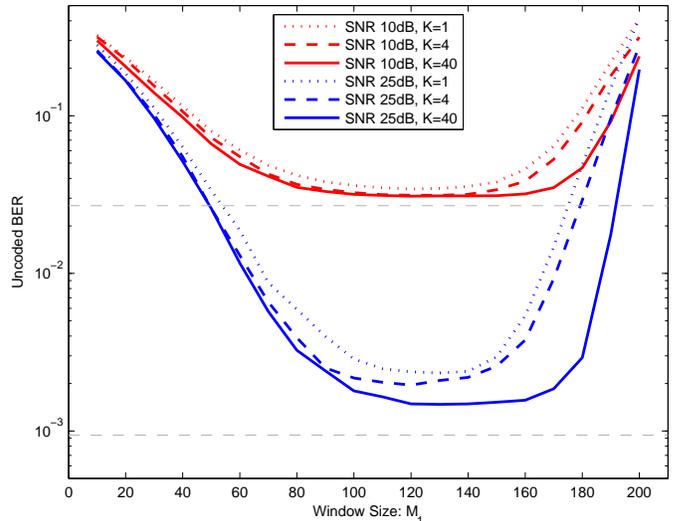


Figure 3. BER performance as a function of  $M_1$ . The two grey-dashed lines indicate the BER performance at 10dB and 25dB of SNR using true/known channel frequency responses.

distributed with mean parameter equal to 15. The maximum excess delay is the same as for the static reference channel and also the power delay profile is similar to that of the static reference channel. Since  $\mathbb{E}[L(t)] = 15$ , then roughly anything from five to twenty-five echoes can be observed in the instantaneous realizations of the channel. In some realizations the delays will tend to cluster while in others tend to be more dispersed. As before, we feed the true number of delays to Unitary ESPRIT such that it always seeks for the instantaneous amount of channel echoes. Figure 3 illustrates how the window size  $M_1$  affects the system performance in this case.

As can be readily seen from Fig. 3, the wide range of window sizes leading to the same degree of performance is not present anymore. The curves are still bathtub shaped, however, notably less steep and edged compared to Fig. 2. Also, none of the curves appear tight along the known channel bound as in the first case considered. This is jointly caused by the fact that more delays have to be estimated on average and since the instantaneous realizations of the channel sometimes trigger the delays more clustered. If for system design purposes we were to select and fix a single value of  $M_1$ , then based on Fig. 2, anything in the range from 90 to 150 would seem appropriate. Based on Fig. 3, however, the optimum value of  $M_1$  seems to appear tightly around 120.

### C. State-of-the-art Comparison

To get a full picture of the BER performance across a wide SNR-range we have fixed  $M_1 = 120$  and conducted another simulation study. We now compare the LMMSE estimator from [2] using Unitary ESPRIT against the robustly designed state-of-the-art channel estimator from [1]. Our comparison is carried out using the dynamical channel with parameters as in the previous section. Figure 4 reports the outcome, where two selected values for  $K$  are shown, namely  $K = 1$  and  $K = 40$ .

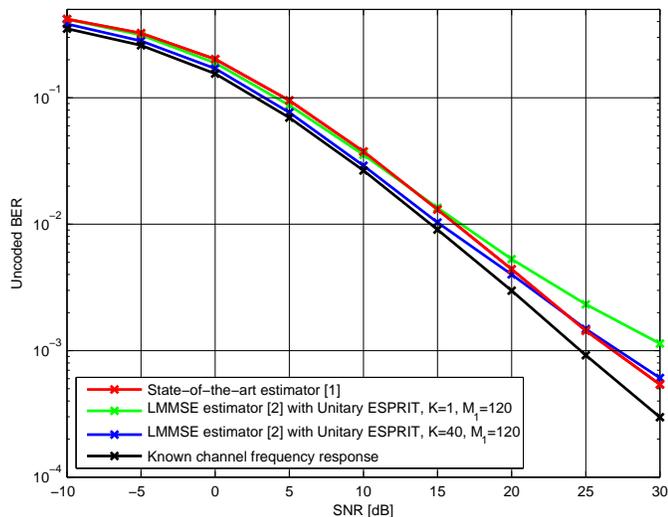


Figure 4. BER-performance as a function of average SNR.

In the SNR-range from  $-10\text{dB}$  to  $15\text{dB}$  the state-of-the-art solution is marginally outperformed with  $K = 1$ . However, when using  $K = 40$  the state-of-the-art solution is more notably outperformed and in a slightly wider SNR-range. That is, better or similar performance can be achieved using the LMMSE estimator from [2] together with Unitary ESPRIT. Yet, the state-of-the-art solution operates on lower computational complexity and this fact directly implies a need for complexity reductions in order to comparably gain the performance enhancements suggested in Fig. 4.

Notice from Fig. 2, where the static channel model was assumed, that a similar study as reported in Fig. 4 would conclude that the state-of-the-art solution could be notably outperformed in the entire SNR-range considered, even with  $K = 1$ . This follows since the BER performance in Fig. 2 with  $K = 1$  and  $M_1 = 120$  is almost as good as using known channel frequency responses, both at  $10\text{dB}$  and  $25\text{dB}$  of SNR. The point here is that the channel model selection can importantly affect the results obtained. In general, validity of the evaluated algorithm performance is achieved through adequate and comprehensive modeling.

## VI. CONCLUSION

In this paper we have considered channel estimation techniques for pilot-aided OFDM systems, where the estimation is grounded on a parametric model of the wireless channel. The multipath delay parameters in the channel model have been estimated via the Unitary ESPRIT algorithm and spatial smoothing techniques have been applied to improve the estimation accuracy. Incorporation of the delay estimates in a LMMSE estimator allows for improved performance compared to the robustly designed state-of-the-art solution. That is, the state-of-the-art channel estimator can be outperformed over a wide SNR-range. Yet, computational complexity and estimation of the instantaneous number of channel echoes remain critical issues for the opposing channel estimator investigated.

In order to provide a rigorous performance assessment of the opposing channel estimation solution, we have compared state-of-the-art channel modeling against a refined channel model of additional dynamical nature. The main additional features comprise a time-varying number of channel echoes together with fluctuating delay positions, i.e. non-constant inter-delay gaps. From simulations we have analyzed the impact of spatial smoothing techniques when used to improve the multipath delay estimation accuracy. Our results indicate that both estimation accuracy and the optimum smoothing parameters are notably affected with increased dynamical behavior of the channel model assumed.

To conclude, our work shows that the selection of appropriate channel models is crucial when assessing the performance of receiver algorithms. Choosing inadequate models may imply misleading comprehension and could therefore yield improper algorithm selection for practical applications.

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