Aalborg Universitet

Dynamic Heat Transfer Model of Refrigerated Foodstuff

Cai, Junping; Risum, Jørgen; Thybo, Claus

Published in:
Proceedings of the 11th International Refrigeration and Air Conditioning Conference

Publication date:
2006

Document Version
Publisher's PDF, also known as Version of record

Link to publication from Aalborg University

Citation for published version (APA):
Cai, J., Risum, J., & Thybo, C. (2006). Dynamic Heat Transfer Model of Refrigerated Foodstuff In *Proceedings of the 11th International Refrigeration and Air Conditioning Conference*

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal -

Take down policy
If you believe that this document breaches copyright please contact us at vbn@aub.aau.dk providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from vbn.aau.dk on: April 19, 2024

Dynamic Heat Transfer Model of Refrigerated Foodstuff

Junping Cai, Jørgen Risum and Claus Thybo

***Abstract*— Traditional control of commercial refrigeration systems focus on controlling the air temperature inside refrigerated display cabinets. It is the product temperature that directly influences the product quality, so if we want to store the foodstuff with an optimal quality, we need to know their temperature relation.**

This paper discusses the dynamic heat transfer model of foodstuff inside the display cabinet, one-dimensional dynamic model is developed, and the Explicit Finite Difference Method is applied, to handle the unsteady heat transfer problem with phase change, as well as time varying boundary condition. The influence of different factors such as air velocity, type of food, size of food, or food package are investigated, the question such as what kind of food are more sensitive to the surrounding temperature change is answered.

This model can serve as a prerequisite for modelling of food quality changes, thus enabling the possibility of improving the control of supermarkets refrigeration system.

I. INTRODUCTION

Traditional control of commercial refrigeration systems focus on controlling the air temperature inside refrigerated display cabinets. It is typically implemented as a hysteresis function that activates and deactivates refrigeration when the temperature reaches the cut-in and cut-out temperature of the hysteresis band, to maintain a desired air temperature around the foodstuff, not an optimal product temperature.

As it is well known, besides the strict legislative control from food authority regarding safety, a customer's perception of food as 'good' plays a vital role in the food retail industry. What characterizes 'good' food? It could be based on appearance, smell, taste, texture, or nutritious facts, etc.so if we want to store the foodstuff with an optimal quality, we need to know their temperature relation.

A well designed optimal control strategy for governing the food temperature can address all these requirements, but it requires some models that are capable of describing the thermal characteristics of food in the display cabinet. One possible solution which is described in the sequence* is to integrate the food thermal model and quality model into the model of a refrigeration system, to find an optimal temperature profile, which can maintain an optimal food quality, at the same time, take the overall energy consumption into consideration. The overall project prospective is depicted as Fig.1, where MPC is the abbreviation of model predicative control.

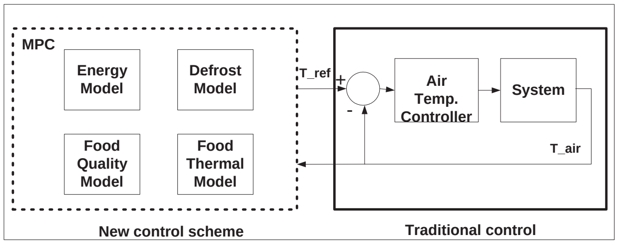


Fig. 1. Overall project prospective

As freezing and cold storage are very important operations in the food industry, there has been carried out a lot of research to predict the freezing time or thawing time [1] [2], mainly concerning is the surface and center temperature profile of the food during the process, which is often step response.

Our approach at this stage is to construct thermal models for some selected typical foodstuff in the supermarket. Here fish is selected as the quality of fish products are very sensitive to the temperature change. A one dimensional model is developed, where the explicit finite different method is used, to deal with the unsteady heat transfer problem with phase change, as well as time varying boundary condition.

This paper is organized as follows: the refrigeration of foodstuff in the display cabinet of supermarket is described in Section 2, where a simple introduction of frost and defrost is included. Since it is a critical and undesired phenomenon that dramatically influences the food quality. The mathematical model is introduced in Section 3, where the thermophysical properties of foodstuff are discussed. In Section 4 we discuss the different methodologies for solving this kind of problems. In Section 5, results are presented. Finally some discussion and conclusion are given in Section 6.

II. REFRIGERATION OF FOODSTUFF IN A SUPERMARKET

The display cabinet depicted in Fig. 2 consists of a food container and an air tunnel, circulating cold air around the food container. An evaporator in the air tunnel cools the passing air which creates a carpet of cold air on top of the food.

The fact that the air carpet is colder than the food and ambient air, will keep the air carpet in place and as it is more dense, enabling the desired effect of heat transfer from the curtain to the container and food. A side effect is that the ambient air will infiltrate into the curtain at the load zone. The display cabinet's temperature is normally controlled by a hysteresis controller which opens and closes the inlet valve, to control the flow of refrigerant into evaporator, thus keeping

---

J. Cai is with Department of Control Engineering, Aalborg University, 9220 Aalborg, Denmark `jc@control.aau.dk`

J. Risum is with BioCentrum, Technical University of Denmark, Copenhagen, Denmark `jr@biocentrum.dtu.dk`

C. Thybo is with Central RD - Refrigeration and Air Conditioning, Danfoss A/S, 6430 Nordborg, Denmark, `thybo@danfoss.com`

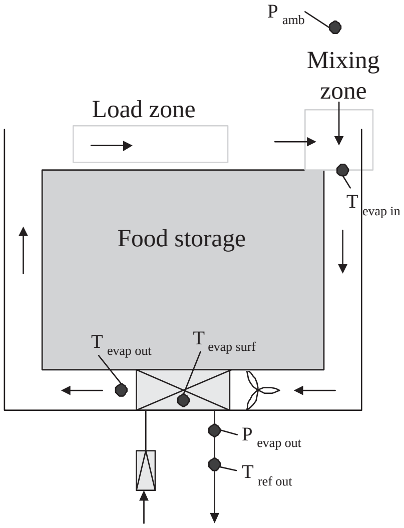


Fig. 2. A simplified display cabinet in the supermarket

the air temperature within the specific bands, see Fig.3 and Fig.4, where the big change in temperature is caused by the defrost cycle.

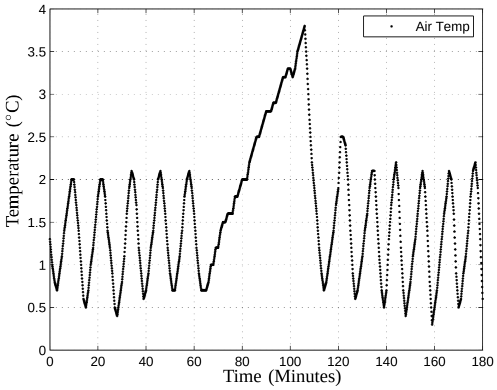


Fig. 3. Fresh fish air temperature profile

A. FROST FORMATION AND DEFROST CYCLE

Frost forms on display cabinet evaporator as water vapor in the air condenses and freezes when it contacts the coil surface which is normally below 0 °C. This is a well known and undesirable phenomenon. It deteriorates the system performance by decreasing the effective air flow area and increasing the thermal resistance between the air and the evaporator coils. Currently there are no clear and reliable measures that can prevent frost formation [3]. When frost accumulates to a certain level, defrost must be done to maintain a satisfactory system performance.

Defrost methods vary depending on the refrigeration application and storage temperature, and initiation and termination of defrost can be controlled by many different parameters, such as timer or temperature, sometimes within up to 3 cycles per day.

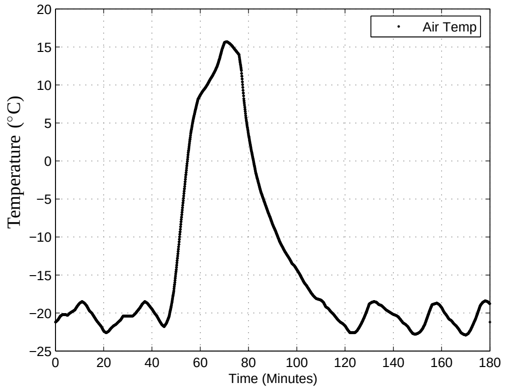


Fig. 4. Frozen fish air temperature profile

During the defrost cycle, the air temperature will increase and remain outside of the normal temperature range for a period of time.

B. LEGAL REQUIREMENTS

In the Danish supermarkets, there are legal requirements regarding the storage temperature for different foodstuff in the display cabinets [4], here the temperature is the air temperature:

- Frozen food, the maximum temperature is -18 °C.
- Fresh fish, the maximum temperature is +2 °C.
- Milk, the maximum temperature is +5 °C.

In general, the requirement is a temperature below +5 °C [5]. There are also some temperature requirements during the food processing and transportation.

III. MATHEMATIC MODEL

A. NUMERICAL SIMULATION OF THE UNSTEADY-STATE CONDUCTION

Unsteady state conduction refers to the class of problems in which the temperature of the conduction region varies with time, such as the case in which the boundary condition varies with time such that neither a steady state nor a periodic behavior is ultimately attained, and this is exactly our case.

Governing equation

$$\nabla(k(T)\cdot\nabla T) = \rho \cdot C_P(T)\cdot\frac{\partial T}{\partial t} \tag{1}$$

where $k(T)$,ρ, $C_P(T)$ are so called thermophysical properties of foodstuff: thermal conductivity, density and specific heat capacity, which will be discussed in details later.

Convective boundary condition

$$k(T)\frac{\partial T}{\partial n} = h(T - T_\infty) \tag{2}$$

where h is the convective heat transfer coefficients, and T_∞ is the ambient temperature.

Initial condition,

$$T = T_0 \tag{3}$$

where T_0 is a known temperature.

B. TEMPERATURE-DEPENDENT THERMOPHYSICAL PROPERTIES

In many engineering problem the variation of the thermal properties is significant over the temperature range concerned and must be taken into consideration [6].

Predictive equation based on the water content φ, actual and initial freezing temperature T_{cr} developed by [7] is applied in the modelling of fish.

The water content of the fish varies with different species of fish as the composition is different. Even for the same fish, its constitute values also change in the different environments and different seasons. Based on the source [8], the water content varies between 53% (for very fatty fishes) and 83% (for lean fishes). Here cod is selected as one example, with water content of 79.3%.

1) SPECIFIC HEAT CAPACITY C_p*:* For the food with T_{cr} between $-2 \sim -0.4°C$, it can be calculated as this.

$$C_p(T) = \begin{cases} Cp_{un} = 2.805\varphi + 1.382 & \text{for } T \geq T_{cr} \\ Cp_{fr} = 1.382 - \varphi A(T) - \varphi B(T) & \text{for } T < T_{cr} \end{cases} \quad (4)$$

where

$$A(T) = \frac{2.286}{1 + 0.7138/\ln(T_{cr} - T + 1)} - 2.805$$

$$B(T) = L\frac{dw(T)}{dt} = \frac{-264.231(T_{cr} - T + 1)^{-1}}{(\ln(T_{cr} - T + 1) + 0.7138)^2}$$

$$-45 \leq T \leq 45°C$$

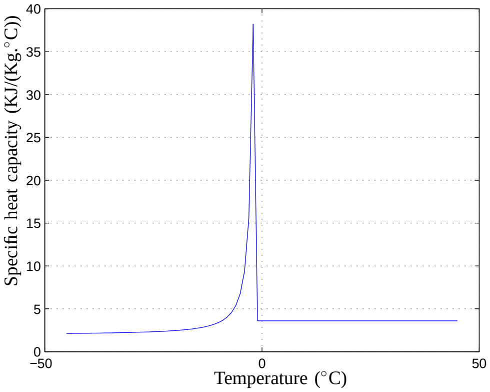


Fig. 5. Dependence of specific heat capacity with temperature

2) THERMAL CONDUCTIVITY k*:* It is an intrinsic property of the material, which depends on its composition and structure. The empirical correction proposed by [9] can be used with satisfactory precision for engineering investigation.

$$k(T) = k_{un}(T) + f\varphi(k_i(T) - k_w(T)) \quad (5)$$

where f is a correction factor, which for meat and fish varies from 0.61 to 0.77, k_{un}, k_i and k_w is the thermal conductivity for that unfrozen food, ice and water respectively.

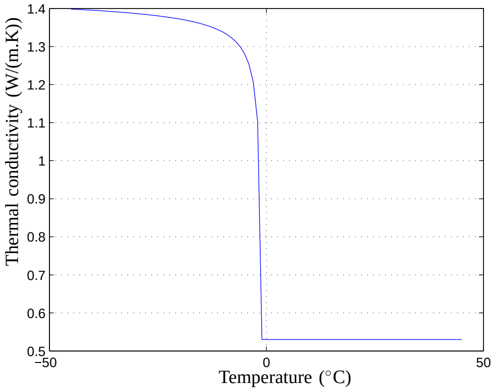


Fig. 6. Dependence of thermal conductivity with temperature

3) DENSITY ρ*:* The density temperature dependence, predetermined mainly by the small difference between water and ice densities, is comparatively weakly expressed. So the average density for frozen and unfrozen fish is used respectively in the model.

IV. METHODOLOGY

The transit heat transfer problems involving melting and solidification are generally refer to as the ”phase change” or the ”moving boundary” problem. Sometimes they are referred as the ”Stefan” problems [10]. There has been developed some methods and algorithms dealing with such kind of problem.

A. ENTHALPY METHOD

It use the mass specific enthalpy, so there is no need to accurately track the phase-change boundary, no need to consider liquid and solid regions separately, example see [11].

B. IMPROVED ENTHALPY METHOD

It combines the volumetric specific enthalpy and Kirchhoff transformation, so that all the non-linearities, caused by the thermal properties depending on temperature, are introduced in the functional relationship between the volumetric specific enthalpy, and Kirchhoff function, details see [7].

Both these methods needs much effort on programming, and often are solved by finite element methods.

C. FINITE DIFFERENCE METHOD

Here finite difference method (FDM) is chosen for simplicity, 1D case is used as an example.

The governing equation (1) can be discretized as:

$$(\rho C_P)_i \frac{T_i^{n+1} - T_i^n}{\Delta t} = \theta\left(k_{i-\frac{1}{2}}\frac{T_{i-1}^{n+1} - T_i^{n+1}}{(\Delta x)^2} + k_{i+\frac{1}{2}}\frac{T_{i+1}^{n+1} - T_i^{n+1}}{(\Delta x)^2}\right) + (1-\theta)\left(k_{i-\frac{1}{2}}\frac{T_{i-1}^n - T_i^n}{(\Delta x)^2} + k_{i+\frac{1}{2}}\frac{T_{i+1}^n - T_i^n}{(\Delta x)^2}\right) \quad (6)$$

where constant $\theta (0 \leq \theta \leq 1)$ is the weight factor which represent the degree of implicitness, the value of $\theta = 0, 1/2, 1$ respectively corresponds to the explicit, Crank-Nicolson and implicit method. In this paper, the explicit FDM is applied, while it has the restriction on time step and space step, in order to ensure the convergence.

D. TEMPERATURE-DEPENDENT PROPERTIES EVALUATION

In these transient problems, the temperature variation in the conduction region is small, so average value of the material properties will be taken at each time interval.

Simplest method to evaluate the property is to lag the evaluation by one time step, such as

$$k^{n+1} = k(T^{n}) \tag{7}$$

Some more complicate methods includes evaluating the properties as or at the average value, or use an exploration scheme, for example,

$$k^{n+1} = k^{n} + \left(\frac{\partial k}{\partial T} \right)^{n} (T^{n} - T^{n-1}) \tag{8}$$

E. INTERPOLATION OF AIR TEMPERATURE IN BOUNDARY CONDITION

The air temperature in the real refrigeration system is sampled every minute, it varies with time. To apply it in the dynamic convective boundary condition, we need to interpolate it for each time step, the most easy way is to assume it is linear within one minute.

V. RESULT

The results are obtained by FDM and based on the parameters in Table I.

TABLE I

PARAMETERS USED IN THE SIMULATION

Specification	Cod	Herring	Unit
Heat transfer coefficient	5	5	$W/(m^2.K)$
Relative water content	0.793	0.53	Kg/Kg
Thermal conductivity, unfrozen	0.53	0.796	$W/(m.K)$
Correction factor	0.70	0.70	
Density, unfrozen	1050	930	Kg/m^3
Density, frozen	960	850	Kg/m^3
Initial freezing point	-1	-1	°C
Thermal conductivity ice	2.18	2.18	$W/(m.K)$
Thermal conductivity water	0.58	0.58	$W/(m.K)$

And following assumptions:

- A infinite plate of minced fish with the thickness of 20mm.
- The fish has the both sides the same convective boundary condition

Since it is symmetric, so only half of it need to be considered.

Fresh fish: we ignore the small change in the thermal properties above the initial freezing point, and assume they are constant, the result is shown in Fig. 7 and 8. In defrost cycle, the surface temperature increase about 0.4 °C, and the maximum temperature difference between the surface and center is less than 0.1 °C.

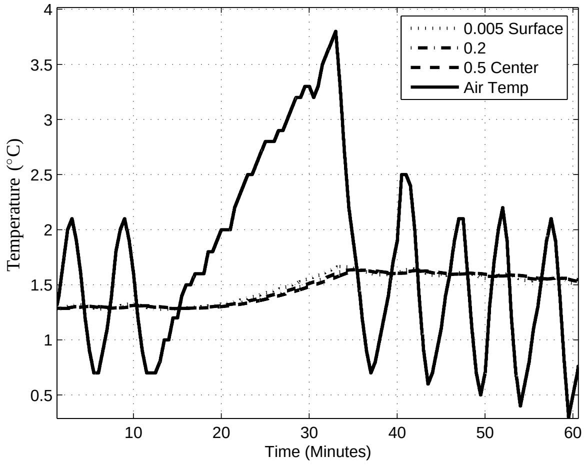


Fig. 7. Fresh fish time temperature profile in different depth

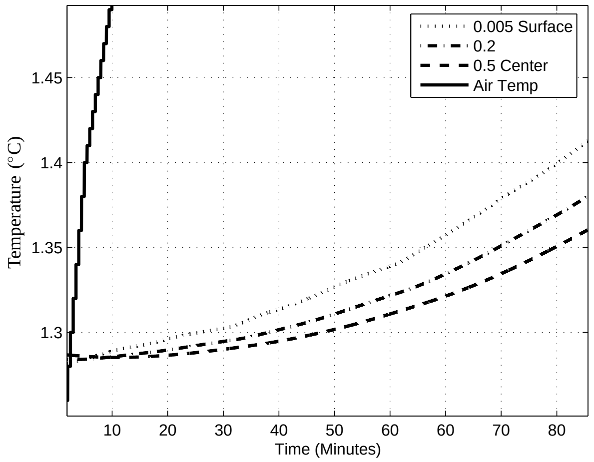


Fig. 8. Fresh fish time temperature profile in different depth-zoomed

Frozen fish: since it has non linear thermal physical properties, which change with temperature, the evaluation of properties needs to be done at each temperature, here at one time step lag. The result is obtained and shown in Fig. 9 and 10. In the defrost cycle, the surface temperature increase 9.9 °C, and the maximum temperature difference between the surface and center is around 0.6 °C.

VI. DISCUSSION AND CONCLUSION

In this section, different factors that influence the heat transfer, such that the storage good temperature are discussed.

A. Air velocity inside the display cabinet

The heat transfer coefficient is very often a difficult parameter to obtain for a process. It depends on the shape

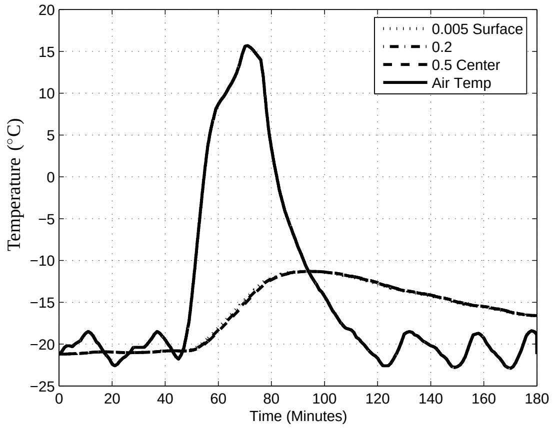


Fig. 9. Frozen fish time temperature profile in different depth

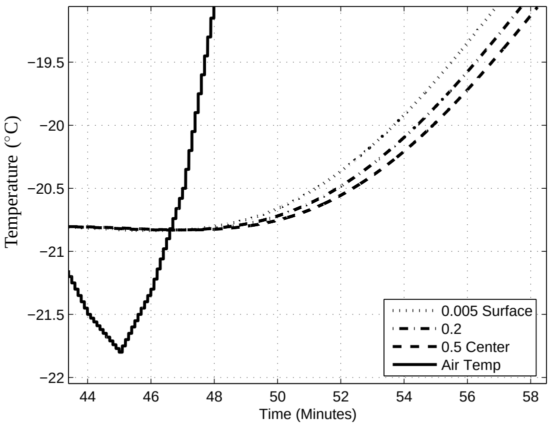


Fig. 10. Frozen fish time temperature profile in different depth-zoomed

of the product and the motion of the gas or liquid used for heating the product. While the measurement of center temperature in a product can provide us with an average heat transfer coefficient for the product investigated [12]. In a normal display cabinet, air velocity is around 0.05∼0.2 m/s, the heat transfer coefficient is around 3∼5 $W/(m^2.K)$. If by disturbance or some other reasons, the air circulates at a much higher speed, the corresponding h will also increase a lot. Here $h = 5$ $W/(m^2.K)$ is compared with $h = 15$ $W/(m^2.K)$, see Fig. 11.

B. Type of fish

The estimation of thermal conductivity k and specific heat capacity C_p is mainly based on the water content of the fish, which varies with different species of fish as the composition is different. Even for the same fish, its constitute values also change in the different environments and different seasons. Besides, the density ρ of different fish is normally different.

The overall effect is reflected in diffusivity α, Fig. 12 shown the conductivity and diffusivity for fatty fish herring and lean fish cod, and the comparison of their time temperature profile is shown in Fig. 13.

C. Packaging of food

In the case of packaging, such as pack the fish with cardboard ($k = 0.007$ $W/(m.K)$), the local heat transfer coefficient h should be replaced with overall heat transfer coefficient U.

Here $h = 5$ $W/(m^2.K)$ is compared with $U = 1$ $W/(m^2.K)$, see Fig. 14.

D. The size of food

When the food is thick, the difference between the surface and center temperature will become more obvious.

E. Boundary condition

In the above calculations, we assume the both sides of food exposing to the convective boundary, this counts for only one case. The other type of boundary condition includes such as upper side exposing to convective boundary condition, and the bottom is isolated, therefore no heat transfer, or more complicated such as radiation type boundary condition or some combinations.

F. Biot number

The calculation result shows in some cases, there is no significant temperature difference between different layers, or even between the center and surface. Estimation of Biot number, can give us the information if we can treat the model as ’lumped’ model, or we need to pay attention the difference in different layers.

Biot number is defined as follows:

$$B_i = \frac{h}{k}R \tag{9}$$

where R is the characteristic length of the system, typically the radius of a cylinder or a sphere and the ’semi thickness’ of a plate. Biot number represents the ratio of heat transfer resistance in the interior of the system to the resistance between the surroundings and the system surface. Two ideal situations are $Bi = 0$, and $Bi \to \infty$, respectively. In practice, negligible Biot number is considered to be $Bi < 0.1$, which represent uniform temperature every where, so called isothermal, or ’lumped’ model; while $Bi > 100$ is considered to be infinite, which indicate that temperature gradient exists, so called, non-isothermal.

In the phase change problem, even thermal conductivity k is changing with temperature, but we can still by using the correlative education to predict the range of the k value, such that estimate B_i.

G. CONCLUSION

The thermal model and its simulation result present in this paper can be used as one tool, to evaluate the effect of the various parameters to the product temperature.

It can be concluded that the defrost cycle in the refrigeration system influences the product temperature dramatically. By simulating the different defrosting scenarios, we can obtain the different product temperature profiles, together with the hereafter developed quality model, we can find out which defrosting scheme is more optimal for product quality during refrigerated storage.

In the later stage, we need also put the overall energy consumption into our control objective. We plan to use the multi-objective optimization to design the new controller, the model predictive control could be one of the approaches.

VII. ACKNOWLEDGMENTS

The authors gratefully acknowledge the sponsorship of the Center for Model Based Control and Monitoring (CMBC), Denmark.

REFERENCES

[1] K. Morgan, “A numerical analysis of freezing and melting with convection,” *Computer Methods in Applied Mechanics and Engineering*, vol. 28, no. 3, pp. 275–284, 1981.

[2] A. R. de Reinick, “Estimation of average freezing and thawing temperature versus time with infinite slab correlations,” *Computers in Industry*, vol. 19, no. 3, pp. 297–305, 1992.

[3] J. Shin, A. V. Tikhonov, and C. Kim, *Experimental Study on Frost Structure on Surface with Different Hydrophilicity: Density and Thermal Conductivity*. Journal of Heat Transfer, 125, pp. 84-94, 2003.

[4] DSK, D. S. Group, and COOP, *Hygiene and Self Control, Regulation for Supermarkets*, Sept.,2004.

[5] *Bekendtgorge nr. 1271 of 13/12/2004, Bekendtgorge om Fodevarehygiejne*.

[6] Y. Jaluria and K. E. Torrance, *Computational Heat Transfer,Series in Computational Methods in Mechanics and Thermal Sciences*. Washington, D.C.: Hemisphere Pub. Corp., 1986.

[7] K. A. Fikiin and A. G. Fikiin, “Predictive equation for thermo physical properties and enthalpy during cooling and freezing of food material,” *Jounral of Food Engineering*, vol. 40, pp. 1–6, 1999.

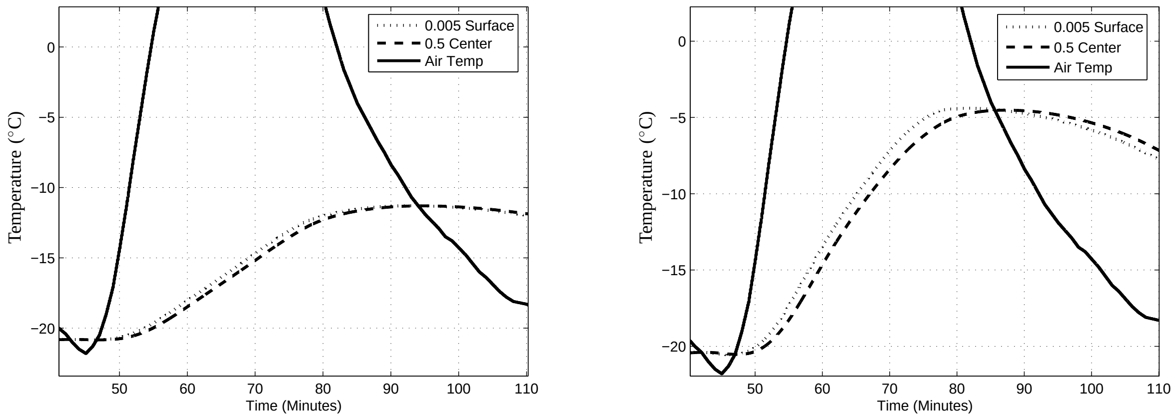
[8] *www.foodcomp.dk*.

[9] C. Miles, *The Thermalphysical Properties of Frozen Food. In W. Bald (Ed.), Food Freezing, Today and Tomorrow*. London: Springer-Verlag, pp 45-65,1991.

[10] M. N. Ozisik, *Finite Difference Methods in Heat Transfer*. Boca Raton:CRC, 1994.

[11] B. Nedjar, “An enthalpy-based finite element method for nonlinear heat problems involving phase change,” *Computers and Structures*, vol. 80, no. 1, pp. 9–21, 2002.

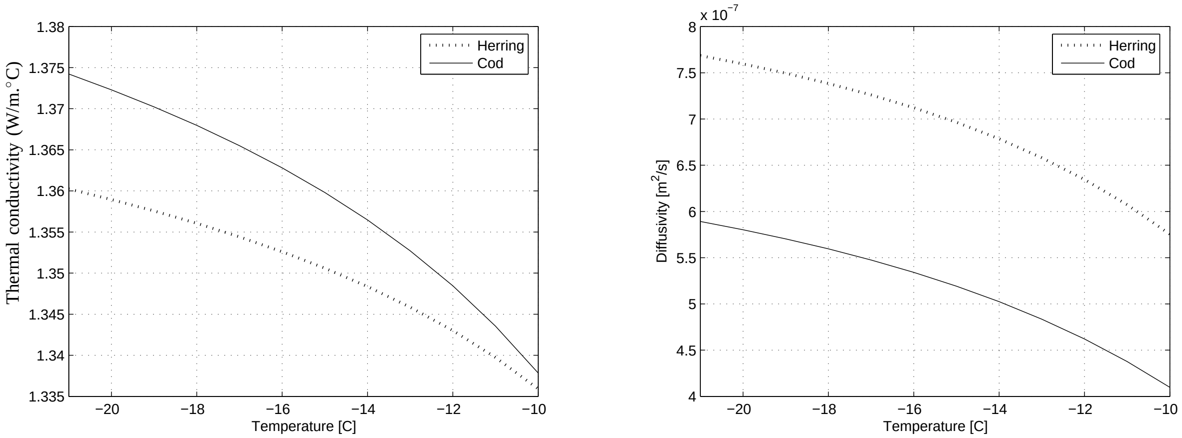
[12] A. Friis, B. B. B. Jensen, and J. Risum, *Food Process Engineering, An Introduction*, 1st ed. Biocentrum-DTU, 2004.



(a) h=5 $W/(m^2.K)$

(b) h=15 $W/(m^2.K)$

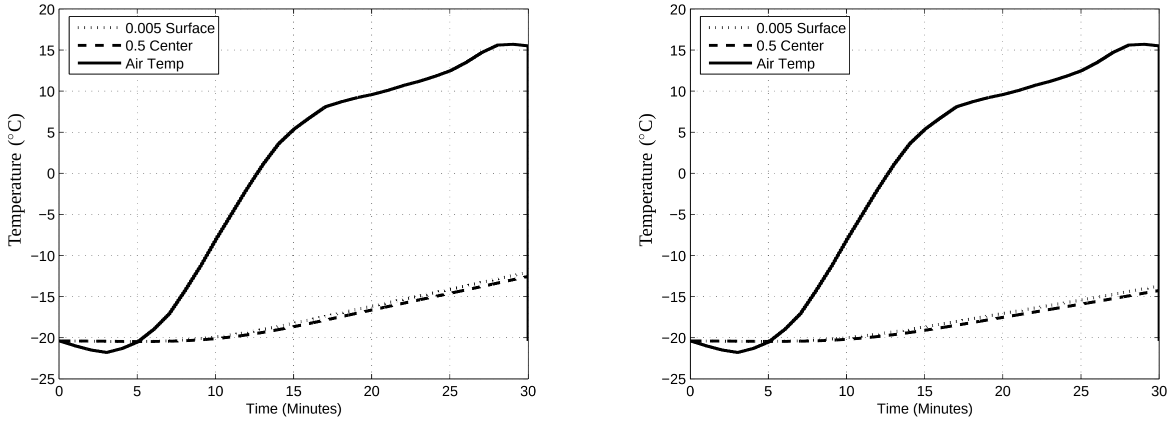
Fig. 11. Frozen fish time temperature profile in different depth for different h



(a) Conductivity

(b) Diffusivity

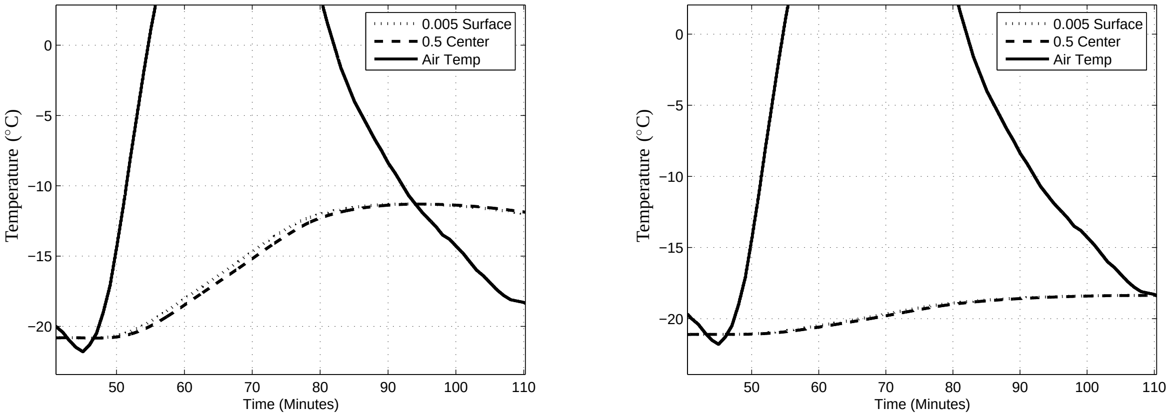
Fig. 12. Conductivity and Diffusivity of different fishes



(a) Fatty fish, Herring

(b) Lean fish, Cod

Fig. 13. Frozen fish time temperature profile in different depth-fat and lean fish



(a) h=5 $W/(m^2.K)$

(b) U=1 $W/(m^2.K))$

Fig. 14. Frozen fish time temperature profile in different depth under different heat transfer coefficient