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Eliminating Oscillations in TRV-Controlled Hydronic Radiators

Fatemeh Tahersima, Jakob Stoustrup and Henrik Rasmussen

***Abstract*— Thermostatic Radiator Valves (TRV) have proved their significant contribution in energy savings for several years. However, at low heat demands, an unstable oscillatory behavior is usually observed and well known for these devices. This instability is due to the nonlinear dynamics of the radiator itself which result in a large time constant and high gain for radiator at low flows. A remedy to this problem is to make the controller of TRVs adaptable with the operating point instead of widely used fixed PI controllers. To this end, we have derived a linear parameter varying model of radiator, formulated based on the operating flow rate, room temperature and the radiator specifications. In order to derive such formulation, the partial differential equation of the radiator heat transfer dynamics is solved analytically. Using the model, a gain schedule controller among various possible control strategies is designed for the TRV. It is shown via simulations that the designed controller based on the proposed LPV model performs excellent and stable in the whole operating conditions.**

I. INTRODUCTION

Efficient control of heating, ventilation and air conditioning (HVAC) systems has a great influence on the thermal comfort of residents. The other important objective is energy savings, mainly because of the growth of energy consumption, costs and also correlated environmental impacts.

Hydronic radiators controlled by thermostatic radiator valves (TRV) provide good comfort under normal operating conditions. Thermal analysis of the experimental results of a renovated villa in Denmark, built before 1950, has demonstrated that energy savings near 50% were achieved by mounting TRVs on all radiators and fortifying thermal envelope insulation [1].

A. System Description

The case study is composed of a room and a radiator with thermostatic valve. Disturbances which excite the system are ambient temperature and heat dissipated by radiator. It is assumed that heat transfer to the ground is negligible having thick layers of insulation beneath the concrete floor. Block diagram of the system is shown in Fig. 1. All of the symbols, subscripts and the parameters value are listed in table I and table II. The chosen values for all parameters are in accordance with the typical experimental and standard values [2]. As mentioned before, the case study is adopted to the one previously studied in [3].

The TRV is driven by a batterized stepper motor. Pressure drop across the radiator valve is maintained constant unlike what is taken as the control strategy in [4]. Instead, flow control is assumed to be feasible by the accurate adjustment of the valve opening. The valve opening is regulated by the stepper motor which allows the concrete adjustment.

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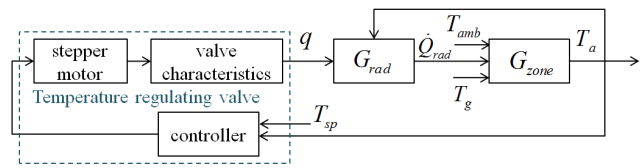

Fig. 1. Closed loop control system of room and radiator

B. Problem Definition

To maintain the temperature set point in a high load situation, TRVs are usually tuned with a high controller gain.The inefficiency appears in the seasons with low heat demand especially when the water pump or radiator are over dimensioned [5]. In this situation, due to a low flow rate, loop gain increases; and as a result oscillations in room temperature may occur. Besides discomfort, oscillations decrease the life time of the actuators. This problem is addressed in [4] for a central heating system with gas-expansion based TRVs. It is proposed to control the differential pressure across the TRV to keep it in a suitable operating area using an estimate of the valve position.

The dilemma between stability and performance arises when TRV is controlled by a fixed linear controller. Designing TRV controller for high demand seasonal condition, usually leads to instability in low demand weather condition. A high loop gain and long time constant are the main reasons of this phenomenon. In contrast, selecting a smaller controller gain to handle the instability situation, will result in a poor radiator reaction while the heat demand is high.

Figures 2 and 3 show the results of a simulation where oscillations and low performance occur. In the shown simulation results, the forward water temperature is at $50°\mathrm{C}$. The proportional integral (PI) controller of TRV is tuned based on Ziegler-Nichols step response method [6].

A remedy to this dilemma is choosing an adaptive controller instead of the current fixed PI controller.

It is, also, worth mentioning that the same problem was investigated via simulation based studies in [3]. The LPV control oriented model of radiator was, however, developed based on simulations.

In order to validate the controller performance, we utilized simulation models of the HVAC components. Two approaches for HVAC systems modeling are the forward, [7],

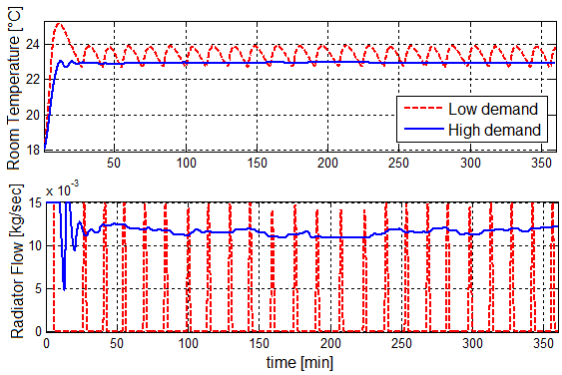

Fig. 2. Undamped oscillations in room temperature and radiator flow which occur in low demand situation while the controller is designed for high demand condition.

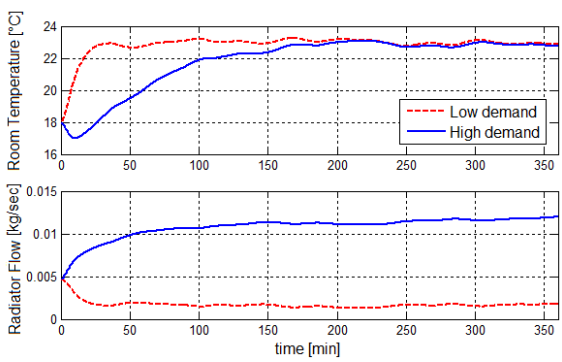

Fig. 3. Poor performance in the cold weather condition, applying the controller designed for the low demand situation

[8], [9] and the data-driven methods [10], [11], [12] indicated by [2].

In this paper, we adopted heat balance equations of the room model in accordance to the analogous electric circuit, described formerly in [13]. Radiator dynamics are formulated as a distributed system in order to analyze the radiator transferred heat.

Rest of the paper is organized as follows: In section II, the radiator transferred heat is derived analytically. Based on the result, control oriented models are developed in section III. Utilizing the models, the control structure based on flow adaptation is proposed in the same section. A simulation-based test is conducted in section IV. Discussion and conclusions are given finally in Sections V.

II. System Modeling

A. Heat Balance Equations

Radiator is modeled as a lumped system with N elements in series. The nth section temperature is given by, [14]:

$$\frac{C_{rad}}{N}\dot{T}_n = c_w q(T_{n-1} - T_n) - \frac{K_r}{N}(T_n - T_a) \tag{1}$$

in which C_{rad} is the heat capacity of the water and radiator material, T_n is the temperature of the radiator's nth element and $n = 1, 2, \ldots, N$. The temperature of the radiator ending points are inlet temperature: $T_0 = T_{in}$, and return temperature: $T_N = T_{out}$. In this formulation, we assumed the same temperature of the radiator surface as the water inside radiator. Besides, heat transfer only via convection is considered. K_r represents the radiator equivalent heat transfer coefficient which is defined based on one exponent formula, [14] in the following:

$$K_r = \frac{\Phi_0}{\Delta T_{m,0}{}^{n_1}}(T_n - T_a)^{n_1 - 1} \tag{2}$$

in which Φ_0 is the radiator nominal power in nominal condition which is $T_{in,0} = 90°C$, $T_{out,0} = 70°C$ and $T_a = 20°C$. $\Delta T_{m,0}$ expresses the mean temperature difference which is defined as $\Delta T_m = \frac{T_{in} - T_{out}}{2} - T_a$ in nominal condition. n_1 is the radiator exponent which varies between 1.2 and 1.4, but 1.3 is the value of the exponent for most radiators. In such case, we can approximate the non fixed, nonlinear term in K_r with a constant between 2.5 and 3.2 for a wide enough range of temperature values. Picking 2.8 as the approximation value, $K_r = 2.8 \times \frac{\Phi_0}{\Delta T_{m,0}{}^{1.3}}$.

The power transferred to the room can be described as:

$$Q_r = \sum_{n=1}^{N} K_r(T_n - T_a) \tag{3}$$

Heat balance equations of the room is governed by the following lumped model [9]:

$$\begin{aligned} C_e\dot{T}_e &= U_e A_e(T_{amb} - T_e) + U_e A_e(T_a - T_e) \\ C_f\dot{T}_f &= U_f A_f(T_a - T_f) \\ C_a\dot{T}_a &= U_e A_e(T_e - T_a) + U_f A_f(T_f - T_a) + Q_r \end{aligned} \tag{4}$$

in which T_e represents the envelop temperature, T_f the temperature of the concrete floor and T_a the room air temperature. Q_r is the heat power transferred to the room by radiator. Each of the envelop, floor and room air are considered as a single lump with uniform temperature distribution.

Assuming a constant pressure drop across the valve, the thermostatic valve is modeled with a static polynomial function mapping the valve opening δ to the flow rate q:

$$q = -3.4 \times 10^{-4}\delta^2 + 0.75\delta \tag{5}$$

The above presented radiator model is highly nonlinear and not suitable for design of controller; thus a simplified control oriented LPV model is developed in the next section.

B. Control Oriented Models

The relationship between room air temperature and radiator output heat can be well approximated by a 1st order transfer function.

$$\frac{T_a}{Q_{rad}}(s) = \frac{K_a}{1 + \tau_a s} \tag{6}$$

The above model parameters can be identified simply via a step response test as well.

Step response simulations and experiments confirm a first order transfer function between the radiator output heat and input flow rate at a specific operating point as

$$\frac{Q_r}{q}(s) = \frac{K_{rad}}{1 + \tau_{rad} s} \tag{7}$$

In the next section, parameters of the above model are formulated based on the closed-form solution of the radiator output heat, $Q_r(t, q, T_a)$.

C. Radiator Dynamical Analysis

In this paper, unlike [3], we found the closed-form map between the radiator heat and operating point which is corresponding flow rate q, and room temperature T_a. We, previously, derived this dependency via a simulative study in the form of two profile curves, [3].

To develop $Q(t, q, T_a)$, a step flow is applied to the radiator, i.e. changing the flow rate from q_0 to q_1, at a constant differential pressure across the valve. Propagating with the speed of sound, the flow shift is seen in a fraction of second all along the radiator. Hence, flow is regarded as a static parameter for $t > 0$, rather than temperature distribution along radiator.

Consider a small radiator section Δx with depth d and hight h as shown in Fig. 4. The temperature of incoming flow to this section is $T(x)$, while the outgoing flow is at $T(x+\Delta x)°C$. Temperature is considered to be constant $T(x)$ in a single partition.

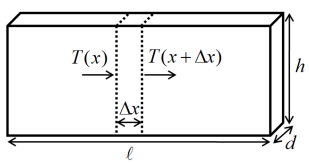

Fig. 4. A radiator section area with the heat transfer equation governed by (8)

The corresponding heat balance equation of this section is given as follows.

$$qc_w\left(T(x) - T(x+\Delta x)\right) + K_r\frac{\Delta x}{\ell}\left(T_a - T(x)\right) = \\ = C_r\frac{\Delta x}{\ell}\frac{\partial T}{\partial t} \tag{8}$$

in which flow rate is q_0 at $t = 0$ and q_1 for $t > 0$. C_r is the heat capacity of water and the radiator material defined as: $C_r = c_w\rho_w V_w$. Dividing both sides by Δx and approaching $\Delta x \to 0$, we have:

$$-qc_w\frac{\partial T(x,t)}{\partial x} + \frac{K_r}{\ell}\left(T_a - T(x,t)\right) = \frac{C_r}{\ell}\frac{\partial T(x,t)}{\partial t} \tag{9}$$

with boundary condition $T(0,t) = T_{in}$, $T(\ell, 0^-) = T_{out,0}$ and $T(\ell,\infty) = T_{out,1}$. If there exists a separable solution, it would be like $T(x,t) = T(t) \times X(x)$. Substituting it into (9), we achieve:

$$T(0,t) = c_1 e^{k_1 t} + c_2 \tag{10}$$

which implies a contradiction.

Before proceed to solve the full PDE (9), we need to find the two boundary conditions $T_{out,0}$ and $T_{out,1}$. For this purpose, take the steady state form of (9) as follows.

$$-qc_w\frac{dT}{dx} + \frac{K_r}{\ell}\left(T_a - T(x)\right) = 0 \tag{11}$$

which can be written as:

$$\frac{dT}{dx} + \frac{\beta}{\gamma}T(x) = T_a \tag{12}$$

with constants $\beta = \frac{K_r}{C_r}$ and $\gamma = \frac{qc_w\ell}{C_r}$. We will be using the two definitions throughout the paper frequently.

Therefore, the steady state temperature, $T(x,t)|_{t\to\infty}$ will be achieved as:

$$T(x) = c_1 e^{-\frac{\beta}{\gamma}x} + c_0 \tag{13}$$

at the specific flow rate q. Substituting the above equation in (12) gives $c_0 = T_a$. Knowing $T(0) = T_{in}$, c_1 is also found. Finally $T(x)$ looks like:

$$T(x) = (T_{in} - T_a)e^{-\frac{\beta}{\gamma}x} + T_a \tag{14}$$

Therefor the two boundary conditions are: $T_{out,0} = (T_{in} - T_a)e^{-\frac{\beta}{\gamma_0}x} + T_a$ and $T_{out,1} = (T_{in} - T_a)e^{-\frac{\beta}{\gamma_1}x} + T_a$ corresponding to the flow rates q_0 and q_1.

Generally solving the full PDE (9) in time domain is a difficult task. However we are interested in the radiator transferred heat to the room rather than temperature distribution along the radiator. Instead of $T(x,t)$, therefore, we will find $Q(t)$ which is independent of x. $Q(t)$ can be formulated as:

$$Q(t) = \int_0^\ell \frac{K_r}{\ell}\left(T(x,t) - T_a\right)\,dx \tag{15}$$

Taking time derivative of the above equation and using (9):

$$\frac{dQ}{dt} = \int_0^\ell \frac{K_r}{C_r}\left(-qc_w\frac{\partial T}{\partial x} + \frac{K_r}{\ell}\left(T_a - T(x,t)\right)\right)dx \tag{16}$$

with $\beta = \frac{K_r}{C_r}$. The above equation can be rewritten as:

$$\frac{dQ}{dt} + \beta Q = \beta qc_w\left(T_{in} - T_{out}\right) \tag{17}$$

in which T_{in} is the constant forward temperature. However T_{out} in the above equation is a function of time. Therefor we need an expression for $T_{out}(t)$ which is attained in the following. To develop $T_{out}(t)$, consider (9) at $x = \ell$:

$$-qc_w\frac{\partial T}{\partial x}\,|_\ell + \frac{K_r}{\ell}\left(T_a - T(\ell,t)\right) = \frac{C_r}{\ell}\frac{dT(\ell,t)}{dt} \tag{18}$$

The first term in the left side of the above equation is an unknown function of time which we call it $f(t)$. Thus the above equation can be rewritten as:

$$\dot{T}_{out} + \beta T_{out} = \beta T_a - \gamma f(t) \tag{19}$$

with $\beta = \frac{K_r}{C_r}$ and $\gamma = \frac{qc_w\ell}{C_r}$. In order to estimate $f(t)$ we take a look at the simulation result for this function which is a position derivative of $T(x,t)$ at the end of radiator. It turns out we can approximate $f(t)$ with an exponential function roughly as shown in Fig.5

We know the initial and final value of $f(t)$. Also, the minimum of $f(t)$ occurs at the transportation time of flow to the end of radiator i.e. $\frac{\rho_w V_w}{q}$. Therefore, we approximate $f(t)$ as bellow:

$$f(t) = (f_0 - f_1)e^{-\tau t} + f_1 \tag{20}$$

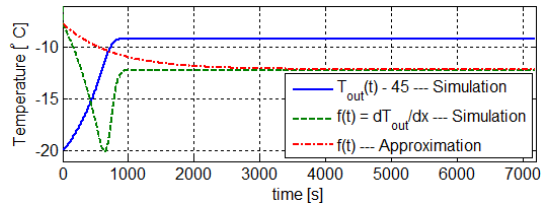

Fig. 5. Simulation results for scaled $T_{out}(t)$, its first position derivative and its approximation are shown. The first position derivative i.e. $f(t)$ is approximated with an exponential function.

with $f_0 = -\frac{\beta}{\gamma_0}(T_{in} - T_a)e^{-\frac{\beta}{\gamma_0}\ell}$, $f_1 = -\frac{\beta}{\gamma_1}(T_{in} - T_a)e^{-\frac{\beta}{\gamma_1}\ell}$ and $\tau = \frac{q}{\rho_w V_w}$.

Substituting $f(t)$ in (18), the return temperature is obtained as follows:

$$T_{out}(t) = c_1 e^{-\beta t} + c_2 e^{-\tau t} + c_0 \tag{21}$$

with $c_0 = T_a - \frac{\gamma_1}{\beta}$, $c_2 = \frac{\gamma_1(f_0 - f_1)}{\tau}$ and $c_1 = T_{out,0} - c_0 - c_2$.

Back to (17), we substitute $T_{out}(t)$ in the equation. $Q(t)$ becomes:

$$\begin{aligned} Q(t) &= (k_1 t + k_0)e^{-\beta t} + k_2 e^{-\tau t} + k_3 \qquad (22)\\ k_1 &= -\beta q c_w c_1 \\ k_2 &= \frac{\beta q c_w c_2}{\tau - \beta} \\ k_3 &= q c_w (T_{in} - c_0) \\ k_0 &= c_w q_0 (T_{in} - T_{out,0}) - k_2 - k_3 \end{aligned}$$

The result is not a precise solution because we have made an approximation while deriving $T_{out}(t)$. But it is still enough for us to extract usefull information regarding the time constant and gain. The analytic solution and simulation for a specific flow rate is shown in Fig.6.

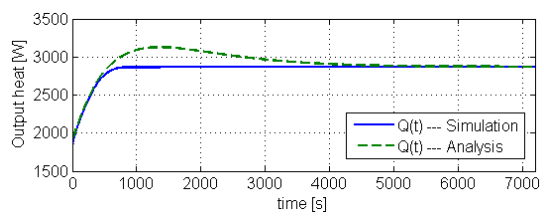

Fig. 6. Simulation and analysis results for Q(t). The analytic solution gives us a good enough approximation of the transient and final behavior of the radiator output heat. We utilize this analytic solution to extract the parameters of a first order approximation of $Q(t)$ step response.

The overshoot in the analytic solution compared to the simulation is due to neglecting an undershoot in $T_{out}(t)$ calculations.

In the next section, we utilize the derived formula to extract the required gain and time constant for the control oriented LPV model.

D. Radiator LPV Model

Parameters K_{rad} and τ_{rad} of the radiator LPV model (7) are derived based on first order approximation of the radiator power step response (22). The steady state gain is:

$$K_{rad} \quad = c_w(T_{in} - T_{out,1}) \tag{23}$$

with $T_{out,1}$ corresponding to the flow rate q_1. Using the tangent to $Q(t)$ at $t = 0$ we can obtain the time constant. The slope of the tangent would be equivalent to the first derivative of $Q_{final} + (Q_0 - Q_{finale})e^{-\frac{t}{\tau_{rad}}}$ at $t = 0$ which gives:

$$\tau_{rad} = \frac{Q_{final} - Q_0}{k_1 - \beta k_0 - \tau k_2} \tag{24}$$

Therefore, at a specific operating point, the radiator gain and time constant can be obtained via (24) and (23). For a set of operating points these parameters are shown as two profile of curves in Fig. 7.

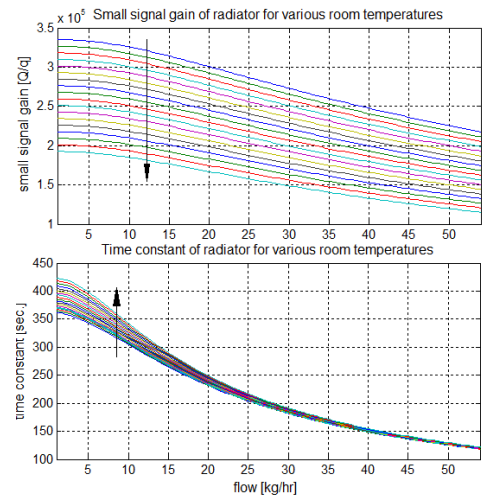

Fig. 7. Steady state gain and time constant variations for various values of the radiator flow and room temperature. The arrows show the direction of room temperature increase. Room temperature is changed between $-10\,^{\circ}\mathrm{C}$ and $24\,^{\circ}\mathrm{C}$ and flow is changed between the minimum and maximum flow

Fig. 7 shows that the radiator gain and time constant of the heat-flow transfer function significantly depend on the flow rate. The high gain and the long time constant in the low heat demand conditions mainly contribute to the oscillatory behavior. The control oriented model of room-radiator can be written as:

$$\frac{T_a}{q}(s) = \frac{K_{rad}K_a}{(1 + \tau_{rad}s)(1 + \tau_a s)} \tag{25}$$

Room parameters, K_a and τ_a can be estimated easily by preforming a simple step response experiment. We obtained these parameters based on [2] assuming specif materials for the components.

III. Gain Scheduling Control Design based on Flow Adaptation

In the previous section, we developed a linear parameter varying model for radiator instead of the high-order nonlinear model (1). To control this system, among various possible control structures, gain scheduling approach is selected which is a very useful technique for reducing the effects of parameter variations [15]. Therefore, the name of flow adaptation indicates to this fact that controller parameters are dependent on the estimated radiator flow.

The main idea for design of adaptive controller is to transform the system model (25) to a system independent of the operating point. Then, the controller would be designed based on the transformed linear time invariant (LTI) system. The block diagram of this controller is shown in Fig. 8.

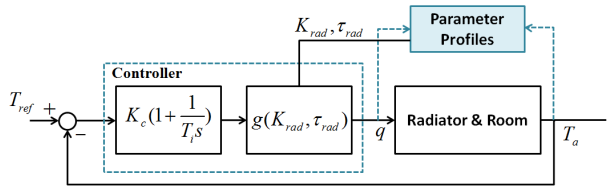

Fig. 8. Block diagram of the closed loop system with linear transformation

Function g is chosen such that to cancel out the moving pole of the radiator and places a pole instead in the desired position. This position corresponds to the farthest position of the radiator pole which happens in high flows or high demand condition. Therefore, the simplest candidate for the linear transfer function g is a phase-lead structure, (26).

$$g(K_{rad}, \tau_{rad}) = \frac{K_{hd}}{K_{rad}} \frac{\tau_{rad} s + 1}{\tau_{hd} s + 1} \tag{26}$$

in which K_{hd} and τ_{hd} correspond to the gain and time constant of radiator in the high demand situation when the flow rate is maximum. Consequently, the transformed system would behave always similar to the high demand situation. By choosing the high demand as the desired situation, we give the closed loop system the prospect to have the dominant poles as far as possible from the origin, and as a result as fast as possible.

The controller for the transformed LTI system is a fixed PI controller then. The parameters of this controller is calculated based on Ziegler-Nichols step response method [6]. To this end, the transformed second order system is approximated by a first-order system with a time delay, (27). The choice of PI controller is to track a step reference with zero steady state error.

$$\frac{T_a}{q}(s) \quad = \quad \frac{k}{1+\tau s} e^{-Ls} \tag{27}$$

The time delay and time constant of the above model can be found by a simple step response time analysis of the transformed second-order model:

$$T_a(t) = K_{hd} K_a (1 + \frac{\tau_{hd}}{\tau_a - \tau_{hd}} e^{\frac{-t}{\tau_{hd}}} + \frac{\tau_a}{\tau_{hd} - \tau_a} e^{\frac{-t}{\tau_a}}) q(t) \tag{28}$$

in which $q(t) = u(t)$ is the unit step input. The apparent time constant and time delay are calculated based on the time when 0.63 and 0.05 of final T_a is achieved, respectively. The positive solution of the following equation gives the time delay when $\chi = 0.95$ and the time constant when $\chi = 0.37$.

$$(\chi + 1)t^2 + 2(\tau_{hd} + \tau_a)(\chi - 1)t^2 + a(\chi - 1)\tau_{hd}\tau_a = 0 \tag{29}$$

Having τ and L calculated, the parameters of the regulator obtained by Ziegler-Nichols step response method would be the integration time $T_i = 3L$ and proportional gain $K_c = \frac{0.9}{a}$ with $a = k\frac{L}{T}$ and $k = K_{hd} \times K_a$ which is the static gain.

A. Simulation Results

The proposed controller parameterized based on radiator parameters, is applied to the simulation models of room and radiator. Parameters of the PI controller are found based on the parameter values in table II as $K_c = 0.01$ and $T_i = 400$. Ambient temperature is considered as the only source of disturbance for the system. In a partly cloudy weather condition, the effect of intermittent sunshine is modeled by a fluctuating outdoor temperature. A random binary signal is added to a sinusoid with the period of two hours to model the ambient temperature.

Simulation results with the designed controller and the corresponding ambient temperature are depicted in Fig. 9 and Fig. 10. The results are compared to the case with fixed PI controllers designed for both high and low heat demand conditions.

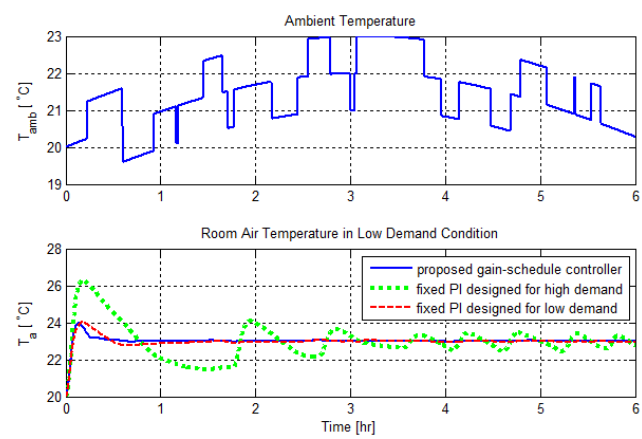

Fig. 9. (Top) ambient temperature, (bottom) room temperature for three controllers. The results of simulation with flow adaptive controller together with two fixed PI controllers are shown. The PI controller designed for the high demand situation encounters instability in the low heat demand condition.

The simulation results of the proposed control structure show significant improvement in the system performance and stability compared to the fixed PI controller.

IV. Discussions

All the gain scheduling control approaches are based on this assumption that all states can be measured and a generalized observability holds [15]. In this study, we also need to clarify if this assumption is valid. The parameters that we need to measure or estimate are room temperature and radiator flow rate. Measuring the first state is mandatory when the goal is seeking a reference for this temperature. However, radiator flow is not easily measurable.

To have an estimation of the radiator flow rate, one possibility is using a new generation of TRVs which drive the valve with a step motor. It is claimed that this TRV can give an estimation of the valve opening. Knowing this fact and assuming a constant pressure drop across the radiator valve, we would be able to estimate the flow rate.

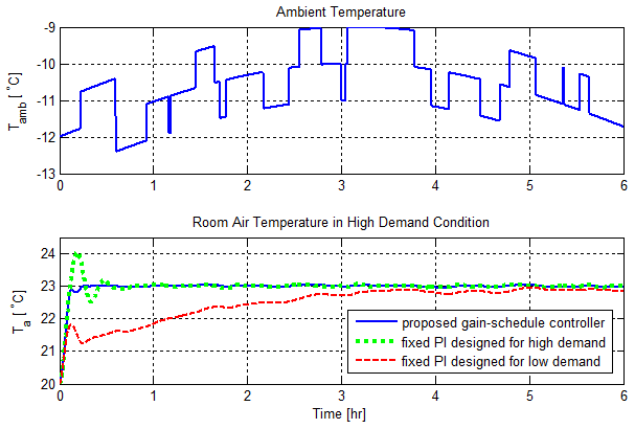

Fig. 10. (Top) ambient temperature, (bottom) room temperature for three controllers. The results of the simulation with flow adaptive controller together with two fixed PI controllers are shown. The PI controller designed for the low demand condition is very slow for the high demand situation.

We have shown through the paper that using the new generation of TRVs, gain scheduling control would guarantee the efficiency of the radiator system.

V. Conclusion

The dynamical behavior of a TRV controlled radiator is investigated. A dilemma between stability and performance for radiator control is presented. We dealt with the dilemma using a new generation of thermostatic radiator valves. With the new TRV, flow estimation and control would be possible. Based on the estimated flow, we have developed a gain schedule controller which guarantees both performance and stability for the radiator system. To this end, we derived low-order models of the room-radiator system. The model is parameterized based on the estimated operating point which is radiator flow rate. Gain schedule controller is designed for the resulted time varying model.

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TABLE I

Symbols and Subscripts

Nomenclature	
A	surface area (m^2)
C	thermal capacitance $(J/kg\,^\circ\mathrm{C})$
c_w	thermal capacitance of both water and radiator material
d	depth of radiator
g	linear transformation function
G	transfer function
h	height of radiator (m)
h_a	air convective heat transfer coefficient
K	DC gain
K_c	controller gain
K_r	equivalent heat transfer coefficient of radiator $(J/sec\,^\circ\mathrm{C})$
L	time delay $(sec.)$
N	total number of radiator distributed elements
n_1	radiator exponent
Q	heat (W)
q	water flow through radiator (kg/sec)
T	temperature $(^\circ\mathrm{C})$
T_i	integration time
T_{in}	inlet water temperature
T_n	temperature of the radiator n^{th} element $(^\circ\mathrm{C})$
T_{out}	outlet water temperature
T_{out}^i	steady state T_{out} corresponding to flow rate q_i
U	thermal transmittance $(kW/m^2\,^\circ\mathrm{C})$
V_w	water capacity of radiator (m^3)
ρ	density (kg/m^3)
τ	time constant $(sec.)$
δ	valve opening
ℓ	length of radiator
Subscripts	
a	room air
amb	ambient temperature (outdoor)
e	envelop
f	floor
hd	high demand
rad	radiator
ref	reference temperature of room

TABLE II

System Parameters

Room Parameters		Radiator Parameters	
A_e	$56\ m^2$	A_r	$1.5\ m^2$
A_f	$20\ m^2$	C_{rad}	$3.1 \times 10^4\ J/kg\,^\circ\mathrm{C}$
C_a	$5.93 \times 10^4\ J/kg\,^\circ\mathrm{C}$	c_w	$4186.8\ J/kg\,^\circ\mathrm{C}$
C_e	$5 \times 10^4\ J/kg\,^\circ\mathrm{C}$	N	45
C_f	$1.1 \times 10^4\ J/kg\,^\circ\mathrm{C}$	n_1	1.3
U_e	$1.2\,kW/m^2\ ^\circ\mathrm{C}$	q_{max}	$0.015\ kg/sec$
U_f	$1.1\,kW/m^2\ ^\circ\mathrm{C}$	T_s	$70\ ^\circ\mathrm{C}$
		V	$5\ Liter$