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Transforming First-Year Calculus Teaching for Engineering Students — Field Specific Examples, Problems, and Exams

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Summary

Many first-year engineering students perceive mathematics courses as being too abstract and far from their chosen study programme. This may lead to a lack of motivation and effort, thus decreasing the learning outcomes. We demonstrate that it is feasible to include field specific problems in introductory mathematics courses to motivate engineering students. This is done in a way that still allows large parts of the course to be common to all students, ensuring economic viability. The courses have been restructured into smaller subunits, each of which has a corresponding workshop treating a real-world problem from the specific field of a given group of students. These workshops are developed in consultation with the relevant fields of study, and they are given a prominent role in the course for instance by forming the basis for the oral exams. Based on the feedback from students, we find that inclusion of field specific problems does help to highlight the applicability and importance of mathematics in engineering. When implementing such a solution, however, there are a number of challenges to keep in mind.

Keywords: Calculus, linear algebra, engineering, motivation, field-specific problems, video, streamed lectures

Type of contribution: Best practice extended abstracts

1 Introduction

Generating student motivation can be especially challenging in introductory mathematics courses for engineering students. Research shows that first-year engineering students often complain about having to study calculus (Härterich et al., 2012). Engineering students furthermore do not develop mathematical concepts the same way as mathematics students, and the concepts have different meaning to them (Maull & Berry, 2000). Studies also show that engineering students see mathematics as a tool and wish to see applications as part of a course (Bingolbali et al., 2007), and engineering students who are at risk of demotivation and drop-out list the level of mathematics and theory as being too high as one of the issues (Baillie & Fitzgerald, 2000). Therefore, integrating mathematics education into the engineering field and work on real-life professional problems may help this student group see the use of mathematics in engineering (Pepin et al., 2021).

In this work, we evaluate a recent change in the structure of the calculus and linear algebra courses at Aalborg University (AAU) in Denmark. These courses, both 5 ECTS, are followed by all engineering students, primarily in their first year of study. Both courses follow a similar structure. To simplify the exposition, we will therefore focus on calculus in the descriptions and examples given in this work. We note, however, that our evaluation (see Section 5) treats both courses, not only calculus.

2 Institutional setting

The education at AAU is based on the principles of problem-based learning (PBL). That is, in any given semester, the students are organized into small groups, writing a project around half of the time. Here, the groups choose an ill-structured and authentic problem, and the students work on a solution or answer to this problem, documenting this in a joint report. This is based on six PBL principles that are common for the whole university: (1) The *problem* is the point of departure and guides the project work, (2) The *projects* are organised in groups of up to eight students, (3) The projects are supported by *courses*, (4) *Collaboration* with a facilitator/supervisor and sometimes an external partner, (5) *Exemplarity*, meaning that the project's learning outcomes are transferable to similar future professional problems, (6) *Students* are responsible for their own learning and organise the work themselves (Askehave et al., 2015). The project groups carry over to other parts of university programmes, meaning that exercises for courses are typically solved in these groups as well. That is, there is no need to form *ad hoc* groups in teaching situations. Instead, teachers simply piggyback on the existing project groups. Another by-product of the group-based education is the continuous feedback from students via the s*teering group* meetings. Each programme typically has two or three such meetings during the semester, and at these meetings, each group sends a representative to discuss any issues that may have arisen with their courses, projects, or practicalities with IT, maintenance etc. The steering group meetings are typically organized by a *semester coordinator*, and teachers are invited to participate as well. These meetings are devoted to continual evaluation of all parts of the semester, ensuring that potential issues can be addressed — and hopefully rectified — quickly.

3 Research methodology

The roll-out of the new course structure happened at the same time as SARS-CoV-2 (i.e. the virus causing COVID-19) spread across the world. In Denmark, this caused large parts of society to be shut down in an attempt to slow down transmission of the virus. Naturally, this also affected universities, and courses had to be digitalized 'on-the-fly'.

The changes caused by COVID-19 are very likely to have had a negative impact on student well-being and student motivation, which is in turn likely to affect dropout rates and overall performance. A US study on students' responses to this situation found that nearly half of the engineering and computer science students in the study listed motivation during lockdown as a challenge that affected their ability to perform (Casper et al., 2022). Another US study on the impact of COVID-19's remote instructions on engineering students' learning, found that students experienced a decline in their engagement and learning of new concepts. However, the study also found "that students' motivation could be the outcome of their learning experiences" (Anwar & Menekse, 2023, p. 7).

Hence, evaluating the new structure based solely on grades, dropout rates etc. would not give meaningful results. Instead, we performed a qualitative analysis based on comments given by the students in the minutes of the steering group meetings. These minutes also contain the students' thoughts on the calculus and linear algebra courses and their structure, so they will help in gauging the students' perception of the revised structure and the use of programme-specific workshops. These minutes are written following a fairly uniform template across the university and although sometimes written by students and other times by faculty, the minutes are always approved. Our analysis is based on all the steering group minutes we have access to from the autumn semester 2021 and the spring semester 2022. This amounts to a total of 57 meeting minutes spread across 11 study programmes.

4 Course structure

To allow the study programmes to tailor the course for their specific needs, they select among topics predefined by the Department of Mathematical Sciences. In this structure, each topic corresponds to a subunit, which we refer to as a *block.* The initial block treats multivariate functions and their role in calculus. This includes the concept of partial derivatives and the basic principles of optimization. This block is mandatory for all students following the calculus course, and to cover the required topics, this block is larger than the others, comprising six lectures. The remaining seven blocks all consist of two lectures and focus on a more specialized topic within the curriculum such as differential equations, complex numbers, or the Laplace transform. Each study programme will only follow three of these blocks, and the choice of blocks is made by the study board responsible for the given programme. At AAU, the lectures in the elective blocks are livestreamed to the group rooms of the students followed by on-site exercises. Using livestreams is not vital to the structure described in this work, however, and other educational institutions seeking to implement a similar course structure can opt for online or on-site lecturing as they see fit. Once a study programme has chosen the blocks for their calculus course, academic staff from the Department of Mathematical Sciences will meet with representatives from the given field of study to discuss potential problems that could form a basis for each of the four workshops.

A suitable problem for a specific block will need to satisfy two criteria. First, it should be something the students might see later in their studies or something that is currently worked on by researchers within the given field of study. Second, it must somehow rely on the mathematical concepts and results contained in the given block. The main purpose of the problem is to provide motivation, so it can exceed the current skill level of the students. The actual exercise problems that students will solve when doing the workshop will, for the most part, only be small 'toy examples' that explore certain parts of the grand solution to the problem as a whole. Moreover, the design of the workshop ensures that solving these exercises will require essential parts of the mathematics concepts and skills.

The course is evaluated through an oral exam. At the exam each student draws at random one of the four workshops that the student has solved during the course. The student is then given approximately five minutes to present exercises of their choice from the given workshop. Afterwards, the examiners have approximately five minutes to ask follow-up questions or other questions to the curriculum in general. Thus, including grade deliberations and other exam logistics, the examination lasts 15 minutes per student. Two examiners grade the student. One examiner is well-known to the students through the course, the other is in most cases a researcher from the students' engineering subject area. Note here that students do not hand in material in relation to the workshops, so the grade is based purely on the oral presentation given by the individual student.

4.1 Scheduling and administration

Before implementation of the new structure, we would have several parallel calculus courses where the study programmes were divided between these to fully utilize the capacity of the lecture halls. With the increased flexibility in the curriculum, this is no longer possible. We now must consider the many different combinations of blocks, each of which is likely to have a small number of students (in some cases as low as ~20 students). Running these as separate courses is not economically sustainable and would entail many duplicate lectures. Consequently, we now organize the course schedule based on the blocks, and then add study programmes as appropriate. As an example, the block on differential equations is conducted for thirteen programmes at the same time. That is, each group of students follow the chosen combination of blocks, but they will share lectures and exercise sessions with different programmes for each block. Note though that they do *not* share workshops since the workshop problems are tailored to each individual programme.

4.2 Example workshop

In this section, we give an example of a problem that is used as a motivating example in the workshops. More examples can be found in Christensen et al. (2023).

The health technology students follow the block on integration. For their workshop, it was decided to consider two different problems related to human health. The first is to determine the total flow of blood through a vein with a given radius and its relation to blood pressure. Using the Hagen-Poiseuille equation, the velocity of flow can be described as the solution to a differential equation, and the students are given this solution in the introduction. Determining the total flow then corresponds to integrating the velocity of flow over a cross section of the vein. The first exercise problems revolve around this as well as computing the flow in the case of a more advanced cross section.

The second problem in this workshop is the estimation of muscle volume when scanning results provide a finite number of cross-sectional images along the muscle in question. The students are then guided through this estimation using truncated cones in the setting where each cross section is circular. Afterwards, they consider an example with rectangular cross sections to see the approach in more general settings. In solving these exercises, they will use solids of revolution as well as triple integrals.

To conclude the workshop, they are asked to consider a muscle with circular cross sections given by a specific function. We provide a MATLAB-script computing the true volume, the estimated volume using truncated cones, and the estimated volume using cylinders. The students are asked to sketch the shapes whose volumes are computed and compare the results of the three calculations. Finally, they are asked to experiment with the parameters to confirm for both estimations that the precision increases as the number of available cross sections grows.

5 Evaluation

To aid the validity of our analysis of the steering group minutes, we used multiple interpreters and explication of the procedures (Kvale, 1996, pp. 207—208). First, the first author analysed all the minutes, coding each utterance as *Workshop*, *Teacher, Structure*, or *General* comments, respectively. The utterances were also tagged as being *Positive*, *Balanced*, or Negative. Next, a random programme was picked for each course using random.org, and the minutes corresponding to these programmes were then coded separately by the two other authors to check for coder reliability. In the first round of samples, we discovered that the four categories were not sufficiently well-defined. In order to improve coder reliability, we decided to do two things. First, reduce the categories to *Workshop*, *General*, and *Teacher*, and defined these more precisely as detailed in (Christensen et al., 2023). Second, we decided that other coders would be given the division of utterances used by the initial coder to circumvent the second type of problem seen in the first round of samples. With these changes, a second round of samples — different from the first round of samples — gave an agreement of 72% with respect to sentiment, 56% with respect to category, and 40% with respect to both at the same time. However, for every utterance at least two coders agreed on both category and sentiment. In addition, we note that any disagreement regarding the sentiments were between balanced and negative or balanced and positive. After computing agreement percentages, we discussed the cases that we had coded differently in order to figure out why we disagreed and to obtain consensus on the most appropriate code. An overview of the comments sorted by their final codes is given in the table below. We now proceed to analyse the overall tendencies in the comments in greater detail.

Table 1: Student comments sorted by codes.

Category	Count	Negative	Balanced	Positive
General	70	49%	16%	36%
Workshops	29	48%	24%	28%
Teacher	23	43%	22%	35%

5.1 Student comments

In the initial minutes from the students in structural engineering, the motivating problems are mentioned explicitly: "*during workshops, [the topics] are pinned to something tangible—works well*"'. Thus, it seems like the students initially recognize the effort put into connecting terms from calculus to applications in their own field of study. At the following meeting, however, they comment that the course "*can sometimes be difficult to relate to, and [to see] how to apply it in other contexts."* This could indicate that even though the students see calculus used in context during the workshops, they still find it difficult to relate to the course as a whole.

A point of critique appearing in the minutes from health technology is that their workshops tended to be too extensive. Along the same lines, students from chemistry and biology state that "*the topic worked on during the workshop seemed to be outside the curriculum […]"*. This highlights a challenge regarding the choice of motivating problem. On the one hand, the problem must be as close as possible to the field of the students, but at the same time, it should not act as a hindrance to solving the underlying calculus problems. Some of the negative comments regarding the course structure were not related to what *had happened* in the course, but to the expectation of what *will happen*. These comments all relate to the streaming of the elective blocks, stating concern with the prospect of online teaching. We see this at the first meeting of 2022 for the Danish programmes in Esbjerg, where they say, "*we are happy that it is not yet virtual"*. Interestingly, at their last meeting where the streamed lectures have taken place, their sentiment has changed: *"[The] online part went well compared to last semester."*

A benefit of the streamed lectures is that students can watch it (or parts of it) again, which we also see in the minutes. For instance, we find comments such as *"the students like that videos stay online from the digital lectures, so that one has a resource to look back on"* and *"Lectures have been recorded, which has been nice"*. A reoccurring theme in many of the minutes is the number of teachers involved in the course. We found comments such as *"[We] do not understand why we need different lecturers"* and *"is it possible to reduce the number of lecturers […]."* Part of the problem seemed to stem from us not providing enough (or sufficiently clear) information about the course structure and the assigned teachers at the beginning of the course. As a result, we now spend more time during the first lecture to explain the structure of the course and which teacher will be responsible for which parts. The latter is especially important with respect to the examiner; it should be clear to students who will conduct their exam.

6 Discussion

Through our reading of all the minutes, we see that students generally see the relevance of the workshop problems, exemplified by the quotes in Section 5.1. While this is not a direct measure of their motivation to learn, studies do suggest that perceived relevance is a major motivating factor (Kember et al., 2008). As the comments also suggest, however, the workshop problems must be chosen carefully to match the study programme, and sufficient support must be provided for the students.

The perceived relevance depends on giving students a sufficient understanding of the engineering context. In many of these workshops, the context is not introduced as an engineering subject until much later in their studies. This balance between what fits the curriculum in the mathematics courses and what engineering topic can be explained well enough for the students to buy into it being their engineering subject is the core of this approach.

What seems to cause more concern for students is the modularized structure of the course, which they in some cases found confusing. With limited funds available to run courses, some level of modularization and common scheduling is likely necessary. Based on the feedback we have collected, though, it is clear that care must be taken to make the structure transparent to students — more-so than what is needed for normal courses.

The study also has some limitations. First and foremost, that it is based on the steering group meeting minutes alone. Questionnaires or interviews of students could have provided triangulation of the results. However, steering groups, at least in principle, is a summary of the experience of all students, since the group representative should not only state their own opinions but present the experience from all the group members. Questionnaires are usually not answered by the whole population. Also, steering groups tend to focus on the negative experience.

7 Conclusion

In this work, we have described the restructuring of the calculus and linear algebra courses at Aalborg University in Denmark. Aiming to motivate students and to give them examples closer to their own field of study, the restructuring divides the courses into blocks, i.e., smaller subunits, chosen by the respective study programmes. Each block has a corresponding workshop treating a 'real-world' problem relevant to the given set of students, and these workshops form the basis of the oral exam. Through evaluation of the comments given by students at the steering group meetings during the semesters of autumn 2021 and spring 2022, we see that students do tend to appreciate the workshop problems if they are sufficiently close to their prospective field of study. One needs to choose problems carefully, though, as problems that veer too far from the lecture contents may overwhelm students. As we also highlight, however, performing such a restructuring is non-trivial, and there are a number of administrative challenges that one needs to keep in mind when implementing a similar structure. In particular, giving each study programme more flexibility in designing their curricula results in less flexibility when scheduling courses.

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