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Five Novel Selection Policies for N2R Network Structures

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***Abstract* — This paper shows how 5 new selection policies can be applied to N2R structures. For each number of nodes, a selection policy determines which topology is chosen. Compared to approaches taken previously, the policies proposed in this paper allow us to choose structures which are significantly easier to implement, while having only slightly longer distances. The 5 policies reflect different trade-offs between distances and ease of implementation, and two of them explore the potentials of using $\mathbf{N2R(p;q;r)}$ instead of $\mathbf{N2R(p;q)}$ structures.**

***Keywords* — Communication Networks, Network Topology, Network Planning, Generalized Petersen Graph, N2R networks.**

1. Introduction

New broadband infrastructures are currently being implemented all over the world. Fiber To The Home (FTTH) is the most promising technology, offering almost unlimited bandwidth to the end users. However, it requires a full wired infrastructure to be implemented, which is a huge and expensive task. While the equipment used in FTTH networks can be upgraded quite easily, and is expected to be so during their lifetime, the physical network topologies are hard to change once the infrastructure is implemeted: complete rewiring should be avoided if at all possible. Therefore physical network topologies must be carefully planned prior to implementation.

Recently, most focus has been put on the bandwidth offered by new technologies. While bandwidth is indeed a key factor, the increasing demands for reliability should not be forgotten. Many new applications that require high levels of reliability, have been introduced recently, and more are under development[1]. At the same time there is an increasing general dependency on computers and computer networks. This is gradually leading to a situtation where even short periods of network outage is becoming critical for business users[2] as well as for normal households[3].

What reliability can be offered in the highest layers of a network depends on the physical network topologies: no algorithm can perform better than what is allowed by the physical infrastructure. Therefore network topologies should be chosen, which offer short distances in the network, even when restoration and protection schemes are used. Furthermore, the topologies should have a high level of symmetry[4], and in order to facilitate embeddings along the road network the node degrees should be kept low.

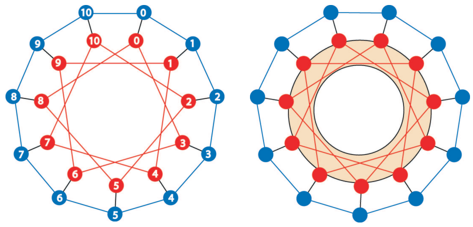

Figure 1. $\mathbf{N2R(11;3)}$ and the tube implementation.

In order to meet these requirements, 3-regular topologies are interesting. $N2R$ networks[5] (a subset of the Generalized Petersen Graphs[6]) have proved to be particularly interesting, with shorter distances than e.g. Double Rings[7] and Degree Three Chordal Rings[8][9]. Given a desired number of nodes in a network, there may exist several $N2R$ structures with different properties. It is crucial to have a selection policy for choosing one structure given the number of nodes, e.g. when comparing $N2R$ structures to other topologies, or when choosing a structure for implementation. In previous studies [7][9] $N2R$ structures were chosen to reduce diameter and average distance, an approach which also minimizes or nearly minimizes other key distance parameters. However, this often leads to highly non-planar structures with many crossing lines, making routing and implementation difficult. Even if implemented by shared ducts, such as the tube in Figure 1[10], huge amounts of fiber are required. Routing studies have indicated that a different selection policy can result in structures which are easier to embed and implement, and have only slightly higher distances[11]. This hypothesis is further investigated in this paper, which contributes to the field by proposing and evaluating five such novel selection policies.

2. Preliminaries

A structure is a set of nodes and a set of lines, where each line interconnects two nodes. Lines are bi-directional, so if a pair of nodes (u, v) is connected, so is (v, u). A structure can be considered a model of a network, abstracting from specific physical conditions such as node equipment, media and wiring, and the definition is similar to that of a simple graph: a path between two distinct nodes u and v is a sequence of nodes and lines: $(u = u_0), e_1, u_1, e_2, u_2, \ldots, u_{n-1}, e_n, (u_n = v)$, such that every line e_i connects the nodes u_{i-1} and u_i. The length of a path equals the number of lines it contains, so in the case above the path is of length n. The distance between a pair of distinct nodes (u, v) equals the length of the shortest path between them and is written $d(u, v)$. This paper considers only connected structures, i.e. between every pair of distinct nodes there exists a path. Two paths between a pair of nodes (u, v) are said to be independent if they share no lines or nodes except for u and v, and a set of paths is said to be independent if the paths are pair wise independent. The size of a structure equals the number of nodes it contains.

$N2R$ structures are defined as follows[5]. Let p and q be positive integers, such that $p \geq 3$, $q < \frac{p}{2}$ and $gcd(p, q) = 1$. p and q then define a structure $N2R(p; q)$, which consists of two rings, an outer ring and an inner ring, each containing p nodes. The nodes of the outer ring are labeled $o_0, o_1, \ldots, o_{p-1}$ and the nodes of the inner ring labeled $i_0, i_1, \ldots, i_{p-1}$. Thus, it contains $2p$ nodes. For each i such that $0 \leq i \leq p - 1$ there exists a line between each of the following pairs of nodes:

- $(o_i, o_{i+1(mod\ p)})$ (lines of the outer ring)
- $(i_i, i_{i+q(mod\ p)})$ (lines of the inner ring)
- (o_i, i_i) (lines connecting the two rings)

The classical double ring with $2p$ nodes obviously corresponds to $N2R(p; 1)$. An example of a $N2R$ structure is shown in Figure 1. One more restriction to q given p applies throughout the paper: given p, let $q_1 < q_2$ fulfill for $i = 1, 2$ that $q_i < \frac{p}{2}$ and $gcd(q_i, p) = 1$. Then $N2R(p; q_1)$ is isomorphic to $N2R(p; q_2)$ if $q_1 q_2 = 1(mod\ p)$ or $q_1 q_2 = p - 1(mod\ p)$. For such two isomorphic structures q_2 is discarded and only q_1 considered a permitted value.

The definition can be expanded to cover a third parameter, r. In this case we write $N2R(p; q; r)$. For a $N2R(p; q; r)$ structure it is additionally assumed that r is a positive integer, that $r < \frac{p}{2}$ and that $gcd(p, r) = gcd(q, r) = 1$. A $N2R(p; q; r)$ structure is defined similar to a $N2R(p; q)$ structure, except for the outer ring: Any node o_i is connected to $o_{i+r(mod\ p)}$ instead of $o_{i+1(mod\ p)}$. It is easily seen that $N2R(p; q; 1)$ is equivalent to $N2R(p; q)$ for all values of p and q.

2.1 Evaluation parameters

Widely used distance measures for network topologies are average distance and diameter, indicating transmission delays as well as traffic load[12].

- Average distance: The average of $d(u, v)$ taken over all pairs of distinct nodes.
- Diameter: The maximum of $d(u, v)$ taken over all pairs of distinct nodes.

For real-time applications where even short transmission outages are not acceptable, protection schemes are used. For this, k paths are established when the connection is set up. Traffic can be sent simultaneously along all these k paths, or along only one path, keeping the last $k - 1$ path(s) ready for immediate use whenever a failure is detected. In both cases, long restoration times are avoided. The k-measures k-average distance and k-diameter reflect the considerations of average distance and diameter, and are considered key parameters:

- k-average distance: For every pair of distinct nodes (u, v), k independent paths between u and v are constructed such that the sum of the lengths of these paths is smallest possible. The k-average distance is the average of these sums over all pairs of distinct nodes.
- k-diameter: For every pair of distinct nodes (u, v), k independent paths between u and v are constructed such that the longest of these paths is shortest possible. The k-diameter is the maximum over the lengths of these longest paths, over all pairs of distinct nodes.

Since $N2R$ structures are 3-regular these parameters are considered for $k = 2, 3$. 1-average distance and 1-diameter equal average distance and diameter. Where not confusing, we will simply write k-average instead of k-average distance.

3. The selection policies

The following selection policies form the base for this paper. Each policy describes how q given p is chosen among the permitted values of q. In the last policy, both q and r are chosen.

- Policy 1 (P1): In P1 q is chosen such that the diameter is smallest possible. If more values of q satisfy this, q is chosen among these such that the average distance is smallest possible. If more values of q still satify the requirements, q is chosen to be smallest possible.
- Policy 2 (P2): In P2 q is the smallest value of $q*$ satisfying the following conditions. Let q_+ be the smallest permitted value of q such that $q* < q_+$ (if it exists) and let q_- be any permitted value of q such that $q_- < q*$ if it exists (the properties listed must hold for all such possible values of q_-). The diameter of $N2R(p; q*)$ is smaller than or equal to the diameter of $N2R(p; q_-)$, and if the diameters of such two structures are equal, the average distance of $N2R(p; q*)$ is strictly smaller than that of $N2R(p; q_-)$. The diameter of $N2R(p; q_+)$ is equal to or higher than the diameter of $N2R(p; q*)$. If the diameters are equal, the average distance of $N2R(p; q_+)$ is equal to or higher than that of $N2R(p; q*)$.
- Policy 3 (P3): First P2 is used to obtain an average distance and diameter of $N2R(p; q*) = N2R(p; q*; 1)$.

Then q and r are chosen such that $q + r$ does not exceed $q * +1$, and such that first diameter and second average distance is smallest possible. If these parameters equal those of $N2R(p; q*)$, $N2R(p; q*; 1)$ is chosen. For additional calculations we also consider the cases where $N2R(p; q; r)$ can be chosen with average distance and diameter as for $N2R(p; q*)$, but allowing for $q \neq q*$ as long as $q + r \leq q * +1$.

P1 is the selection policy applied in previous studies, and corresponds to selecting a global minimum of diameter and average distance. P2 is the first novel selection policy proposed in this paper, corresponding to selecting the first local minimum of diameter and average distance. P3 is similar to P2, but slightly more advanced, since it allows for using $N2R(p; q; r)$ instead of $N2R(p; q)$.

All these policies minimize diameter and average distance, globally or locally. Sometimes, smaller values of q may have only slightly larger distances than those found by the selection policies, and thus be a better choice. Since the distance characteristics vary greatly with p, it is hard to present this trade-off in a general manner. It is our hypothesis however, that for structures with equal diameters, the average distances vary only slightly. Therefore we also test 3 alternative policies, P1x, P2x, and P3x. These correspond to P1, P2, and P3, except that only diameter is considered.

All the selection policies ensure that for each value of p, only single values of q and r are chosen. For $N2R(p; q)$, the value of q can be used to indicate how non-planar a structure is, and since q is the number of "parallel" lines of the inner ring, it also indicates how much fiber is needed for the tube implementation compared to the double ring. When $N2R(p; q; r)$ is used, it makes more sense to compare the values of $q + r$, since the parallel lines can be found in both outer and inner rings.

4. Methods

Calculations are performed for structures with $3 \leq p \leq 500$, i.e. for structures with up to 1000 nodes. For each value of p, q and r are found according to the various selection policies (for P1, P1x, P2, and P2x we set $r = 1$), and the 6 distance parameters determined for each of these $N2R$ structures.

Due to the symmetries, it is for each structure sufficient to calculate the distance parameters from one node in the outer ring and from one node in the inner ring. Average distance and diameter are easily calculated while the other parameters are more difficult to determine; in this study they are all basically calculated brute-force using an integrated algorithm in order to improve efficiency.

5. Results

An overview of the results is provided in Table 1, where each of the new selection policies introduced in the paper are compared to the P1 policy. For each policy, the percentage of values of p for which it yields a different value of q than P1 is listed. Then, over these cases, the average of the differences for each evaluation parameter is also listed.

Table 1. The percentage of values of p for which the proposed selection policies yield a different structure than using P1, and the average differences of the evaluation parameters taken over the values of p, where the structures differ.

Pol	Pct. Diff	Average of differences in % of P1 values						
		avg	2avg	3avg	dia	2dia	3dia	q (q+r)
P1x	52.7	1.03	0.84	1.01	0	2.33	2.26	49.1(47.9)
P2	77.1	5.62	5.12	5.15	13.8	9.48	8.92	62.3(60.8)
P3	77.1	5.47	5.00	5.00	13.3	9.43	8.87	(61.0)
P2x	92.0	10.7	9.81	9.48	22.2	18.4	17.4	64.5(62.6)
P3x	92.0	10.6	9.81	9.48	22.2	18.4	17.4	(62.6)

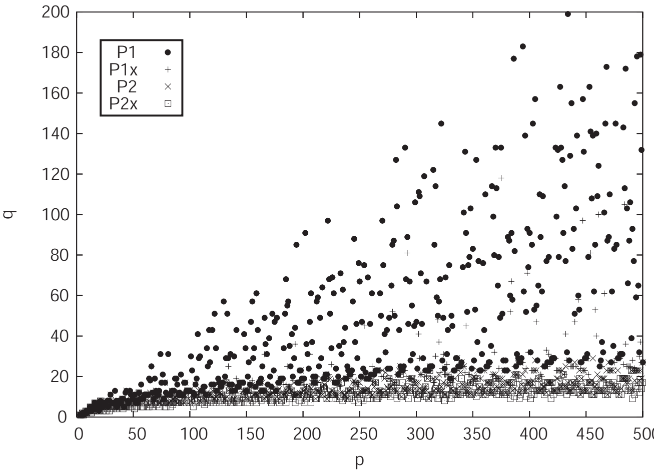

Figure 2. The values of q obtained using the selection policies P1, P1x, P2 and P2x.

In most cases, P2 and P3 as well as P2x and P3x result in the same structures, so when representing the results as in Table 1 the differences between these policies seem very small. For this reason, the reminder of this section is divided in two, so that first P1x, P2, and P2x are compared to P1, and then P3 and P3x are compared to P2 and P2x.

5.1 Comparison of P1, P1x, P2, and P2x

The values of q resulting from the four selection policies are shown in Figure 2, and the indexes related to the other parameters are shown in Figures 3-8. Only values different from one are shown: In order to support a visual presentation of the results, an index value is for each structure calculated for each parameter. Assume that $N2R$ is chosen according to selection policy P, and that a parameter $\text{Parameter}_{\text{P}}$ is obtained. Then the index for this parameter is obtained by $\frac{\text{Parameter}_{\text{P}}}{\text{Parameter}_{\text{P1}}}$, where $\text{parameter}_{\text{P1}}$ is the parameter calculated for $N2R$ chosen according to P1.

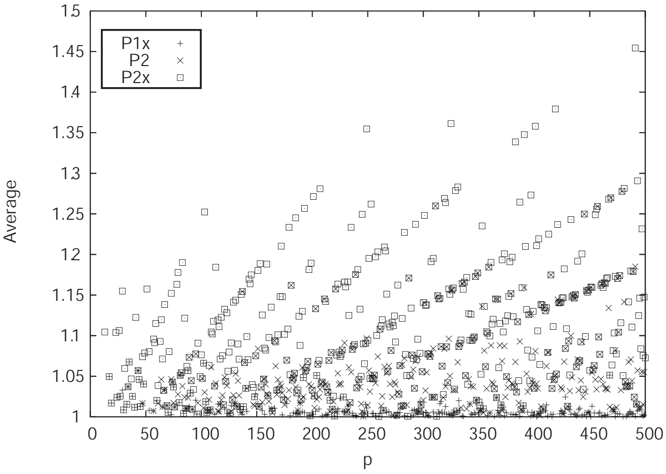

Figure 3. Average distances for P1x, P2 and P2x (indexes) compared to P1.

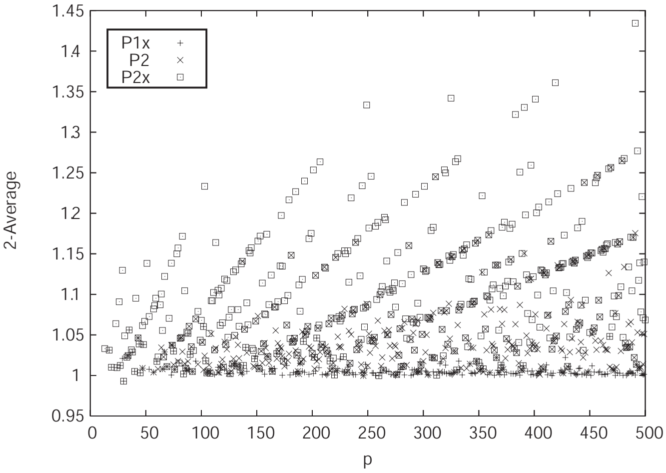

Figure 4. 2-Average distances for P1x, P2 and P2x (indexes) compared to P1.

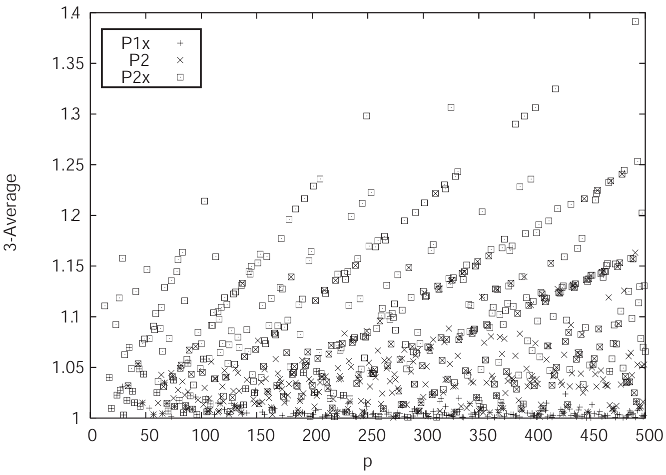

Figure 5. 3-Average distances for P1x, P2 and P2x (indexes) compared to P1.

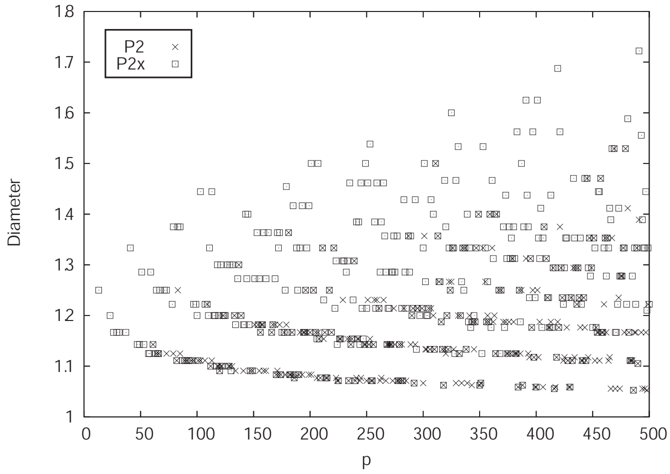

Figure 6. Diameters for (P1x,) P2 and P2x (indexes) compared to P1.

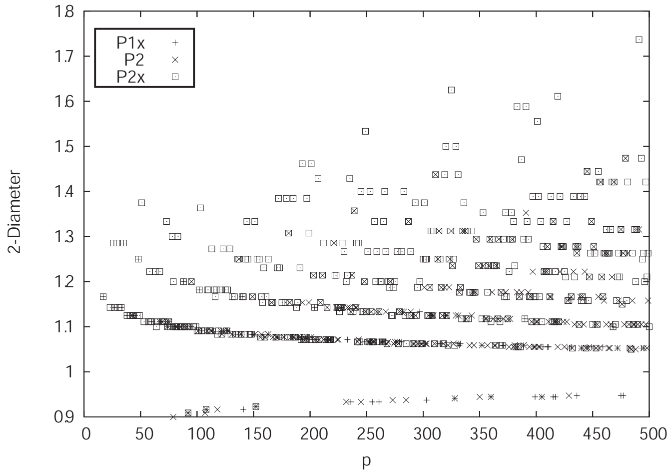

Figure 7. 2-Diameters for P1x, P2 and P2x (indexes) compared to P1.

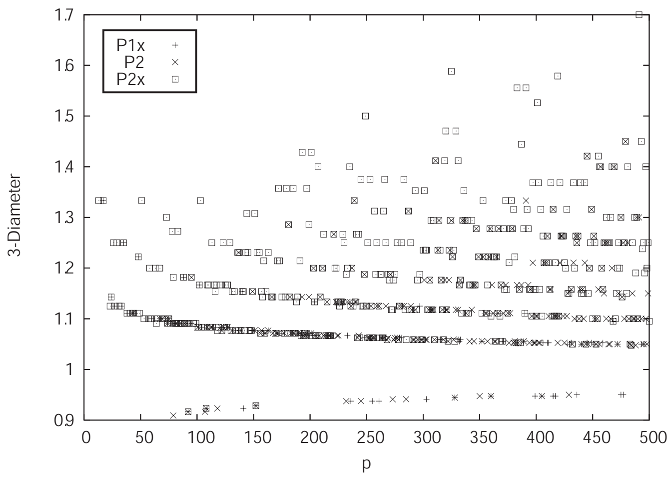

Figure 8. 3-Diameters for P1x, P2 and P2x (indexes) compared to P1.

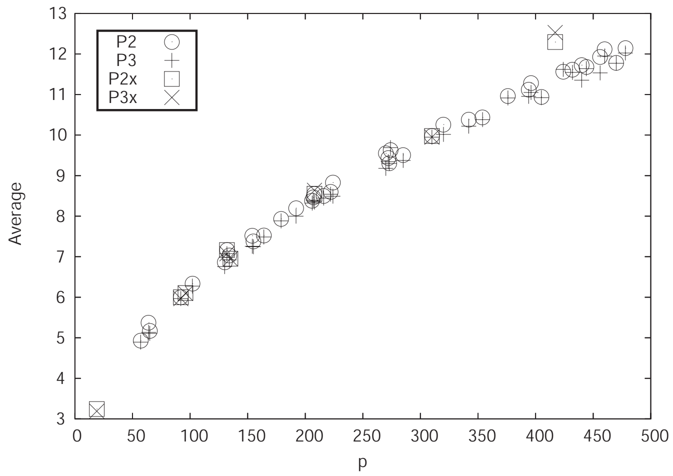

Figure 9. Average distances for P2, P2x, P3, and P3x when P2 and P2x respectively P3 and P3x yield different structures.

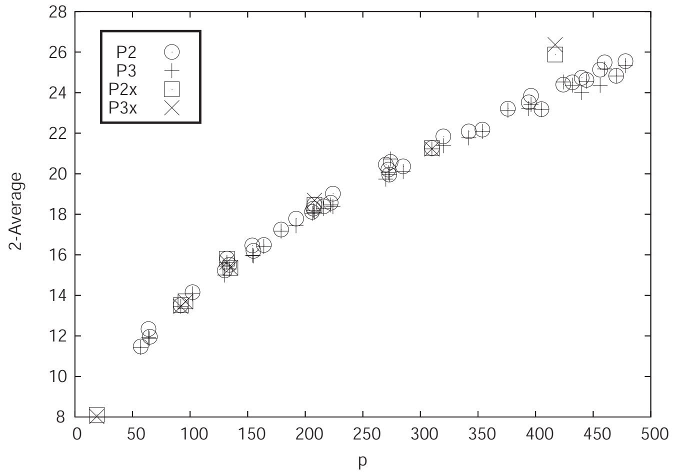

Figure 10. 2-Average distances for P2, P2x, P3, and P3x when P2 and P2x respectively P3 and P3x yield different structures.

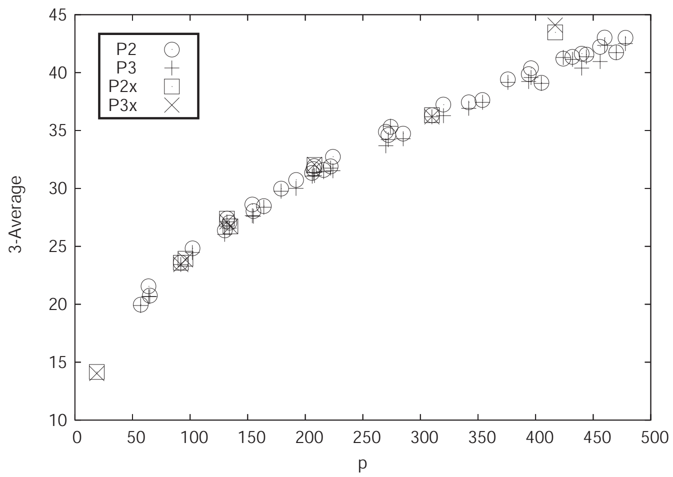

Figure 11. 3-Average distances for P2, P2x, P3, and P3x when P2 and P2x respectively P3 and P3x yield different structures.

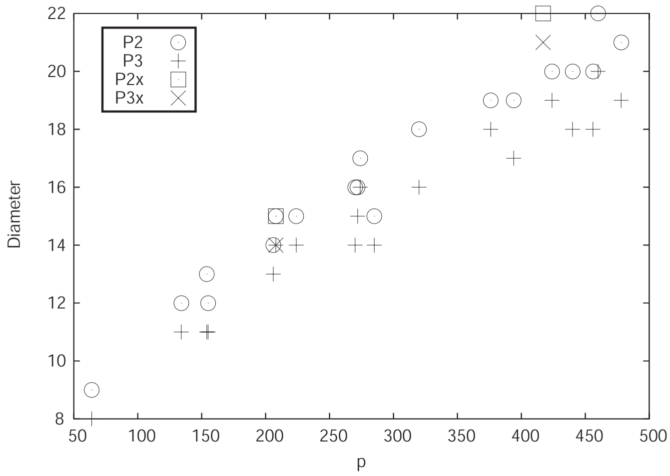

Figure 12. Diameters for P2, P2x, P3, and for P3x when P2 and P2x respectively P3 and P3x yield different structures.

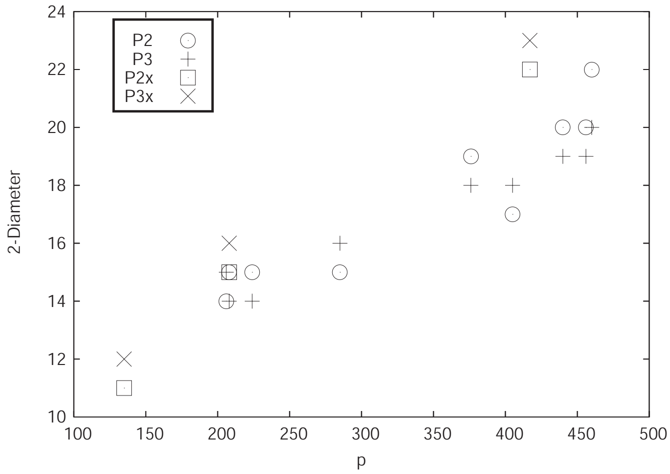

Figure 13. 2-Diameters for P2, P2x, P3, and P3x when P2 and P2x respectively P3 and P3x yield different structures.

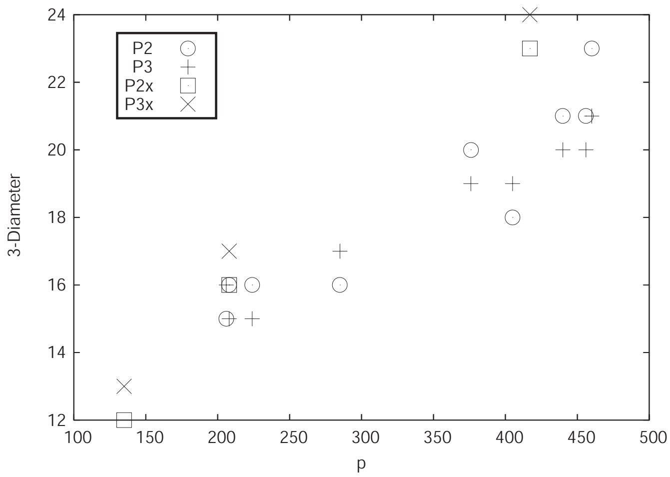

Figure 14. 3-Diameters for P2, P2x, P3, and P3x when P2 and P2x respectively P3 and P3x yield different structures.

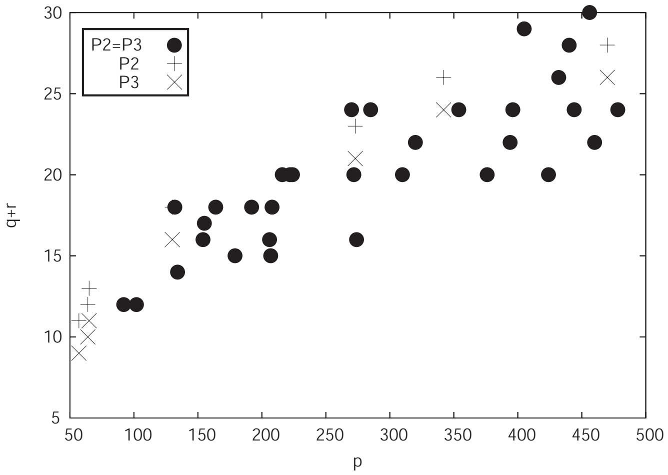

Figure 15. The values of $\mathbf{q} + \mathbf{r}$ obtained using the selection policies P2 and P3, when they lead to different structures.

5.2 Comparison of P3 to P2 and P3x to P2x

When using P3 instead of P2, the value of $(q_{\mathrm{P3}}+r_{\mathrm{P3}})$ may be equal to or lower than that of $(q_{\mathrm{P2}}+1)$, where q_{P2} is the value of q given P2. See Figure 15. P2x and P3x usually result in the same structure, and in the 8 cases where different structures are chosen, only two lead to $(q_{\mathrm{P3x}}+r_{\mathrm{P3x}}) < (q_{\mathrm{P2x}}+1)$: for $p = 135$ $q_{\mathrm{P3x}}+r_{\mathrm{P3x}} = 11+2 = 13$ whereas $q_{\mathrm{P2x}} = 13$, and for $p = 208$ $q_{\mathrm{P3x}}+r_{\mathrm{P3x}} = 11+3 = 14$ whereas $q_{\mathrm{P2x}} = 15$.

In addition to these cases, where a better average distance/diameter was obtained using P3(X) instead of P2(X), it was in 65 cases possible to obtain the same average distance and diameter, while having $(q_{\mathrm{P3}}+r_{\mathrm{P3}}) < (q_{\mathrm{P2}}+1)$ (33 cases) or $(q_{\mathrm{P3}}+r_{\mathrm{P3}}) = (q_{\mathrm{P2}}+1)$ (33 cases). In the former 33 cases, the difference between $(q_{\mathrm{P3}}+r_{\mathrm{P3}})$ and $(q_{\mathrm{P2}}+1)$ is on average 2. Using P3x instead of P2x, it was similarly possible to obtain $(q_{\mathrm{P3x}}+r_{\mathrm{P3x}}) < (q_{\mathrm{P2x}}+1)$ in 2 cases (the difference being 1 and 2 respectively) and $(q_{\mathrm{P3x}}+r_{\mathrm{P3x}}) = (q_{\mathrm{P2x}}+1)$ in 15 cases.

For the values of p, where P2 and P3 respectively P2x and P3x result in differences in the other evaluation parameters, the differences are shown in Figures 9 - 14.

6. Conclusion and discussion

We showed that the proposed selection policies can be used to significantly reduce the number of parallel lines in $N2R$ networks, while only slightly affecting the distances. The resulting structures are less complex and easier to implement.

P1x was surprisingly efficient as it reduced the value of q in more than 50% of the cases, and in these cases the values were on average reduced by 49.1%. The distance parameters were on average increased only by a few percent, and the diameters were not affected at all. P2 and P2x also turned out to be efficient, with more and larger reductions of q than P1x. The price to pay was that the distances were larger, especially for P2x. While the choice of selection policy is a matter of trade-offs, we believe that P2 is in general a good policy: it avoids the largest of q-values as shown in Figure 2, while the distances shown in Figures 3-8 are kept satisfactory low.

In general, P3 and P3x did not yield significantly lower distances than P2 and P2x. However, P3 seems to be a good alternative to P2, since the number of "parallel" lines can be reduced: It was possible to obtain the same average distance and diameter as with P2, but with $(q_{P3}+r_{P3}) \leq (q_{P2}+1)$. This can facilitate implementation, and probably also reduce the drawbacks of using shared ducts for outer and inner rings when making tube implementations.

The policies allow us to reduce the number of parallel lines, but further research is needed to explore the exact impact on the problems which occur when multiple lines are cut simultaneously. Using traditional tube implementations of $N2R$ networks inevitably makes it difficult to offer short independent protection paths. Therefore, we also suggest further research to explore more robust ways of implementation. The results of this paper form an interesting base for such further studies.

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