A Leakage-Based MMSE Beamforming Design for a MIMO Interference Channel

Sun, Fan; De Carvalho, Elisabeth

Published in:
I E E E Signal Processing Letters

DOI (link to publication from Publisher):
10.1109/LSP.2012.2196040

Publication date:
2012

Document Version
Early version, also known as pre-print

A Leakage-Based MMSE Beamforming Design for a MIMO Interference Channel

Fan Sun and Elisabeth de Carvalho

***Abstract*—We propose a low complexity design of the linear transmit filters for a MIMO interference channel. This design is based on a minimum mean squared error (MMSE) approach incorporating the signal and the interference leakage for each transmitter. Unlike the previous methods, it allows a closed-form expression of the regularization factor for the MMSE transmit filter. Hence, it requires a lower computational complexity compared to the conventional MMSE approach that is optimally achieved by solving a polynomial equation to find the regularization factor. Furthermore, the mean squared error (MSE) performance of the proposed design is verified by simulations to have nearly no loss compared to the conventional MMSE approach.**

***Index Terms*—MMSE, MIMO interference channel, linear beamforming.**

I. Introduction

We consider the K-user multiple input multiple output (MIMO) interference channel, where each transmitter delivers data to its desired user while creating interference to the users served by the other transmitters. We propose a design of the transmit filters based on a minimum mean squared error (MMSE) approach. An MMSE design is desirable for several reasons. First, it addresses finite SNR regions which are critical to practical wireless systems, unlike designs based on matched filtering or zero-forcing filtering. Second, it generally offers low complexity solutions. More importantly, a recent paper [1] has proven that weighted mean squared error (MSE) minimization is equivalent to sum-rate maximization for the MIMO broadcast channel if the MSE weights are optimally adjusted. The weighted MMSE design is extended to the interference channel and the interfering broadcast channel in [2].

The design of the MMSE linear filters was done for a point-to-point MIMO channel in [3]–[5]. For the MIMO broadcast channel, [6] solves the joint transceiver design problem based on MMSE. The extension to the MIMO interference channel was presented in [7], which contains a step where the Lagrange multiplier corresponding to the power per transmitter is calculated via solving a polynomial equation.

We propose a modified MMSE approach of the transmit filters that avoids solving the polynomial equation. It thus offers a lower computational complexity while maintaining similar MSE performance. Our approach contains two main ingredients. First, the modified MMSE cost function, named as minimum mean squared error for signal and interference leakage (MMSE-SL), considers only the signal and the interference leakage delivered from one transmitter: the signal for the desired user and the leakage interfering with other users. Note that this approach is related to [8], where the signal-to-leakage-and-noise ratio (SLNR) is used as the maximization criterion. However, the SLNR criterion takes simply the suboptimal matched filtering at the receivers. The second ingredient is an additional degree of freedom as done in [5] for the point-to-point MIMO channel. This degree of freedom is a scalar that scales the received signal and is incorporated in the MMSE optimization. This design results in a closed-form expression of the Lagrange multiplier and hence also a closed-form solution of the transmit filter. The design of the transmit filters assumes the receive filters as known. For evaluation purposes, the proposed transmit filter design is incorporated into an iterative transceiver design alternating between transmit and receive filter optimization. Although the proposed transmit filter design is suboptimal because it is based on a modified version of the MMSE cost function, simulations show nearly no performance loss compared to [7].

II. System model

We consider the K-user MIMO interference channel as shown in Figure 1. The kth transmitter is equipped with M_k antennas and serves its desired user with N_k antennas. Transmitter k proceeds to transmit data vector $\mathbf{s}_k \in \mathbb{C}^{[d_k \times 1]}$

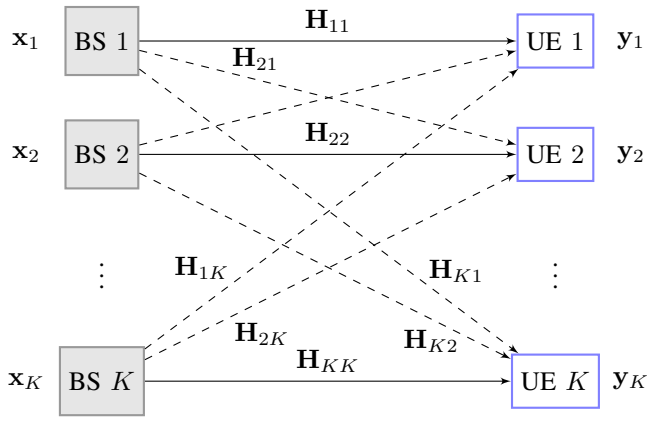

Fig. 1: System model: K-user MIMO interference channel.

to its user with $\mathbb{E}\left[\mathbf{s}_k\mathbf{s}_k^{\mathrm{H}}\right] = \mathbb{I}$, where d_k is the number of data streams to be delivered to user k. The value of d_k is chosen to fulfill the degrees of freedom (DOF) requirement in [9]. The data is pre-processed as $\mathbf{x}_k = \mathbf{B}_k\mathbf{s}_k$ where $\mathbf{B}_k \in \mathbb{C}^{[M_k \times d_k]}$ is the transmit filter for user k.

The complex-valued signal $\mathbf{x}_k \in \mathbb{C}^{[M_k \times 1]}$ fulfills the individual transmit power constraint $\mathbb{E}\left[\mathbf{x}_k^{\mathrm{H}}\mathbf{x}_k\right] = \mathrm{Tr}\left(\mathbf{B}_k\mathbf{B}_k^{\mathrm{H}}\right) \leq P_{tx}^k$. We denote $\mathbf{H}_{kk'}$ to be the narrowband MIMO channel from transmitter k' to user k. The signal received at user k is

$$\mathbf{y}_k = \mathbf{H}_{kk}\mathbf{B}_k\mathbf{s}_k + \sum_{k \neq k'} \mathbf{H}_{kk'}\mathbf{B}_{k'}\mathbf{s}_{k'} + \mathbf{n}_k$$

The authors are with the Department of Electronic Systems, Aalborg University, Denmark (e-mail: fs, edc@es.aau.dk). This work is supported by the Danish Research Council for Technology and Production, grant nr. 09-065920.

where $\mathbf{n}_k \in \mathbb{C}^{[N_k \times 1]}$ is a vector containing circularly symmetric white Gaussian noise with covariance $\mathbf{R}_{\mathbf{n}_k\mathbf{n}_k} = \mathbb{I}$. The effective noise covariance matrix is an important quantity accounting for the noise and the inter-user interference

$$\mathbf{R}_{\mathbf{I}_k} = \sum_{k' \neq k} \mathbf{H}_{kk'}\mathbf{B}_{k'}\mathbf{B}_{k'}^{\mathrm{H}}\mathbf{H}_{kk'}^{\mathrm{H}} + \mathbb{I}.$$

III. MMSE FOR TRANSCEIVER FILTERS

The MMSE criterion considers a joint design of the transceiver filters. Denoting $\mathbf{A}_k \in \mathbb{C}^{[d_k \times N_k]}$ as a linear receiver at user k, the MMSE cost function under individual power constraints is

$$\arg\min_{\mathbf{A}_k,\mathbf{B}_k} \sum_{k=1}^{K} \mathrm{Tr}\left(\mathcal{E}_k\right) \quad \text{s.t. } \mathrm{Tr}\left(\mathbf{B}_k\mathbf{B}_k^{\mathrm{H}}\right) \leq P_{tx}^k \quad \forall k \tag{1}$$

where $\mathcal{E}_k = \mathbb{E}\left[(\mathbf{A}_k\mathbf{y}_k - \mathbf{s}_k)(\mathbf{A}_k\mathbf{y}_k - \mathbf{s}_k)^{\mathrm{H}}\right]$ is the MSE. One advantage of this approach is that it leads to a low complexity alternating minimization procedure. Fixing the transmit filters and minimizing the MMSE cost function (1), we can find the well-known MMSE receive filter for user k as

$$\mathbf{A}_k^{\mathrm{MMSE}} = \mathbf{B}_k^{\mathrm{H}}\mathbf{H}_{kk}^{\mathrm{H}}\left(\mathbf{H}_{kk}\mathbf{B}_k\mathbf{B}_k^{\mathrm{H}}\mathbf{H}_{kk}^{\mathrm{H}} + \mathbf{R}_{\mathbf{I}_k}\right)^{-1}. \tag{2}$$

Then the optimization problem remains on how to find the transmit filters when the receive filters are fixed.

IV. COMPUTATION OF THE TRANSMIT FILTERS

Different techniques exist to determine the MMSE transmit filter for the MIMO interference channel (such as the methods described in [2] and [7]). We compare our method to the method in [7] as it offers the lowest computational complexity among the existing MMSE based methods, to the best of our knowledge. We first describe the reference method and then our proposed method.

A. Transmit Filter Design for MMSE

The problem of computing the MMSE transmit filters in the MIMO interference channel was treated in [7] when the receive filters are fixed. For this convex quadratic problem, the transmit filter for transmitter k can be found from the Karush-Kuhn-Tucker (KKT) conditions as

$$\mathbf{B}_k = \left(\sum_{k'=1}^{K} \mathbf{H}_{k'k}^{\mathrm{H}}\mathbf{A}_{k'}^{\mathrm{H}}\mathbf{A}_{k'}\mathbf{H}_{k'k} + \lambda_k\mathbb{I}\right)^{-1}\mathbf{H}_{kk}^{\mathrm{H}}\mathbf{A}_k^{\mathrm{H}}. \tag{3}$$

where λ_k is computed from the power constraint $\mathrm{Tr}\left(\mathbf{B}_k\mathbf{B}_k^{\mathrm{H}}\right) = P_{tx}^k$. This equation results in solving a polynomial equation of degree $2M_k$. When no $\lambda_k \in \mathbb{R}_+$ is found, λ_k is set to 0 in [7].

B. MMSE for signal and interference leakage (MMSE-SL)

In this section, we introduce a modified MMSE criterion which leads to a closed form expression of the Lagrange multiplier λ_k and hence save some significant computational complexity. For the MIMO point-to-point channel, an answer to this issue was provided in [5]. [5] modifies the MMSE cost function and introduces an additional degree of freedom that allows for a simple expression of the regularizing factor and a simple multiplicative scaling of the transmit filter to comply with the transmit power constraint. Unfortunately, this technique cannot be directly extended to the MIMO interference channel because of the inter-user interference.

1) Rewriting the MMSE cost function: Traditionally, the MMSE approach in (1) considers the weighted squared error at each user and takes the sum of the weighted errors over all the users to get the final MMSE cost function. We take another view point. The channel from transmitter k to all users can be seen as a broadcast channel where user k receives the desired signal and other users receive the interference leakage.

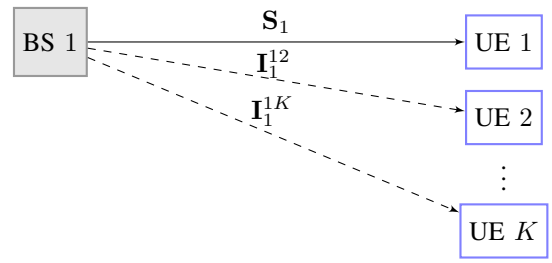

Fig. 2: Signal and interference leakage for transmitter 1.

- *Signal:* the desired signal received at user k is $\mathbf{H}_{kk}\mathbf{B}_k\mathbf{s}_k$. We use $\mathbf{S}_k = \mathbf{H}_{kk}\mathbf{B}_k\mathbf{s}_k + \mathbf{n}_k$ to denote the summation of the desired signal and the noise.
- *Interference Leakage to user k'*: the interference seen by user k' is $\mathbf{I}_k^{kk'} = \mathbf{H}_{k'k}\mathbf{B}_k\mathbf{s}_k$ $(k' \neq k)$.

Figure 2 illustrates these two types of flows delivered from transmitter 1. We denote

$$\mathbf{SI}_k = [\ (\mathbf{A}_k\mathbf{S}_k)^{\mathrm{H}} \quad (\mathbf{A}_1\mathbf{I}_k^{k1})^{\mathrm{H}} \quad \cdots \quad (\mathbf{A}_{k-1}\mathbf{I}_k^{k(k-1)})^{\mathrm{H}} \quad (\mathbf{A}_{k+1}\mathbf{I}_k^{k(k+1)})^{\mathrm{H}} \quad \cdots \quad (\mathbf{A}_K\mathbf{I}_k^{kK})^{\mathrm{H}}\ ]^{\mathrm{H}}$$

and $\mathbf{s}_k^{'} = [\ \mathbf{s}_k^{\mathrm{H}} \quad \mathbf{0}^{\mathrm{H}}\ ]^{\mathrm{H}}$ where $\mathbf{0}$ is a zero vector with proper size. The MSE expression applied to the signal and the interference leakage for transmitter k is defined as

$$\begin{aligned}\mathbf{MSE}_k^{SL} &= \mathrm{Tr}\left\{\mathbb{E}\left[(\mathbf{A}_k\mathbf{S}_k - \mathbf{s}_k)(\mathbf{A}_k\mathbf{S}_k - \mathbf{s}_k)^{\mathrm{H}}\right]\right\} \\ &\quad + \mathrm{Tr}\left\{\sum_{k' \neq k}\mathbb{E}\left[(\mathbf{A}_{k'}\mathbf{I}_k^{kk'}(\mathbf{I}_k^{kk'})^{\mathrm{H}}\mathbf{A}_{k'}^{\mathrm{H}}\right]\right\} \\ &= \mathrm{Tr}\left\{\mathbb{E}\left[\left(\mathbf{SI}_k - \mathbf{s}_k^{'}\right)\left(\mathbf{SI}_k - \mathbf{s}_k^{'}\right)^{\mathrm{H}}\right]\right\}.\end{aligned} \tag{4}$$

2) Modified MMSE cost function and derivation of the Lagrange multiplier: We describe the design of the transmit filters using the MMSE-SL criterion with fixed receive filters. Inspired by [5], an additional weighting β_k is introduced into the MMSE transmit filter design. The modified MSE expression for transmitter k is:

$$\begin{aligned}\mathbf{MSE}_k^{SL,\beta} &= \mathrm{Tr}\left\{\mathbb{E}\left[\left(\beta_k^{-1}\mathbf{SI}_k - \mathbf{s}_k^{'}\right)\left(\beta_k^{-1}\mathbf{SI}_k - \mathbf{s}_k^{'}\right)^{\mathrm{H}}\right]\right\} \\ &= \mathrm{Tr}\left\{\mathbb{E}\left[\left(\beta_k^{-1}\mathbf{A}_k\mathbf{S}_k - \mathbf{s}_k\right)\left(\beta_k^{-1}\mathbf{A}_k\mathbf{S}_k - \mathbf{s}_k\right)^{\mathrm{H}}\right]\right\} \\ &\quad + \mathrm{Tr}\left\{\sum_{k' \neq k}\beta_k^{-2}\mathbb{E}\left[(\mathbf{A}_{k'}\mathbf{I}_k^{kk'}(\mathbf{I}_k^{kk'})^{\mathrm{H}}\mathbf{A}_{k'}^{\mathrm{H}}\right]\right\}.\end{aligned} \tag{5}$$

We can find the KKT conditions for the transmit filter of the modified MMSE problem $f_k = \mathbf{MSE}_k^{SL,\beta} + \lambda_k\left[\mathrm{Tr}\left(\mathbf{B}_k\mathbf{B}_k^{\mathrm{H}}\right) - P_{tx}^k\right]$. Taking the partial derivatives:

$$\begin{aligned}\frac{\partial f_k}{\partial \mathbf{B}_k^*} &= \beta_k^{-2}\left(\sum_{k'=1}^{K}\mathbf{H}_{k'k}^{\mathrm{H}}\mathbf{A}_{k'}^{\mathrm{H}}\mathbf{A}_{k'}\mathbf{H}_{k'k}\right)\mathbf{B}_k \\ &\quad - \beta_k^{-1}\mathbf{H}_{kk}^{\mathrm{H}}\mathbf{A}_k^{\mathrm{H}} + \lambda_k\mathbf{B}_k = 0\end{aligned} \tag{6}$$

$$\beta_k^2 \mathrm{Tr}\left(\mathbf{B}_k^{\mathrm{H}} \frac{\partial f_k}{\partial \mathbf{B}_k^*}\right) = \mathrm{Tr}\left(\sum_{k'=1}^{K} \mathbf{H}_{k'k}^{\mathrm{H}} \mathbf{A}_{k'}^{\mathrm{H}} \mathbf{A}_{k'} \mathbf{H}_{k'k} \mathbf{B}_k \mathbf{B}_k^{\mathrm{H}}\right) - \beta_k \mathrm{Tr}\left(\mathbf{B}_k^{\mathrm{H}} \mathbf{H}_{kk}^{\mathrm{H}} \mathbf{A}_k^{\mathrm{H}}\right) + \alpha_k \mathrm{Tr}\left(\mathbf{B}_k \mathbf{B}_k^{\mathrm{H}}\right) = 0 \tag{8}$$

$$-\frac{1}{2} \ \beta_k^3 \ \frac{\partial f_k}{\partial \beta_k} = \mathrm{Tr}\left(\sum_{k'=1}^{K} \mathbf{H}_{k'k}^{\mathrm{H}} \mathbf{A}_{k'}^{\mathrm{H}} \mathbf{A}_{k'} \mathbf{H}_{k'k} \mathbf{B}_k \mathbf{B}_k^{\mathrm{H}}\right) - \beta_k \mathrm{Tr}\left(\mathbf{B}_k^{\mathrm{H}} \mathbf{H}_{kk}^{\mathrm{H}} \mathbf{A}_k^{\mathrm{H}}\right) + \mathrm{Tr}\left(\mathbf{A}_k \mathbf{A}_k^{\mathrm{H}}\right) = 0 \tag{9}$$

$$\begin{aligned} \frac{\partial f_k}{\partial \beta_k} &= 2\beta_k^{-2} \mathrm{Tr}\left(\mathbf{B}_k^{\mathrm{H}} \mathbf{H}_{kk}^{\mathrm{H}} \mathbf{A}_k^{\mathrm{H}}\right) - 2\beta_k^{-3} \mathrm{Tr}\left(\mathbf{A}_k \mathbf{A}_k^{\mathrm{H}}\right) \\ &\quad - 2\beta_k^{-3} \mathrm{Tr}\left(\sum_{k'=1}^{K} \mathbf{H}_{k'k}^{\mathrm{H}} \mathbf{A}_{k'}^{\mathrm{H}} \mathbf{A}_{k'} \mathbf{H}_{k'k} \mathbf{B}_k \mathbf{B}_k^{\mathrm{H}}\right) = 0 \end{aligned} \tag{7}$$

From (6), the transmit filter is $\mathbf{B}_k = \beta_k \tilde{\mathbf{B}}_k^{SL}$ with

$$\tilde{\mathbf{B}}_k^{SL} = \left(\sum_{k'=1}^{K} \mathbf{H}_{k'k}^{\mathrm{H}} \mathbf{A}_{k'}^{\mathrm{H}} \mathbf{A}_{k'} \mathbf{H}_{k'k} + \lambda_k \beta_k^2 \mathbb{I}\right)^{-1} \mathbf{H}_{kk}^{\mathrm{H}} \mathbf{A}_k^{\mathrm{H}}$$

where $\beta_k = \sqrt{P_{tx}^k / \mathrm{Tr}\left[\tilde{\mathbf{B}}_k^{SL} (\tilde{\mathbf{B}}_k^{SL})^{\mathrm{H}}\right]}$ sets the transmit power to P_{tx}^k. Using this method, all transmitters transmit at full power. Although full power transmission is not necessarily the optimal strategy[1] in the MIMO interference channel, we observe through simulations that this assumption results in no performance loss when compared to the reference method in [7]. By multiplying (6) with $\mathbf{B}_k^{\mathrm{H}}$ and introducing $\alpha_k = \lambda_k \beta_k^2$, (6) and (7) are transformed into (8) and (9), respectively. From the equality $\mathrm{Tr}\left(\mathbf{B}_k \mathbf{B}_k^{\mathrm{H}}\right) = P_{tx}^k$, $\alpha_k = \mathrm{Tr}\left(\mathbf{A}_k \mathbf{A}_k^{\mathrm{H}}\right) / P_{tx}^k$ is obtained. The solution is given as

$$\mathbf{B}_k = \beta_k \left(\sum_{k'=1}^{K} \mathbf{H}_{k'k}^{\mathrm{H}} \mathbf{A}_{k'}^{\mathrm{H}} \mathbf{A}_{k'} \mathbf{H}_{k'k} + \alpha_k \mathbb{I}\right)^{-1} \mathbf{H}_{kk}^{\mathrm{H}} \mathbf{A}_k^{\mathrm{H}}. \tag{10}$$

V. Alternating Optimization

The transmit filters depend on the optimal receive filters of all users, and vice versa. This naturally points towards iterative algorithms based on alternating minimization.

A. Alternating Minimization

We incorporate the transmit filter expression from (10) into an iterative algorithm for joint transceiver design. The iterative MMSE for signal and interference leakage (I-MMSE-SL) algorithm is as follows:

> ALGORITHM: **I-MMSE-SL**
> *set* $n = 0$ and $\mathbf{B}_k^{(0)} = \mathbf{B}_k^{(\mathrm{init})} \ \ \forall k$
> *iterate*
> *update* $n = n + 1$
> I. *compute* $\mathbf{A}_k^{(n)} | \mathbf{B}_k^{(n-1)} \ \forall k$ using (2)
> II. *compute* $\mathbf{B}_k^{(n)} | \mathbf{A}_k^{(n-1)} \ \forall k$ using (10)
> *until MSE convergence*

In Step II of I-MMSE-SL, we compute α_k, β_k and $\mathbf{B}_k$ sequentially. As a benchmark, we then substitute (10) with (3) in Step II to obtain the iterative MMSE (I-MMSE) algorithm. As highlighted in Section VI, the proposed I-MMSE-SL solution gives similar MSE results compared to I-MMSE.

[1]Power optimization is out of the scope of this work.

B. Convergence Analysis

For I-MMSE, the alternating minimization ensures convergence to a fixed point (also a stationary point of MSE). Indeed, the MSE is reduced each time the receive filters and the transmit filters are updated. Therefore, the MSE decreases monotonically after each iteration and as the MSE is lower bounded, I-MMSE is convergent. For the heuristic I-MMSE-SL algorithm, convergence is not straightforward to determine as the optimization of the transmit filters and the receive filters comes from two different cost functions. The convergence behavior in terms of the MSE is shown via simulation results in Section VI. The modified MSE optimization of the transmit filters is valid in a broadcast scenario when considering the MSE minimization of the error on the signal of interest and the interference leakage. However, the modified MSE optimization of the receive filters is invalid. At user k, signals from multiple transmitters correspond to multiple scaling factors $\beta_{k'}^{-1}$ $(1 \leq k' \leq K)$. But the receiver lacks of a sufficient number of degrees of freedom to compensate for all of them. In order to minimize the MSE, the MMSE receivers are applied.

C. Complexity Analysis

For updating $\mathbf{A}_k$, both algorithms require the same computational complexity: in $O[\sum_{k'=1}^{K}(2M_{k'}N_{k'}d_{k'} + M_{k'}^2 d_{k'})]$ for the matrix multiplications inside the inverse, in $O(N_k^3)$ for the inversion and in $O(N_k^2 M_k + M_k N_k d_k)$ for the external matrix multiplications with the inverse. When not accounting for the computation of the regularizing factor, both algorithms require the same computational complexity to update $\mathbf{B}_k$ in $O[\sum_{k'=1}^{K}(2M_{k'}N_{k'}d_{k'} + N_{k'}^2 d_{k'}) + M_k^3 + M_k^2 N_k + M_k N_k d_k]$. For I-MMSE-SL, the complexity to compute the regularizing factor in $\mathbf{B}_k$ is in $O(d_k N_k)$. For I-MMSE, the complexity to determine λ_k is evaluated based on the root search method relying on the computation of the eigenvalues of a companion matrix[2] of size $2M_k$ [10]: this complexity is of order $O(8M_k^3)$. This polynomial equation related complexity brought by the calculation of λ_k is obviously non-negligible and this complexity increases dramatically with the increase of the number of transmit antenna M_k for each communication pair. Therefore, we can see a clear reduction in complexity from using I-MMSE-SL especially when M_k is large.

D. Initialization Methods

We use two initialization methods for the transmit filters[3]: (1) Random initialization: Initialize all the transmit filters with

[2]Iterative methods such as the bisection method and the Newton method can also be applied. The complexity is difficult to determine as the convergence speeds depend on the polynomial coefficients and the required precision.

[3]We can also initialize the algorithms with interference alignment (IA) solutions [9] for high SNR performance, but this method is only applicable for certain cases.

i.i.d. Gaussian random variables; (2) Singular initialization: Initialize the kth transmit filter with the first d columns of the right singular matrix of $\mathbf{H}_{kk}$ [7]. The initial transmit filters are normalized to satisfy the individual power constraints.

VI. Numerical Evaluations

We focus on the 3-user MIMO interference channel with each transmitter and user having the same number of antennas $M_k = N_k = 4\ \forall k$. The number of data streams delivered by each transmitter is the same and the transmit power constraint is the same for all transmitters, $d_k = 2, P_{tx}^k = P\ \forall k$. A quasi-static flat Rayleigh fading channel model is used. The average energy of the channel between a transmitter and its desired user is $\sigma_h^{2\,\text{intra}}$; the average energy of the cross links is $\sigma_h^{2\,\text{inter}}$ with $\rho^{\text{gap}} = \sigma_h^{2\,\text{intra}}/\sigma_h^{2\,\text{inter}}$.

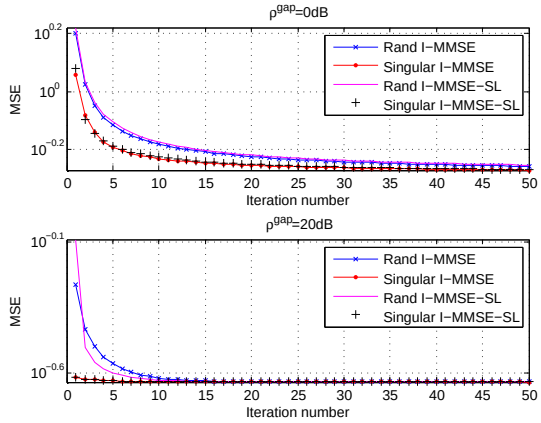

Fig. 3: MSE vs iteration number, $\rho^{\text{intra}} = 10$dB.

In Figure 3, we show the convergence behavior where the MSE is plotted against the iteration number for $\rho^{\text{gap}} = 0$dB and $\rho^{\text{gap}} = 20$dB. The MSE is averaged over sufficient channel realizations and the average SNR is fixed to be $\rho^{\text{intra}} = P\sigma_h^{2\,\text{intra}} = 10$dB. The plots indicate that the convergence speed is comparable for the I-MMSE and the I-MMSE-SL algorithms: 30 iterations appear to be sufficient. From the simulations, I-MMSE and I-MMSE-SL with the singular initialization have similar convergence speed; when ρ^{gap} increases, I-MMSE-SL converges faster compared to I-MMSE. Furthermore, the singular initialization results in better convergence compared to the random initialization; convergence becomes faster as ρ^{gap} increases. In the following, we use the singular initialization in both I-MMSE and I-MMSE-SL and stop at 30 iterations.

In Figure 4, we show the MSE performance of the different algorithms. The distributed max-SINR algorithm in [11] is also included as a benchmark. For a fair comparison, all the algorithms use the linear MMSE receiver in the simulations. We can also see at both $\rho^{\text{gap}} = 0$dB and $\rho^{\text{gap}} = 20$dB, I-MMSE-SL performs approximately the same as I-MMSE. Therefore, we can conclude that the low-complexity I-MMSE-SL algorithm has nearly no MSE performance loss compared to the I-MMSE algorithm. Furthermore, I-MMSE-SL performs better than max-SINR: there is an MSE error floor for the distributed max-SINR algorithm with singular initialization, which is also observed in [7].

In the high SNR region, if the transmit filters are initialized with IA solutions, the MSE results of both algorithms do not saturate. This is because the setup fulfills the DOF requirement from [9] and IA design for the transmission together with MMSE receivers achieves MSE minimization at high SNR in simulations.

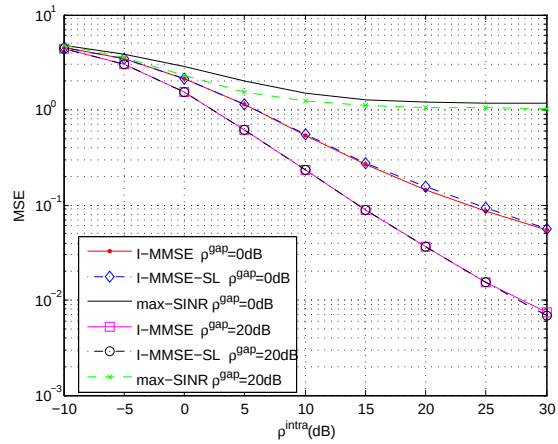

Fig. 4: MSE performance of the different algorithms.

VII. Conclusion

We have proposed a low-complexity design for the linear transmit filters in the MIMO interference channel. MMSE-SL relies on a modified MMSE cost function which considers the signal and the leakage interference for each transmitter. The proposed I-MMSE-SL algorithm guarantees convergence while ensuring a similar performance compared to the more complex I-MMSE algorithm.

References

[1] S. Christensen, R. Agarwal, E. Carvalho, and J. Cioffi, "Weighted sum-rate maximization using weighted MMSE for MIMO-BC beamforming design," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 4792–4799, Dec. 2008.

[2] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He, "An Iteratively Weighted MMSE Approach to Distributed Sum-Utility Maximization for a MIMO Interfering Broadcast Channel," *IEEE Trans. Signal Process.*, vol. 59, no. 9, pp. 4331–4340, Sept. 2011.

[3] H. Sampath, P. Stoica, and A. Paulraj, "Generalized linear precoder and decoder design for MIMO channels using the weighted MMSE criterion," *IEEE Trans. Commun.*, vol. 49, no. 12, pp. 2198–2206, Dec. 2001.

[4] D. Palomar, J. Cioffi, and M. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: a unified framework for convex optimization," *IEEE Trans. Signal Process.*, vol. 51, no. 9, pp. 2381–2401, Sept. 2003.

[5] M. Joham, W. Utschick, and J. Nossek, "Linear Transmit Processing in MIMO Communications Systems," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 2700–2712, Aug. 2005.

[6] S. Shi, M. Schubert, and H. Boche, "Downlink MMSE Transceiver Optimization for Multiuser MIMO Systems: Duality and Sum-MSE Minimization," *IEEE Trans. Signal Process.*, vol. 55, no. 11, pp. 5436–5446, Nov. 2007.

[7] H. Shen, B. Li, M. Tao, and X. Wang, "MSE-Based Transceiver Designs for the MIMO Interference Channel," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3480–3489, Nov. 2010.

[8] M. Sadek, A. Tarighat, and A. Sayed, "A Leakage-Based Precoding Scheme for Downlink Multi-User MIMO Channels," *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1711–1721, May 2007.

[9] T. Gou and S. A. Jafar, "Degrees of Freedom of the K User M x N MIMO Interference Channel," *IEEE Trans. Inform. Theory*, vol. 56, no. 12, pp. 6040–6057, Dec. 2010.

[10] A. Edelman and H. Murakami, "Polynomial roots from companion matrix eigenvalues," *Math. Comp*, vol. 64, pp. 763–776, 1995.

[11] K. Gomadam, V. Cadambe, and S. Jafar, "A Distributed Numerical Approach to Interference Alignment and Applications to Wireless Interference Networks," *IEEE Trans. Inform. Theory*, vol. 57, no. 6, pp. 3309–3322, June 2011.