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Computer game design for playful math-learning.

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***Abstract*—Creating playful games to support domain-specific learning is a complex task. This paper presents the design and development of Prime Slaughter, a computer game to play with abstract mathematical concepts, like factorization and primality. The target group is composed of primary and early secondary school students. Following the findings of a participatory design study about children interaction in museums, our game maps operational aspects of prime factorization onto a gameplay inspired by 2D action-adventure games, where primality and factorizaton are experienced in a visual and direct way. The museum study also suggests to support learning by multiple play styles and goal-directed activities. A new participatory design study is about to start, to validate and improve the latest prototype of the game.**

Keywords-playful learning; prime factorization; computer games; forms of play.

In order to investigate the transposition of complex processes from a domain into games, we decided to design and implement a computer game about mathematics. The chosen domain is prime numbers and factorization, as they present a complex topic for late primary school children, yet they have great relevance in further studies. Factorization can be considered an advanced topic related to fractions; in fact it is related to the procedures to find *greatest common divisor* (GCM) and *least common multiple* (LMC), needed to simplify and add fractions. Moreover, factorization, GCM and LMC algorithms, together with algorithms to test primality are standard part of the discrete math in computer science curricula. The dynamics of factorization of a number has to do with trying different divisors, and leads to discover more and more primes (like in algorithm known as Eratosthenes' sieve).

A survey of existing math-related games showed that few applications offer playful learning of complex math concepts, such as prime factorization; therefore we adopted a theoretical framework based on studies about the role of goals directed activities in learning. The findings of a 1 year participatory design study (conducted by one of the authors) about children interaction in museums, provided inspiration for the design of our new math-related game: Prime Slaughter (PS for short).

In PS mathematical concepts and operations are transposed into game elements and become integral part of the gameplay. In specific, abstract concepts related to factorization and primality are presented and experienced in a direct way, in an action-adventure game setting.

The rest of the paper presents a survey of math-related games (section I), the theoretical framework and findings of the museum project (section II). The design of the game is explained in section III; sections IV and V discuss future work and conclusions.

I. Math games

This study started with a survey about math-related games and e-learning applications, we analyzed (among the others):

- Number ninjas[1], a free online game
- BBC bitesize[2] games, a collection of games online
- a few titles from the Danish Pixeline[3] PC game series, in particular *Pixeline – Regning - Talmesterens labyrint* (translated as: *Pixeline – counting – number's master labyrinth*) by Krea
- Toon University: Prime Factoring[4], online game
- ABCya! Fraction Tiles[5], a virtual-manipulative online application

In *number ninjas* the players is a number and move around in a side-scroller game; the goal is to collect shuriken (ninja's throwing stars) and use them to kill other numbers. Each shuriken is an operation, so for example you are number 1 in the beginning, and by killing number 3 with a *plus* shuriken, 3 dies and you become number 4. In advanced levels there might be multiple ways to finish, by adding, subtracting or multiplying your initial value with the available numbers disseminated in the level. The general rule is that if you become more than 9 or less than 0, you will die. This rule contributes to make the solution of the levels more

[1] http://armorgames.com/play/11790/number-ninjas
[2] http://www.bbc.co.uk/schools/ks1bitesize/numeracy
[3] http://www.kreagames.dk/laering-spil
[4] http://www.toonuniversity.com
[5] http://www.abcya.com/fraction_tiles.htm

challenging. Decisions involving application of operations to numbers are essential parts of the game, which makes it very interesting and inspiring for us.

BBC bitesize are small games with nice graphics, sound effects and even voice recordings, clearly designed with occasional players in mind. However, such games are mostly a simple transposition of traditional math exercises into computer-supported exercises. The game about division, in the KS1 section (for smaller children), asks explicitly to find the result of a division (e.g. 10 / 2) and 3 possible answers are displayed, for the player to click on. When the answer is selected, an animation follows, where many diamonds are sorted into wagons. Hence, the action of dividing 10 by 2 will result in an animation of 10 diamonds arranged between 2 wagons. In case of wrong answers, some of the wagons remain empty or some diamonds fall on the ground after all wagons have being used, then a voice signals the error. The user interface is developed for children with little reading capabilities, so moving the mouse close to a button triggers a voice that reads the text on the button aloud: this feature cannot be disabled and could be annoying in the long run. The most relevant feature *BBC bitesize* games have is that exercises sometimes offer a concrete context for an abstract mathematical operation: dividing numbers for example is proposed as the action of splitting a bunch of diamonds on many wagons.

Pixeline games are technically well realized and engaging, in particular they offer:

- a playful environment (a houses of village or the rooms of a house) where Pixeline, the player's avatar, can roam around and play mini-games
- exercises are usually taken from school curriculum
- provide a good context to make exercises more concrete, understandable and fun. Still the goal is to solve exercises and there is no theory
- include collaborative and competitive environment for exercises

The *Toon University: Prime Factoring* game is based on questions, and the player needs to shoot the right answer with a cannon. The goal of the game is finding the right answer among 3 choices, after performing mental calculation. This game is very distant from our idea of mapping problem-solving dynamics into the gameplay, in fact none of the steps in solving factorization problems are represented in the game. The visual quality of the game is rather low, but in this *Toon University: Prime Factoring* is a good representative of many free online math games.

A different type of application is represented by *ABCya! Fraction Tiles*. This online program offers a vitual manipulative environment, where students explore how parts make up a whole. In *Fraction Tiles* the player interact through a desktop-like user interface and a palette of fraction tiles, from 1 to 1/12; tiles can be taken from the palette and arranged as construction blocks. The width of each tile is proportional to the size of the fraction it represents, so for example two ¼ fractions one aside of the other are as long as a single ½ fraction. The goal here is to let the children explore fractions, freely or under supervision of teachers that could provide challenges or tasks.

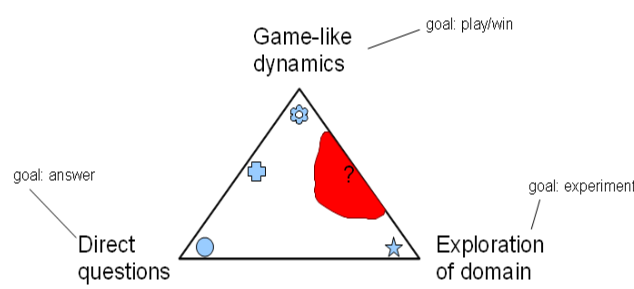
Various Facebook games present similar exploratory feature, as some of them focus on arranging the environment in a certain way (e.g. *Farmville*[6] by Zynga). Finally another interesting case of a game supporting players in acquiring new knowledge and skills, is represented by *Code Hero*[7] by Primer, in which players are supposed to learn how to make video games through direct manipulation of abstract concepts. The player can alter the environment of the game itself, by direct manipulation of code, in the form of scripts, that can be copied, then altered and re-used.

Examples of games that successfully embed subject-related elements and concepts in their gameplay, are classical strategy games, like SimCity (by Maxis) [1]. Unfortunately similar games are usually not related to math: the domain of SimCity is economics and city planning. Number ninjas is probably the best example we could find of math-related game, in which the numbers and the operations between them are integral parts of the game and the player directly manipulates them to play.

Our conclusion, after the survey, is that in most cases math is not effectively transposed into gameplay; in particular games are typically unconcerned with abstract operations like prime factorization or primality testing. The most common approach seems to be porting math exercises into a nice, possibly fun graphical environment.

A. A space of the math games

These games and applications cover a large spectrum of possible features; to summarize and help visualize features and goals, we propose to consider a space of math applications, as in figure 1.



§Figure 1. Space of the math-related applications we surveyed. The 3 corners represent stereotypical application. The red area is the one that we would like to investigate with our new game.

The triangular shape of this space is based on the observation that applications in the math-learning domain mainly focus on one of 3 possible mechanisms: asking math questions, provide a visual environment to freely explore

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6 http://www.farmville.com

7 http://primerlabs.com/codehero

math concepts, or (more rarely) embed math-related dynamics into a game.

In figure 1, the blue circle is an application like those in *BBC bitesized*, where the approach is to transpose math questions in the computer medium; the game elements mostly provide graphics, while the goal in these applications is clearly answering correctly to questions. Games like *Pixeline* are represented by the blue cross, in figure 1, since they are more game-like; still the game elements are used mainly to contextualize mathematical exercises. *Number ninjas* is represented by the blue flower, and games similar to *SimCity*, but with math as primary focus, would also be clustered in the top corner of the triangle. The blue star in the triangle marks the location of *Fraction Tiles* and other virtual-manipulative environments, in the math applications space.

Our research focuses on the red area in figure 1, between math games and exploratory computer-based environments. A central problem in the development of PS is therefore to transpose the factorization dynamics in gameplay and find ways to visually represent primality.

II. Theory and observations

The PS game is based on findings and reflections from another ongoing research project (conducted by one of the authors [2]) about playful learning and the transposition of abstract notions into playful interactions. The aim of that project is to enrich museum learning practice in relation to historical processes, specifically urban development of rural settlements. The museum project is addressed to primary school children around 10 years old. A one year Participatory Design (PD) process has been conducted in cooperation with an after school institution, so that a group of 25 children around 10 years old were involved in co-designing the prototype of an educational game [3]. Insights collected through this process and related theoretical framework provided inspiration to try the same approach in factorization and primality. An analogy was identified between the two projects, as in both games the children are supposed to experience theoretical notions and problem solving activities, playing as characters within the provided narrative framework.

During the PD process the children were invited to co-design a game about the Viking Age, focusing on urban development. During the first 4 sessions, design materials were presented to them such as: construction bricks, papers and colored pencils, and modeling clay. In the further two sessions the children were invited to test and complete a low-fidelity prototype of the game, by playing and designing new game pieces or introducing new game rules whenever they felt something was missing.

The theoretical framework adopted for the museum project is based on Rogoff's theory of apprenticeship [4]. According to Rogoff children learn by engaging in goal-directed activities within informal contexts, together with adults, whose guide is needed when reaching the boundary "zone of proximal development", situated between the already known and the unknown [4]. In such case adult experts support the children in going beyond their current knowledge. Play is then supposed to mediate between the children and the learning content, also in case they would like to explore the learning domain by themselves. Play is then intended, as in [5], a self-driven activity, which could take different forms according to individual interests.

Figure 2 Different play styles, on the front the mixed group.

In this respect observations conducted during two PD sessions have concretely showed to the researchers how the children could manifest different, self-driven forms of play, although playing with the same platform and within the same group of players. Some children adopted a *military* board games play style, usually related to games like Stratego by Milton Bradley Board Games and Monopoly by Hasbro. They placed their tangibles on the board and challenged each other, as they were fighting a war to win control over the other player's land. They also engaged in forms of theatrical improvisation, in which they teased each other pretending that the game situation was real. Other children instead preferred a *designerly* play style, as they were playing construction bricks or games like SimCity [1]. They spent all their time making new game pieces and buildings for their own settlement, as they were urban planners. These children made buildings, such as houses and shops, humans and animals, but also landscape elements, such as trees and different plants. Some other children instead started playing in a designerly style and later shifted to the military one. For instance a group of four children, two girls and two boys, provided an interesting case of mixed play style. Both girls and boys started by planning their settlement, but the girls, especially one, used a lot of time to plan her settlement, caring for all the details. The boys instead used little time for that; they just placed the tangibles, as they would have done to play a board game, so that they engaged as soon as possible in the military style. The four children agreed on playing together, in the military play style, but the girl, who was very active in urban planning, insisted to have the "whole settlement ready before" (figure 2). Hence the boys started to play by themselves, while the girls completed their settlement, afterward they were all able to play together. At the same time other children played with each other on a parallel basis, as urban planners or fighting landowners, almost as they were in another room.

Furthermore, the combination of a narrative framework and a goal-oriented activity within it seem to provide support for individual needs. The narrative framework provided a sort of improvisational score, both for designerly and military players. Then the children who preferred the latter, saw in the narrative a sort of theatrical improvisation score, so that they pretended to be competing landowners. Instead the children who played in the designerly style, interpreted the narrative as a theme and found an intrinsic motivation, in creating their tangibles and settlements in a certain way.

These observations provided a valuable grounding to determine requirements for the museum project and also inspired us to create Prime Slaughter for a similar domain. Therefore, combining these data and the theoretical framework provided by Rogoff, this study is aimed at exploring a scenario in which abstract mathematic concepts are transposed into playful goal directed activities embedded in the game. Such activities should embody both theory and problem solving techniques. Hence children may explore mathematic concepts they learned from their teachers in school, in a more tangible and playful way by themselves. In this sense both learning and playing are intended as self-driven activities, tangential to adults' involvement.

The learners are then supposed to acquire domain-specific and procedural knowledge through the play, as sense-making of the learning content is fundamental to collect points in the game, as it is very common in game players (see [6] and [1]).

III. The Prime Slaughter game

In subjects like math and computer science, abstractions are very common, but they are sometimes difficult to understand for beginners. However, in the field of algorithm animation, direct manipulation is often used to support learning, by allowing beginners to intuitively interact with abstract concepts [7]. Typically a visual and spatial representation of data structures and sometimes of algorithms is provided, so that the learners can experience the semantics of their actions without being forced to adopt a textual syntax, which is complex and language-specific.

Similarly, in PS direct manipulation of natural numbers and of the operations involved in factorization is seen as a central aspect in the design of our game, as it should allow the players to directly experience the meaning of these abstract notions in a concrete and engaging way.

In our view it is fundamental that PS is playable as a real game, and not just as computer-augmented exercises. Therefore, children are supposed to be able to play, according to their perception of "play". In this respect inspiration came from the data collected in the museum study, in integrating different forms of play. Specifically the game should afford for a designerly, generative play style, based on some children's interest in modeling the natural landscape, and for a military play style, inspired by classic action-adventure 2D games (such as The Legend of Zelda, by Nintendo). Moreover, the game should also allow for a mixed play style, in which the children could freely shift from one style to the other.

Finally the players should be able to play and have fun also without having a solid knowledge of factorization and primality, so to elicit a motivation to play more and gain new knowledge. This is the exploratory goal, also found in virtual manipulatives and, to some extent, in math-related games.

Game Design

In order to design our game, we have to find out about the typical elements and mechanics in action-adventure games. Then we need to analyze the actions involved in prime factorization, such as division, multiplication, and the possible ways to visualize abstract concepts like the primality of a natural number or divisibility of a number by another. Finally we should define mappings between math and game dynamics, creating an engaging gameplay.

Classic 2D action-adventure games couple the exploration of the hero's world with the killing of enemies, usually monsters. Explorations allow the players to acquire meaningful domain-specific knowledge, in the form of tricks and hidden artifacts, to defeat more effectively different kinds of enemies. These games have nice graphics and compelling audio, incremental difficulty of the successive levels and engaging narrative elements disseminated throughout the game. Narrative elements and the increasing complexity of levels or of quests, act as short term goals, guiding the player and offering rewards that increase her ability to progress in the game. Usually there is a hero, defined by energy level, experience points, sometimes magic or special abilities, and the items collected so far in the game. An interesting feature is that typically short term progress in the game depends on the player's current state and knowledge of the game environment, while long term progress and reaching of high scores requires some strategy in managing experience points or bonuses, and a deeper understanding of the rules of the game (i.e. of what is possible to do, including unexpected side-effects of certain actions or artifacts). Many games have special items, sometimes weapons or magical spells, that can be collected to be used in particular situations. The use of such items might require the player to study game manuals or systematically experiment with her avatar in the game, to acquire an adequate level of knowledge.

Our game should provide a framework to learn factorization and primality, in which the player focuses on goal-directed activities, such as killing enemies or exploring. Mathematical concepts and algorithms should be integrated as narrative elements, characters and as dynamics, and should facilitate the players toward succeeding in her goal-directed activities. In this sense our game is envisioned as supporting forms of tangential learning, in which learning is not the main focus of players' actions and occurs indirectly [8].

Understanding the built-it mathematical concepts should make you a better player, but you should enjoy playing even with little previous knowledge. Moreover, mathematical concepts should be easily recognizable by player that already studied them in school. Finally, conceptual (mathematical) errors should be represented as damage points (e.g. player's character energy decreases). Players' motivation and reward should come from a captivating gameplay and nice graphics.

A. *Factorization and gameplay*

Factorization has to do with learning to see a natural number also as a collection of its prime factors: for instance 12 is also *2 x 2 x 3*. The fundamental theorem of arithmetic ensures that the factorization of a natural number is unique (ignoring the order of the prime factors). A simple algorithm to find the prime factors of an integer number N, is to start from 2 and try to divide N by ever larger primes. And this, in turn, requires to be able to identify prime numbers. Therefore, integer division and multiplication are central concepts in this domain. Divisibility offers yet another way to perceive a number: a number is related to all its factors. For example, the factors of 12 are {1,2,3,4,6,12}.

At a conceptual level we would like our players to discover or consolidate the idea that a natural number has multiple representations, and that when some properties are not clearly visible, we can manipulate the number to make these properties more directly evident. This idea is present in both mathematics and computer science, where rewriting is often used to transform expressions, not to change their value, but to come closer to a normal or simpler form (e.g. *derivation* in calculus and *reduction* in lambda-calculus [9]. Moreover, in computation theory natural numbers are sometimes expressed in unary (i.e. tally notation), for example to help visualize the behavior of Turing machines [10]. When a natural number is expressed in unary, division becomes a matter of re-organizing, clustering rows of **1**s into grids. Dividing 12 by 3 for instance, could be visualized as in figure 2. This suggests that division by N could be expressed as folding N times the digits of a number (written in unary). Prime numbers will be "unfoldable", therefore visually different from non-primes.

12 = IIIIIIIIIIII
12 / 2 = (II II II II II II) / 2

$$6 = \begin{matrix} \boldsymbol{I} & I & I \\ \boldsymbol{I} & I & I \end{matrix}$$

Figure 2. Number 12 expressed in unary notation. The number is divided by clustering the individual digits. Primes will not cluster, they will remain a single line of **I**s, while a number like 6 can be folded in 3 piles of 2 **I**s.

In the early design phases of PS, many different visualizations of the division operation were explored and then only a few were selected. Taking as inspiration the 2D adventure genre, we created concept arts of the different renditions of numbers, the division and multiplication operations. Figure 3, left panel, shows division as splitting an object (a cube of jello in the figure) into 2 or 3 parts. Multiplication is instead depicted (in figure 3 on the right) as the process of pruning a young tree, as it grows. A tree initially has a value of 1, and a single branch; the top of this tree will have a single leaf of value 1. Cutting the tree in 3 would generate 3 new branches, each with a leaf of value 3. Cutting the tree again in 2, would result in a single trunk, 3 branches and 6 smaller branches (as visible in figure 3, on the right). Interestingly, this form of pruning enables players to freely explore the space of the possible tree shapes.

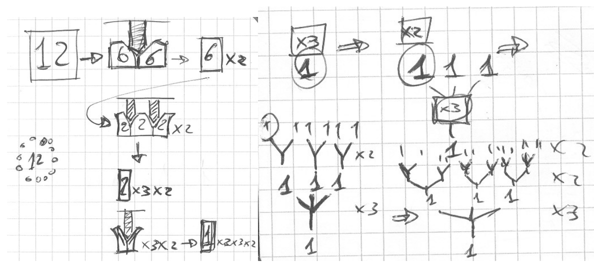

Figure 3. Splitting numbers could represent division; pruning a tree can be like multiplication: each new cut creates many new branches.

Division as splitting reminds of a typical element of action-adventure games: the slaying of monsters. Hence, we decided that in PS numbers are monsters, that the hero has to fight with his sword, representing a prime number by which the monsters can be split. The rule for PS is that a monster of value N (or N-monster) can only be sliced by a sword of value M, where M is a factor of N. The sliced monster divides into many smaller monsters; for instance, a sword of value 2 can slice a 12-monster into 2 monsters of value 6 (see figure 4). Trying to slice a monster with the wrong sword (e.g. attacking a 9-monster with a 2-sword) results in damage for the hero, who loses energy points, and ultimately dies. Monsters reduced to value 1 are short lived, and spontaneously disappear.

The primary play style of PS is, therefore, to explore a level, while killing as many monsters as possible, collecting points.

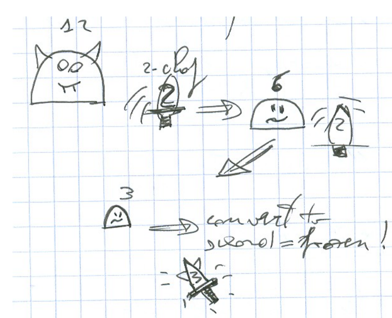

Figure 4. Early sketch of the gameplay. A 2-sword can divide (chop) a 12-monster. The result is that there will be 2 smaller 6-monsters. When a monster is a prime, in the figure a 3-monster, it can be frozen and captured; the 3-monster will become a 3-sword in the hand of the player's character, and the sword that was held before will be lost.

At the beginning of the game, the player's character has a 2-sword (sword of value 2) and monsters are reduced to prime numbers after a few hits, it would be impossible to kill them without acquiring new proper prime-swords. Even further in the slicing process, every number-monster will eventually be reduced to a 1-monster, impossible to kill, since 1 is only divisible by 1. Taking all these points into consideration, it was decided that when a number-monster is prime (for example an 11-monster) it can be frozen by the hero through magic, and transformed into a sword. This double role of the monsters as enemies and material to construct new swords, is intended to enforce the idea that prime numbers are a single fundamental concept throughout the game, and monsters and swords have ultimately the same nature, i.e. they are primes.

Since the levels of PS contain monsters with values from 1 to 20, the hero needs to collect a number of swords, with all the values of the primes from 1 to 20: 2, 3, 5, 7, 11, 13, 17, 19 (1 is not prime by definition). The hero in PS can only hold one sword at any given time, so all other swords are kept in an armory (as illustrated in the sketch in figure 5). In case the hero dies, he will be re-born in his armory, and all swords collected before dying will still be available. The levels of the current prototype of the game are visible in figure 6; the armory is called "home" (snapshot 2 in figure 6). To win the game, the player can collect at least 400 points (by slicing as many monsters as possible) or she can collect all 8 prime-swords in her armory.

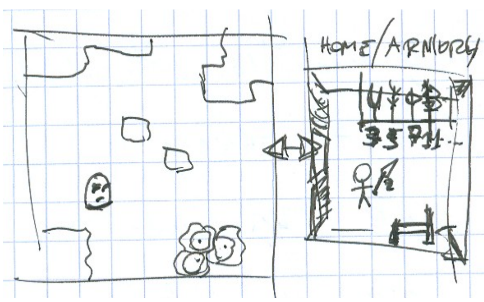

Figure 5. The first level: the player has a home where she can keep her swords; each sword is a prime number.

Finally, number-monsters have different sizes in the game; the size is proportional to their number of prime factors of the monster. Hence, a 12-monster has size 3, while a 17-monster has size 1, and 1-monsters have size 0, the minimal possible. In this way the player can use size as a visual indicator of how close the monster is to being prime.

B. *Allowing for multiple play styles*

This *kill-them-all* play style might appeal to players fond of action, military or strategy genres (as discussed in section III), while others may prefer alternative styles. Therefore, based on findings from the museum project, it was decided to support also a designerly play style. Hence, a level without monsters was introduced, a forest of number-trees (snapshot 4 in figure 6). These trees are called *natural bonsais* as they represent each a natural number, and like bonsais can be shaped in various forms. If in PS division is transposed into slicing monsters, then multiplication corresponds to growing trees, by appropriately pruning them: we decided to map the duality division/multiplication onto the duality growing/killing. Monsters cannot cross levels, and in this way levels act like rooms to physically segregate play styles, so that a player can go to the bonsai forest when being tired of slicing monsters and vice versa.

In the current prototype of the game, a bonsai starts as a seed with a natural value associated, for example 12; then it can be pruned by the hero's sword, but only if the value of the sword is a factor of the value of the bonsai, in this case 2 or 3. If a newly born 12-bonsai is pruned by a 3-sword, it grows 3 more branches, on top of the existing tree trunk; each of the new branches will have a leaf of value 12/3, i.e. 4. The process of pruning and growth continues until the leaves are primes, then the leaves are replaced by fruits, each with the same value as the leaf that generated it (apples in the current prototype, as visible in figure 6, snapshot 4). When the hero collects a number-fruit, his sword is changed accordingly. Points are given during the pruning process, but not when fruits are collected. Old trees can be felled by freezing them, i.e. using the same magic that freezes prime-monsters. When an old tree disappears from the level, a new one is planted at the current position of the hero; this set of rules are meant to suggest players to often fell and re-plant their trees, in order to acquire more points. Moreover, since the bonsai level has no enemies and the player has plenty of time to reflect before acting, making mistakes is punished more severely than in the other levels: for example trying to prune a 12-bonsai with a 5-sword results in reduction of 25% of the hero's current energy.

C. *Graphics and sound effects*

The game is currently at a prototypical stage and graphics is not complete; the main graphics-related activity was the creation of concept arts, so to explore possible looks for the characters and the elements of the background. It was decided to use watercolors to experiment with shades of color and create an atypical visual style. Characters and landscape elements were sketched on paper with pencils and watercolors, then scanned. The drawings were then digitally retouched in gimp2[8], so to enhance contrast and contours. The monsters are designed as masses of jello, which are being reduced to smaller blobs when sliced by the hero's sword.

The landscape contains many mathematical references, for instance some trees are shaped as Serpinsky triangles, others have fractal roots and logarithmic spirals as leaves (figure 6, snapshot 3). At this stage the natural bonsai forest (figure 6, snapshot 4) is based on vectorial graphics, as it is easy to represent their growth and pruning. Another option, currently being explored, is to use hand-made sketches of portions of bonsais branches, and then stitch them together programmatically. The background of each level is at the moment a single large image; in future prototypes backgrounds should be composed out of smaller tiles, as typical in 2D adventure games.

[8] http://www.gimp.org

Together with graphics, sound effects are very important when designing an engaging game. The latest prototype of PS has a sound track looping at a low volume, and special-effect sounds are triggered when slaying a monster or when the hero uses the wrong sword on a monster or a tree, thereby losing energy. All sounds in PS are obtained freely via Freeaudio[9]. In order to create the main theme we downloaded and combined loops from Freeaudio, which were composed with Audacity[10], a free audio editing tool.

D. Greenfoot as an agile development tool

PS was intended to be a 2D action-adventure game, so instead of writing specifications and discussing the gameplay on paper, we decided to start implementing a very simple prototype and proceed according to agile development, in very short time-boxes (with daily releases), using working code to incrementally converge on the most appropriate gameplay.

In the first prototype, which was developed in a few hours, the hero could already move around in a single level, and slice monsters, represented simply by rectangles of different size, with a number written in the center. When the hero sliced a monster, the latter was removed from the level, and many smaller monsters were added. The slicing rule of PS was already implemented, so that (as explained in section III, B) a 2-sword could slice a 6-monster into 2 3-monsters.

Java was selected as a programming language, as that provided the opportunity to deploy the game as an applet; moreover, it was a priority for us to be able to rapidly develop simple but playable prototypes, so we opted for the Greenfoot environment. Greenfoot "*is a framework and environment to create interactive, simulation-like applications in a two-dimensional plane.*" [11]. Greenfoot provides API for 2D graphics and collision detection and support to play audio files. A Greenfoot project consists of a world and many actors, so it was natural to model each level of our game as a world in itself, while hero, monsters and bonsais, became subclasses of the actor class.

Typically Greenfoot projects have a single world, however this would create a strong limitation for PS, as we consider the presence of multiple levels central to support exploratory play styles. Luckily from version 2.1 of Greenfoot it is possible to swap worlds within the same project; but this new feature still has a few issues, as every world (i.e. level in our case) is an object, and when the game changes from a world to the next, the old will be garbage-collected and re-instantiated when needed. All dynamic data are lost in this way, so the effect would be that walking from one level to the next in Prime Slaughter all monsters would be deleted and recreated anew, quite the opposite of the persistence that is common in modern computer games. To overcome this problem, we have implemented a simple serialization mechanism that converts the relevant part of the state of a world object into a string. The mechanism is inspired by JavaScript's JSON, and allows each world object to be serialized into a global table. Hence, instead of creating new instances of the same world, the serialized state can be restored.

The choice of Greenfoot as an agile development tool was very productive for us: we were able to develop 6 versions of the game prototypes in 4 weeks, including discussions, testing and re-design in between prototypes.

IV. On-going and future work

A participatory study is being started to evaluate the game, both through online tests and as participatory design workshops in local schools and student clubs. We expect to observe if and how the game can actually support killing and growing play styles, and what might be missing. Furthermore, we intend to observe if new play styles, we did not consider, may emerge. This study is based on qualitative methods, so to allow the children to play as they like and to freely express their impressions about how the game could be improved, through interviews and prototyping sessions.

Since we can deploy PS online, Facebook and other social media could also be interesting when looking for large and heterogeneous groups multi-cultural participants.

In the future we would also like to involve high-school students in the development of our game; by sharing the code of PS via Greenfoot's site, young programmers as well as math teachers could create more levels and experiment with variations of the gameplay.

Finally, a multi-player option could help extend the play styles and allow the study of cooperation and competition within the game.

V. Conclusions

This paper describes the design and development of Prime Slaughter, a computer game in which mathematical concepts and operations are transposed into the game elements and integrated in the gameplay. In particular, abstract concepts related to factorization and primality are presented and experienced in a direct way, in an action-adventure game setting. The players are children in late classes of primary school.

A classification of existing math-related games is proposed, based on their goals and features; we found very few math-related applications that are game-like and support exploration and direct manipulation of abstract concepts. Therefore, we based the design of our game on the results and reflections of an ongoing research project about playful learning in museums, and transposition of abstract notions into playful interactions. The museum project supported our intuition circa the need for multiple play styles, and suggested to map math processes into gameplay. The development followed an agile approach, with Java and Greenfoot as main tools.

A participatory design study is currently being conducted with local schools, to investigate the capabilities and limitations of that current prototype.

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[9] http://www.freeaudio.com

[10] http://audacity.sourceforge.net

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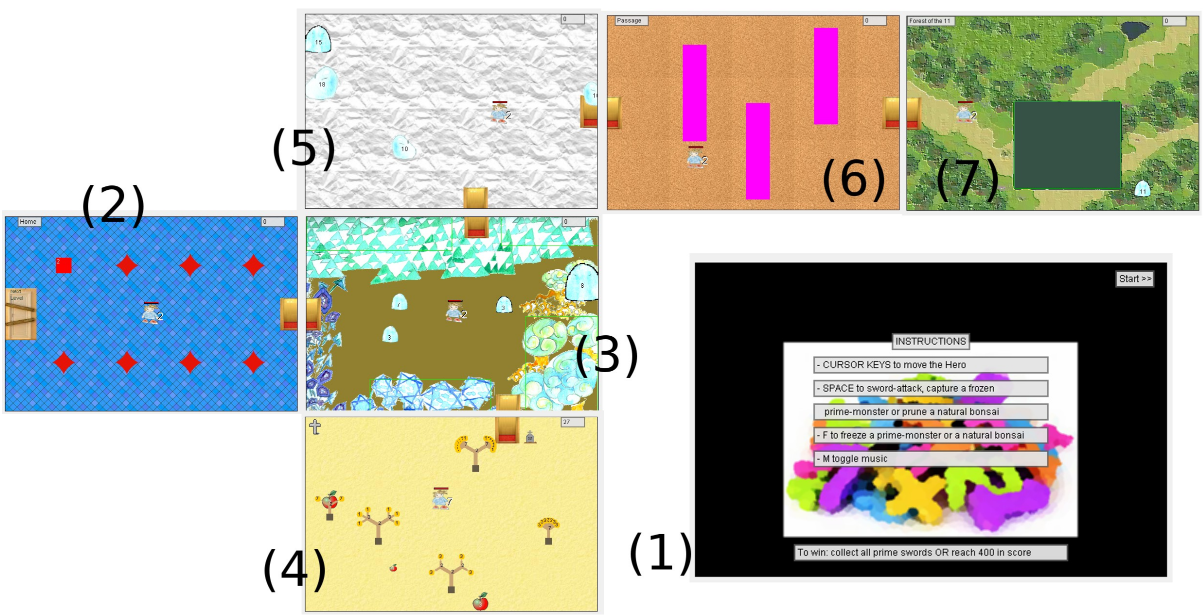

Figure 6. Map of the levels. On the bottom-right part of the figure, the initial menu is visible (1). The player will start her exploration from the far left of the figure, in the “Home” level (2). Through various doors, the player can reach any of the 5 other levels. The “bonsai graveyard” level (4) offers a different gameplay: the trees that grow there can be pruned using the player's swords. The shape and size of the trees will depend on the value in their seed, and on the way they are pruned.