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### Comment on ‘‘Fault tolerance analysis for stochastic systems using switching diffusion processes’’ by Yang, Jiang and Cocquempot

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## Comment on ‘Fault tolerance analysis for stochastic systems using switching diffusion processes’ by Yang, Jiang and Cocquempot

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Results are given in Yang, Jiang and Cocquempot (Yang, H., Jiang, B., and Cocquempot, V. (2009), ‘Fault Tolerance Analysis for Stochastic Systems using Switching Diffusion Processes’, *International Journal of Control*, 82, 1516–1525) regarding the overall stability of switched diffusion processes based on stability properties of separate processes combined through stochastic switching. This article argues two main results to be empty, in that the presented hypotheses are logically inconsistent.

**Keywords:** stochastic system; switching diffusion; stability

Stability results are presented in Yang, Jiang, and Cocquempot (2009) for the so-called *switching diffusion processes* (SDP). Such systems evolve in a hybrid state space, i.e. including both continuous and discrete state variable components. In the sequel we follow the notation in Yang et al. (2009). In each discrete state  $\sigma \in \mathcal{M}$  evolution of the continuous state  $x \in \mathbb{R}^n$  is governed by a diffusion process, i.e.

$$dx = f_\sigma(x, u)dt + g_\sigma(x, u)dW, \quad (1)$$

where  $W$  is a Brownian motion,  $u \in \mathbb{R}^m$  is a control and  $f_\sigma$  and  $g_\sigma$  are appropriate mappings satisfying suitable smoothness conditions to ensure unique solutions to (1).

Evolution of the discrete state  $\sigma$  is governed by a continuous-time Markov chain with an infinitesimal generator matrix  $\Gamma = \{\rho_{ij}, i, j \in \mathcal{M}\}$  modelling stochastic transition between nominal and various faulty states. As in Yang et al. (2009), we refer to (1) as an SDP.

The concept of input-to-state stability (ISS) is used in the presented analysis, where the SDP (1) is said to be ISS w.r.t. the input  $u$  iff

$$\begin{aligned} &\exists \beta \in \mathcal{KL}, \exists \alpha, \gamma \in \mathcal{K}_\infty \text{ such that} \\ &\mathbf{E}[\alpha(|x(t)|)] \leq \beta(|x(0)|, t) + \gamma(\|u\|_{[0,t]}) \quad \forall t \geq 0, \forall x(0) \in \mathbb{R}^n. \end{aligned} \quad (2)$$

In Yang et al. (2009) stochastic Lyapunov analysis is applied to obtain sufficient conditions under which the SDP given by (1) is ISS. Combining a finite number of diffusion processes through Markovian switching naturally poses the question about overall stability.

In Yang et al. (2009) three different situations are analysed: all separate processes are ISS, only some are ISS, and lastly no processes are ISS. Our comment pertains primarily to results given for the first and last cases.

We repeat the main result for the first case given in Theorem 2 of Yang et al. (2009).

**Theorem 1:** *Suppose that there exists  $\alpha_1, \alpha_2, \chi \in \mathcal{K}_\infty$ ,  $\lambda_0 > 0$ ,  $\mu > 1$  and  $V_k \in \mathcal{C}^2(\mathbb{R}^n, \mathbb{R}_+)$ ,  $k \in \mathcal{M}$ , such that*

$$\alpha_1(|x|) \leq V_q \leq \alpha_2(|x|) \quad \forall q \in \mathcal{M} \quad (3)$$

$$\mathcal{L}V_q(x(t)) \leq -\lambda_0 V_q(x(t)) + \gamma_1(|u|) \quad \forall q \in \mathcal{M} \quad (4)$$

$$V_q(x) \leq \mu V_p(x) \quad \forall p, q \in \mathcal{M} \quad (5)$$

and

$$\mu < \frac{\max_{ij}\{\rho_{ij}\}}{\max_i\{|\rho_{ii}|\}}. \quad (6)$$

Then the SDP given by (1) is ISS

In the sequel we argue that the hypothesis of the above theorem (i.e. Theorem 2 in Yang et al. (2009)) is false.

From the definitions of the generator matrix  $\Gamma$ , we get (since all  $\rho_{ii} \leq 0$  and all  $0 \leq \rho_{ij}$ ,  $i \neq j$ )

$$\begin{aligned} \max_i\{|\rho_{ii}|\} &= \max_i \sum_{j \neq i} \{\rho_{ij}\} \geq \max_i \{\max_{j \neq i} \{\rho_{ij}\}\} \\ &= \max_{i, j \neq i} \{\rho_{ij}\} = \max_{i, j} \{\rho_{ij}\}. \end{aligned} \quad (7)$$

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Hence from (6) we conclude that  $\mu < 1$ . However it is also assumed that  $\mu > 1$ . This proves that the hypothesis of Theorem 2 in Yang et al. (2009) is false.

Inspecting relations (12) and (13) in Yang et al. (2009) reveals that a less restrictive criterion for ISS may replace inequality (6), namely

$$\mu < \frac{\lambda_0 + \max_{ij}\{\rho_{ij}\}}{\max_i\{|\rho_{ii}|\}}, \quad (8)$$

which can be consistent with  $\mu > 1$ . With (8), Theorem 2 of Yang et al. (2009) becomes a straightforward generalisation of Theorem 5 in Chatterjee and Liberzon (2006) for diffusion processes.

In the accompanying interpretation in Yang et al. (2009) it is stated: ‘Roughly speaking, if each mode is ISS, and the fault occurrence transition rate  $\max_{ij}\{\rho_{ij}\}$  is large enough, then the ISS of the stochastic system is guaranteed’. We find this statement highly counter intuitive, since stability arguments for switched systems under (4) and (5) would rely on long dwell times of separate systems to ensure sufficient decay of individual Lyapunov functions in between shifts as also pointed out in Chatterjee and Liberzon (2006). Rewriting (8) into

$$\max_i\{|\rho_{ii}|\} < \frac{\lambda_0 + \max_{ij}\{\rho_{ij}\}}{\mu} \quad (9)$$

and recognising  $\max_i\{|\rho_{ii}|\}$  as the maximal transition rate out of any state, (8) calls for an interpretation opposite to Yang et al. (2009), i.e. transition rates should be low for high values of  $\mu$ .

Turning to the last situation, where no separate systems are assumed ISS, condition (4) is in Theorem 5 of Yang et al. (2009) replaced by

$$\mathcal{L}V_q(x(t)) \leq \lambda_1 V_q(x(t)) + \gamma_1(|u|) \quad (10)$$

where  $\lambda_1 > 0$  and (6) by

$$\mu < \frac{\max_{ij}\{|\rho_{ij}|\} - \lambda_1}{\max_i\{|\rho_{ii}|\}}, \quad (11)$$

which is equally inconsistent since  $\lambda_1 > 0$ .

In the accompanying interpretation it is stated: ‘Theorem 5 shows that if the fault occurrence transition rate  $\max_{ij}\rho_{ij}$  is larger than that of any previous cases (all ISS modes, partial ISS modes) then the ISS of SDP is achieved without any ISS mode. This result implies that, under the condition (11), we do not need to design the stabilising controller even if the stochastic system is not stable separately in the healthy and faulty situations’.

This statement can be met by a simple counter-example, where two identical unstable systems are combined by stochastic switching. (In this case we may choose  $V_q = V_p$ , so that  $u=1$  can be used and since there are only 2 states  $\max_{ij}\{\rho_{ij}\} = \max_i\{|\rho_{ii}|\}$ , so we are as close as possible to fulfilling (11)). However, switching between identical unstable systems does not make the overall system stable.

## References

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